Exercise 1: Performance Evaluation metrics

a-/ We can see that when $x=3$ or $x=4$ there is a greater probability that X is from C_1. So we can take

$$g(x) = \frac{h_1(x)}{h_1(x)+h_2(x)}$$ to represent this fact.

and $$d(g(x)) = \begin{cases} 1 & \text{if } g(x) >= \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

b-/ $g(x) + B_x = 0 \Rightarrow B_x = \frac{-h_1(x)}{h_1(x)+h_2(x)} \Rightarrow B_1 = -\frac{1}{5}, B_2 = \frac{-2}{5}$

$B_3 = -\frac{3}{5}, B_4 = \frac{-4}{5}$

c-/ $$TPR = \frac{\#TP}{\#TP+\#FN} = \frac{\sum_{g(x)+B_x>=0} h_1(x)}{\sum_{g(x)+B_x>=0} h_1(x) + \sum_{g(x)+B_x<0} h_1(x)} = \frac{\sum_{g(x)+B_x>=0} h_1(x)}{\sum h_1(x)}$$

$$FPR = \frac{\#FP}{\#FP+\#TN} = \frac{\sum_{g(x)+B_x>=0} h_2(x)}{\sum_{g(x)+B_x>=0} h_2(x) + \sum_{g(x)+B_x<0} h_2(x)} = \frac{\sum_{g(x)+B_x>=0} h_2(x)}{\sum h_2(x)}$$

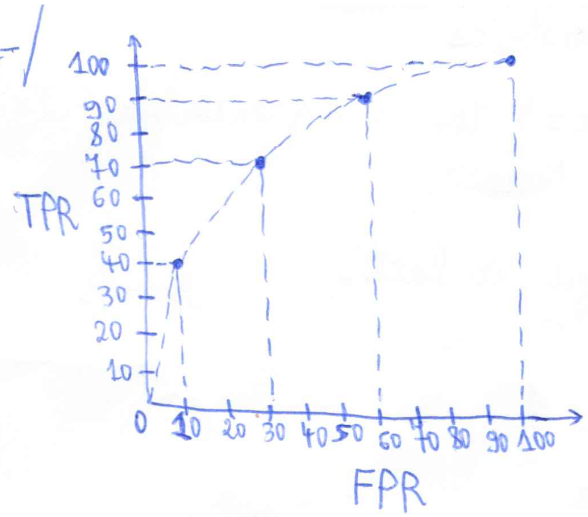
$TPR_1 = 1$ $\quad$ $TPR_2 = 0,9$ $\quad$ $TPR_3 = 0,7$

$FPR_1 = 1$ $\quad$ $FPR_2 = 0,6$ $\quad$ $FPR_3 = 0,3$

$TPR_4 = 0,4$

$FPR_4 = 0,1$

d-/



e-/ $PP = \frac{TP}{TP+FP}$

$$= \frac{\sum_{g(x)+B_x \geq 0} h_1(x)}{\sum_{g(x)+B_x \geq 0} h_1(x) + \sum_{g(x)+B_x \geq 0} h_2(x)}$$

$\Rightarrow$ $PP_1 = \frac{1}{2} = 0{,}5$

$PP_2 = \frac{18}{18+12} = \frac{6}{10} = 0{,}6$

$PP_3 = \frac{14}{14+6} = 0{,}7$

$PP_4 = \frac{8}{8+2} = 0{,}8$

$PP = \frac{14}{14+6} = 0{,}7$ (when Bias is $\frac{1}{2}$)

$S = TPR = \frac{14}{20} = 0{,}7$

Exercice 2:

$$g(\vec{X}, \vec{W}) = \sum_{m=1}^{M} a_m y_m f\left(\|\vec{X} - \vec{X_m}\|\right) + w_0$$

$b = w_0 = 0$

1- /a/ $g(\vec{X}, \vec{w}) = a_3 y_3 f(\|\vec{X} - \vec{X}_3\|) + a_4 y_4 f(\|\vec{X} - \vec{X}_4\|) +$
$a_9 y_9 f(\|\vec{X} - \vec{X}_9\|)$

$$= f(\|\vec{X} - \vec{X}_3\|) - f(\|X - \vec{X}_4\|) - f(\|\vec{X} - \vec{X}_9\|)$$

$$= e^{-\frac{\|\vec{X} - \vec{X}_3\|^2}{2}} - e^{-\frac{\|\vec{X} - \vec{X}_4\|^2}{2}} - e^{-\frac{\|\vec{X} - \vec{X}_9\|^2}{2}}$$

Let $\vec{X} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\|\vec{X} - \vec{X}_3\|^2 = (x-2)^2 + (y-2)^2 = x^2 + y^2 - 4x - 4y + 8$$
$$\|\vec{X} - \vec{X}_4\|^2 = (x-3)^2 + (y-5)^2 = x^2 + y^2 - 6x - 10y + 34$$
$$\|\vec{X} - \vec{X}_9\|^2 = (x-5)^2 + (y-3)^2 = x^2 + y^2 - 10x - 6y + 34$$

$$g(\vec{X}, \vec{w}) = e^{-\frac{(x^2+y^2)}{2}} \left(e^{+2x+2y-4} - e^{+3x+5y-17} - e^{5x+3y-17}\right)$$

b/ I have to check if for each $\vec{X_m}$, $y_m\, g(\vec{X_m}) > 0$.
You will find the python program joined to this email.

Exercise 3 :

a-/ Using the formula $a_j^{(\ell)} = f(z_j^{(\ell)}) = f\left(\sum_{i=1}^{N^{(\ell-1)}} W_{ji}^{(\ell)} a_i^{(\ell-1)} + b_j\right)$

we obtain : $a_1^{(0)} = 1$, $a_2^{(0)} = 0$ (because $\vec{a}^{(0)} = \vec{X}$)

$$\vec{a}^{(1)} = \begin{pmatrix} 0,5 \\ 0,26894 14 \\ 0,73 \end{pmatrix} \qquad \vec{a}^{(2)} = (0,77)$$

b-/ $$\delta_{j,1}^{(\ell)} = a_{j,m}^{(\ell)}\left(1 - a_{j,m}^{(\ell)}\right) \cdot \sum_{k=1}^{N^{(\ell+1)}} W_{kj}^{(\ell+1)} \delta_{k,1}^{(\ell+1)}$$

when $\ell < L$

$$\delta_{j,1}^{(L)} = z_j^{(L)}\left(1 - z_j\right)\left(a_{j,1}^{(L)} - y_1\right)$$

and we obtain

$$\vec{\delta}_1^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{(it is always the case)}$$

$$\vec{\delta}_1^{(1)} = \begin{pmatrix} 0,016 \\ -0,013 \\ 0 \end{pmatrix}$$

$$\vec{\delta}_1^{(2)} = (0,064)$$

c-/ $\Delta W_{ji,1}^{(\ell)} = a_i^{(\ell-1)} \delta_{j,1}^{(\ell)}$ $\quad \Delta b_{j,m}^{(\ell)} = \delta_{j,1}^{(\ell)}$

Using these formulas we obtain

$$\overrightarrow{\Delta W}^{(1)} = \begin{pmatrix} 0,016 & 0 \\ -0,013 & 0 \\ 0 & 0 \end{pmatrix} \qquad \overrightarrow{\Delta_W}^{(2)} = \begin{pmatrix} 0,032 & 0,017 & 0,047 \end{pmatrix}$$

d-/

We apply the corrections in this way:

$$w_{ji}^{(\ell)} \leftarrow w_{ji}^{(\ell)} - \eta \, \Delta w_{ji}^{(\ell)} \qquad b_j^{(\ell)} \leftarrow b_j^{(\ell)} - \eta \cdot \Delta b_j^{(\ell)}$$

The resulting parameters are:

$$\vec{W}_{ji}^{(1)} = \begin{pmatrix} 1{,}0095 & 0{,}0211 \\ -1{,}0079 & -0{,}0185 \\ -0{,}0002 & 0{,}9999 \end{pmatrix} \qquad W_{ji}^{(2)} = \begin{pmatrix} 1{,}014 & -0{,}998 & 0{,}029 \end{pmatrix}$$

$$b_j^{(1)} = \begin{pmatrix} -0{,}994 \\ -0{,}003 \\ 0{,}999 \end{pmatrix} \qquad b_j^{(2)} = \begin{pmatrix} 1{,}021 \end{pmatrix}$$

e-/ this time we obtain:

$$W_{ji}^{(1)} = \begin{pmatrix} 1{,}0097 & 0{,}0165 \\ -1{,}0077 & -0{,}0143 \\ 0 & 1 \end{pmatrix} \qquad W_{ji}^{(2)} = \begin{pmatrix} 1{,}0377 & -0{,}9942 & 0{,}0673 \end{pmatrix}$$

Intelligent systems

Exercise 4:

- Explain Baye's rule for someone who has no mathematical training. What is it? How is it used?

Baye's rule is about how we should update or revise our theories as new evidence emerges.
When you believe something is 20% likely and then you get a new piece of information, it can tell you wether you now should think it's 30% likely or 40% likely.
~~For that you need to know~~ Then probability is the confidence we have in our beliefs. Then to apply Bayes rules, you need the probability of the theories, the probility of the new evidences and the probability of the new evidences, when the theories are true.

- Can Baye's rule be used with symbolic features such as hair color or nationality?

Yes by using a hash and use the symbolic value as an adress.

- Under what circumstances will Bayes Rule give incorrect results?

When the value of a feature is dependent of the value of an other one.