Dirichlet o escencial

$$\frac{d^2\phi}{dx^2} - \phi - 2\operatorname{sen}(x) = 0 \qquad \phi(\pi/2) = 1 \qquad \frac{d\phi}{dx}(\pi) = -1$$

Newman

Natural

$\hat{\phi} = \sum a_m N_m$

$N_m = x^m$

$\cancel{\psi(x)}$

$$R_\Omega = A(\hat{\phi}) = \mathcal{L}(\hat{\phi}) + p$$

$$\hookrightarrow \mathcal{L}(\hat{\phi}) = \frac{d^2\hat{\phi}}{dx} - \hat{\phi}$$

$$p(x) = -2\operatorname{sen}(x)$$

$\beta(\hat{\phi}) - \cancel{\beta(\phi)}$ 0

$$R_\Gamma = \beta(\hat{\phi}) \begin{cases} \hat{\phi} - 1 & \text{en } \pi/2 \\ \frac{d\hat{\phi}}{dx} + 1 & \pi \end{cases}$$

$$\phi\big|_{\pi/2} = 1 \rightarrow \phi - 1 = 0$$

$$\underbrace{(\cdot)}_{M} \underbrace{-1}_{r} = 0$$

$$\beta(\cdot) = M(\cdot) + r$$

Si usos $\phi \Rightarrow R_\Gamma = 0$

$$\int_{\pi/2}^{\pi} W_e A(\hat{\phi})\,dx + \overbrace{\bar{W}_e(\hat{\phi} - 1)\Big|_{\pi/2} + \bar{W}_e\left(\frac{d\hat{\phi}}{dx} + 1\right)\Big|_{\pi}}^{\text{Si usos } \phi \Rightarrow R_\Gamma = 0} = 0$$

$$\int_{\pi/2}^{\pi} W_e\left(\frac{d^2\hat{\phi}}{dx^2} - \hat{\phi} - 2\operatorname{sen}(x)\right)dx + \bar{W}_e(\hat{\phi} - 1)\Big|_{\pi/2} + \bar{W}_e\left(\frac{d\hat{\phi}}{dx} + 1\right)\Big|_{\pi} = 0$$

$$\int_{\pi/2}^{\pi} W_e \frac{d^2\hat{\phi}}{dx^2}dx = W_e\frac{d\hat{\phi}}{dx}\Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} \frac{dW_e}{dx}\cdot\frac{d\hat{\phi}}{dx}\,dx$$

$\bar{W}_e \cdot 1\big|_{\pi}$

$$\cancel{W_e\frac{d\hat{\phi}}{dx}\Big|_{\pi}} - W_e\frac{d\hat{\phi}}{dx}\Big|_{\pi/2} - \int_{\pi/2}^{\pi}\frac{dW_e}{dx}\frac{d\hat{\phi}}{dx}\,dy - \int_{\pi/2}^{\pi} W_e\hat{\phi}\,dx - \int_{\pi/2}^{\pi} W_e 2\operatorname{sen}(x)\,dx + \bar{W}_e(\hat{\phi} - 1)\Big|_{\pi/2} + \bar{W}_e\left(\cancel{\frac{d\hat{\phi}}{dx}} + 1\right)\Big|_{\pi} = 0$$

$$W_e\big|_{\pi} = -\bar{W}_e\big|_{\pi}$$

$$\underbrace{-W_e\frac{d\hat{\phi}}{dx}\Big|_{\pi/2} - \int_{\pi/2}^{\pi}\frac{dW_e}{dx}\frac{d\hat{\phi}}{dx}\,dx - \int_{\pi/2}^{\pi} W_e\hat{\phi}\,dx}\ \underbrace{-\int_{\pi/2}^{\pi} W_e 2\operatorname{sen}(x)\,dx}_{f_e}\ \underbrace{+\bar{W}_e(\hat{\phi} - 1)\Big|_{\pi/2} - W_e\Big|_{\pi}}_{R} = 0$$

$$W_e = N_e \qquad \hat{\phi} = \sum a_m N_m$$

$$\sum a_m \underbrace{\left(-N_e\frac{dN_m}{dx}\Big|_{\pi/2} - \int_{\pi/2}^{\pi}\frac{dN_e}{dx}\frac{dN_m}{dx}\,dx - \int_{\pi/2}^{\pi} N_e N_m\,dx\right)}_{K_{em}}$$

$$N_m = x^m$$

$$dN_m = m x^{m-1}$$

$$K_{em} = -x^e m x^{m-1}\Big|_{\pi/2} - \int_{\pi/2}^{\pi} e x^{e-1} m x^{m-1}\,dx - \int_{\pi/2}^{\pi} x^e x^m\,dx$$

$$f_e = -\int N_e 2\operatorname{sen}(x)\,dx = -\int x^e 2\operatorname{sen}(x)\,dx$$

$$\cancel{\begin{bmatrix} K \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} f \end{bmatrix} + \begin{bmatrix} \pi \\ \pi^2 \\ \pi^3 \end{bmatrix}} + \cancel{0} = 0$$

$$-W_e\big|_{\pi} = -N_e\big|_{\pi} = \pi^e = \begin{bmatrix} \pi \\ \pi^2 \\ \pi^3 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}} + \cancel{0} = 0$$

$$\phi(\pi/2) = 1$$

$$\hat{\phi}(\pi/2) = 1 \rightarrow a_1 N_1(\pi/2) + a_2 N_2(\pi/2) + a_3 N_3(\pi/3) = 1$$

$$a_1 \pi/2 + a_2\frac{\pi^2}{4} + a_3\frac{\pi^3}{8} = 1$$

$$\bar{W}_e(\cancel{\phi} - 1)\Big|_{\pi/2} \quad 0$$

$$\boxed{a_1 N_1(x_1) + a_2 N_2(x_2) = \bar{\phi}}$$

$$\begin{bmatrix} N_1(x_1) & N_2(x_2) \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} -\bar{\phi} \\ 2 \end{bmatrix} = 0$$

$$K_{11} = -x^1\cdot(1)x^0\Big| - \int_{\pi/2}^{\pi} 1\,dx - \int_{\pi/2}^{\pi} x\,x\,dx$$