```
In [1]:
        import numpy as np
         import pandas as pd
         import tensorflow as tf
         import keras
         from keras.layers import Dense, Dropout, Conv2D, MaxPooling2D, Flatten
         from tensorflow.keras import regularizers
         from tensorflow.keras.optimizers import Optimizer
         import matplotlib.pyplot as plt
         import time
         import keras.backend as K
         import keras
         # import the necessary packages
         from pyimagesearch.minigooglenet import MiniGoogLeNet
         from pyimagesearch.clr callback import CyclicLR
         from pyimagesearch import config
         from pyimagesearch import learningratefinder
         from sklearn.preprocessing import LabelBinarizer
         from sklearn.metrics import classification report
         from tensorflow.keras.layers import AveragePooling2D, Input
         from tensorflow.keras.regularizers import 12
         from tensorflow.keras.preprocessing.image import ImageDataGenerator
         from tensorflow.keras.optimizers import SGD, Adam
         from tensorflow.keras.datasets import fashion mnist
         from tensorflow.keras.layers import Dense, Conv2D
         from tensorflow.keras.layers import BatchNormalization, Activation
         from tensorflow.keras.layers import AveragePooling2D, Input
         from tensorflow.keras.layers import Flatten, add
         from tensorflow.keras.optimizers import Adam
         from tensorflow.keras.callbacks import ModelCheckpoint, LearningRateScheduler
         from tensorflow.keras.callbacks import ReduceLROnPlateau
         from tensorflow.keras.preprocessing.image import ImageDataGenerator
         from tensorflow.keras.regularizers import 12
         from tensorflow.keras.models import Model
         from tensorflow.keras.datasets import cifar10
         from tensorflow.keras.utils import plot model
         from tensorflow.keras.utils import to categorical
         # from tensorflow.keras.datasets import cifar10
         import matplotlib.pyplot as plt
         import numpy as np
         import cv2
```

Problem 1

question 1

import sys

• Write the weight update equations for the five adaptive learning rate methods. Explain each term clearly. What are the hyper-parameters in each policy? Explain how AdaDelta and Adam are different from RMSProp.

Adagrad

$$w_{t+1} = w_t - \frac{\alpha}{\sqrt{S_t + \epsilon}} \cdot \frac{\partial L}{\partial w_t}$$
$$S_t = S_{t-1} + \left[\frac{\partial L}{\partial w_t}\right]^2$$

- Here the $\frac{\partial L}{\partial w_t}$, is the gradient for each update step, control this value by multiplying it with $\frac{1}{\sqrt{S_t+\epsilon}}$, will let actural learning rate become smaller and smaller, thus good for final covergence. So adagrad is a optimizer that decaying learning rate by S_t
- hyper-parameter:
 - base learning rate: α .
 - ϵ : none-zero denominator.

RMSProp

$$w_{t+1} = w_t - \frac{\alpha}{\sqrt{S_t + \epsilon}} \cdot \frac{\partial L}{\partial w_t}$$
$$S_t = \beta S_{t-1} + (1 - \beta) \left[\frac{\partial L}{\partial w_t} \right]^2$$

- RMSProp is very similar to Adagrad, they both controling learning rate by decaying it with multiplying it by $\frac{1}{\sqrt{S_t+\epsilon}}$, the difference is that RMSprop have additional hyper-parameter β controling the decaying speed, if we let $\beta=1$, the learning rate will remain steady and don't change during learning.
- hyper-parameter:
 - base learning rate : α
 - decaying speed : β
 - \bullet : none-zero denominator.

RMSProp + Nesterov

Nesterov

$$egin{aligned} w_{t+1} &= w_t - rac{lpha}{\sqrt{S_t + \epsilon}} V_t \ V_t &= eta_v V_{t_1} + (1 - eta_v) rac{\partial L}{\partial w^*} \ S_t &= eta_d S_{t-1} + (1 - eta_d) [rac{\partial L}{\partial w_t}]^2 \ w^* &= w_t - lpha V_{t-1} \end{aligned}$$

- Very similar to RMSprop, adding Nesterov just introduces momentum and predicting next position instead of directly using velosity.
- Here we have to hyper-parameters combine RMSprop with Nesterov:
 - β_v is a hyper-parameter controlling the gradient using momentum mechanism, if β_v = 0, previous velocity will not affect following learning gradient.
 - β_d is a hyper-parameter controlling the decaying in RMSprop, if we let β_d = 1, the learning rate will remain steady and don't change during learning.
 - ullet ϵ : none-zero denominator.

Adadelta

\mathbf{A} dadelta

$$w_{t+1} = w_t - \frac{\sqrt{D_{t-1} + \epsilon}}{\sqrt{S_t + \epsilon}} \cdot \frac{\partial L}{\partial w_t}$$
$$D_t = \beta D_{t-1} + (1 - \beta) [\Delta w_t]^2$$
$$S_t = \beta S_{t-1} + (1 - \beta) \left[\frac{\partial L}{\partial w_t} \right]^2$$

- As shown above, Adadelta has one hyper-parameters.
- In the denominator, it's the same learning rate decay as **RMSprop** and **Adagrad**, it controls the decaying speed of the learning rate.
- The nominator is different, Since Adagrad will finally reduce the learning rate to 0 as iteration increases, the nominator is actually an improvement of this weakness, so instead of traditional learning rate α , here Adadelta uses the accumulation of changes of weight as the learning rate. So in the early stage, the gradient is huge, so the change of weights is huge, thus we get a larger D_t , and finally, when the network is close to convergence, the change of weights is small, thus we get a smaller D_t .

Adam

Adam

$$w_{t+1} = w_t - \frac{\alpha}{\sqrt{\hat{S}_t} + \epsilon} \cdot \hat{V}_t$$

$$\hat{V}_t = \frac{V_t}{1 - \beta_1^t}$$

$$\hat{S}_t = \frac{S_t}{1 - \beta_2^t}$$

$$V_t = \beta_1 V_{t-1} + (1 - \beta_1) \frac{\partial L}{\partial w_t}$$

$$S_t = \beta_2 S_{t-1} + (1 - \beta_2) \left[\frac{\partial L}{\partial w_t} \right]^2$$

- Adam combine the decaying mechanism in RMSprop and momentum.
- Adam have three hyper-paramters:
 - ullet α : base learning rate
 - β_1 : control the momentum. If $beta_1 = 0$, then previous velocity will not influence following gradients.
 - β_2 : control the weight decay speed. If β_2 = 0, the weight decay will not accumulate.

The difference between AdaDelta and Adam

• Instead of using Δw_t to control the learning rate, Adam uses momentum instead, and both Adadelta and Adam are using S_t to control weight's scale.

• Train the neural network using all the five methods with L2-regularization for 200 epochs each and plot the training loss vs number of epochs. Which method performs best (lowest training loss)?

```
In [2]:
         #load data
         (x train, y train), (x test, y test) = keras.datasets.cifar10.load data()
         y train = tf.keras.utils.to categorical(y train)
         y test = tf.keras.utils.to categorical(y test)
In [3]:
         def make an optimizer(optimizer type):
             if optimizer type == 'Adagrad':
                 return tf.keras.optimizers.Adagrad()
             elif optimizer type == 'Adadelta':
                 return tf.keras.optimizers.Adadelta()
             elif optimizer type == 'RMSprop':
                 return tf.keras.optimizers.RMSprop()
             elif optimizer_type == 'RMSprop nesterov':
                 return tf.keras.optimizers.Nadam()
             elif optimizer_type == 'Adam':
                 return tf.keras.optimizers.Adam()
         def train a neural network (regularization type, optimizer type, epoch, batch size):
             # define model
             if regularization type == "L2 regularization":
                 model = keras.models.Sequential([
                     Flatten (input shape = (32, 32, 3)),
                     Dense(1000, activation = 'relu', kernel regularizer='12', kernel initializer=
                     Dense(1000, activation = 'relu', kernel regularizer='12', kernel initializer=
                     Dense(10, activation = 'softmax')
             if regularization type == "drop out":
                 model = keras.models.Sequential([
                     Flatten (input shape = (32, 32, 3)),
                     Dropout (0.2),
                     Dense(1000, activation = 'relu', kernel initializer='HeNormal'),
                     Dropout(0.5),
                     Dense(1000, activation = 'relu', kernel initializer='HeNormal'),
                     Dropout (0.5),
                     Dense(10, activation = 'softmax')
             optimizer = make an optimizer(optimizer type)
             model.compile(loss = 'categorical crossentropy', optimizer = optimizer, metrics = ['ac
             time start = time.time()
             history = model.fit(x train, y train, epochs = epoch, batch size = batch size,
                                 validation data=(x test, y test), verbose = 0)
             time end = time.time()
             return model, history, time end - time start
         optimizer lost = ['Adagrad', 'Adadelta', 'RMSprop', 'RMSprop nesterov', 'Adam']
In [4]:
         model 2 1, history 2 1, time 2 1 = train a neural network('L2 regularization', optimizer 3
In [5]:
         model 2 2, history 2 2, time 2 2 = train a neural network('L2 regularization', optimizer 3
In [6]:
         model 2 3, history 2 3, time 2 3 = train a neural network('L2 regularization', optimizer 3
In [7]:
        model 2 4, history 2 4, time 2 4 = train a neural network('L2 regularization', optimizer ]
```

```
In [8]:
    model_2_5, history_2_5, time_2_5 = train_a_neural_network('L2_regularization', optimizer_]
In [10]:
#Loss PLOT
    epoch_vis = list(range(len(history_2_1.history['loss'])))
    plt.plot(epoch_vis, history_2_1.history['loss'], label = optimizer_lost[0], c = 'r')
    plt.plot(epoch_vis, history_2_2.history['loss'], label = optimizer_lost[1], c = 'g')
    plt.plot(epoch_vis, history_2_3.history['loss'], label = optimizer_lost[2], c = 'b')
    plt.plot(epoch_vis, history_2_4.history['loss'], label = optimizer_lost[3], c = 'y')
    plt.plot(epoch_vis, history_2_5.history['loss'], label = optimizer_lost[4], c = 'c')
    plt.legend()
    plt.title('L2 regularization loss plot')
    plt.show()
```

L2 regularization loss plot 160 Adagrad Adadelta 140 RMSprop 120 RMSprop_nesterov Adam 100 80 60 40 20 0 25 50 75 100 125 150 175 200

Out[21]: (15.68968391418457, 37.67438888549805, 2.7793161869049072, 2.302692413330078, 2.302698850631714)

- Compared with other optimizers, RMSprop, Adam and RMSprop+nesterov have the lowest loss. Adagrad and Adadelta have a higher loss.
- RMSprop_nesterov's loss is 2.302692413330078 is the lowest loss.

question 3

• Add dropout (probability 0.2 for input layer and 0.5 for hidden layers) and train the neural network again using all the five methods for 200 epochs. Compare the training loss with that in part 2. Which method performs the best? For the five methods, compare their training time (to finish 200 epochs with dropout) to the training time in part 2 (to finish 200 epochs without dropout).

```
In [11]: model_3_1, history_3_1, time_3_1 = train_a_neural_network('drop_out', optimizer_lost[0], 2
```

```
model 3 2, history 3 2, time 3 2 = train a neural network('drop out', optimizer lost[1],
In [12]:
In [13]:
          model 3 3, history 3 3, time 3 3 = train a neural network('drop out', optimizer lost[2],
In [14]:
          model 3 4, history 3 4, time 3 4 = train a neural network('drop out', optimizer lost[3],
In [15]:
          model 3 5, history 3 5, time 3 5 = train a neural network('drop out', optimizer lost[4],
In [16]:
          #LOSS PLOT
          epoch vis = list(range(len(history 2 1.history['loss'])))
          plt.plot(epoch vis, history 3 1.history['loss'], label = optimizer lost[0], c = 'r')
          plt.plot(epoch vis, history 3 2.history['loss'], label = optimizer lost[1], c = 'g')
          plt.plot(epoch vis, history 3 3.history['loss'], label = optimizer lost[2], c = 'b')
          plt.plot(epoch vis, history 3 4.history['loss'], label = optimizer lost[3], c = 'y')
          plt.plot(epoch vis, history 3 5.history['loss'], label = optimizer lost[4], c = 'c')
          plt.legend()
          plt.title('L2 regularization loss plot')
          plt.show()
                         L2 regularization loss plot
                                            Adagrad
          500
                                            Adadelta
                                            RMSprop
          400
                                            RMSprop_nesterov
          300
          200
          100
           0
                                        125
                    25
                         50
                              75
                                   100
                                                  175
                                                       200
                                             150
In [20]:
          (history 3 1.history['loss'][-1],
          history 3 2.history['loss'][-1],
          history 3 3.history['loss'][-1],
          history 3 4.history['loss'][-1],
          history 3 5.history['loss'][-1])
         (2.3035354614257812,
Out[20]:
          2.4348690509796143,
          2.3067286014556885,
          2.302696943283081,
          2.3027024269104004)
In [22]:
          # time difference between part 2 and part 3
          (time 2 1 - time 3 1, time 2 2 - time_3_2, time_2_3 - time_3_3, time_2_4 - time_3_4, time
          (22.345446825027466,
Out[22]:
          26.96778964996338,
          23.584877967834473,
          24.263986825942993,
          26.715335607528687)
```

- After adding dropout, all 5 optimizers decrease loss to similar level, and the oen with the lowest loss is RMSprop with nesterov.
- After adding dropout, the training is much faster, around 25 seconds faster than L2 regularization.

```
In [32]:
          print('test accuracy for 10 models: ')
          (history 2 1.history['val accuracy'][-1],
          history 2 2.history['val_accuracy'][-1],
          history 2 3.history['val accuracy'][-1],
          history 2 4.history['val accuracy'][-1],
          history 2 5.history['val accuracy'][-1],
          history_3_1.history['val_accuracy'][-1],
          history 3 2.history['val accuracy'][-1],
          history 3 3.history['val accuracy'][-1],
          history 3 4.history['val accuracy'][-1],
          history 3 5.history['val accuracy'][-1])
         test accuracy for 10 models:
         (0.4440000057220459,
Out[32]:
         0.37059998512268066,
          0.20630000531673431,
          0.10000000149011612,
          0.10000000149011612,
          0.09989999979734421,
          0.10019999742507935,
          0.10000000149011612,
          0.10000000149011612,
          0.10000000149011612)
```

- According to the result, adding dropout can let loss decrease loss at start, but can't let model converge. All models with dropout finally have test accuracy around 0.1(random guess). (maybe need to fine tune the learning rate for dropout for a better performance).
- The best model is L2 regularization with Adagrad optimizer, which has test accuracy around 44%.

problem 2

question 1

• Fix batch size to 64 and start with 10 candidate learning rates between 10^{-9} and 10^{1} and train your model for 5 epochs. Plot the training loss as a function of learning rate. You should see a curve like Figure 3 in reference below. From that figure identify the values of lr_{min} and lr_{max} .

```
In [2]: # prepare data
    img_witdth, img_height = 32, 32
    ((trainX, trainY), (testX, testY)) = fashion_mnist.load_data()

    trainX = trainX.astype("float")
    testX = testX.astype("float")
    # apply mean subtraction to the data
    mean = np.mean(trainX, axis=0)
    trainX -= mean
    testX -= mean

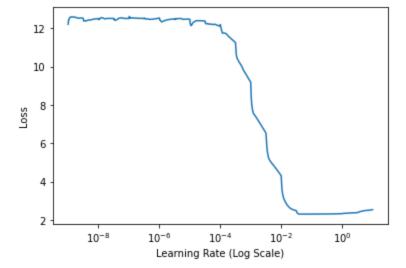
# Fashion MNIST images are 28x28 but the network we will be training
# is expecting 32x32 images
trainX = np.array([cv2.resize(x, (img_witdth, img_height)) for x in trainX])
```

```
testX = np.array([cv2.resize(x, (img witdth, img height))  for x in testX])
        # scale the pixel intensities to the range [0, 1]
        trainX = trainX.astype("float") / 255.0
        testX = testX.astype("float") / 255.0
        # reshape the data matrices to include a channel dimension (required
        # for training)
        trainX = trainX.reshape((trainX.shape[0], img witdth, img height, 1))
        testX = testX.reshape((testX.shape[0], img witdth, img height, 1))
        # convert the labels from integers to vectors
        lb = LabelBinarizer()
        trainY = lb.fit transform(trainY)
        testY = lb.transform(testY)
        # construct the image generator for data augmentation
        aug = ImageDataGenerator(
           width shift_range=0.1,
           height shift range=0.1,
           horizontal flip=True,
           fill mode="nearest",
In [49]:
        min lr = 1e-9
        \max lr = 10
        opt = SGD(min lr, momentum=0.9)
        model = MiniGoogLeNet.build(width=img witdth, height=img height, depth=1, classes=10)
        model.compile(loss="categorical crossentropy", optimizer=opt, metrics=["accuracy"])
        clr = CyclicLR(
           mode=config.CLR METHOD,
           base lr=min lr,
           max lr=max lr,
           step size=config.STEP SIZE * (trainX.shape[0] // config.BATCH SIZE),
        )
            x=aug.flow(trainX, trainY, batch size=config.BATCH SIZE),
            validation data=(testX, testY),
           steps per epoch=trainX.shape[0] // config.BATCH SIZE,
           epochs=config.NUM EPOCHS,
           callbacks=[clr],
           verbose=1,
        )
        Epoch 1/5
         5/937 [.....] - ETA: 24s - loss: 2.7100 - accuracy: 0.0844 WARN
        ING:tensorflow:Callback method `on train batch end` is slow compared to the batch time (ba
        tch time: 0.0139s vs `on train batch end` time: 0.0396s). Check your callbacks.
        - val loss: 4.4435 - val accuracy: 0.6254
        Epoch 2/5
        - val loss: 8.0529 - val accuracy: 0.5188
        Epoch 3/5
        - val loss: 12.0776 - val accuracy: 0.4553
```

- val loss: 19.7525 - val accuracy: 0.0990

Epoch 5/5

```
Epoch 1/20
Epoch 2/20
Epoch 3/20
Epoch 4/20
Epoch 5/20
Epoch 6/20
Epoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
Epoch 11/20
Epoch 12/20
Epoch 13/20
Epoch 14/20
Epoch 15/20
Epoch 16/20
Epoch 17/20
Epoch 18/20
Epoch 19/20
Epoch 20/20
```



Use the cyclical learning rate policy (with exponential decay) and train your network using batch size 64 and Irmin and Irmax values obtained in part 1. Here you will train till convergence and not just 5 epochs as in part 1. Plot train/validation loss and accuracy curve (similar to Figure 4 in reference)

• According to previous findings, the min max learning rate is around (10^{-4} , 10^{-1})

```
In [70]:
          min lr = 10e-4
          \max lr = 10e-1
          # Set learning rate to the best one find in question 1
          opt = SGD(lr = min lr, momentum=0.9)
          clr = CyclicLR(
              mode='exp range',
              base lr=min lr,
              max lr=min lr,
              step size=config.STEP SIZE * (trainX.shape[0] // config.BATCH SIZE),
          lr = lrf.lrs[lrf.losses.index(min(lrf.losses))]
          model = MiniGoogLeNet.build(width=img witdth, height=img height, depth=1, classes=10)
          model.compile(loss="categorical crossentropy", optimizer=opt, metrics=["accuracy"])
          H = model.fit(
              x=aug.flow(trainX, trainY, batch size=config.BATCH SIZE),
              validation data=(testX, testY),
              steps per epoch=trainX.shape[0] // config.BATCH SIZE,
              epochs=20,
              callbacks=[clr],
              verbose=1
          )
```

```
- val loss: 0.4313 - val accuracy: 0.8458
Epoch 5/20
- val loss: 0.3967 - val accuracy: 0.8566
Epoch 6/20
- val loss: 0.4190 - val accuracy: 0.8448
Epoch 7/20
- val loss: 0.3538 - val accuracy: 0.8725
Epoch 8/20
- val loss: 0.3349 - val accuracy: 0.8800
Epoch 9/20
- val loss: 0.3525 - val accuracy: 0.8745
Epoch 10/20
- val loss: 0.3155 - val accuracy: 0.8865
Epoch 11/20
- val loss: 0.3373 - val accuracy: 0.8780
Epoch 12/20
- val loss: 0.3255 - val accuracy: 0.8863
Epoch 13/20
- val loss: 0.2936 - val accuracy: 0.8918
Epoch 14/20
- val loss: 0.2881 - val accuracy: 0.8981
- val loss: 0.2866 - val accuracy: 0.8960
Epoch 16/20
- val loss: 0.2851 - val accuracy: 0.8986
Epoch 17/20
- val loss: 0.2751 - val accuracy: 0.8986
Epoch 18/20
- val loss: 0.2920 - val accuracy: 0.8955
Epoch 19/20
- val loss: 0.2915 - val accuracy: 0.8939
Epoch 20/20
- val loss: 0.2731 - val accuracy: 0.9015
# model is still not converged, train 20 more epochs
H 1 = model.fit(
  x=aug.flow(trainX, trainY, batch size=config.BATCH SIZE),
  validation data=(testX, testY),
  steps per epoch=trainX.shape[0] // config.BATCH SIZE,
  epochs=20,
  callbacks=[clr],
  verbose=1
)
Epoch 1/20
```

In [71]:

```
- val loss: 0.3322 - val accuracy: 0.8843
Epoch 2/20
- val loss: 0.2595 - val accuracy: 0.9083
Epoch 3/20
- val loss: 0.2787 - val accuracy: 0.9037
Epoch 4/20
- val loss: 0.2677 - val accuracy: 0.9034
Epoch 5/20
- val loss: 0.2897 - val accuracy: 0.8949
Epoch 6/20
- val loss: 0.2844 - val accuracy: 0.8924
Epoch 7/20
- val loss: 0.2611 - val accuracy: 0.9072
Epoch 8/20
- val loss: 0.2397 - val accuracy: 0.9129
Epoch 9/20
- val loss: 0.2902 - val accuracy: 0.8991
Epoch 10/20
- val loss: 0.2990 - val accuracy: 0.8961
Epoch 11/20
- val loss: 0.2673 - val accuracy: 0.9083
- val loss: 0.2556 - val accuracy: 0.9107
Epoch 13/20
- val loss: 0.2535 - val accuracy: 0.9120
Epoch 14/20
- val loss: 0.2478 - val accuracy: 0.9146
Epoch 15/20
- val loss: 0.2926 - val accuracy: 0.8999
Epoch 16/20
- val loss: 0.2421 - val accuracy: 0.9121
Epoch 17/20
- val loss: 0.2752 - val accuracy: 0.9058
Epoch 18/20
- val loss: 0.2456 - val accuracy: 0.9133
Epoch 19/20
- val loss: 0.2363 - val accuracy: 0.9169
Epoch 20/20
- val loss: 0.2300 - val accuracy: 0.9208
```

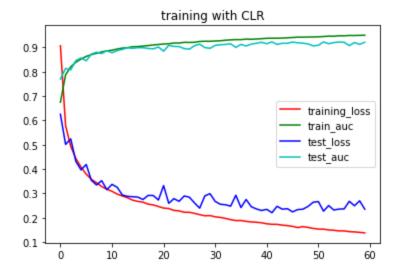
```
In [72]:
```

```
# still not converged, train 20 more epochs
# model is still not converged, train 20 more epochs
H_2 = model.fit(
    x=aug.flow(trainX, trainY, batch_size=config.BATCH_SIZE),
    validation_data=(testX, testY),
    steps_per_epoch=trainX.shape[0] // config.BATCH_SIZE,
```

```
epochs=20,
  callbacks=[clr],
  verbose=1
Epoch 1/20
5/937 [.....] - ETA: 29s - loss: 0.2446 - accuracy: 0.9250WARNI
NG:tensorflow:Callback method `on train batch end` is slow compared to the batch time (bat
ch time: 0.0180s vs `on train batch end` time: 0.0410s). Check your callbacks.
- val loss: 0.2347 - val accuracy: 0.9150
Epoch 2/20
- val loss: 0.2208 - val accuracy: 0.9224
Epoch 3/20
- val loss: 0.2472 - val accuracy: 0.9122
Epoch 4/20
- val loss: 0.2355 - val accuracy: 0.9167
- val loss: 0.2374 - val accuracy: 0.9167
Epoch 6/20
- val loss: 0.2243 - val accuracy: 0.9217
Epoch 7/20
- val loss: 0.2337 - val accuracy: 0.9187
Epoch 8/20
- val loss: 0.2352 - val accuracy: 0.9172
Epoch 9/20
- val loss: 0.2467 - val accuracy: 0.9141
Epoch 10/20
- val loss: 0.2639 - val accuracy: 0.9061
Epoch 11/20
- val loss: 0.2667 - val accuracy: 0.9087
Epoch 12/20
- val loss: 0.2273 - val accuracy: 0.9222
Epoch 13/20
- val loss: 0.2502 - val accuracy: 0.9151
Epoch 14/20
- val loss: 0.2320 - val accuracy: 0.9195
Epoch 15/20
- val loss: 0.2358 - val accuracy: 0.9218
Epoch 16/20
- val loss: 0.2367 - val accuracy: 0.9208
Epoch 17/20
- val loss: 0.2678 - val accuracy: 0.9073
Epoch 18/20
- val loss: 0.2493 - val accuracy: 0.9197
Epoch 19/20
- val loss: 0.2697 - val accuracy: 0.9123
```

• utill 60 epochs, although the training loss is still decreasing, the test accuracy is steady, so I think 60 epochs is enough for the training.

```
In [77]:
    epochs_vis = list(range(60))
    train_loss = np.concatenate([H.history['loss'], H_1.history['loss'], H_2.history['loss']],
    train_auc = np.concatenate([H.history['accuracy'], H_1.history['accuracy'], H_2.history['extest_loss = np.concatenate([H.history['val_loss'], H_1.history['val_loss'], H_2.history['val_aucuracy'], H_1.history['val_accuracy'], H_2.history['val_aucuracy'], H_1.history['val_accuracy'], H_2.history['val_aucuracy'], H_1.history['val_aucuracy'], H_2.history['val_aucuracy'], H_2.history['val_aucuracy'], H_1.history['val_aucuracy'], H_2.history['val_aucuracy'], H_2.history['val_aucur
```



question 3

We want to test if increasing batch size for a fixed learning rate has the same effect as decreasing learning rate for a fixed batch size. Fix learning rate to Irmax and train your network starting with batch size 32 and incrementally going upto 16384 (in increments of a factor of 2; like 32, 64...). You can choose a step size (in terms of number of iterations) to increment the batch size. If your GPU cannot handle large batch sizes, you need to employ effective batch size approach as discussed in Lecture 3 to simulate large batches. Plot the training loss as a function of batch size. Is the generalization of your final model similar or different than cyclical learning rate policy?

```
In [3]:
    def run_batch_experiment(batch_size, num_epochs):
        all_loss = []
        optimizer = SGD(learning_rate=0.1, momentum=0.9)
        model = MiniGoogLeNet.build(width=img_witdth, height=img_height, depth=1, classes=10)
        batch_size = batch_size
        train_dataset = tf.data.Dataset.from_tensor_slices((trainX, trainY))
        train_dataset = train_dataset.shuffle(buffer_size=1024).batch(batch_size)
        epochs = num_epochs
        for epoch in range(epochs):
            print('current working epoch:' + str(epoch), end='\r')
        if batch_size<=512:
            for step, (x_batch_train, y_batch_train) in enumerate(train_dataset):</pre>
```

```
print('current working epoch:'+ str(epoch)+ ' current working batch:' +
                               flush=True, end='\r')
                         with tf.GradientTape() as tape:
                             logits = model(x batch train, training=True) # Logits for this minibe
                             loss value = keras.losses.categorical crossentropy(y batch train, logi
                         grads = tape.gradient(loss value, model.trainable weights)
                         final grad = []
                         for grad layer in grads:
                             final grad.append(grad layer/batch size)
                         optimizer.apply gradients(zip(final grad, model.trainable weights))
                         all loss.append(loss value)
                 else:
                     for step, (x batch train, y batch train) in enumerate(train dataset):
                         print('current working epoch:'+ str(epoch)+ ' current working batch:' +
                               flush=True, end='\r')
                         start = 0
                         end = 512
                         grads list = []
                         loss list this micro batch = []
                         for i in range(int(batch size/512)):
                             if start+512 > x batch train.shape[0]:
                                 batch x = x batch train[start:,:,:]
                                 batch y = y batch train[start:,:]
                             else:
                                 batch x = x batch train[start:end,:,:,:]
                                 batch y = y batch train[start:end,:]
                             if batch x.shape[0] > 0:
                                 with tf.GradientTape() as tape:
                                     logits = model(batch x, training=True) # Logits for this min:
                                     loss value = keras.losses.categorical crossentropy(batch y, 10
                                 grads = tape.gradient(loss value, model.trainable weights)
                                 grads list.append(grads)
                                 loss list this micro batch.append(loss value)
                                 start += 512
                                 end += 512
                             else:
                         # calculate mean grads for each layer
                         final layer grads = []
                         loss list this micro batch = np.concatenate(loss list this micro batch,axi
                         all loss.append(loss list this micro batch)
                         for layer index in range(len(grads list[0])):
                             grads this layer = []
                             for micro batch index in range(len(grads list)):
                                 grads this layer.append(grads list[micro batch index][layer index]
                             final layer grads.append(sum(grads this layer)/((len(grads list)*512))
                         optimizer.apply gradients(zip(final layer grads, model.trainable weights))
             return all loss
In [4]:
         loss 32 = run batch experiment(32,10)
        current working epoch:9 current working batch:1874/1875
In [5]:
         loss 64 = run batch experiment(64,10)
        current working epoch:9 current working batch:937/938
In [6]:
        loss 128 = run batch experiment(128,10)
        current working epoch:9 current working batch:468/469
In [7]:
        loss 256 = run batch experiment(256,10)
```

```
current working epoch:9 current working batch:234/235
 In [8]:
          loss 512 = run batch experiment(512,10)
         current working epoch:9 current working batch:117/118
In [9]:
          loss 1024 = run batch experiment(1024, 10)
         current working epoch:9 current working batch:58/59
In [10]:
          loss 2048 = run batch experiment (2048, 10)
         current working epoch:9 current working batch:29/30
In [11]:
          loss 4096 = run_batch_experiment(4096,10)
         current working epoch:9 current working batch:14/15
In [12]:
          loss 8192 = run batch experiment(8192,10)
         current working epoch:9 current working batch:7/8
In [13]:
          loss 16384 = run batch experiment (16384, 10)
         current working epoch:9 current working batch:3/4
In [40]:
          losses = [loss 32,loss 64,loss 128,loss 256,loss 512,loss 1024,loss 2048,loss 4096,loss 81
          losses mean = []
          for i in losses:
              losses mean.append(np.mean(i[-1]))
In [41]:
          vis x = ['32','64','128','256','512','1024','2048','4096','8192','16384']
          vis x.reverse()
          losses mean.reverse()
          plt.plot(vis x, losses mean)
         [<matplotlib.lines.Line2D at 0x17f60fe9c70>]
Out[41]:
         0.4
         0.3
         0.2
         0.1
             16384 8192 4096 2048 1024 512
```

- From the plot we can see.
 - Large learning rate = small batch size => having lower training loss
 - We see a similar curve as CLR algorithm

Problem 3

question 1

Calculate the number of parameters in Alexnet. You will have to show calculations for each layer and then sum it to obtain the total number of parameters in Alexnet. When calculating you will need to account for all the filters (size, strides, padding) at each layer. Look at Sec. 3.5 and Figure 2 in Alexnet paper (see reference). Points will only be given when explicit calculations are shown for each layer.

```
conv1: (11 \times 11) \times 3 \times 96 + 96 = 34944

conv2: (5 \times 5) \times 96 \times 256 + 256 = 614656

conv3: (3 \times 3) \times 256 \times 384 + 384 = 885120

conv4: (3 \times 3) \times 384 \times 384 + 384 = 1327488

conv5: (3 \times 3) \times 384 \times 256 + 256 = 884992

fc1: (6 \times 6) \times 256 \times 4096 + 4096 = 37752832

fc2: 4096 \times 4096 + 4096 = 16781312

fc3: 4096 \times 1000 + 1000 = 4097000
```

total_parameters: conv1+conv2+conv3+conv4+conv5+fc1+fc2+fc3 = 62378344

question 2

VGG (Simonyan et al.) has an extremely homogeneous architecture that only performs 3x3 convolutions with stride 1 and pad 1 and 2x2 max pooling with stride 2 (and no padding) from the beginning to the end. However VGGNet is very expensive to evaluate and uses a lot more memory and parameters. Refer to VGG19 architecture on page 3 in Table 1 of the paper by Simonyan et al. You need to complete Table 1 below for calculating activation units and parameters at each layer in VGG19 (without counting biases). Its been partially filled for you.

```
In [28]:
          col 1 = ['input', 'Conv3-64', 'Conv3-64', 'POOL2',
                   'CONV3-128', 'CONV3-128', 'POOL2',
                   'CONV3-256', 'CONV3-256', 'CONV3-256', 'CONV3-256', 'POOL2',
                  'CONV3-512', 'CONV3-512', 'CONV3-512', 'CONV3-512', 'POOL2',
                  'CONV3-512', 'CONV3-512', 'CONV3-512', 'CONV3-512', 'POOL2',
                  'FC', 'FC', 'FC', 'TOTAL']
          col 2 = ['224*224*3 = 150K','224*224*64 = 3.2M','224*224*64 = 3.2M','112*112*64 = 800K',
                  '112*112*128 = 1.6M', '112*112*128 = 1.6M', '56*56*128 = 400K',
                  '56*56*256 = 800K','56*56*256 = 800K','56*56*256 = 800K','56*56*256 = 800K','28*2
                  '28*28*512 = 400K', '28*28*512 = 400K', '28*28*512 = 400K', '14*14
                  '14*14*512 = 100K', '14*14*512 = 100K', '14*14*512 = 100K', '14*14*512 = 100K', '7*7*5
                  '4096','4096','1000','sum(all)']
          col 3 = ['0', '(3*3*3)*64=1728', '(3*3*64)*64 = 36864', '0',
                   (3*3*64)*128 = 73728', (3*3*128)*128 = 147456', 0',
                   '(3*3*128)*256 = 294912','(3*3*256)*256 = 589824','(3*3*256)*256 = 589824','(3*3*
                  '(3*3*256)*512 = 117948','(3*3*512)*512 = 2358296','(3*3*512)*512 = 2358296','(3*3
                  '(3*3*512)*512 = 2358296','(3*3*512)*512 = 2358296','(3*3*512)*512 = 2358296','(3*3*512)*512
                  '7*7*512*4096','4096*4096','4096*1000','sum(all)']
          table = pd.DataFrame([col 1, col 2, col 3]).transpose()
          table.rename({0:'Layer',1:'Number of Activations (Memory)',2:'Parameters(Compute)'}, axis
```

Out[28]:		Layer	Number of Activations (Memory)	Parameters (Compute)
	0	input	224*224*3 = 150K	0
	1	Conv3-64	224*224*64 = 3.2M	(3*3*3)*64=1728
	2	Conv3-64	224*224*64 = 3.2M	(3*3*64)*64 = 36864
	3	POOL2	112*112*64 = 800K	0
	4	CONV3-128	112*112*128 = 1.6M	(3*3*64)*128 = 73728
	5	CONV3-128	112*112*128 = 1.6M	(3*3*128)*128 = 147456
	6	POOL2	56*56*128 = 400K	0
	7	CONV3-256	56*56*256 = 800K	(3*3*128)*256 = 294912
	8	CONV3-256	56*56*256 = 800K	(3*3*256)*256 = 589824
	9	CONV3-256	56*56*256 = 800K	(3*3*256)*256 = 589824
	10	CONV3-256	56*56*256 = 800K	(3*3*256)*256 = 589824
	11	POOL2	28*28*256 = 200K	0
	12	CONV3-512	28*28*512 = 400K	(3*3*256)*512 = 117948
	13	CONV3-512	28*28*512 = 400K	(3*3*512)*512 = 2358296
	14	CONV3-512	28*28*512 = 400K	(3*3*512)*512 = 2358296
	15	CONV3-512	28*28*512 = 400K	(3*3*512)*512 = 2358296
	16	POOL2	14*14*512 = 100K	0
	17	CONV3-512	14*14*512 = 100K	(3*3*512)*512 = 2358296
	18	CONV3-512	14*14*512 = 100K	(3*3*512)*512 = 2358296
	19	CONV3-512	14*14*512 = 100K	(3*3*512)*512 = 2358296
	20	CONV3-512	14*14*512 = 100K	(3*3*512)*512 = 2358296
	21	POOL2	7*7*512 = 25K	0
	22	FC	4096	7*7*512*4096
	23	FC	4096	4096*4096

25

FC

TOTAL

VGG architectures have smaller filters but deeper networks compared to Alexnet (3x3 compared to 11x11 or 5x5). Show that a stack of N convolution layers each of filter size F \times F has the same receptive field as one convolution layer with filter of size (N F -N + 1) \times (N F -N + 1). Use this to calculate the receptive field of 3 filters of size 5x5.

4096*1000

sum(all)

1000

sum(all)

- N_{th} output size: 11, input_size = (1+F-1) (1+F-1)
- $N-1_{th}$ output size: (1+F-1) * (1+F-1), input_size = (1+2F-2)(1+2F-2)
- $N-2_{th}$ output size: (1+2F-2)(1+2F-2), input_size = (1+3F-3)(1+3F-3)
-
-

- $input_layer$: input_size = (1+NF-N)(1+NF-N)
- Receptive field of 3 filters of size 5x5:

```
• 3_{rd} Filter: output size = 1*1, input size = (1+5-1)(1+5-1)
```

- 2_{nd} Filter: output size = (1+5-1)(1+5-1), input size = (1+10-2)(1+10-2)
- 1_{st} Filter: output size = (1+10-2)(1+10-2), input size = (1+15-3)(1+15-3)
- receptice field is (1+15-3)(1+15-3) = 13*13

The original Googlenet paper (Szegedy et al.) proposes two architectures for Inception module, shown in Figure 2 on page 5 of the paper, referred to as naive and dimensionality reduction respectively.

(a)

What is the general idea behind designing an inception module (parallel convolutional filters of different sizes with a pooling followed by concatenation) in a convolutional neural network?

By using different size of filters on the same activation maps and then concatenate them togather will
provide model the ability to capture fine grind features as well as coarse grind features, by using the
combined features will improve final model performance.

(b)

Assuming the input to inception module (referred to as "previous layer" in Figure 2 of the paper) has size 32x32x256, calculate the output size after filter concatenation for the naive and dimensionality reduction inception architectures with number of filters given in Figure 1.

- (Naive version):
 - input: 32 * 32 * 256
 - 1*1 conv output: 32*32*128
 - 3*3 conv output: 32*32*192
 - 5*5 conv output: 32*32*96
 - 3*3 maxpooling output: 32*32*256
 - concatenate: 32 * 32 * 672
- (dimension reduction version):
 - \blacksquare input: 32 * 32 * 256
 - 1*1 conv output: 32*32*128
 - 1*1+3*3 conv output: 32*32*192
 - 1*1+5*5 conv output: 32*32*96
 - 3*3+1*1 pooling *conv*: \$3232*64\$
 - concatenate: 32 * 32 * 480

(c)

Next calculate the total number of convolutional operations for each of the two inception architecture again assuming the input to the module has dimensions 32x32x256 and number of filters given in Figure 1.

- (Naive version):
 - 1*1 conv op: 32*32*128*1*1*256 = 33554432

```
* 3*3 conv op: 32*32*192*3*3*256 = 452984832 
* 5*5 conv op: 32*32*96*5*5*256 = 629145600 
* total ops: 1115684864
```

- total ops. 1110004004

• (dimension reduction version):

```
 \begin{array}{c} \bullet \quad 1*1*64 \text{ conv op: } 32*32*64*1*1*256*3 = 50331648 \\ \bullet \quad 1*1*128 \text{ conv op: } 32*32*128*1*1*256 = 33554432 \\ \bullet \quad 3*3 \text{ conv op: } 32*32*192*3*3*64 = 113246208 \\ \bullet \quad 5*5 \text{ conv op: } 32*32*96*5*5*64 = 157286400 \\ \bullet \quad \text{total ops: } 354418688 \\ \end{array}
```

(d)

Based on the calculations in part (c) explain the problem with naive architecture and how dimensionality reduction architecture helps (Hint: compare computational complexity). How much is the computational saving?

- Problem of naive architecture: naive architecture need too many operations (need 1115684864 ops), training can be very computional intense.
- Dimensionality reduction first use 1*1 conv layers to reduce the input channel from 256 to 64, and do 3*3 and 5*5 conv layers on the 64 channel input instead the total of 256 channels, thus greatly reduced computational complexity.
- \bullet Total saved ops: 1115684864 354418688 = 761266176

Problem 4

```
In [6]:
    def apply_mask(image, size=12, n_squares=1):
        h, w, channels = image.shape
        new_image = image
        for _ in range(n_squares):
            y = np.random.randint(h)
            x = np.random.randint(w)
            y1 = np.clip(y - size // 2, 0, h)
            y2 = np.clip(y + size // 2, 0, h)
            x1 = np.clip(x - size // 2, 0, w)
            x2 = np.clip(x + size // 2, 0, w)
            return new_image
```

question 1

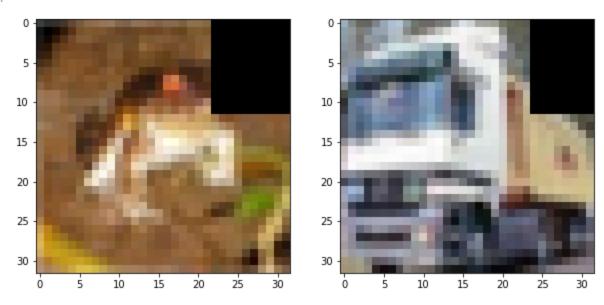
Explain cutout regularization and its advantages compared to simple dropout (as argued in the paper by DeVries et al) in your own words. Select any 2 images from CIFAR10 and show how does these images look after applying cutout. Use a square-shaped fixed size zero-mask to a random location of each image and generate its cutout version. Refer to the paper by DeVries et al (Section 3) and associated github repository.

• As stated in the paper, traditional drop out works fine for fully connected model, but it's less powerful for training a convlutional model. Two main reason for this is: 1, Conv layers have fewer parameters than fully-connected, thus require less regularization. 2, Input of conv layers is images, images have

positional factor, information dropped can be repersented by near pixles, thus drop out algorithm is less effective in reducing co-adaptation.

```
fig, ax = plt.subplots(1,2,figsize = (10,5))
ax[0].imshow(apply_mask(X_train[0].copy()))
ax[1].imshow(apply_mask(X_train[1].copy()))
```

Out[106... <matplotlib.image.AxesImage at 0x195f8de7df0>



question 2

Using CIFAR10 datasest and Resnet-44 we will first apply simple data augmentation as in He et al. (look at Section 4.2 of He et al.) and train the model with batch size 64. Note that testing is always done with original images. Plot validation error vs number of training epochs.

```
In [3]:
         1.1.1
         ResNet-44 code from
         https://github.com/PacktPublishing/Advanced-Deep-Learning-with-Keras/blob/master/chapter2-
         n = 7
         depth = n * 6 + 2
         input shape = X train[0].shape
         def lr schedule(epoch):
             """Learning Rate Schedule
             Learning rate is scheduled to be reduced after 80, 120, 160, 180 epochs.
             Called automatically every epoch as part of callbacks during training.
             # Arguments
                 epoch (int): The number of epochs
                 lr (float32): learning rate
             lr = 1e-3
             if epoch > 180:
                 1r *= 0.5e-3
```

```
elif epoch > 160:
        lr *= 1e-3
    elif epoch > 120:
       lr *= 1e-2
    elif epoch > 80:
       lr *= 1e-1
    return lr
def resnet layer(inputs,
                 num filters=16,
                 kernel size=3,
                 strides=1,
                 activation='relu',
                 batch normalization=True,
                 conv first=True):
    """2D Convolution-Batch Normalization-Activation stack builder
    Arguments:
        inputs (tensor): input tensor from input image or previous layer
        num filters (int): Conv2D number of filters
        kernel size (int): Conv2D square kernel dimensions
        strides (int): Conv2D square stride dimensions
        activation (string): activation name
       batch normalization (bool): whether to include batch normalization
        conv first (bool): conv-bn-activation (True) or
            bn-activation-conv (False)
    Returns:
        x (tensor): tensor as input to the next layer
    conv = Conv2D(num filters,
                  kernel size=kernel size,
                  strides=strides,
                  padding='same',
                  kernel initializer='he normal',
                  kernel regularizer=12(1e-4))
    x = inputs
    if conv first:
        x = conv(x)
        if batch normalization:
            x = BatchNormalization()(x)
        if activation is not None:
           x = Activation(activation)(x)
    else:
        if batch normalization:
            x = BatchNormalization()(x)
        if activation is not None:
           x = Activation(activation)(x)
        x = conv(x)
    return x
def resnet v1(input shape, depth, num classes=10):
    """ResNet Version 1 Model builder [a]
    Stacks of 2 x (3 x 3) Conv2D-BN-ReLU
    Last ReLU is after the shortcut connection.
   At the beginning of each stage, the feature map size is halved
    (downsampled) by a convolutional layer with strides=2, while
    the number of filters is doubled. Within each stage,
    the layers have the same number filters and the
    same number of filters.
    Features maps sizes:
    stage 0: 32x32, 16
    stage 1: 16x16, 32
    stage 2: 8x8, 64
    The Number of parameters is approx the same as Table 6 of [a]:
```

```
ResNet20 0.27M
    ResNet32 0.46M
    ResNet44 0.66M
    ResNet56 0.85M
    ResNet110 1.7M
   Arguments:
        input shape (tensor): shape of input image tensor
        depth (int): number of core convolutional layers
       num classes (int): number of classes (CIFAR10 has 10)
    Returns:
       model (Model): Keras model instance
    if (depth - 2) % 6 != 0:
       raise ValueError('depth should be 6n+2 (eq 20, 32, in [a])')
    # start model definition.
    num filters = 16
    num res blocks = int((depth - 2) / 6)
    inputs = Input(shape=input shape)
    x = resnet layer(inputs=inputs)
    # instantiate the stack of residual units
    for stack in range(3):
        for res block in range(num res blocks):
            strides = 1
            # first layer but not first stack
            if stack > 0 and res block == 0:
                strides = 2 # downsample
            y = resnet layer(inputs=x,
                             num filters=num filters,
                             strides=strides)
            y = resnet layer(inputs=y,
                             num filters=num filters,
                             activation=None)
            # first layer but not first stack
            if stack > 0 and res block == 0:
                # linear projection residual shortcut
                # connection to match changed dims
                x = resnet layer(inputs=x,
                                 num filters=num filters,
                                 kernel size=1,
                                 strides=strides,
                                 activation=None,
                                 batch normalization=False)
            x = add([x, y])
            x = Activation('relu')(x)
        num filters *= 2
    # add classifier on top.
    # v1 does not use BN after last shortcut connection-ReLU
    x = AveragePooling2D(pool size=8)(x)
    y = Flatten()(x)
    outputs = Dense(num classes,
                    activation='softmax',
                    kernel initializer='he normal') (y)
    # instantiate model.
    model = Model(inputs=inputs, outputs=outputs)
    return model
model = resnet v1(input shape=input shape, depth=depth)
model.compile(loss='sparse categorical crossentropy',
```

```
optimizer=Adam(lr=lr schedule(0)),
               metrics=['acc'])
      C:\Learning\SoftWare\anaconda\lib\site-packages\keras\optimizer v2\adam.py:105: UserWarnin
      g: The `lr` argument is deprecated, use `learning rate` instead.
       super(Adam, self). init (name, **kwargs)
In [87]:
      lr scheduler = LearningRateScheduler(lr schedule)
      lr reducer = ReduceLROnPlateau(factor=np.sqrt(0.1),
                          cooldown=0,
                          patience=5,
                          min lr=0.5e-6)
      callbacks = [lr reducer, lr scheduler]
      history nocutoff = model.fit(X train, y train,
               batch size=64,
               epochs=100,
               validation data=(X test, y test),
               shuffle=True,
               callbacks=callbacks)
      Learning rate: 0.001
      Epoch 1/100
      l loss: 1.6173 - val acc: 0.5398 - lr: 0.0010
      Learning rate: 0.001
      Epoch 2/100
      l loss: 1.3426 - val acc: 0.6387 - lr: 0.0010
      Learning rate: 0.001
      Epoch 3/100
      l loss: 1.9748 - val acc: 0.5295 - lr: 0.0010
      Learning rate: 0.001
      Epoch 4/100
      l loss: 1.2534 - val acc: 0.6730 - lr: 0.0010
     Learning rate: 0.001
     Epoch 5/100
      l loss: 1.5322 - val acc: 0.6149 - lr: 0.0010
     Learning rate: 0.001
     Epoch 6/100
      l loss: 1.4247 - val acc: 0.6707 - lr: 0.0010
     Learning rate: 0.001
      Epoch 7/100
      l loss: 0.9942 - val acc: 0.7537 - lr: 0.0010
     Learning rate: 0.001
     Epoch 8/100
      l loss: 1.0600 - val acc: 0.7434 - lr: 0.0010
     Learning rate: 0.001
```

Epoch 9/100

Epoch 10/100

Epoch 11/100

Learning rate: 0.001

Learning rate: 0.001

l loss: 1.2419 - val acc: 0.6924 - lr: 0.0010

l loss: 1.1271 - val acc: 0.7357 - lr: 0.0010

```
l loss: 1.3101 - val acc: 0.7193 - lr: 0.0010
Learning rate: 0.001
Epoch 12/100
l loss: 1.1446 - val acc: 0.7297 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 13/100
l loss: 1.3804 - val acc: 0.6985 - lr: 0.0010
Learning rate: 0.001
Epoch 14/100
l loss: 1.0397 - val acc: 0.7829 - lr: 0.0010
Learning rate: 0.001
Epoch 15/100
l loss: 0.9681 - val acc: 0.7997 - lr: 0.0010
Learning rate: 0.001
Epoch 16/100
l loss: 1.1980 - val acc: 0.7527 - lr: 0.0010
Learning rate: 0.001
Epoch 17/100
l loss: 1.5867 - val acc: 0.6770 - lr: 0.0010
Learning rate: 0.001
Epoch 18/100
l loss: 1.1873 - val acc: 0.7612 - lr: 0.0010
Learning rate: 0.001
Epoch 19/100
l loss: 1.2333 - val acc: 0.7551 - lr: 0.0010
Learning rate: 0.001
Epoch 20/100
l loss: 1.3614 - val acc: 0.7613 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 21/100
l loss: 2.0787 - val acc: 0.6301 - lr: 0.0010
Learning rate: 0.001
Epoch 22/100
l loss: 1.0557 - val acc: 0.7967 - lr: 0.0010
Learning rate: 0.001
Epoch 23/100
l loss: 1.1290 - val acc: 0.7908 - lr: 0.0010
Learning rate: 0.001
Epoch 24/100
l loss: 1.5849 - val acc: 0.6999 - lr: 0.0010
Learning rate: 0.001
Epoch 25/100
l loss: 1.0643 - val acc: 0.7975 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 26/100
l loss: 1.1088 - val acc: 0.7989 - lr: 0.0010
Learning rate: 0.001
Epoch 27/100
```

l loss: 1.2352 - val acc: 0.7657 - lr: 0.0010

```
Learning rate: 0.001
Epoch 28/100
l loss: 1.2456 - val acc: 0.7760 - lr: 0.0010
Learning rate: 0.001
Epoch 29/100
l loss: 1.1476 - val acc: 0.7680 - lr: 0.0010
Learning rate: 0.001
Epoch 30/100
l loss: 1.3683 - val acc: 0.7578 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 31/100
l loss: 1.3922 - val acc: 0.7457 - lr: 0.0010
Learning rate: 0.001
Epoch 32/100
l loss: 1.0145 - val acc: 0.8084 - lr: 0.0010
Learning rate: 0.001
Epoch 33/100
l loss: 1.0878 - val acc: 0.7963 - lr: 0.0010
Learning rate: 0.001
Epoch 34/100
l loss: 1.1551 - val acc: 0.7879 - lr: 0.0010
Learning rate: 0.001
Epoch 35/100
l loss: 1.3335 - val acc: 0.7570 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 36/100
l loss: 1.1011 - val acc: 0.7955 - lr: 0.0010
Learning rate: 0.001
Epoch 37/100
l loss: 1.5619 - val acc: 0.7062 - lr: 0.0010
Learning rate: 0.001
Epoch 38/100
l loss: 1.1393 - val acc: 0.7810 - lr: 0.0010
Learning rate: 0.001
Epoch 39/100
l loss: 1.1955 - val acc: 0.7781 - lr: 0.0010
Learning rate: 0.001
Epoch 40/100
l loss: 1.2685 - val acc: 0.7662 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 41/100
l loss: 1.2871 - val acc: 0.7605 - lr: 0.0010
Learning rate: 0.001
Epoch 42/100
l loss: 1.3699 - val acc: 0.7502 - lr: 0.0010
Learning rate: 0.001
Epoch 43/100
l loss: 1.1185 - val acc: 0.7956 - lr: 0.0010
```

Learning rate: 0.001

Epoch 44/100

```
l loss: 1.7085 - val acc: 0.7234 - lr: 0.0010
Learning rate: 0.001
Epoch 45/100
l loss: 1.4187 - val acc: 0.7606 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 46/100
l loss: 1.4183 - val acc: 0.7643 - lr: 0.0010
Learning rate: 0.001
Epoch 47/100
l loss: 1.3729 - val acc: 0.7549 - lr: 0.0010
Learning rate: 0.001
Epoch 48/100
l loss: 1.2230 - val acc: 0.7851 - lr: 0.0010
Learning rate: 0.001
Epoch 49/100
l loss: 1.2709 - val acc: 0.7801 - lr: 0.0010
Learning rate: 0.001
Epoch 50/100
l loss: 1.2202 - val acc: 0.7738 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 51/100
l loss: 1.1678 - val acc: 0.7869 - lr: 0.0010
Learning rate: 0.001
Epoch 52/100
l loss: 1.2070 - val acc: 0.7814 - lr: 0.0010
Learning rate: 0.001
Epoch 53/100
l loss: 1.4276 - val acc: 0.7531 - lr: 0.0010
Learning rate: 0.001
Epoch 54/100
l loss: 1.3229 - val acc: 0.7718 - lr: 0.0010
Learning rate: 0.001
Epoch 55/100
l loss: 1.2855 - val acc: 0.7777 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 56/100
l loss: 1.2299 - val acc: 0.7853 - lr: 0.0010
Learning rate: 0.001
Epoch 57/100
l loss: 1.4009 - val acc: 0.7630 - lr: 0.0010
Learning rate: 0.001
Epoch 58/100
l loss: 1.3599 - val acc: 0.7709 - lr: 0.0010
Learning rate: 0.001
Epoch 59/100
l loss: 1.5912 - val acc: 0.7366 - lr: 0.0010
Learning rate: 0.001
Epoch 60/100
```

l loss: 1.2113 - val acc: 0.7795 - lr: 3.1623e-04

```
Learning rate: 0.001
Epoch 61/100
782/782 [============= ] - 23s 30ms/step - loss: 0.3725 - acc: 0.9675 - va
l loss: 1.1424 - val acc: 0.7958 - lr: 0.0010
Learning rate: 0.001
Epoch 62/100
l loss: 1.1426 - val acc: 0.7996 - lr: 0.0010
Learning rate: 0.001
Epoch 63/100
l loss: 1.4362 - val acc: 0.7681 - lr: 0.0010
Learning rate: 0.001
Epoch 64/100
l loss: 1.2137 - val acc: 0.7718 - lr: 0.0010
Learning rate: 0.001
Epoch 65/100
l loss: 1.1948 - val acc: 0.7770 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 66/100
l loss: 1.2301 - val acc: 0.7741 - lr: 0.0010
Learning rate: 0.001
Epoch 67/100
l loss: 1.7558 - val acc: 0.7163 - lr: 0.0010
Learning rate: 0.001
Epoch 68/100
l loss: 1.4243 - val acc: 0.7509 - lr: 0.0010
Learning rate: 0.001
Epoch 69/100
l loss: 1.2507 - val acc: 0.7835 - lr: 0.0010
Learning rate: 0.001
Epoch 70/100
l loss: 1.3538 - val acc: 0.7600 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 71/100
l loss: 1.3749 - val acc: 0.7570 - lr: 0.0010
Learning rate: 0.001
Epoch 72/100
l loss: 1.1880 - val acc: 0.7897 - lr: 0.0010
Learning rate: 0.001
Epoch 73/100
l loss: 1.3043 - val acc: 0.7806 - lr: 0.0010
Learning rate: 0.001
Epoch 74/100
l loss: 1.4826 - val acc: 0.7464 - lr: 0.0010
Learning rate: 0.001
Epoch 75/100
l loss: 1.2425 - val acc: 0.7740 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 76/100
782/782 [============ ] - 23s 30ms/step - loss: 0.3631 - acc: 0.9670 - va
l loss: 1.2678 - val acc: 0.7683 - lr: 0.0010
```

Learning rate: 0.001

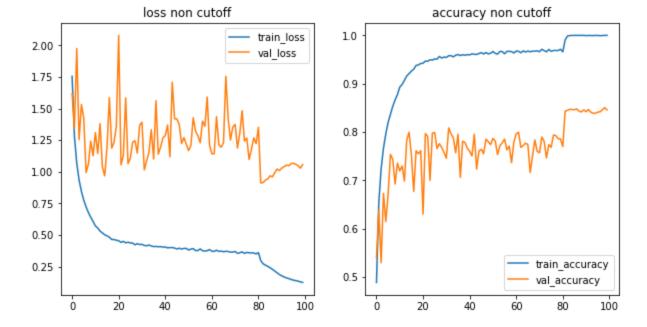
Epoch 77/100

```
l loss: 1.0978 - val acc: 0.7938 - lr: 0.0010
Learning rate: 0.001
Epoch 78/100
l loss: 1.1778 - val acc: 0.7927 - lr: 0.0010
Learning rate: 0.001
Epoch 79/100
l loss: 1.2689 - val acc: 0.7860 - lr: 0.0010
Learning rate: 0.001
Epoch 80/100
l loss: 1.2231 - val acc: 0.7858 - lr: 3.1623e-04
Learning rate: 0.001
Epoch 81/100
l loss: 1.3517 - val acc: 0.7699 - lr: 0.0010
Learning rate: 0.0001
Epoch 82/100
l loss: 0.9125 - val acc: 0.8431 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 83/100
l loss: 0.9146 - val acc: 0.8448 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 84/100
l loss: 0.9346 - val acc: 0.8470 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 85/100
l loss: 0.9445 - val acc: 0.8465 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 86/100
l loss: 0.9683 - val acc: 0.8461 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 87/100
l loss: 0.9598 - val acc: 0.8477 - lr: 3.1623e-05
Learning rate: 0.0001
Epoch 88/100
l loss: 0.9930 - val acc: 0.8431 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 89/100
l loss: 1.0220 - val acc: 0.8416 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 90/100
l loss: 1.0098 - val acc: 0.8457 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 91/100
l loss: 1.0290 - val acc: 0.8419 - lr: 1.0000e-04
Learning rate: 0.0001
Epoch 92/100
l loss: 1.0400 - val acc: 0.8465 - lr: 3.1623e-05
Learning rate: 0.0001
Epoch 93/100
```

l loss: 1.0535 - val acc: 0.8411 - lr: 1.0000e-04

```
Learning rate: 0.0001
      Epoch 94/100
      l loss: 1.0478 - val acc: 0.8387 - lr: 1.0000e-04
      Learning rate: 0.0001
      Epoch 95/100
      l loss: 1.0645 - val acc: 0.8387 - lr: 1.0000e-04
      Learning rate: 0.0001
      Epoch 96/100
      l loss: 1.0677 - val acc: 0.8411 - lr: 1.0000e-04
      Learning rate: 0.0001
      Epoch 97/100
      l loss: 1.0598 - val acc: 0.8417 - lr: 3.1623e-05
      Learning rate: 0.0001
      Epoch 98/100
      l loss: 1.0491 - val acc: 0.8461 - lr: 1.0000e-04
      Learning rate: 0.0001
      Epoch 99/100
      l loss: 1.0289 - val acc: 0.8503 - lr: 1.0000e-04
      Learning rate: 0.0001
      Epoch 100/100
      l loss: 1.0565 - val acc: 0.8457 - lr: 1.0000e-04
In [89]:
      epoch vis = list(range(100))
      train loss noncutoff = history nocutoff.history['loss']
      train auc noncutoff = history nocutoff.history['acc']
      test loss noncutoff = history nocutoff.history['val loss']
      test auc noncutoff = history nocutoff.history['val acc']
      fig, ax = plt.subplots(1, 2, figsize = (10, 5))
      ax[0].plot(epoch vis, train loss noncutoff, label = 'train loss')
      ax[1].plot(epoch vis, train auc noncutoff, label = 'train accuracy')
      ax[0].plot(epoch vis, test loss noncutoff, label = 'val loss')
      ax[1].plot(epoch vis, test auc noncutoff, label = 'val accuracy')
      ax[0].legend()
      ax[1].legend()
      ax[0].set title('loss non cutoff')
      ax[1].set title('accuracy non cutoff')
      Text(0.5, 1.0, 'accuracy non cutoff')
```

Out[89]:



Next use cutout for data augmentation in Resnet-44 as in Hoffer et al. and train the model and use the same set-up in your experiments. Plot validation error vs number of epochs for different values of M (2,4,8,16) where M is the number of instances generated from an input sample after applying cutout M times effectively increasing the batch size to M ·B, where B is the original batch size (before applying cutout augmentation). You will obtain a figure similar to Figure 3(a) in the paper by Hoffer et al. Also compare the number of epochs and wallclock time to reach 92% accuracy for different values of M. Do not run any experiment for more than 100 epochs. If even after 100 epochs of training you did not achieve 92% then just report the accuracy you obtain and the corresponding wallclock time to train for 100 epochs. Remember to use the same hyperparameters for training as used with Resnet44 training in He et al (look at the third paragraph in Sec. 4.2 of He et al for the hyperparameter values). Before attempting this question it is advisable to read the paper by Hoffer et al. and especially Section 4.1.

```
In [4]:
         def cut off train data(M):
             train x cutoff = []
             train y cutoff = []
             for index, img in enumerate(X train):
                 for m in range(M):
                     train x cutoff.append(apply mask(img.copy()))
                     train y cutoff.append(y train[index])
             return np.array(train x cutoff),np.array(train y cutoff)
         def train a cut off model(M):
             # def model
             model cutoff = resnet v1(input shape=input shape, depth=depth)
             model cutoff.compile(loss='sparse categorical crossentropy',
                           optimizer=Adam(lr=lr schedule(0)),
                           metrics=['acc'])
             # training data
             train with m x, train with m y = cut off train data(M)
             # trainmodel
             lr scheduler = LearningRateScheduler(lr schedule)
             lr reducer = ReduceLROnPlateau(factor=np.sqrt(0.1),
                                             cooldown=0,
                                             patience=5,
```

```
In [16]:
```

```
history_cutoff_1, time_1 = train_a_cut_off_model(1)
```

```
Epoch 1/20
val loss: 1.7007 - val acc: 0.5169 - lr: 0.0010
Epoch 2/20
val loss: 1.2922 - val acc: 0.6451 - lr: 0.0010
Epoch 3/20
val loss: 1.3092 - val acc: 0.6473 - lr: 0.0010
Epoch 4/20
val loss: 1.2850 - val acc: 0.6672 - lr: 0.0010
Epoch 5/20
val_loss: 1.1702 - val_acc: 0.7001 - lr: 0.0010
Epoch 6/20
val loss: 1.0027 - val acc: 0.7463 - lr: 0.0010
Epoch 7/20
val loss: 1.1535 - val acc: 0.7163 - lr: 0.0010
Epoch 8/20
val loss: 1.2956 - val acc: 0.6923 - lr: 0.0010
Epoch 9/20
val loss: 1.2702 - val acc: 0.6910 - lr: 0.0010
Epoch 10/20
val loss: 1.0899 - val acc: 0.7396 - lr: 0.0010
Epoch 11/20
val loss: 0.9606 - val acc: 0.7772 - lr: 0.0010
Epoch 12/20
val loss: 1.1112 - val acc: 0.7594 - lr: 0.0010
Epoch 13/20
val loss: 0.9798 - val acc: 0.7823 - lr: 0.0010
Epoch 14/20
val loss: 1.2200 - val acc: 0.7187 - lr: 0.0010
Epoch 15/20
val loss: 1.2267 - val acc: 0.7473 - lr: 0.0010
Epoch 16/20
```

```
val loss: 1.4111 - val acc: 0.6998 - lr: 3.1623e-04
   Epoch 17/20
   val loss: 1.1516 - val acc: 0.7725 - lr: 0.0010
   Epoch 18/20
   val loss: 0.9879 - val acc: 0.8013 - lr: 0.0010
   Epoch 19/20
   val loss: 1.0601 - val acc: 0.7809 - lr: 0.0010
   Epoch 20/20
   val loss: 1.0400 - val acc: 0.7999 - lr: 0.0010
In [17]:
   history cutoff 2, time 2 = train a cut off model(2)
   Epoch 1/20
   val loss: 1.5763 - val acc: 0.5705 - lr: 0.0010
   Epoch 2/20
   val loss: 1.0923 - val acc: 0.7161 - lr: 0.0010
   Epoch 3/20
   val loss: 0.9938 - val acc: 0.7513 - lr: 0.0010
   Epoch 4/20
   val loss: 0.9377 - val acc: 0.7559 - lr: 0.0010
   Epoch 5/20
   val loss: 1.0307 - val acc: 0.7403 - lr: 0.0010
   Epoch 6/20
   val loss: 1.2214 - val acc: 0.7131 - lr: 0.0010
   Epoch 7/20
   val loss: 1.1379 - val acc: 0.7460 - lr: 0.0010
   Epoch 8/20
   val loss: 1.1653 - val acc: 0.7504 - lr: 0.0010
   Epoch 9/20
   val loss: 1.0267 - val acc: 0.7672 - lr: 3.1623e-04
   Epoch 10/20
   val loss: 1.1807 - val acc: 0.7574 - lr: 0.0010
   Epoch 11/20
   val loss: 1.0686 - val acc: 0.7872 - lr: 0.0010
   Epoch 12/20
   val loss: 1.0318 - val acc: 0.7925 - lr: 0.0010
   Epoch 13/20
   val loss: 1.0127 - val acc: 0.8002 - lr: 0.0010
   Epoch 14/20
   val loss: 1.0421 - val acc: 0.8007 - lr: 3.1623e-04
```

Epoch 15/20

Epoch 16/20

val loss: 1.0787 - val acc: 0.8112 - lr: 0.0010

val loss: 1.3073 - val acc: 0.7564 - lr: 0.0010

```
Epoch 17/20
   val loss: 1.4237 - val acc: 0.7357 - lr: 0.0010
   Epoch 18/20
   val loss: 1.2861 - val acc: 0.7707 - lr: 0.0010
   Epoch 19/20
   val loss: 1.1247 - val acc: 0.7836 - lr: 3.1623e-04
   Epoch 20/20
   val loss: 1.4280 - val acc: 0.7497 - lr: 0.0010
In [18]:
    history cutoff 4, time 4 = train a cut off model(4)
   Epoch 1/20
   val loss: 1.4764 - val acc: 0.5846 - lr: 0.0010
   val loss: 1.0704 - val acc: 0.7329 - lr: 0.0010
   Epoch 3/20
   val loss: 1.1984 - val acc: 0.7157 - lr: 0.0010
   Epoch 4/20
   val loss: 1.0041 - val acc: 0.7695 - lr: 0.0010
   Epoch 5/20
   val loss: 1.1104 - val acc: 0.7634 - lr: 0.0010
   Epoch 6/20
   val loss: 1.2633 - val acc: 0.7558 - lr: 0.0010
   Epoch 7/20
   val loss: 1.2725 - val acc: 0.7553 - lr: 0.0010
   Epoch 8/20
   val loss: 1.3441 - val acc: 0.7467 - lr: 0.0010
   Epoch 9/20
   val loss: 1.3189 - val acc: 0.7522 - lr: 3.1623e-04
   Epoch 10/20
   val loss: 1.3841 - val acc: 0.7494 - lr: 0.0010
   Epoch 11/20
   val loss: 1.3027 - val acc: 0.7695 - lr: 0.0010
   Epoch 12/20
   val loss: 1.3163 - val acc: 0.7728 - lr: 0.0010
   val loss: 1.2604 - val acc: 0.7522 - lr: 0.0010
   Epoch 14/20
   val loss: 1.4126 - val acc: 0.7476 - lr: 3.1623e-04
   Epoch 15/20
```

val loss: 1.3457 - val acc: 0.7603 - lr: 0.0010

val loss: 1.1831 - val acc: 0.7898 - lr: 0.0010

Epoch 16/20

Epoch 17/20

```
val loss: 1.2335 - val acc: 0.7951 - lr: 0.0010
    Epoch 18/20
   val loss: 1.3593 - val acc: 0.7608 - lr: 0.0010
   Epoch 19/20
   val loss: 1.1133 - val acc: 0.8030 - lr: 3.1623e-04
   Epoch 20/20
    val loss: 1.2262 - val acc: 0.7833 - lr: 0.0010
In [19]:
    history cutoff 8, time 8 = train a cut off model(8)
    Epoch 1/20
    val loss: 1.3204 - val acc: 0.6608 - lr: 0.0010
   val loss: 1.2154 - val acc: 0.7099 - lr: 0.0010
   Epoch 3/20
    val loss: 1.0629 - val acc: 0.7585 - lr: 0.0010
   Epoch 4/20
   val loss: 0.9881 - val acc: 0.7836 - lr: 0.0010
   val loss: 1.2083 - val acc: 0.7541 - lr: 0.0010
   Epoch 6/20
   val loss: 2.0251 - val acc: 0.6670 - lr: 0.0010
   Epoch 7/20
   val loss: 1.6866 - val acc: 0.7049 - lr: 0.0010
    Epoch 8/20
   val loss: 1.1605 - val acc: 0.7934 - lr: 0.0010
   Epoch 9/20
   val loss: 1.3453 - val acc: 0.7659 - lr: 3.1623e-04
    Epoch 10/20
    val loss: 1.3324 - val acc: 0.7485 - lr: 0.0010
   Epoch 11/20
   val loss: 1.2246 - val acc: 0.7820 - lr: 0.0010
   Epoch 12/20
   val loss: 1.2731 - val acc: 0.7820 - lr: 0.0010
   Epoch 13/20
   val loss: 1.1112 - val acc: 0.7969 - lr: 0.0010
   Epoch 14/20
```

val loss: 1.3165 - val acc: 0.7864 - lr: 3.1623e-04

val loss: 1.3896 - val acc: 0.7680 - lr: 0.0010

val loss: 1.0102 - val acc: 0.8130 - lr: 0.0010

val loss: 1.2303 - val acc: 0.7818 - lr: 0.0010

Epoch 15/20

Epoch 16/20

Epoch 17/20

Epoch 18/20

```
val loss: 1.4652 - val acc: 0.7730 - lr: 0.0010
   Epoch 19/20
   val loss: 1.0964 - val acc: 0.8116 - lr: 3.1623e-04
   Epoch 20/20
   val loss: 1.5218 - val acc: 0.7562 - lr: 0.0010
In [7]:
   history cutoff 16, time 16 = train a cut off model(16)
   Epoch 1/20
   - val loss: 1.6258 - val acc: 0.6408 - lr: 0.0010
   - val loss: 1.3549 - val acc: 0.7339 - lr: 0.0010
   Epoch 3/20
   - val loss: 1.4744 - val acc: 0.7359 - lr: 0.0010
   Epoch 4/20
   - val loss: 2.2739 - val acc: 0.6582 - lr: 0.0010
   Epoch 5/20
   - val loss: 1.7049 - val acc: 0.7021 - lr: 0.0010
   Epoch 6/20
   - val loss: 1.8806 - val acc: 0.7054 - lr: 0.0010
   Epoch 7/20
   - val loss: 2.0385 - val acc: 0.7102 - lr: 3.1623e-04
   Epoch 8/20
   - val loss: 1.2951 - val acc: 0.7706 - lr: 0.0010
   Epoch 9/20
   - val loss: 1.4359 - val acc: 0.7581 - lr: 0.0010
   Epoch 10/20
   - val loss: 1.2327 - val acc: 0.7777 - lr: 0.0010
   Epoch 11/20
   - val loss: 1.5153 - val acc: 0.7524 - lr: 0.0010
   Epoch 12/20
   - val loss: 1.7484 - val acc: 0.7442 - lr: 0.0010
   Epoch 13/20
   - val loss: 1.5708 - val acc: 0.7548 - lr: 0.0010
   Epoch 14/20
   - val loss: 1.4176 - val acc: 0.7664 - lr: 0.0010
   Epoch 15/20
   - val loss: 1.2585 - val acc: 0.7820 - lr: 3.1623e-04
```

Epoch 16/20

Epoch 17/20

Epoch 18/20

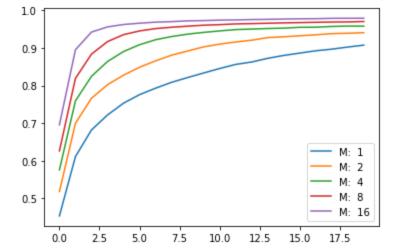
- val loss: 1.2565 - val acc: 0.7808 - lr: 0.0010

- val loss: 1.5072 - val acc: 0.7766 - lr: 0.0010

- val loss: 1.4070 - val acc: 0.7799 - lr: 0.0010

```
- val loss: 1.0858 - val acc: 0.7965 - lr: 0.0010
        Epoch 20/20
        - val loss: 1.1449 - val acc: 0.8045 - lr: 0.0010
In [8]:
         import pickle
         #history list = [history cutoff 1.history, history cutoff 2.history, history cutoff 4.history
         #time list = [time 1,time 2,time 4,time 8]
         #with open('Q4 trained data.pkl','wb') as handle:
           pickle.dump((history list,time list), handle ,protocol = pickle.HIGHEST PROTOCOL)
         with open('Q4 trained data.pkl','rb') as handle:
             previous result = pickle.load(handle)
In [14]:
         history_list_final = [previous_result[0][0], previous_result[0][1], previous_result[0][2],
                             previous result[0][3], history cutoff 16.history]
         time list final = [previous result[1][0],previous result[1][1],previous result[1][2],previ
         with open('Q4 final result data.pkl','wb') as handle:
             pickle.dump((history list final,time list final), handle ,protocol = pickle.HIGHEST PRO
In [15]:
         with open('Q4 final result data.pkl','rb') as handle:
             final result = pickle.load(handle)
In [29]:
         for i in range(5):
             epoch vis = list(range(20))
             plt.plot(epoch vis,final result[0][i]['loss'], label = 'M: '+str(2**i))
        <matplotlib.legend.Legend at 0x1b17caa2160>
Out[29]:
        1.8
                                             - M: 1
                                             M: 2
        1.6
                                              M: 4
                                              M: 8
        14
                                              M: 16
        1.2
        1.0
        0.8
        0.6
        0.4
        0.2
                          7.5
                               10.0
                                   12.5
                                             17.5
            0.0
In [31]:
         for i in range(5):
             epoch vis = list(range(20))
             plt.plot(epoch vis,final result[0][i]['acc'], label = 'M: ' +str(2**i))
         plt.legend()
        <matplotlib.legend.Legend at 0x1b15aff2070>
Out[31]:
```

Epoch 19/20



```
In [32]: # training_time
    final_result[1]

Out[32]: [815.8369281291962,
```

Out[32]: [815.8369281291962, 1008.5819070339203, 1361.3079934120178, 2330.293242454529, 3645.6672925949097]

• due to time limitation, I only ran 20 epochs here