Lec06

Thursday, September 24, 2020 9:52 AM

## Lecture Objectives

Equilibrium for a single particle

Equilibrium for a system of particles

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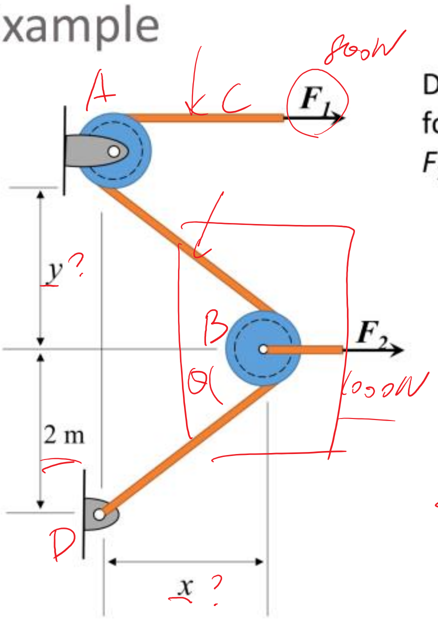
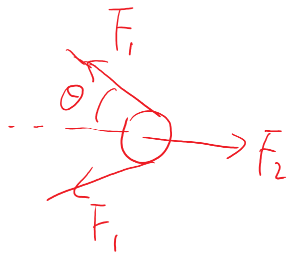
## Example

Determine the distances $x$ and $y$ for equilibrium if $F_1$ = 800 N and $F_2$ = 1000 N.

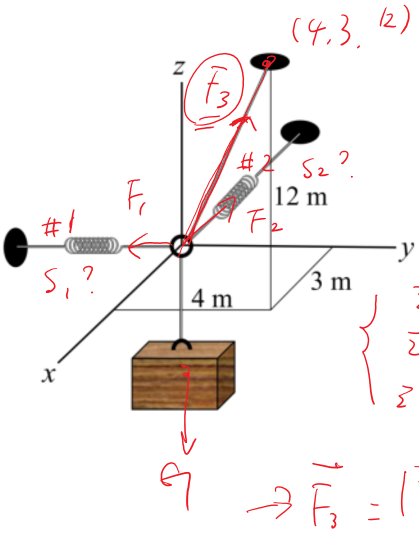



$$2 F_1 \cdot \cos\theta = F_2$$

$$\cos\theta = \frac{1000}{2 \times 800}$$

# Example – 3D




Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k$ = 360 N-m.

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases} \Rightarrow \begin{cases} F_{3x} = F_2 \\ F_{3y} = F_1 \\ F_{3z} = G \end{cases} \Rightarrow \begin{cases} |\vec{F}_3| \cdot \frac{4}{13} = F_2 & (1) \\ |\vec{F}_3| \cdot \frac{3}{13} = F_1 & (2) \\ |\vec{F}_3| \frac{12}{13} = G & (3) \end{cases}$$

$$\rightarrow \vec{F}_3 = |\vec{F}_3| \cdot \hat{u}$$

$$\boxed{\hat{u} = \frac{(4,3,12)}{13}}$$

$$F_1 = k \cdot S_1$$

$$F_2 = k\, S_2$$

# i-Clicker Time

In 3-D, when you know the magnitude of a force but not its direction, how many independent unknowns corresponding to that force remain?

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} = |\vec{F}|\, \hat{u}$$

A) One

✓ $\boxed{\text{B) Two}}$

C) Three

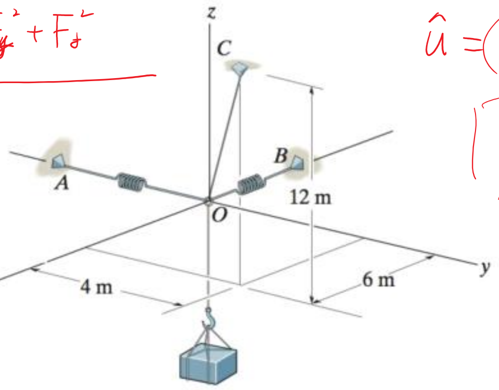
D) Four ✗

E) Five

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

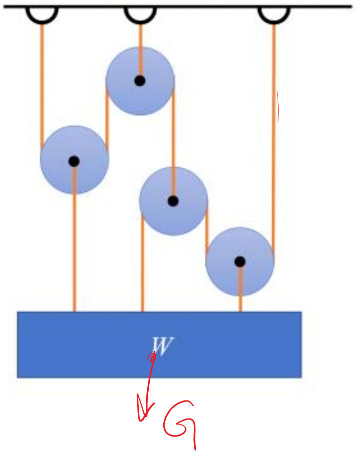
$$\hat{u} = \cos\alpha\, \hat{i} + \cos\beta\, \hat{j} + \cos\gamma\, \hat{k}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$



# Equilibrium of a system of particles

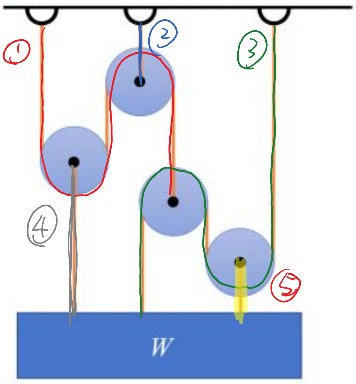
Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law: $\Sigma\mathbf{F} = \mathbf{0}$
on selected multiple free-body diagrams of particles or groups of particles.




Example: Which cable will have the most tension in it?

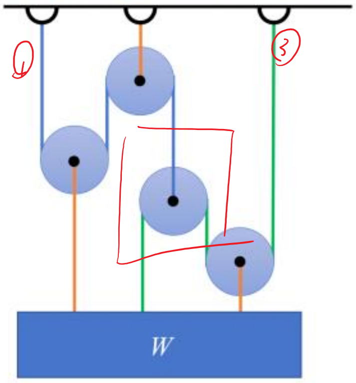
# i-Clicker Time

How many different ~~tensions~~ Cables should be taken into consideration when designing the pulley system in the bottom schematic?




(A) 4
$\boxed{\text{(B) 5}}$ ✓
(C) 6
(D) 7
(E) None of the above

## i-Clicker Time

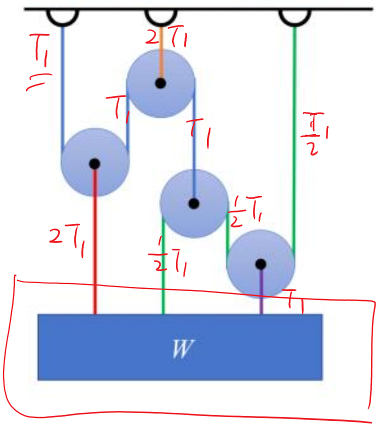



How do the tensions in cable 1 and cable 3 relate to each other?

(A) $1T_1 = 1T_3$
(B) $1T_1 = 2T_3$
(C) $1T_1 = 3T_3$
(D) $3T_1 = 1T_3$
(E) None of the above

$T_1 = 2\,T_3$

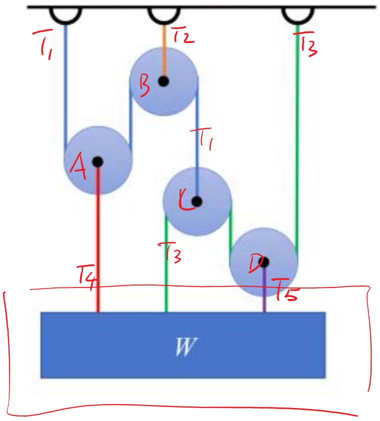
## i-Clicker Time




What is your guess for the cable with the most tension inside

(A) $T_1$
(B) $T_2$ ✓
(C) $T_3$
(D) $T_4$ ✓
(E) $T_5$

# Example




The complete system of equations would be:

A: $T_4 = 2T_1$ (1)

B: $T_2 = 2T_1$ (2)

C: $T_1 = 2T_3$ (3)

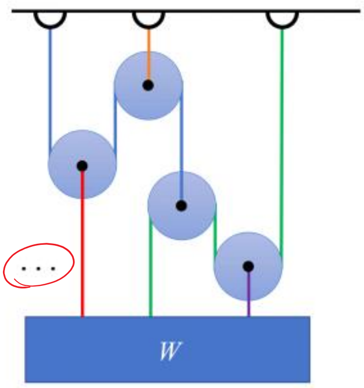
D: $T_5 = 2T_3$ (4)

$T_4 + T_3 + T_5 = W$ (5)

$T_1 \; ... \; T_5$

# MATLAB code

```
%% This MATLAB code solves the pulley system problem

W = 100; %Assume W is 100 N

syms T1 T2 T3 T4 T5;

eq1=0==2*T1-T4;
eq2=0==T2-2*T1;
eq3=0==T1-2*T3;
eq4=0==2*T3-T5;
eq5=0==T4+T3+T5-W;

sol = solve([eq1;eq2;eq3;eq4;;eq5], ...
    [T1 T2 T3 T4 T5]);

T1 = double(sol.T1)
T2 = double(sol.T2)
T3 = double(sol.T3)
T4 = double(sol.T4)
T5 = double(sol.T5)
```



## Summary

Multiple free-body diagrams is necessary when:

1. given parameters and unknown parameter(s) of interest do not relate directly
2. number of unknown parameters exceeds the number of equilibrium equations per free-body-diagram