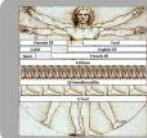


Lecture Objectives



Couple moments



Equivalent systems



Moment about an axis

1

F_1 and F_2 form a couple. The moment of the couple is given by:

A) $\vec{r}_1 \times \vec{F}_1$ ~~x~~

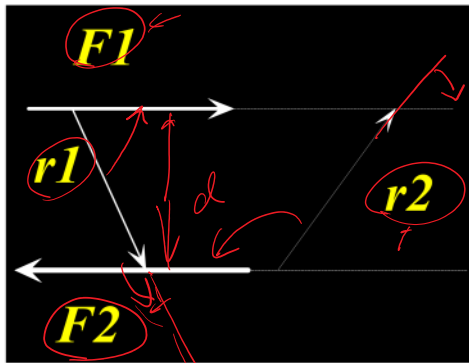
✓ B) $\vec{r}_2 \times \vec{F}_1$

C) $\vec{F}_2 \times \vec{r}_1$

~~D) $\vec{r}_2 \times \vec{F}_2$~~

$\vec{M} = \vec{r} \times \vec{F}$

✓ $\vec{r}_1 \times \vec{F}_2$

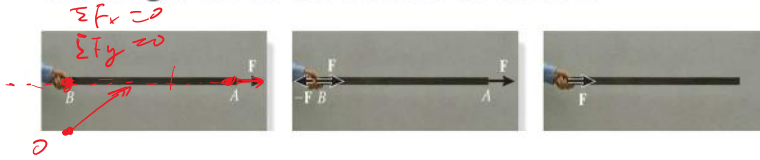


$|\vec{M}| = |\vec{F}|d$

$\vec{M} = \vec{r} \times \vec{F}$

(\vec{M})
 \hat{u}

Moving a force on its line of action



Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect. } force ←
moment

Hence, a force vector is called a sliding vector. ←

However, the internal effect of the force on the body does depend on where the force is applied.

$$\boxed{\epsilon, \tau} \leftarrow$$

3

Moving a force off of its line of action



What if point B is not on the line of action of vector **F**?

External effect:

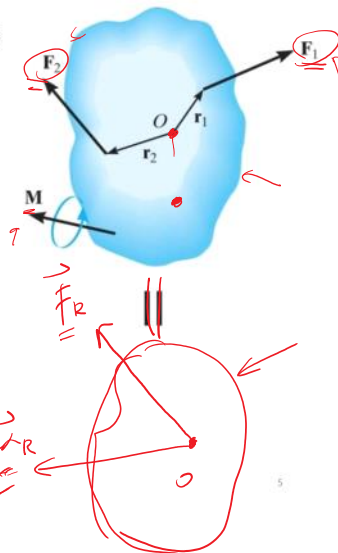
$$\begin{aligned} \sum \vec{F} &= 0 & F \cdot d \\ \sum \vec{M} &= 0 & M = 0 \end{aligned}$$

4

Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

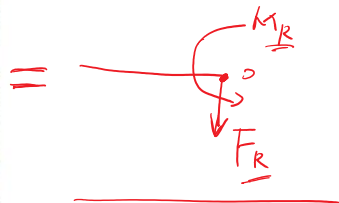
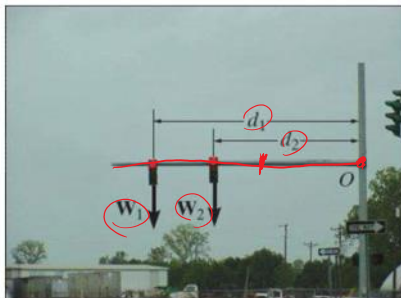
Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point P .



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

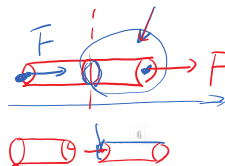
$$(\vec{M}_R)_O = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}$$

What is the equivalent system?



$$F_R = W_1 + W_2$$

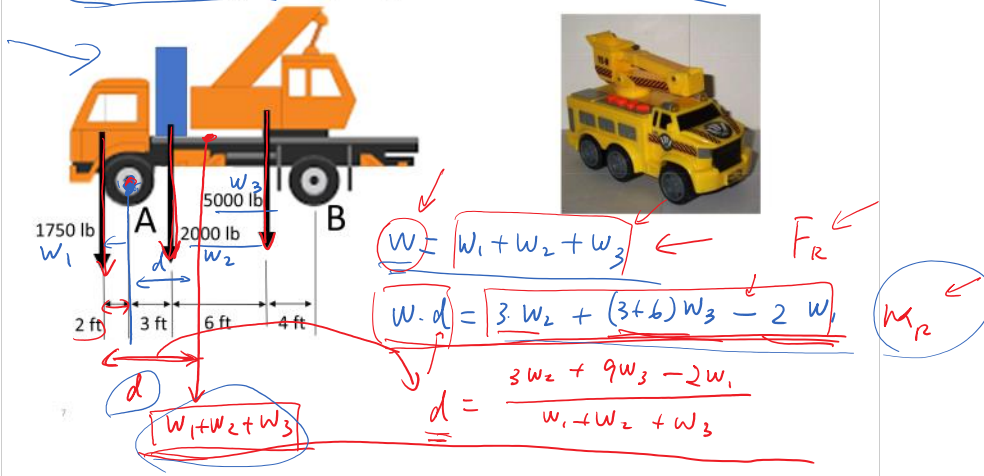
$$M_R = W_1 d_1 + W_2 d_2$$



$$C = \frac{F}{A}$$

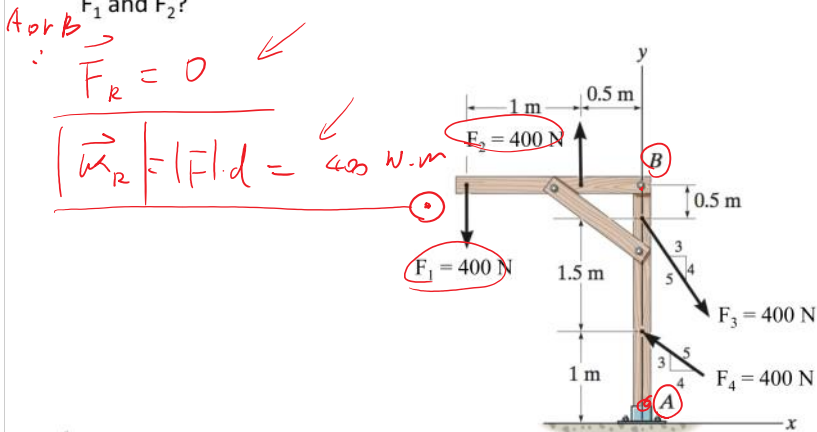
Example – 2D Equivalent System

Replace weights of the components of the truck with a single equivalent weight and specify its location measured from A.



Example – 2D Equivalent System

What is the resultant force and moment on point A and B from F_1 and F_2 ?



Example – 2D Equivalent System

What is the resultant force and moment on point A and B from F_3 and F_4 ?

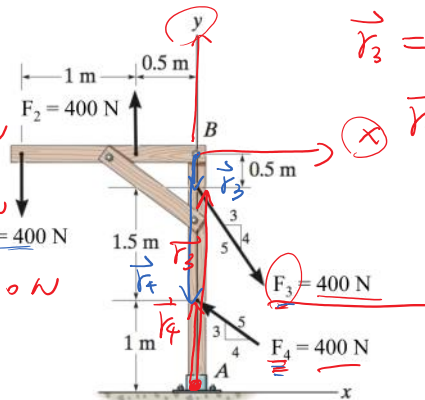
$$\vec{F}_R = \vec{F}_3 + \vec{F}_4$$

$$\vec{F}_3 = \left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right) 400 \text{ N}$$

$$\vec{F}_4 = \left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right) 400 \text{ N}$$

$$\vec{F}_R = \left(-\frac{1}{5}\hat{i} - \frac{1}{5}\hat{j}\right) 400 \text{ N}$$

for both A and B.



$$\vec{r}_3 = (0, 2.5, 0)$$

$$\vec{r}_4 = (0, 1, 0)$$

$$A: \vec{M}_R = \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

$$B: \vec{M}_R = \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

Example – 2D Equivalent System

Replace the loading on the frame by a single resultant force.

Specify where its line of action intersects a vertical line along member AB, measured from A.

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$(\vec{M}_R)_A =$$

$$\vec{r} \times \vec{F}_R = (\vec{M}_R)_A$$

