

Announcements



PrairieLearn quiz 0:

- ☐ Only practice
- ☐ Thursday 8:10-9:10 pm
- ☐ Location: D326



<http://tam2xx.intl.zju.edu.cn/tam211/sched.html>



1

Lecture Objectives



Cartesian Vectors



Position Vector

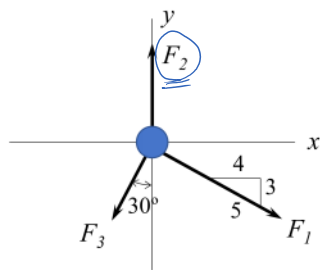


Force Vector along a Line



2

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Given that $F_2 = 40 \text{ N}$, determine the unit vector that represents the direction of \mathbf{F}_2 .

(A) $(0\mathbf{i} + 1\mathbf{j})$ ✓

✓ (B) $(0\mathbf{i} + 1\mathbf{j}) \text{ N}$ =

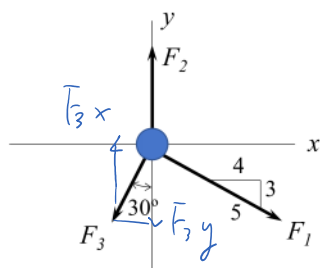
(C) $(0\mathbf{i} + 40\mathbf{j})$

✓ (D) $(0\mathbf{i} + 40\mathbf{j}) \text{ N}$ ✗

(E) None of the above

$$|\hat{u}| = 1$$

i-Clicker



Given that $F_3 = 20 \text{ N}$, express force \mathbf{F}_3 in Cartesian vector form.

(A) $(20\sin 30^\circ \mathbf{i} + 20\cos 30^\circ \mathbf{j}) \text{ N}$

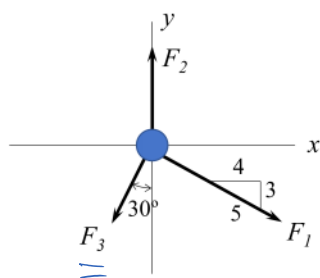
(B) $(-20\sin 30^\circ \mathbf{i} - 20\cos 30^\circ \mathbf{j}) \text{ N}$

(C) $(20\cos 30^\circ \mathbf{i} + 20\sin 30^\circ \mathbf{j}) \text{ N}$

(D) $(-20\cos 30^\circ \mathbf{i} - 20\sin 30^\circ \mathbf{j}) \text{ N}$

(E) None of the above

Example



Given that $F_1 = 50 \text{ N}$ and $F_3 = 20 \text{ N}$, determine of resultant force of \vec{F}_1 and \vec{F}_3 in Cartesian vector form.

$$\vec{R} = \vec{F}_1 + \vec{F}_3$$

$$\vec{F}_1 = 50 \text{ N} \hat{u} = 50 \text{ N} \left(\frac{4}{5} \hat{i} - \frac{3}{5} \hat{j} \right)$$

$$= 40 \hat{i} - 30 \hat{j} \text{ N}$$

$$\vec{F}_3 = 20 \text{ N} \cdot \hat{u} = 20 \text{ N} (-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j})$$

$$\vec{R} = (40 - 20 \sin 30^\circ) \hat{i} - (30 + 20 \cos 30^\circ) \hat{j}$$

Numerical Calculations

Significant figures

- The number of significant figures contained in any number determines the accuracy of the number.
- Use 3 or > significant figures for final answers. 1%
- For intermediate steps, use symbolic notation, store numbers in calculators or use more significant figures, in order to maintain precision.

Practice:

- How many significant figures are in each of the following numbers?

a) 0.0035 --- 2 sig. fig. (leading zeros are not significant). (Rule #4).

b) 1.080 --- 4 sig. fig. (zeros after the decimal & interior zeros are significant). (Rules #2 & 3).

c) 2371 --- 4 sig. fig. (all non zeros digits are significant). (Rule #1).

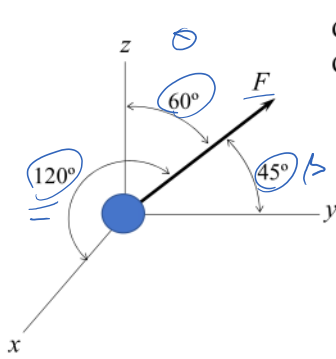
d) 2.97 $\times 10^5$ --- 3 sig. fig. (all non zeros digits are significant) (Rule #1).

0.00135428 /

0.0013 (X)

0.00135 (✓)

Example



Given that $F = 30 \text{ lb}$, express force vector \mathbf{F} using the Cartesian vector form.

$$F_x = F \cdot \cos \alpha = 30 \cdot \cos 120^\circ \text{ lb}$$

$$F_y = F \cdot \cos \beta = 30 \cdot \cos 45^\circ \text{ lb}$$

$$F_z = F \cdot \cos \theta = 30 \cdot \cos 60^\circ \text{ lb}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

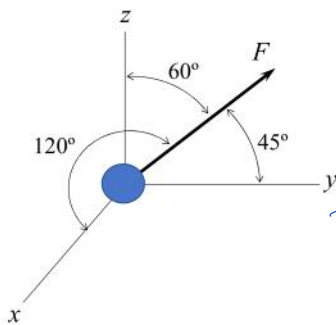
$$\vec{F} = F \hat{u}$$

$$\hat{u} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \theta \hat{k}$$

Approximate Conversion Facts

Metric Unit	Imperial Unit
1 Kilogram	2.2 pounds
1 Litre	1.75 pints
4.5 Litres	1 Gallon
8km	5 Miles
30 cm	1 Foot
2.54 cm	1 Inch

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Given that $F = 30 \text{ lb}$, what is the correct expression for F_x ?

(A) $(30 \sin 120^\circ \mathbf{i}) \text{ lb}$ ✗

(B) $(30 \cos 120^\circ \mathbf{i}) \text{ lb}$

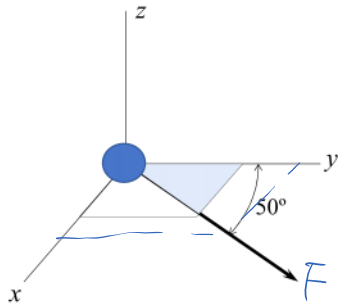
(C) $(30 \cos 45^\circ \mathbf{j}) \text{ lb}$

(D) $(30 \cos 60^\circ \mathbf{k}) \text{ lb}$

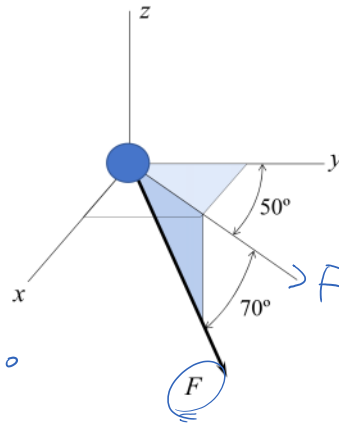
(E) None of the above

3-D Vectors

If the vector lies on the plane of xy, yz, xz, it is a 2D vector.



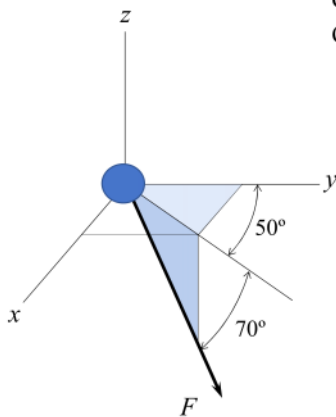
$$\begin{aligned} F_x &= F \cdot \sin 50^\circ \\ F_y &= F \cdot \cos 50^\circ \\ F_z &= 0 \end{aligned}$$



$$|F_z| = F \cdot \cos 70^\circ$$

$$\begin{aligned} F_x &= + F \cdot \cos 70^\circ \cdot \sin 50^\circ \\ F_y &= + F \cdot \cos 70^\circ \cdot \cos 50^\circ \\ F_z &= - F \cdot \sin 70^\circ \end{aligned}$$

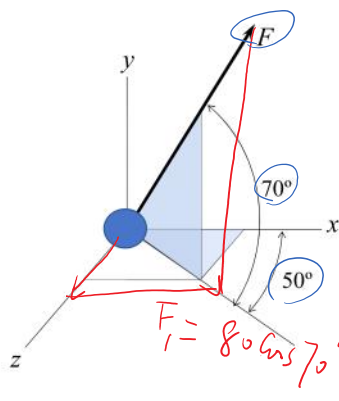
Example



Given that $F = 80 \text{ lb}$, express force vector \mathbf{F} using the Cartesian vector form.

$$\vec{F} = \underline{F_x} \hat{i} + \underline{F_y} \hat{j} + \underline{F_z} \hat{k}$$

i-Clicker



Given that $F = 80$ lb, what is the correct expression for F_z ?

(A) $(80 \sin 70^\circ \mathbf{k})$ lb

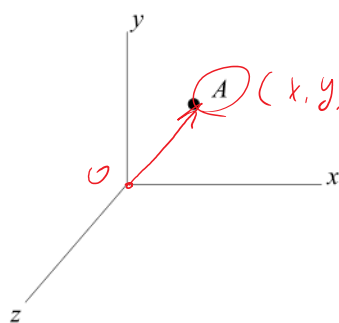
(B) $(80 \cos 70^\circ \mathbf{k})$ lb

✓ (C) $(80 \cos 70^\circ \sin 50^\circ \mathbf{k})$ lb

(D) $(80 \cos 70^\circ \cos 50^\circ \mathbf{k})$ lb

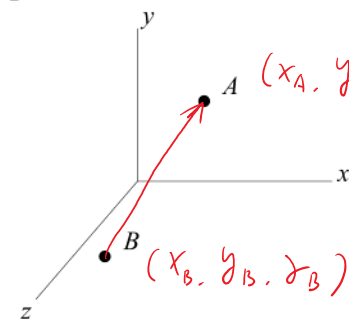
(E) None of the above

Position vectors



\vec{r}_A = from origin to the point

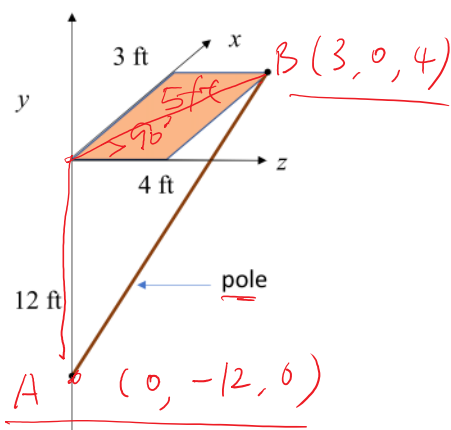
$$\vec{r}_A = (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k} \\ = x\hat{i} + y\hat{j} + z\hat{k}$$



\vec{r}_{BA} : from point B to point A

$$\vec{r}_{BA} = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$$

Force vector directed along a line



Given that the force on the pole = 80 lb, represent the force as a cartesian vector?

$$\vec{F} = 80 \hat{u}_{AB}$$

$$AB (3, 12, 4)$$

$$\hat{u}_{AB} = \frac{(3, 12, 4)}{\sqrt{9 + 144 + 16} = 13}$$

$$\vec{F} = 80 \left(\frac{3}{13} \hat{i} + \frac{12}{13} \hat{j} + \frac{4}{13} \hat{k} \right) \text{ lb}$$

Summary

You should now be able to...

- break down vectors into Cartesian components in 2D and 3D
- express a vector using the starting and ending positions of a vector
- determine the Cartesian representation of a force vector given the line along with the force is acting

$$\begin{matrix} F_x \\ F_y \\ F_z \end{matrix} \quad |\vec{F}| \hat{u} = \vec{F}$$

$$\vec{r}_{AB} =$$

