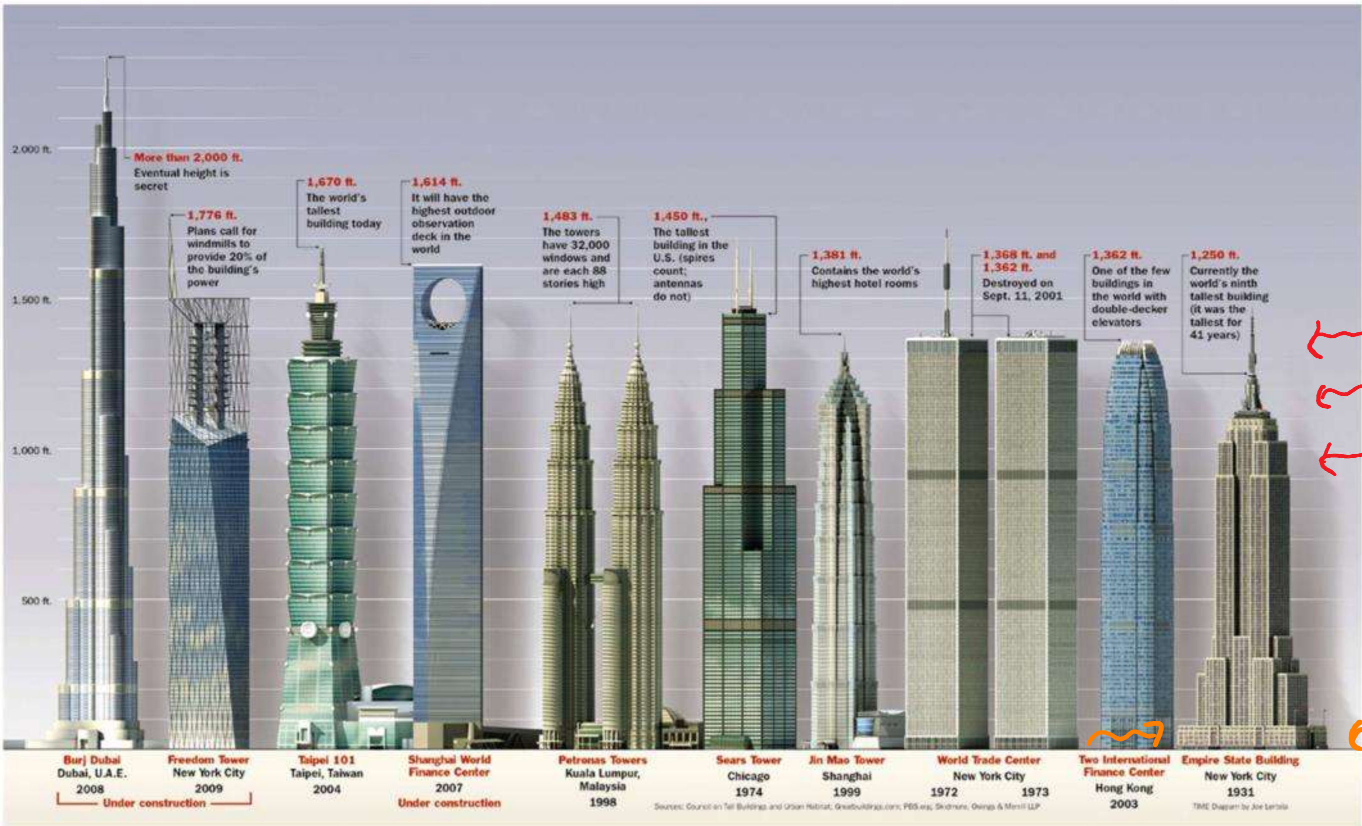
Lecture Objectives

- What is "statics"?
- Newton's laws of motion
- Force vectors and vector operations

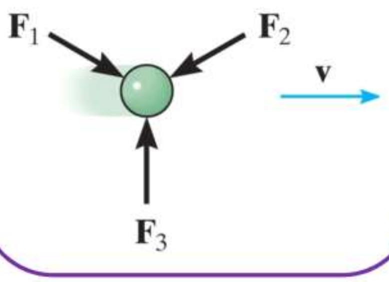
What is Statics?

"The branch of mechanics concerned with bodies at rest and forces in equilibrium."



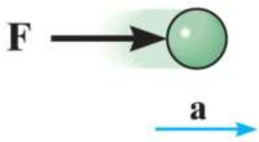
Newton's laws of motion

First law:
Particle at rest (or moving in a straight line with constant velocity) stays that way unless another force comes in.

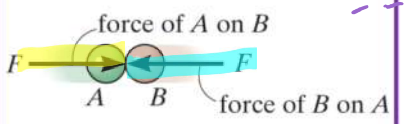


Second law: a particle acted upon by an unbalanced force **F** experiences an acceleration **a** that is proportional to the particle mass *m*:

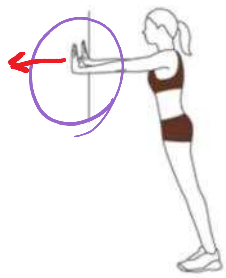
$$F = ma$$

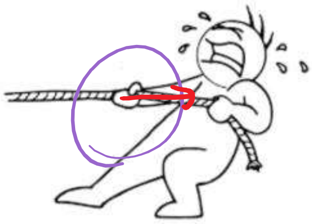


Third law: the mutual forces of action and reaction between two particles are __equal__, __opposite__ and __collinear__.



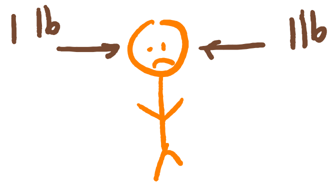
Force vectors





Scalars and vectors

	Scalar	Vector
Examples	mass, volume	weight, acceleration
Characteristics	Magnitude	Magnitude & direction
Notation	a	$\vec{A}$, **A**



total force : 2 lb.

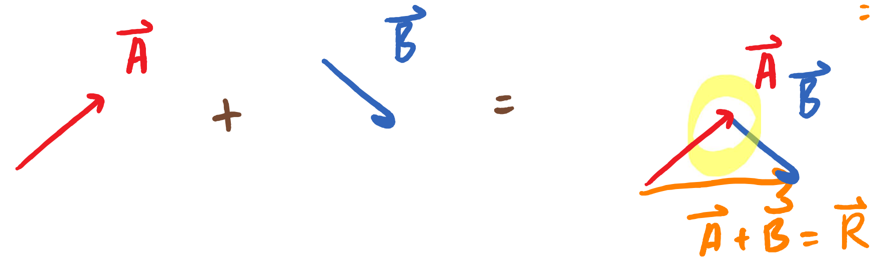
vector provides direction:

$\xrightarrow{1lb} + \xleftarrow{1lb} = 0$ net force

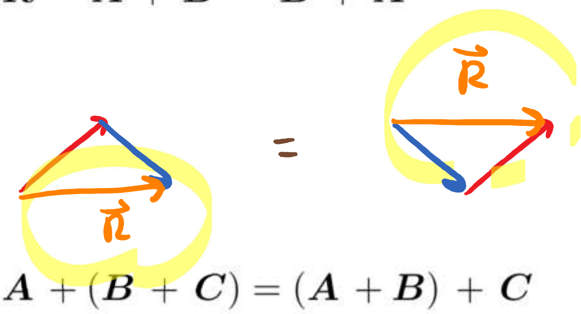
→ no accelerati

Vector addition (graphical rep.)

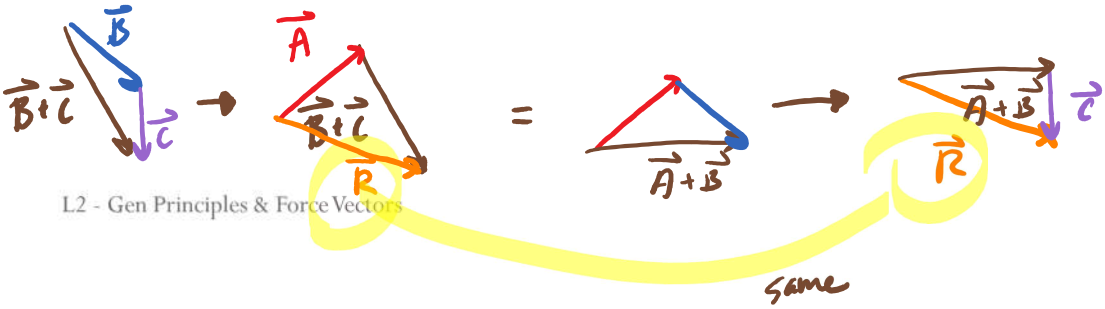
All vector quantities obey the parallelogram law of addition $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$



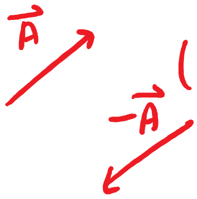
Commutative law: $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$



Associative law: $\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$

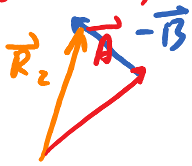


Vector subtraction:



(same magnitude as $\vec{A}$, but opposite direction)

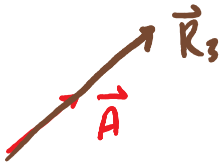
$$\vec{R}_2 = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



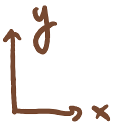
Scalar/Vector multiplication:

$$\vec{R}_3 = a\vec{A}$$

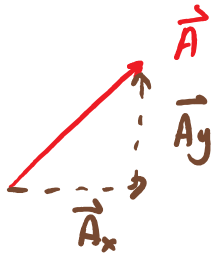
- for "a" = 2, the magnitude $\vec{R}_3$ is 2 times the magnitude of $\vec{A}$



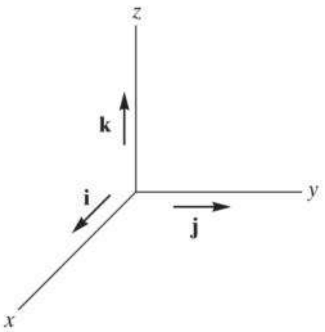
Cartesian vectors (2D and 3D)

2D 

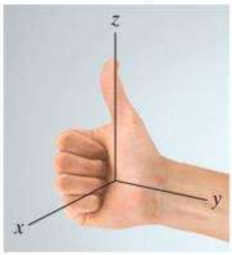
$\vec{A} = A_x \hat{i} + A_y \hat{j}$



$\hat{i}$ & $\hat{j}$ are unit vector



Right-handed
coordinate system



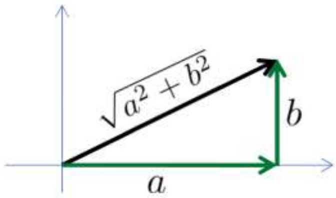
L2 - Gen Principles & Force Vectors

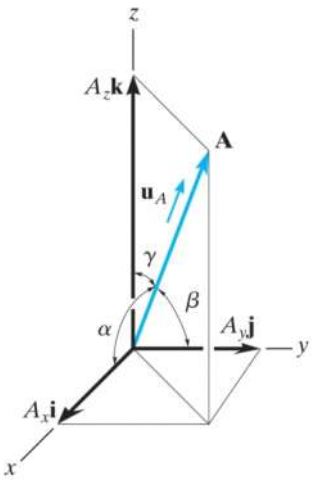
- Unit vector provides directions
- Unit vector has a magnitude of 1.

$\hat{i}$ is in the x-direction
$\hat{j}$ is in the y-direction

Magnitude of Cartesian vectors

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$





Magnitude & unit vector form

$$\vec{A} = A\,\hat{u}_A$$

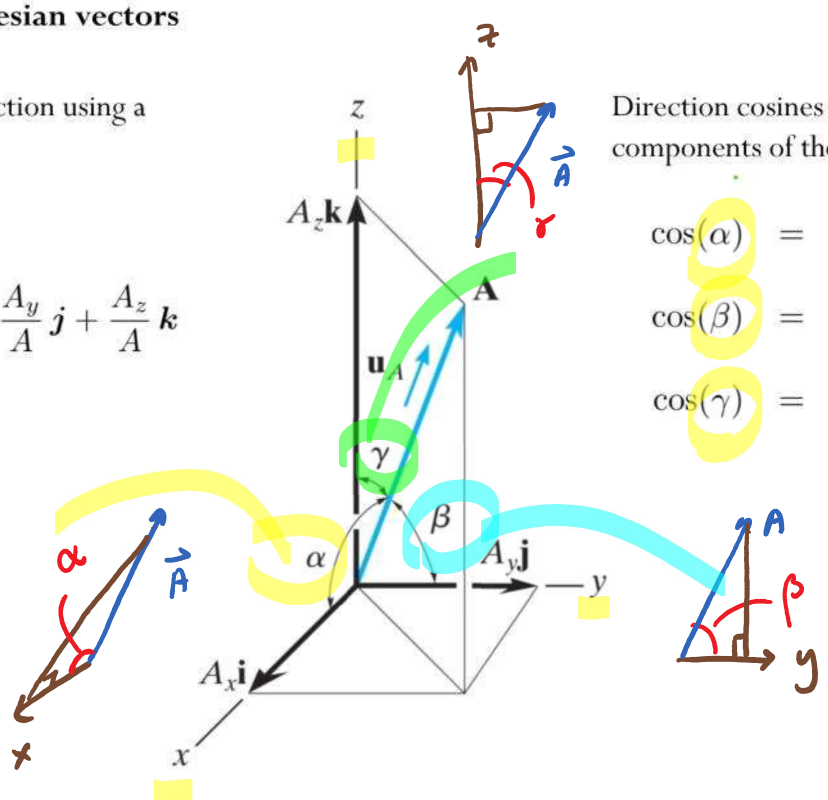
notation for unit vector.

$$\hat{u}_A = \frac{A_x}{A}\hat{i} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k}$$

Direction of Cartesian vectors

Expressing the direction using a unit vector:

$$\boldsymbol{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\boldsymbol{i} + \frac{A_y}{A}\boldsymbol{j} + \frac{A_z}{A}\boldsymbol{k}$$



Direction cosines are the components of the unit vector:

$$\cos(\alpha) = \frac{A_x}{A}$$

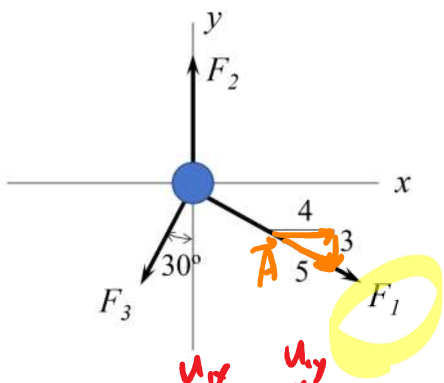
$$\cos(\beta) = \frac{A_y}{A}$$

$$\cos(\gamma) = \frac{A_z}{A}$$

L2 - Gen Principles & Force Vectors

Example

Given that $F_1 = 50$ N, express force vector $\mathbf{F}_1$ using the Cartesian vector form.



$$\vec{F}_1 = \vec{F}_{1x} + \vec{F}_{1y} = F_{1x}\hat{i} + F_{1y}\hat{j} = F_1\hat{u}_1$$

→ Find : F_{1x} , F_{1y}

$$F_{1x} = F_1 u_{Ax} = 50\text{ N}\left(\tfrac{4}{5}\right) = 40\text{ N}$$

$$F_{1y} = F_1 u_{Ay} = 50\text{ N}\left(\tfrac{-3}{5}\right) = -30\text{ N}$$

$$\hat{u}_1 = \underbrace{\tfrac{4}{5}}_{u_{1x}}\hat{i} - \underbrace{\tfrac{3}{5}}_{u_{1y}}\hat{j}$$

$$\vec{A} = 4\hat{i} - 3\hat{j}$$

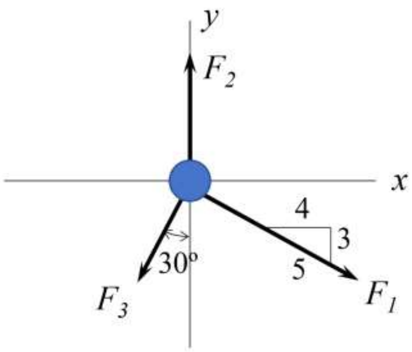
$$A = \sqrt{3^2 + 4^2} = 5$$

$$\hat{u}_1 = \frac{\vec{A}}{A} = \frac{4\hat{i} - 3\hat{j}}{5}$$

L2 - Gen Principles & Force Vectors

Example

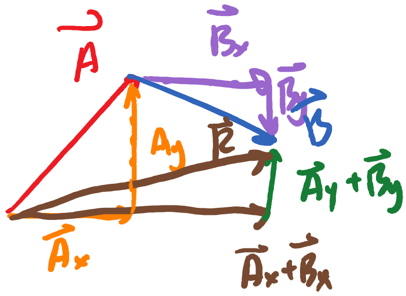
Given that $F_2 = 40$ N, determine the unit vector that represents the direction of $\mathbf{F_2}$.



$\hat{u}_2 = \hat{j}$

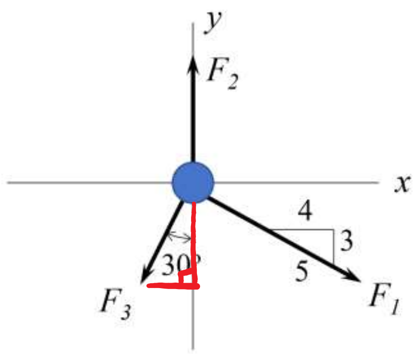
Addition of Cartesian vectors

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = (A_x + B_x)\,\boldsymbol{i} + (A_y + B_y)\,\boldsymbol{j} + (A_z + B_z)\,\boldsymbol{k}$$



L2 - Gen Principles & Force Vectors

Example



Given that $F_1 = 50$ N and $F_3 = 20$ N, determine of $\underbrace{\text{resultant force}}_{\vec{R}}$ of $\mathbf{F}_1$ and $\mathbf{F}_3$ in Cartesian vector form.

$$\vec{F}_1 = (40\hat{i} - 30\hat{j})\ N$$

$$\vec{F}_3 = 20\,(-\sin 30°\,\hat{i} - \cos 30°\,\hat{j})$$

$$\vec{R} = \vec{F}_1 + \vec{F}_3 = (40 + (-20\sin 30°))\hat{i} + (-30 + (-20\cos 30°))\hat{j}$$