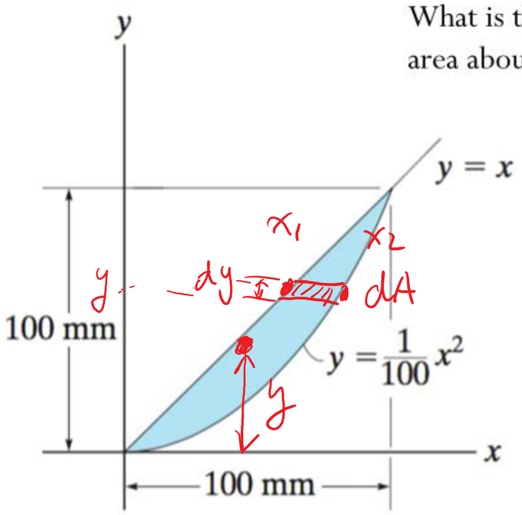
Lec31

Wednesday, December 2, 2020 7:58 AM

Example



What is the moment of inertia of the shaded area about the *x*-axis?

$$I_x = \int y^2\, dA$$

$$dA = dy\,(x_2 - x_1) = (10\sqrt{y} - y)\,dy$$

$$x_1 = y, \quad x_2 = 10\sqrt{y}$$

$$I_x = \int_0^{100} y^2 (10\sqrt{y} - y)\cdot dy$$

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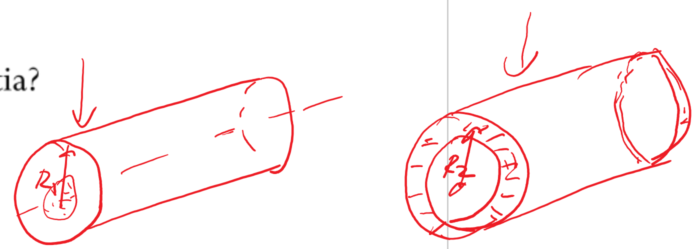
Which one has ~~high~~ higher mass moment of inertia?

A. Solid cylinder

✓ B. Hallow cylinder

Note, mass and R are the same.

J_0 -



$$m_1 = m_2$$

$$R_1 = R_2$$

$$\boxed{J_o = \int (x^2 + y^2)\, dm}$$

$I_x = \int y^2\, dA \ (m^4)$

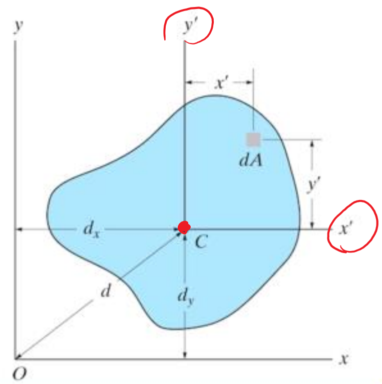
Parallel axis theorem

- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., x' and y':
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

$I_{x'}$, $I_{y'}$

$$I_x = \underline{I_{x'}} + \underline{A \cdot d_y^2}$$

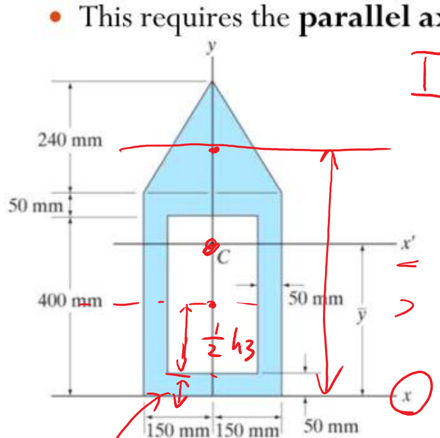
$$I_y = I_{y'} + A \cdot d_x^2$$



Note: the integral over y' gives zero *when done through the centroid axis.*

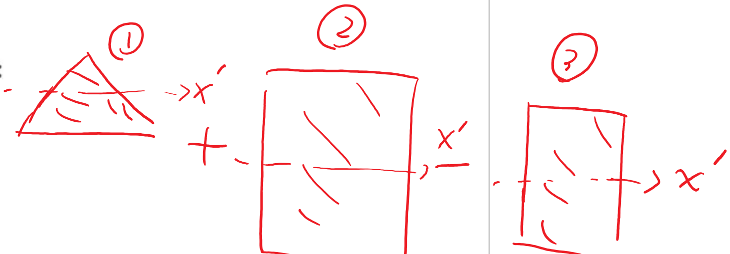
Moment of inertia of composite

- If individual bodies making up a **composite** body have individual areas A and moments of inertia I computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions.
- This requires the **parallel axis theorem**:



$\frac{1}{2}(h_2 - h_3)$

$I_x \Leftarrow$



$\Rightarrow x'$

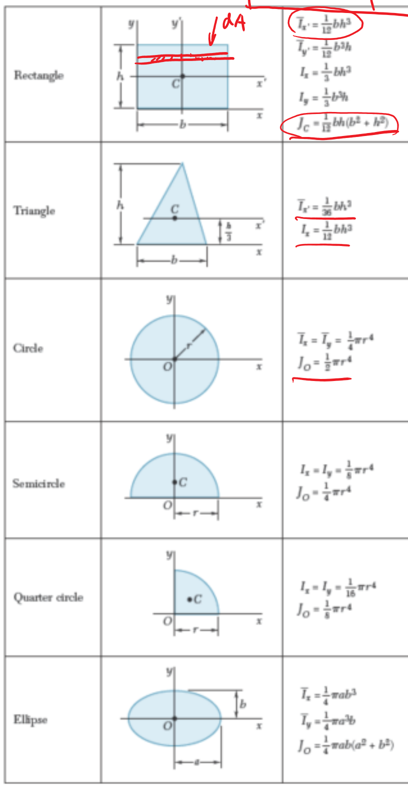
$I_{1,x} + I_{2,x} - I_{3,x}$

$I_x = I_{x'} + Ad^2$

	1	2	3
$I_{x'}$	$\frac{1}{36} b \cdot h_1^3$	$\frac{1}{12} b \cdot h_2^3$	$\frac{1}{12} b_3 h_3^3$
A	$\frac{1}{2} b \cdot h$	$b \cdot h_2$	$b_3 h_3$
d	$h_2 + \frac{1}{3} h_1$	$\frac{1}{2} h_2$	$\frac{1}{2} h_3 + (h_2 - h_3)\frac{1}{2}$
I_x			

y_i y_i dA $\bar{I}_x = \frac{1}{12} bh^3$

h	$h_2 + \frac{1}{3}h_1$	$\frac{1}{2}h_2$	$\frac{1}{2}h_3 + (h_2 - h_3)\frac{1}{2}$
I_x			

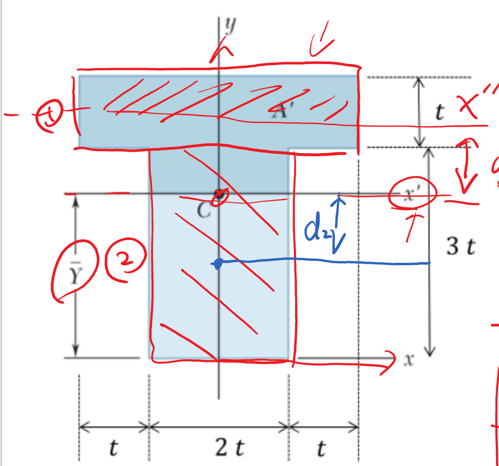


Rectangle	$\bar{I}_{x'} = \frac{1}{12}bh^3$, $\bar{I}_{y'} = \frac{1}{12}b^3h$, $I_x = \frac{1}{3}bh^3$, $I_y = \frac{1}{3}b^3h$, $J_C = \frac{1}{12}bh(b^2+h^2)$
Triangle	$\bar{I}_{x'} = \frac{1}{36}bh^3$, $I_x = \frac{1}{12}bh^3$
Circle	$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$, $J_O = \frac{1}{2}\pi r^4$
Semicircle	$I_x = I_y = \frac{1}{8}\pi r^4$, $J_O = \frac{1}{4}\pi r^4$
Quarter circle	$I_x = I_y = \frac{1}{16}\pi r^4$, $J_O = \frac{1}{8}\pi r^4$
Ellipse	$\bar{I}_x = \frac{1}{4}\pi ab^3$, $\bar{I}_y = \frac{1}{4}\pi a^3b$, $J_O = \frac{1}{4}\pi ab(a^2+b^2)$

$dA = b \cdot dy$

$$I_{x'} = \int y^2\, dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} b\, y^2 \cdot dy = b \cdot \frac{1}{3} y^3 \Big|_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{1}{12} b h^3$$

Find the moment of inertia about its centroid:



$$\bar{Y} = \frac{4t^2\,(3.5t) + 6t^2\,(1.5t)}{4t^2 + 6t^2} = \frac{23t}{10}$$

$$\bar{y} = \frac{\sum \tilde{y}_i A_i}{\sum A_i} = \frac{(3t + \frac{1}{2}t)\cdot t \cdot 4t + (\frac{3}{2}t)\, 3t \cdot 2t}{4t^2 + 6t^2} =$$

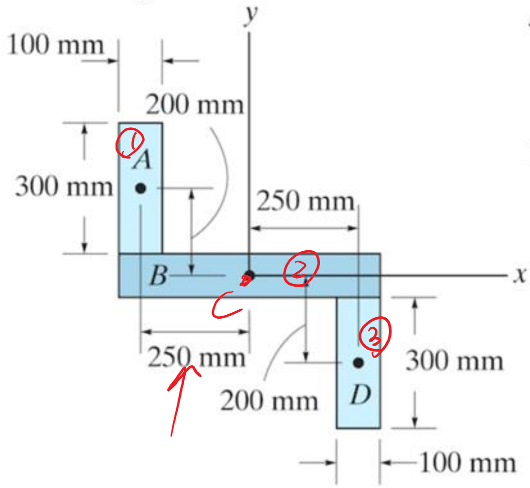
$I_{x'}$

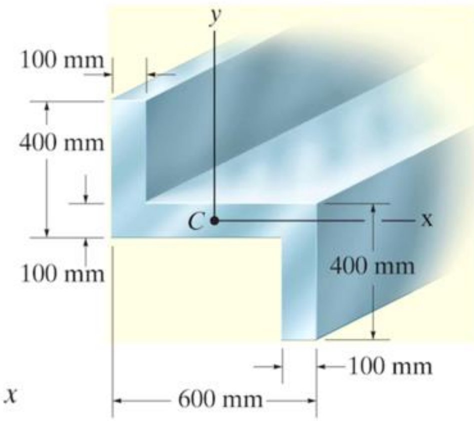
	①	②
$I_{x''}$	$\frac{1}{12} 4t \cdot t^3$	$\frac{1}{12} \cdot 2t\,(3t)^3$
A	$4t \cdot t$	$2t \cdot 3t$
d	$\frac{t}{2} + (3t - \bar{y})$	$\bar{y} - \frac{3}{2}t$
$I_{x'}$		

$I_{x'_i} = I_{x''_i} + A \cdot d^2$

$I_{x'} = \sum_i I_{x'_i}$

Determine the moment of inertia for the cross-sectional area about the centroidal y-axes.





$$I_y = I_{y_1} + I_{y_2} + I_{y_3}$$

$$\bar{I}_{y_1} = I_{y_1'} + d^2 A \quad (d = 250\,\text{mm})$$

$$I_{y_2} = \bar{I}_{y'_2}$$

$$I_{y_3} = I_{y'_3} + d^2 A$$

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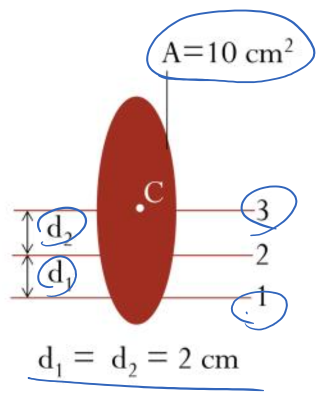
For the given area, the moment of inertia about axis 1 is 200 cm^4 . What is the MoI about axis 3 (the centroidal axis)?

A) 90 cm^4 B) 110 cm^4

C) 60 cm^4 D) 40 cm^4



$$I_x = I_{x'} + d^2 A$$

$$I_{x'} = I_x - d^2 A$$

$$200 - 4^2 \cdot 10 = 40$$

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The moment of inertia of the rectangle about the x-axis equals

A) 8 cm ^{4}. B) 56 cm ^{4}.

C) 24 cm ^{4}. D) 26 cm ^{4}.

$I_{x'} = \frac{1}{12} b h^3 = \frac{1}{12} \cdot 3 \cdot 2^3 = 2$

$I_x = I_{x'} + d^2 A = 2 + 9 \cdot 6 = 56\ cm^4$

$2 + 3^2 \cdot 6$