Lec33

Wednesday, December 9, 2020 7:55 AM

Virtual Work

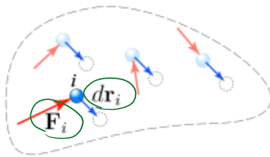
Main goals and learning objectives

- Introduce the principle of virtual work
- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

Definition of Work

Work of a force

A force does work when it undergoes a displacement in the direction of the line of action.



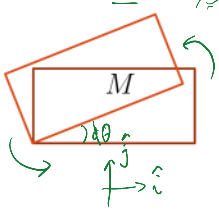
The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$\underline{dU_i} = \underline{\vec{F}_i} \cdot \underline{\vec{dr}_i}$$

Definition of Work

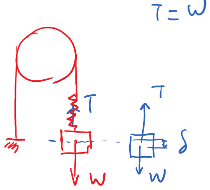
Work of a couple $\underline{dU} = M\cancel{\mathbf{k}} \cdot d\theta\,\cancel{\mathbf{k}} = \underline{M\,d\theta}$



Virtual Displacements

A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist.

$T = W$



δ

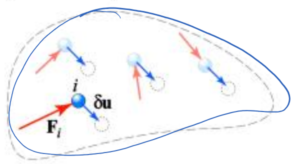
$T \cdot \delta = W\delta$

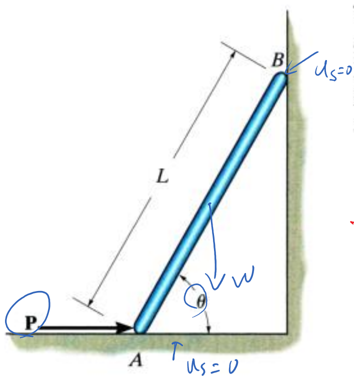
Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

$dU_i = \vec{F}_i \cdot d\vec{r}_i$

$\sum_i dU_i = 0$





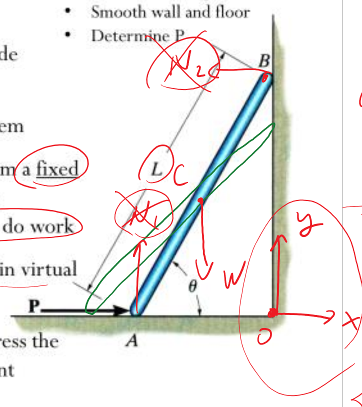
The thin rod of weight *W* rests against the smooth wall and floor. Determine the magnitude of force *P* needed to hold it in equilibrium.

$P = \frac{W}{2} \cdot \cot\theta$

Procedure for Analysis

- Thin rod of weight *W*
- Smooth wall and floor
- Determine P

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a fixed point and select the parallel line of action component and remove forces that do no do work
4. Differentiate position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/couple moment
6. Factor out the common virtual displacement term and solve



$dU_B = 0$

$d\cos\theta = -\sin\theta \, d\theta$

$dU_i = \vec{F}_i \cdot d\vec{r}_i$

$\vec{r}_A = -L \cdot \cos\theta \, \hat{i}$

$\cancel{\vec{r}_B = L \cdot \sin\theta \, \hat{j}}$

$\vec{r}_C = -\frac{1}{2}L \cdot \cos\theta \, \hat{i} + \frac{1}{2}L \cdot \sin\theta \, \hat{j}$

$d\vec{r}_A = +L \cdot \sin\theta \, d\theta \, \hat{i}$

$d\vec{r}_B = L\cos\theta \, d\theta \, \hat{j}$

$d\vec{r}_C = +\frac{1}{2}L \cdot \sin\theta \, d\theta \, \hat{i} + \frac{1}{2}L\cos\theta \, d\theta \, \hat{j}$

$\sum dU_i = 0$

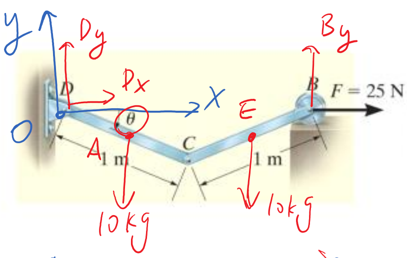
$+P \cdot L\sin\theta \, d\theta - W \cdot \frac{1}{2}L \cdot \cos\theta \, d\theta = 0$

U ↑ D.   R.

$$\sum \delta U_i = 0$$

$$+P \cdot \ell \sin\theta \, d\theta - W \cdot \tfrac{1}{2}\ell \cdot \cos\theta \, d\theta = 0$$

$$P = \frac{W}{2} \cdot \cot\theta$$

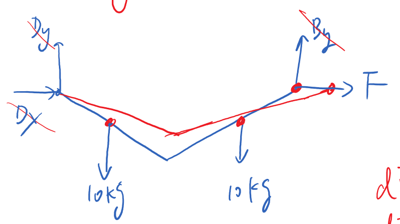


Determine the angle for equilibrium of the two-member linkage. Each member has a mass of 10 kg.

$$\vec{r}_A = \tfrac{1}{2} \cdot \cos\theta \, \hat{i} - \tfrac{1}{2} \sin\theta \, \hat{j}$$

$$\vec{r}_E = \left(\cos\theta + \tfrac{1}{2} \cdot \cos\theta\right)\hat{i} - \tfrac{1}{2}\sin\theta \, \hat{j}$$

$$\vec{r}_B = 2\cos\theta \, \hat{i}$$



$$d\vec{r}_A = -\tfrac{1}{2}\sin\theta \cdot d\theta \, \hat{i} - \tfrac{1}{2}\cos\theta \cdot d\theta \, \hat{j}$$

$$d\vec{r}_E = -\tfrac{3}{2}\sin\theta \, d\theta \, \hat{i} - \tfrac{1}{2}\cos\theta \, d\theta \, \hat{j}$$

$$d\vec{r}_B = -2\sin\theta \, d\theta \, \hat{i}$$

F_B

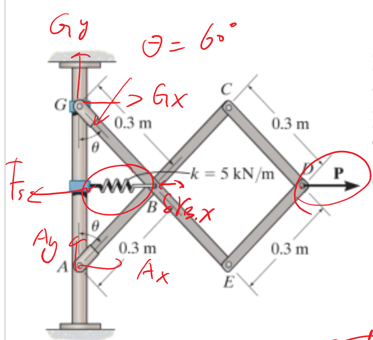
$$\sum \delta U_i = 0$$

$$-\underset{mg}{\cancel{10 \times 9.8}}\left(-\tfrac{1}{2}\cos\theta \cdot d\theta\right) - \underset{mg}{\cancel{10 \cdot 9.8}} \cdot \left(-\tfrac{1}{2}\cos\theta \, d\theta\right) + \underset{F_B}{\cancel{25}} \cdot \left(-2\sin\theta \, d\theta\right) = 0$$

$$mg \cos\theta \, \cancel{d\theta} - 2F_B \cdot \sin\theta \, \cancel{d\theta} = 0$$

$$\tan\theta = \frac{mg}{2F_B} \qquad \theta = \tan^{-1}\left(\frac{mg}{2F_B}\right)$$

$\theta = 60°$

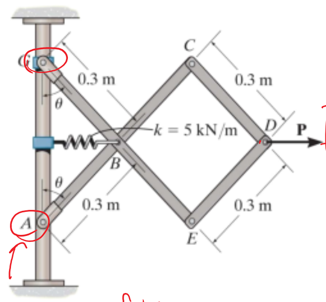


Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.

$$P \cdot 0.9 \cdot \cancel{\cos\theta \, d\theta} - F_s \cdot 0.3 \cdot \cancel{\cos\theta \, d\theta} = 0$$

$$P = \tfrac{1}{3} F_s$$

$$F_s = k \cdot s = 5 \text{ kN/m} \cdot \left(0.3 \cdot \sin 60° - 0.3 \sin 30°\right) \text{m}$$



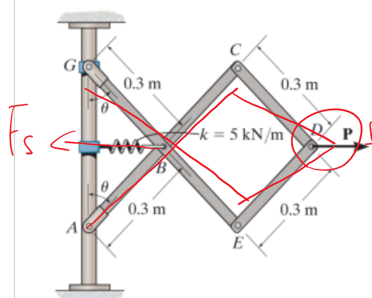
Where should the “origin” from which position coordinates are defined be located?

A) A
B) B
C) C
D) D
E) G

δr_D

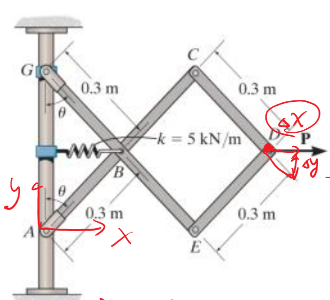
δr_B

virtual



How many forces will do work?

A) 1
B) 2
C) 3
D) 4
E) 5

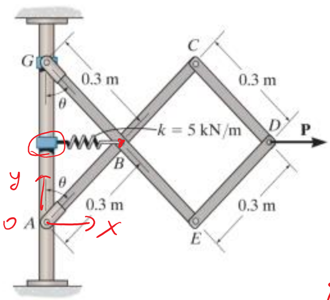


Which coordinate component should be selected/retained for calculating virtual work from **P**?

A) ~~x~~
B) y
C) x and y
D) x or y

$\vec{r}_D = (x , y)$

$\delta r_{D,x}$



Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.

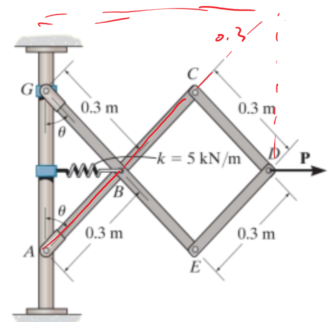
What is the virtual displacement for calculating virtual work from the spring?

A) $0.3\cos\theta \, d\theta$
B) $0.3\sin\theta \, d\theta$
C) $0.3\cos\theta$
D) $0.3\sin\theta$
E) None of the above

$\gamma_{B,x} = 0.3 \cdot \sin\theta$

$\delta\gamma_{B,x} = 0.3\cos\theta \cdot d\theta$

$0.9 \cdot \sin\theta$



What is the virtual displacement associated with **P**?

A) $0.3\sin\theta \, d\theta$
B) $0.3\cos\theta \, d\theta$
C) $0.9\sin\theta \, d\theta$
D) $0.9\cos\theta \, d\theta$
E) None of the above

$0.9 \cdot \sin\theta$

$0.9 \cdot \cos\theta \cdot d\theta$