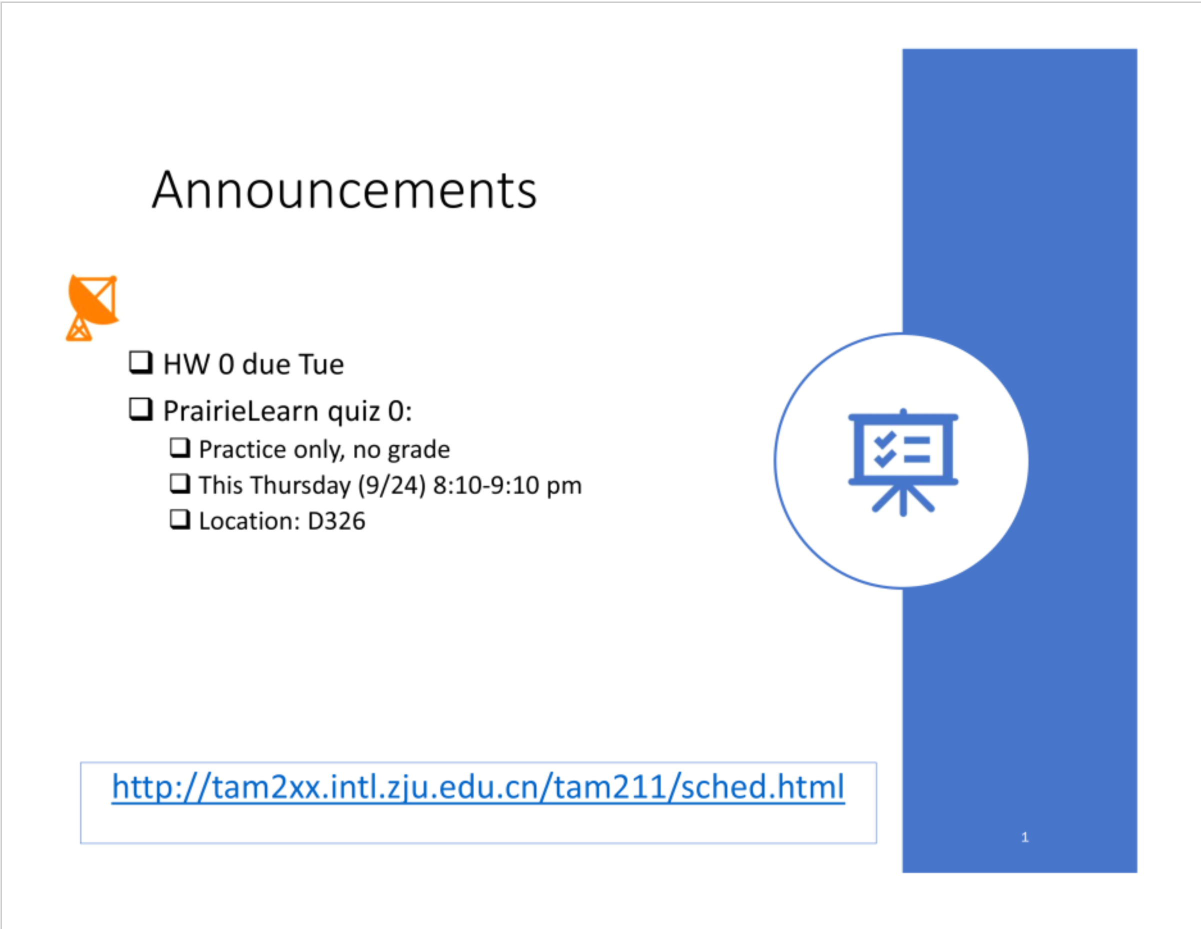
Lec04

Monday, September 21, 2020 7:56 AM



Lecture Objectives

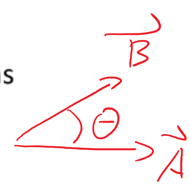
- Force vector along a line
- Dot product
  Vector projections
- Cross Product

2

Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as

$$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$



Cartesian vector formulation:

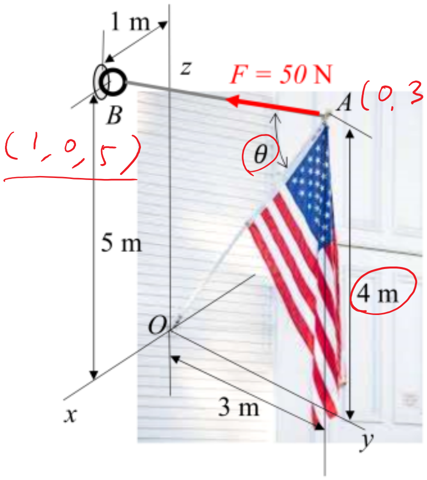
$$\vec{A}\cdot\vec{B} = x_A\cdot x_B + y_A\cdot y_B + z_A\cdot z_B$$

$$\vec{A} = (x_A, y_A, z_A)$$

$$\vec{B} = (x_B, y_B, z_B)$$

3

Example



The wall-mounted flag pole has added cable support *AB*. Determine the angle between *AB* and the axis *AO* of the flag pole.

$$\cos\theta = \frac{\vec{r}_{AO} \cdot \vec{r}_{AB}}{|\vec{r}_{AO}| \cdot |\vec{r}_{AB}|}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (1, -3, 1)$$

$$\vec{r}_{AO} = \vec{r}_O - \vec{r}_A = (0, -3, -4)$$

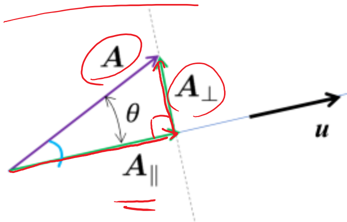
$$\cos\theta = \frac{0 + 9 - 4}{\sqrt{1+9+1} \cdot \sqrt{9+16}} = \frac{5}{5\sqrt{11}} = \frac{\sqrt{11}}{11}$$

$$\theta = \cos^{-1} \frac{\sqrt{11}}{11}$$

4

Vector Projections

The scalar component $A_{\parallel}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



$$A_{\parallel} = \boldsymbol{A} \cdot \boldsymbol{u} = |\boldsymbol{A}| \cos(\theta)$$

And thus the vector components $\boldsymbol{A}_{\parallel}$ and $\boldsymbol{A}_{\perp}$ are given by:

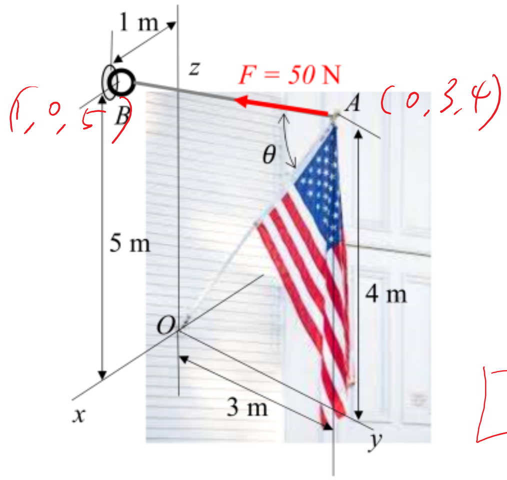
$$\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp}$$

$$\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}$$

$$\vec{A}_{\parallel} = |A_{\parallel}| \cdot \hat{u} = |A| \cdot \cos\theta \cdot \hat{u} = (\vec{A} \cdot \hat{u})\, \hat{u}$$

5

i>clicker time



Which Cartesian components of force exist in cable *AB*?

A) **i** and **j**

B) **j** and **k**

C) **i** and **k**

D) **i, j,** and **k** ✓

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{r}_B - \vec{r}_A = (1, -3, 1)$$

6

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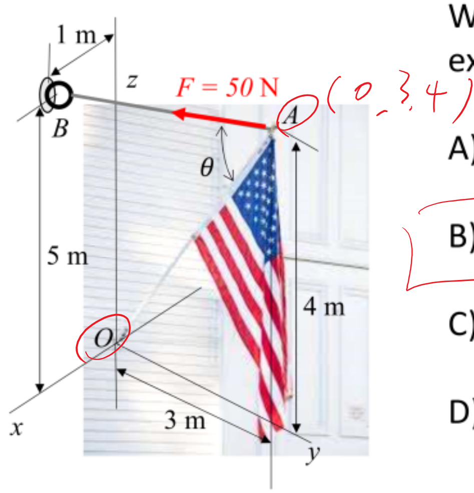


Which Cartesian components of force exist in strut *AO*?

A) **i** and **j**

B) **j** and **k**

C) **i** and **k**

D) **i, j,** and **k**

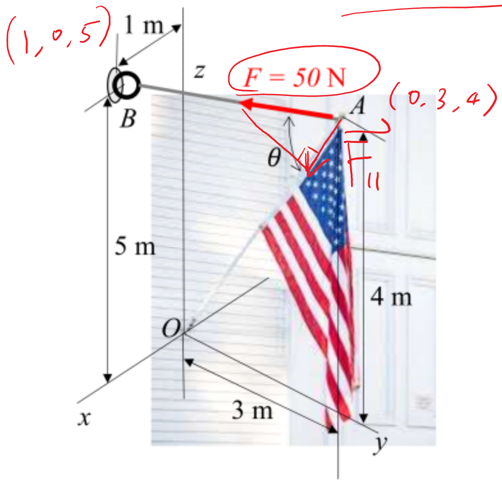
$$\vec{r}_{AO} = \vec{r}_O - \vec{r}_A$$

$$=$$

7

Example

a) Determine the projected component of the force vector ***F*** along the axis *AO* of the flag pole. Express your result as a Cartesian vector.



$$\vec{F}_{\parallel} = (\vec{F} \cdot \hat{u}_{AO})\, \hat{u}_{AO}$$

$$\vec{F} = |\vec{F}| \cdot \hat{u}_{AB} = 50 \cdot \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = 50 \frac{(-1,3,-1)}{\sqrt{1+9+1}}\ N$$

$$= \frac{50}{\sqrt{11}}(-1,3,-1)\ N$$

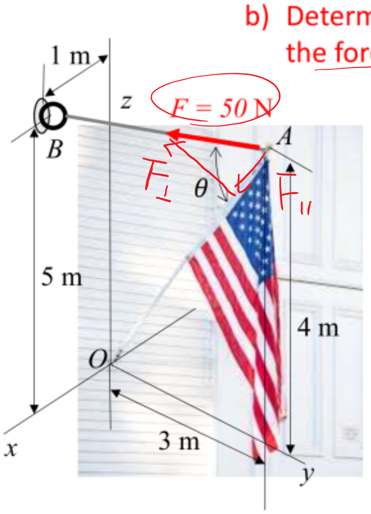
$$\hat{u}_{AO} = \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = \frac{0-(0,3,4)}{5} = \frac{(0,-3,-4)}{5}$$

8

Example

a) Determine the projected component of the force vector ***F*** along the axis *AO* of the flag pole.

b) Determine the perpendicular component from the pole of the force vector ***F***.



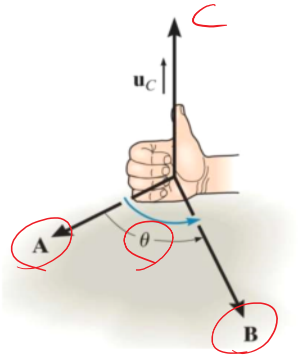
$$\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel}$$

9

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written

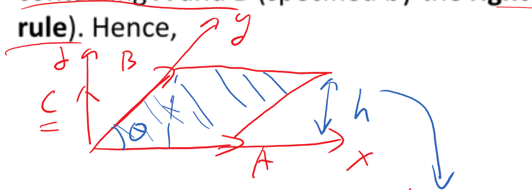


$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$

The magnitude of vector **C** is given by:

$|\vec{C}| = |\vec{A}||\vec{B}| \cdot \sin\theta$

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,



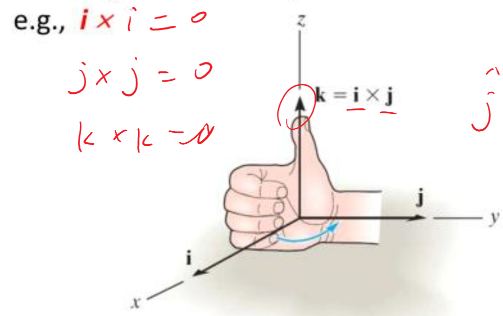
$|\vec{C}| = |\vec{A}||\vec{B}| \cdot \sin\theta$

Cross (or vector) product

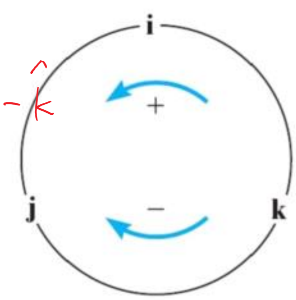
The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$

$j \times j = 0$

$k \times k = 0$



$\hat{j} \times \hat{i} = -\hat{k}$



Considering the cross product in Cartesian coordinates

$$\begin{aligned}
\boldsymbol{A} \times \boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\
&= \cancel{+A_x B_x(\boldsymbol{i} \times \boldsymbol{i})} + A_x B_y(\boldsymbol{i} \times \boldsymbol{j}) + A_x B_z(\boldsymbol{i} \times \boldsymbol{k}) \\
&\quad + A_y B_x(\boldsymbol{j} \times \boldsymbol{i}) + \cancel{A_y B_y(\boldsymbol{j} \times \boldsymbol{j})} + A_y B_z(\boldsymbol{j} \times \boldsymbol{k}) \\
&\quad + A_z B_x(\boldsymbol{k} \times \boldsymbol{i}) + A_z B_y(\boldsymbol{k} \times \boldsymbol{j}) + \cancel{A_z B_z(\boldsymbol{k} \times \boldsymbol{k})} \\
&= (A_y B_z - A_z B_y)\boldsymbol{i} - (A_x B_z - A_z B_x)\boldsymbol{j} + (A_x B_y - A_y B_x)\boldsymbol{k}
\end{aligned}$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\text{cross}(\vec{A}, \vec{B})$

Each component can be determined using 2 × 2 determinants.

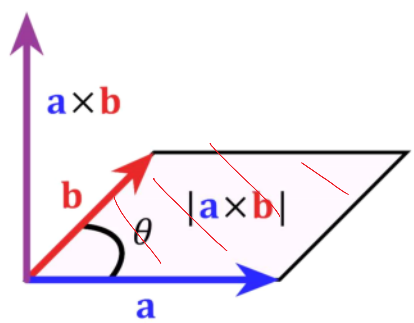
$$\vec{A} \times \vec{B} = + \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$= (A_y \cdot B_z - A_z \cdot B_y)\hat{i} - (A_x B_z - A_z \cdot B_x)\hat{j} + (A_x \cdot B_y - A_y \cdot B_x)\hat{k}$$

12

Example

Determine the area of the parallelogram spanned by the vectors **a** = (3,−3,1) and **b** =(4,9,2).



$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 1 \\ 4 & 9 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 1 \\ 9 & 2 \end{vmatrix} \hat{i} - \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} \hat{j} + \begin{vmatrix} 3 & -3 \\ 4 & 9 \end{vmatrix} \hat{k}$$

$|\vec{a} \times \vec{b}|$

$\boxed{\text{cross}(\vec{a}, \vec{b})}$

$\boxed{\text{Solve}}$

13