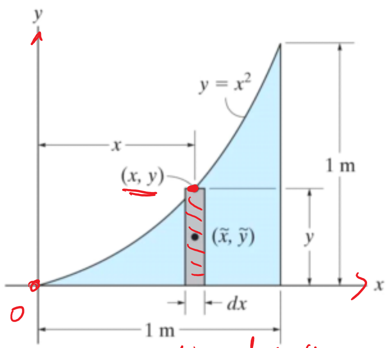
Lec26

Thursday, November 19, 2020 9:58 AM

Lecture Objectives

Centroid



Locate the centroid of the area.

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$dA = dx \cdot y$$

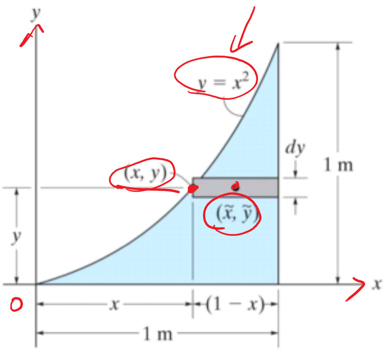
$$\tilde{x} = x$$

$$\tilde{y} = \frac{1}{2} y$$

$$y = x^2$$

$$\bar{x} = \frac{\int \tilde{x}\, dx\, y}{\int y\, dx} = \frac{\int_0^1 x\, x^2 dx}{\int_0^1 x^2 dx} = \frac{\frac{1}{4} x^4 \big|_0^1}{\frac{1}{3} x^3 \big|_0^1} = \frac{3}{4}$$

$$\bar{y} = \frac{\int \frac{1}{2} y \cdot dx\, y}{\int y\, dx} = \frac{\int_0^1 \frac{1}{2} x^2\, x^2 dx}{\int_0^1 x^2 dx} = \frac{\frac{1}{10} x^5 \big|_0^1}{\frac{1}{3} x^3 \big|_0^1} = \frac{3}{10}$$



Locate the centroid of the area.

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$x = \sqrt{y}$$

$$\tilde{x} = \frac{1-x}{2} + x$$

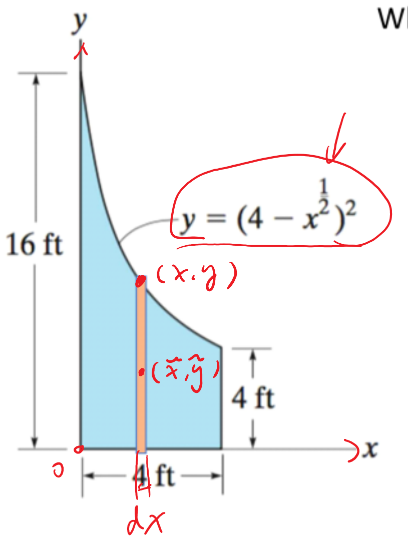
$$\tilde{y} = y$$

$$dA = (1-x)\cdot dy$$

$$\bar{x} = \frac{\int_0^1 \left(\frac{1-x}{2} + x\right)(1-x)\,dy}{\int_0^1 (1-x)\,dy} =$$

$$\bar{y} = \frac{\int y(1-x)\,dy}{\int (1-x)\,dy} =$$

Example



Where is the centroid of the area?

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} = \frac{\int x \cdot y\, dx}{\int y\, dx} = \frac{\int_0^4 x\cdot(4-\sqrt{x})^2\, dx}{\int_0^4 (4-\sqrt{x})^2\, dx}$$

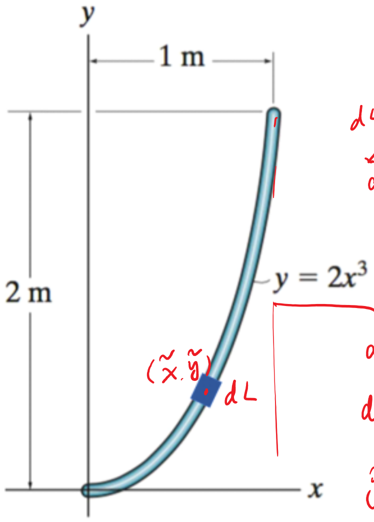
$$\tilde{x} = x$$

$$dA = dx \cdot y$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int_0^4 \frac{1}{2}(4-\sqrt{x})^4\, dx}{\int_0^4 (4-\sqrt{x})^2\, dx}$$

$$\tilde{y} = \frac{y}{2} = \frac{1}{2}(4-\sqrt{x})^2$$

Example



Where is the centroid of the bar?

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL}, \quad \bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

dL, dy, dx

$$dL = \sqrt{dx^2 + dy^2}, \quad \tilde{x} = x$$

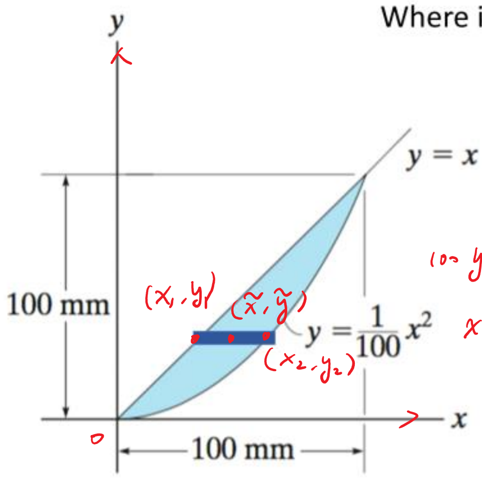
$$\bar{x} = \frac{\int x \cdot \sqrt{dx^2+dy^2}}{\int \sqrt{dx^2+dy^2}} = \frac{\int_0^1 x \cdot \sqrt{1+36x^4}\, dx}{\int_0^1 \sqrt{1+36x^4}\, dx}$$

$$dy = 6 \cdot x^2 dx \qquad \bar{y} = \frac{\int_0^1 2x^3 \sqrt{1+36x^4}\, dx}{\int_0^1 \sqrt{1+36x^4}\, dx}$$

$$dy^2 = 36x^4 dx^2$$

$$\tilde{y} = y = 2x^3$$

Example



Where is the centroid of the area?

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} \qquad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$dA = dy \cdot (x_2 - x_1) = dy(10\sqrt{y} - y)$$

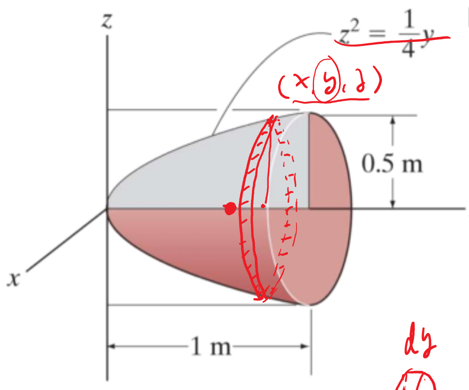
$$100\, y_2 = x_2^2 \qquad \tilde{x} = \frac{x_1 + x_2}{2} = \frac{y + 10\sqrt{y}}{2}$$

$$x_2 = 10\sqrt{y_2}$$

$$y_1 = y_2 = y \qquad \bar{x} = \frac{\int_0^{100} \frac{(y+10\sqrt{y})}{2} \cdot (10\sqrt{y} - y)\, dy}{\int_0^{100} (10\sqrt{y} - y)\, dy}$$

$$\tilde{y} = y$$

$$\bar{y} = \frac{\int_0^{100} y \cdot (10\sqrt{y} - y)\, dy}{\int_0^{100} (10\sqrt{y} - y)\, dy}$$



Locate the centroid of the volume.

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV} \qquad \bar{y} = \frac{\int \tilde{y}\, dV}{\int dV} \qquad \bar{z} = \frac{\int \tilde{z} \int dV}{\int dV}$$

$$\tilde{x} = \tilde{z} = 0, \qquad \tilde{y} = y \qquad dV =$$

$$\bar{y} = \frac{\int_0^1 \frac{\pi}{4} y^2 \cdot dy}{\int_0^1 \frac{1}{4}\pi y\, dy}$$

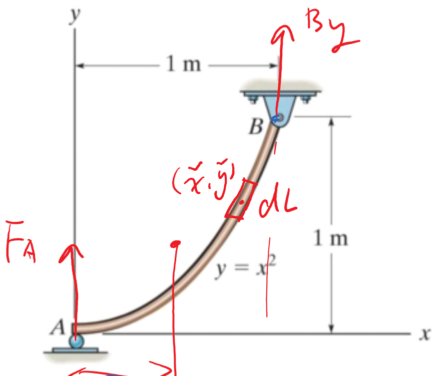
$$z^2 = \frac{1}{4}y$$

dy

$(\tilde{x}, \tilde{y}, \tilde{z})$

$$dV = A \cdot dy = \frac{1}{4}\pi y \cdot dy$$

$$A = \pi r^2 = \pi z^2 = \frac{1}{4}y \cdot \pi$$



Locate the center of gravity of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the reaction supports at *A* and *B*.

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} \qquad dL = \sqrt{dx^2 + dy^2}$$

$$dL = \sqrt{1 + 4x^2}\, dx$$

dL, dy, dx

$$\bar{x} = \frac{\int_0^1 x \cdot \sqrt{1+4x^2} \cdot dx}{\int_0^1 \sqrt{1+4x^2}\, dx}$$

$$\tilde{x} = x \qquad dy^2 = 4x^2 \cdot dx^2$$

$$\tilde{y} = y$$

$$W = 100 L = 100 \int dL = 100 \int_0^1 \sqrt{1+4x^2}\, dx$$

$$\Sigma M_B = W \cdot (1 - \bar{x}) - F_A \cdot 1 = 0 \qquad \Sigma F_y = 0$$

$$F_A = W \cdot (1 - \bar{x}) \qquad W = F_A + B_y$$

$$B_y = ?$$