Lec10

Monday, October 12, 2020 7:52 AM

Lecture Objectives

- Moment about an axis
- Reduction of distributed loading

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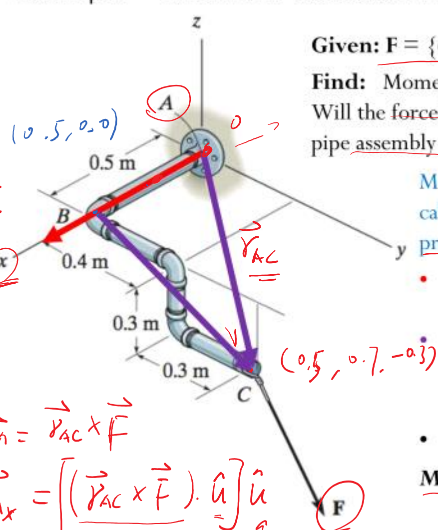
Example – Moment about an Axis



(0.5, 0, 0)

O

$\hat{u}$

$\vec{r}_{AC}$

(0.5, 0.7, -0.3)

Given: **F** = {600**i** + 800**j** - 500**k**} N

Find: Moment of the force about the x-axis. Will the force be tightening or loosening the pipe assembly at A?

Moment about an axis can be calculated using the triple scalar product formula that requires:

$\to (\vec{A}\times\vec{B})\cdot\vec{C}$

- Unit vector that represents the moment axis direction (**u**)
- Position vector from the moment axis to the line of action of the force (**r**)
- The force vector (**F**)

M = [**u** • (**r** X **F**)] **u**

$\vec{m} = \vec{r}_{AC}\times\vec{F}$

$$\vec{m}_x = \left[(\vec{r}_{AC}\times\vec{F})\cdot\hat{u}\right]\hat{u}$$

$$\vec{M} = \vec{r}_{AC}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$\vec{m}_x = 0.7\cdot(-500) - (-0.3\cdot 800)\,\hat{i}$$

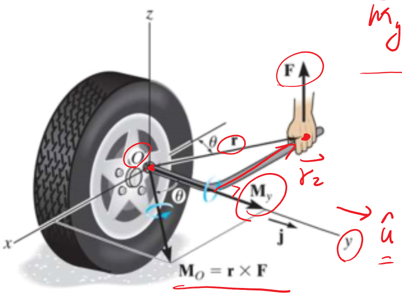
$$-350 + 240 = -110\,\hat{i}$$

$$\vec{M}_x = \left[(\vec{r}_{BA}\times\vec{F})\cdot\hat{u}\right]\hat{u}$$

Moment about a Specific Axis

Remember, the component of a vector, **A**, along the direction of another, **B**, can be determined using the dot product:

$$\vec{M}_y = \left[(\vec{r}\times\vec{F})\cdot\hat{u}\right]\hat{u}$$

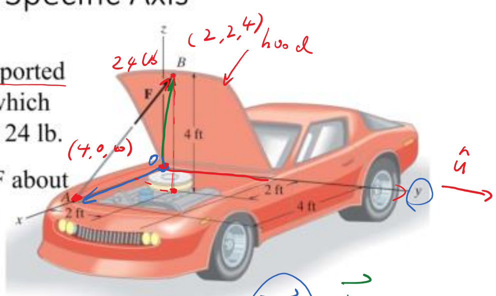


$$\vec{M}_y = \left[\left((\vec{r}_2\times\vec{F})\cdot\hat{u}\right]\hat{u}$$

Moment about a Specific Axis

Given: The hood of the automobile is supported by the strut AB, which exerts a force F = 24 lb.

Find: The moment of F about the hinged axis y.



$$\vec{M}_y = \left[\left((\vec{r}\times\vec{F})\cdot\hat{u}\right]\cdot\hat{u}$$

$\vec{r}_{OA}$ ⇒ (4,0,0)

$\vec{r}_{OB}$

$\vec{F} = |\vec{F}|\,\hat{u}_{AB}$

24 lb

$\vec{r}_{AB} = (-2, 2, 4)$

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{(-2,2,4)}{\sqrt{4+4+16}} \Rightarrow 4\sqrt{2}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ & & \end{vmatrix}$$

i-Clicker Time

The force **F** is acting along DC. Using the triple scalar product to determine the moment of F about the bar BA, you could use any of the following position vectors except:

A) $\mathbf{r}_{BC}$ ✓

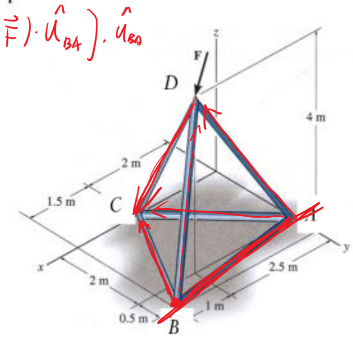
B) $\mathbf{r}_{AD}$ ✓

C) $\mathbf{r}_{AC}$ ✓

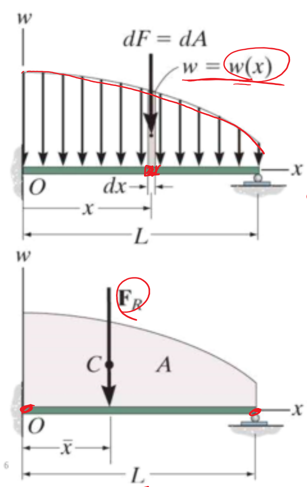
D) $\mathbf{r}_{DB}$ ×

E) $\mathbf{r}_{BD}$ ✓

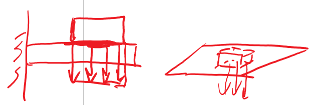
$$\vec{M}_{BA} = \left[(\vec{r}\times\vec{F})\cdot\hat{u}_{BA}\right]\cdot\hat{u}_{BA}$$



Distributed Loading



A common case of distributed loading is a uniform load along one axis of a flat rectangular body.



In such cases, w is a function of x and has units of

$dF = w \cdot dx$, w: W/m,

$dF = w \cdot dx\,dy$

$w(x,y)$: N/m^2

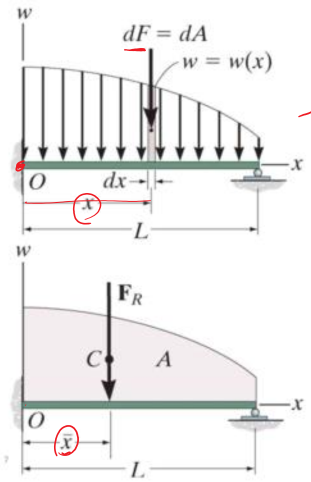
Consider an element of length dx. The force magnitude dF acting on it is given as

$$\int dF = \int w(x)\,dx$$

The net force on the beam is given by

$$F_R = \int dF = \int_0^L w(x)\,dx$$

Location of the Resultant Force



The force *dF* will produce a moment about *O* of

$$\int dM = dF \cdot x = \int x w \cdot dx$$

The total moment about point O is

$$M_O = \int dM = \int_0^L x \cdot w(x) \cdot dx$$

Assuming that $\mathbf{F}_R$ acts at $\bar{x}$, it will produce the moment about point *O* as

$$F_R \cdot \bar{x} = \int_0^L x \, w(x) \, dx$$

Hence, $\bar{x} = \dfrac{\int_0^L x \, w(x) \cdot dx}{F_R} = \dfrac{\int_0^L x \, w(x) \, dx}{\int_0^L w(x) \, dx}$

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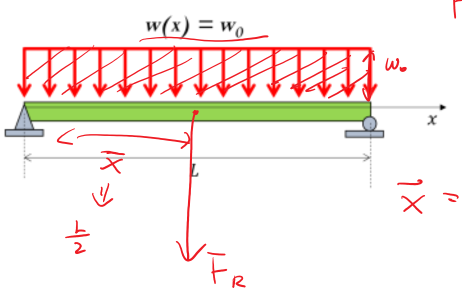
Rectangle Loading



$$F_R = \int_0^L w(x) \, dx$$

$$= w_0 \, x \Big|_0^L$$

$$= w_0 L$$

$$\bar{x} = \frac{\int_0^L x \cdot w_0 \cdot dx}{w_0 L}$$

$$= \frac{w_0 \int_0^L x \cdot dx}{w_0 L}$$

$$= \frac{\cancel{w_0} \, \frac{1}{2} x^2 \Big|_0^L}{\cancel{w_0} \, L} = \frac{1}{2} L$$

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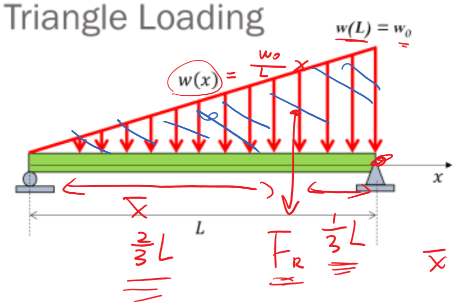
Triangle Loading



$$F_R = \int_0^L w(x)\cdot dx$$

$$= \frac{w_0}{L}\int_0^L x\, dx$$

$$= \frac{w_0}{L}\cdot \frac{1}{2}x^2\Big|_0^L$$

$$= \frac{1}{2}w_0 L$$

$$\bar{x} = \frac{\int_0^L x\cdot w(x)\cdot dx}{F_R} = \frac{\frac{w_0}{L}\int_0^L x^2\cdot dx}{\frac{1}{2}w_0 L}$$

$$= \frac{2}{L^2}\cdot\frac{1}{3}x^3\Big|_0^L$$

$$= \frac{2}{3}\cdot L$$

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