Lec27

Monday, November 23, 2020 7:54 AM

Lecture Objectives

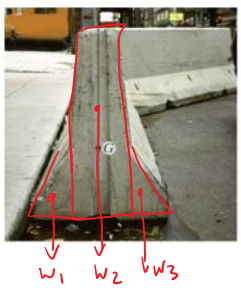
Centroid

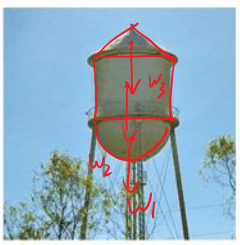
1

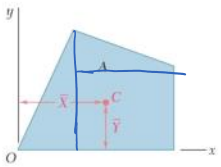
Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

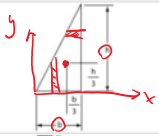
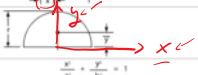
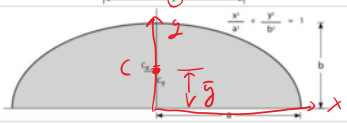
Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.







Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

http://en.wikipedia.org/wiki/List_of_centroids

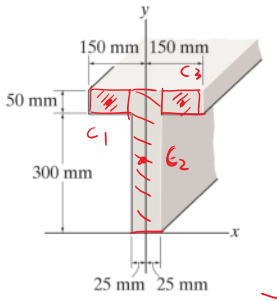
Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \quad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

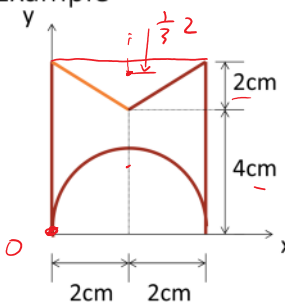
$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \quad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \quad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

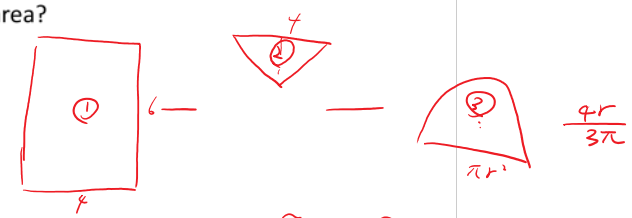


$$\bar{X} = \frac{\tilde{X}_{c_1} \cdot A_1 + \tilde{X}_{c_2} A_2 + \tilde{X}_{c_3} A_3}{A_1 + A_2 + A_3}$$

Example



What is the centroid of the resultant area?



$$\bar{X} = \frac{\tilde{X}_1 A_1 - \tilde{X}_2 A_2 - \tilde{X}_3 A_3}{A_1 - A_2 - A_3}$$

	①	②	③
A	24	4	$2\cdot\pi$
$\bar{x}$	2	2	2

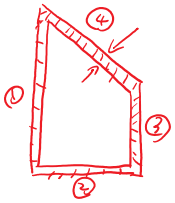
$$\bar{y} = \frac{\tilde{y}_1 A_1 - \tilde{y}_2 A_2 - \tilde{y}_3 A_3}{\quad}$$

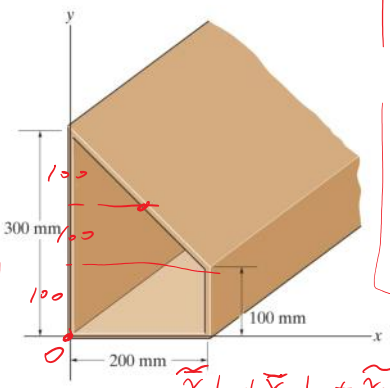
A	2π	4	$2\cdot\pi$
$\bar{x}$	2	2	2
$\bar{y}$	3	$6-\frac{2}{3}$	$\frac{8}{3\pi}$

$$\bar{y} = \frac{\tilde{y}_1 A_1 - \tilde{y}_2 A_2 - \tilde{y}_3 A_3}{A_1 - A_2 - A_3}$$

Example

Locate the centroid of the cross section area.

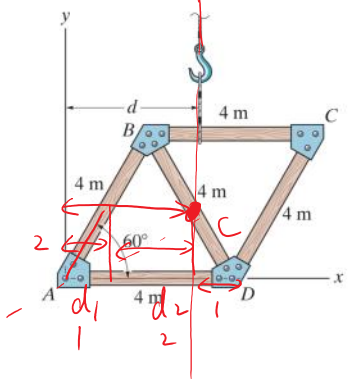




	1	2	3	4
L	300	200	100	$200\sqrt{2}$
$\tilde{x}$	0	100	200	100
$\tilde{y}$	150	0	50	200

$$\bar{x} = \frac{\tilde{x}_1 L_1 + \tilde{x}_2 L_2 + \tilde{x}_3 L_3 + \tilde{x}_4 L_4}{L_1 + L_2 + L_3 + L_4}$$

$$\bar{y} =$$



The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

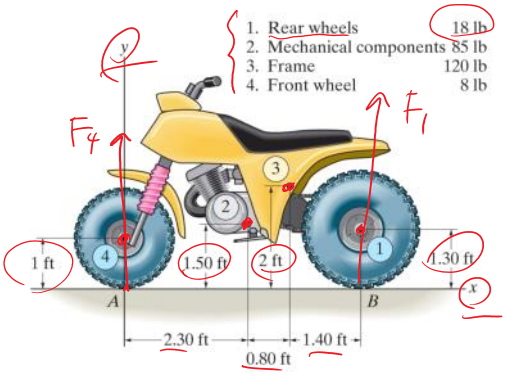
$d_1 = 1$ m

$d_2 = 2$ m

$d = 1 + 2 = 3$ m

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.

1. Rear wheels 18 lb
2. Mechanical components 85 lb
3. Frame 120 lb
4. Front wheel 8 lb



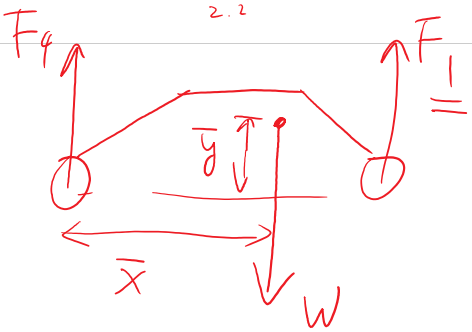
2.2

$$\bar{x} = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3 + x_4 w_4}{w_1 + w_2 + w_3 + w_4}$$

$$\bar{y} = \frac{}{}$$

$$\bar{z} = 0$$

	①	②	③	④
w	18	85	120	8
$\bar{x}$	4.5	2.3	3.1	0
$\bar{y}$	1.3	1.5	2	1



$$F_r = \frac{1}{2} F_1$$

$$F_f = F_4$$