

Lecture Objectives



Moment about a point



Couple moments

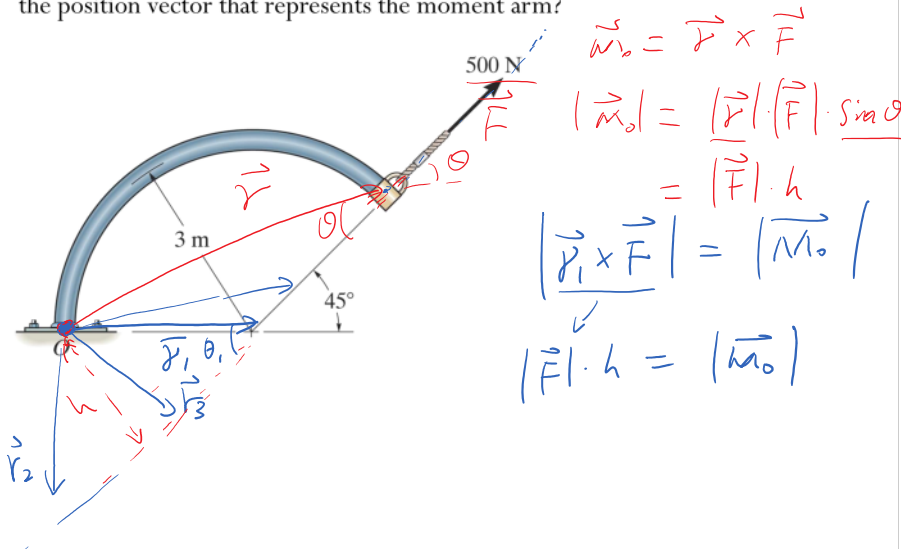


Equivalent systems

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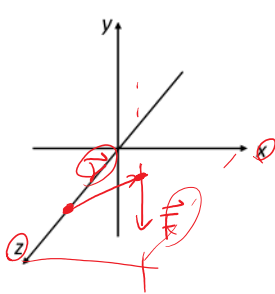
Moment about a Point from a Force

To find the moment about point O by the 500 N force shown below, what is the position vector that represents the moment arm?



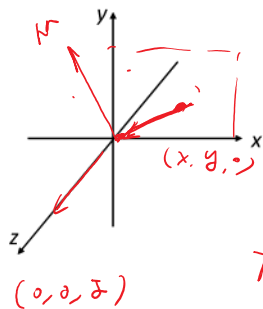
Determine the direction of a moment

If force vector \mathbf{F} is on the xz-plane, what is the direction of the moment vector about a point on the z-axis?



i-Clicker

If force vector \mathbf{F} goes along the z-axis, what is(are) the direction component(s) of the moment vector about a point on the xy-plane?



- (A) i
- (B) j
- (C) k
- (D) i and j**
- (E) j and k

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & -y & 0 \\ 0 & 0 & z \end{vmatrix} = \underline{-y \cdot z} \hat{i} + \underline{xz} \hat{j} + \underline{0} \hat{k}$$

i-Clicker

If force vector \mathbf{F} is on the yz-plane, what is the direction of the moment vector about a point on the x-axis?

(A) i
(B) i and j
(C) j and k
(D) i and k
(E) i, j and k

$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & y & z \\ 0 & y & z \end{vmatrix} = \frac{1}{r} \hat{i} + \frac{1}{r} \hat{j} + \frac{1}{r} \hat{k}$

$\vec{r} = (-x, y, z)$

Moment of a couple

A **couple** is defined as **two parallel forces** that have the **same magnitude**, but **opposite directions**, and are separated by a **perpendicular distance d** .

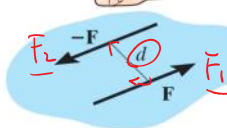
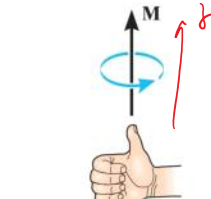
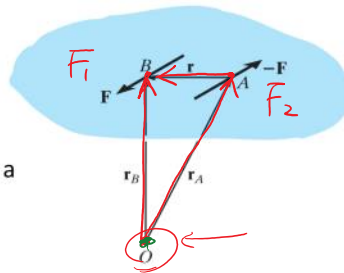
Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

The moment produced by a couple is called **couple moment**.

Let's determine the sum of the moments of both couple forces about **any** arbitrary point:

$$\vec{M} = \vec{r}_B \times \vec{F}_1 + \vec{r}_A \times \vec{F}_2$$

$$\vec{r}_B \times \vec{F} - \vec{r}_A \times \vec{F}$$



$$|F_1| = |F_2|$$

$$\vec{F}_1 = -\vec{F}_2$$

$$\sum F_x = 0$$

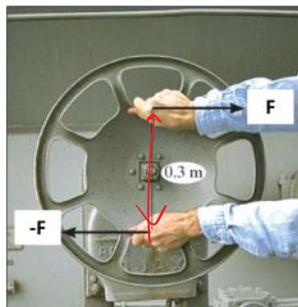
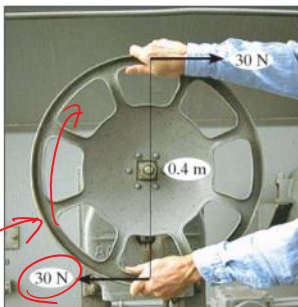
$$\sum F_y = 0$$

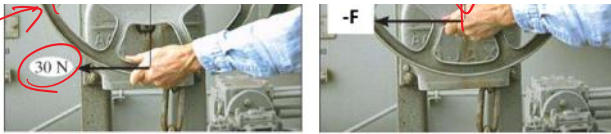
$$\vec{M} = \vec{r} \times \vec{F}$$

$$|\vec{M}| = |\vec{r}| \cdot |\vec{F}| \sin \theta = |\vec{F}| \cdot d$$



$$(\vec{r}_B - \vec{r}_A) \times \vec{F} = \vec{r} \times \vec{F}$$

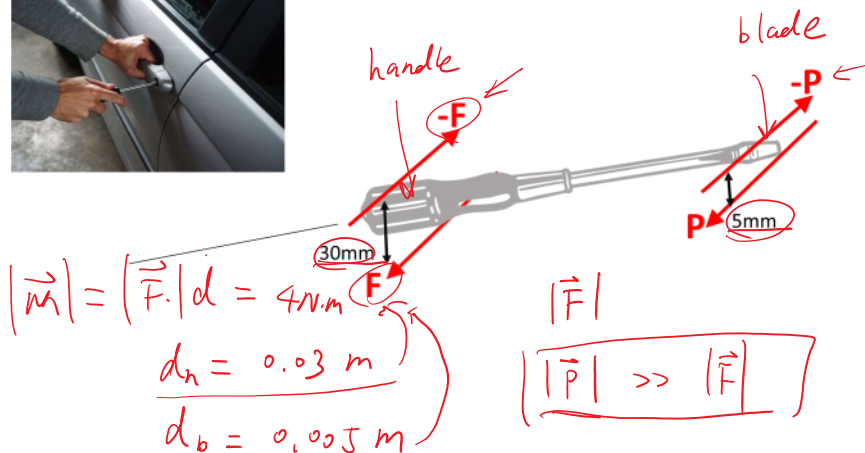




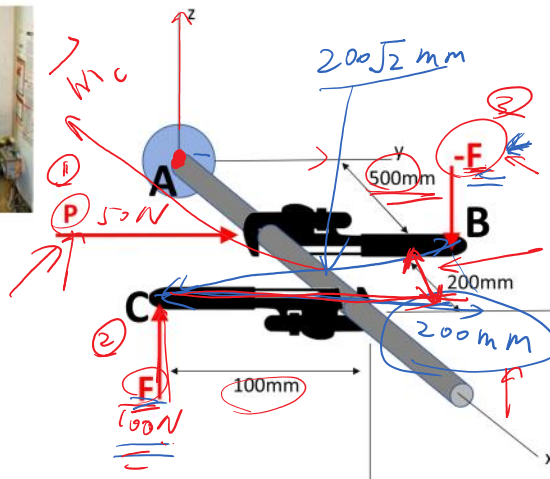
A torque or moment of $12 \text{ N}\cdot\text{m}$ is required to rotate the wheel.
Would F be greater or less than 30 N ?

$$|\vec{M}| = |\vec{F}|d = 30 \times 0.4 = 12 \text{ N}\cdot\text{m}$$

A twist of $4 \text{ N}\cdot\text{m}$ is applied to the handle of the screwdriver.
Resolve this couple moment into a pair of couple forces F exerted on the handle and P exerted on the blade.



Find the moment about the support at A? $F = 100 \text{ N}$, $P = 50 \text{ N}$.



$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3$$

$$\vec{M}_1 = \vec{r}_1 \times \vec{P}$$

$$\vec{M} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

$$= |F| \cdot d \hat{i} - |F| \cdot d \hat{j} + |P| \cdot |r| \hat{k}$$

$$= 100 \cdot 0.2 \hat{i} - 100 \cdot 0.2 \hat{j} + 50 \cdot 0.5 \hat{k}$$