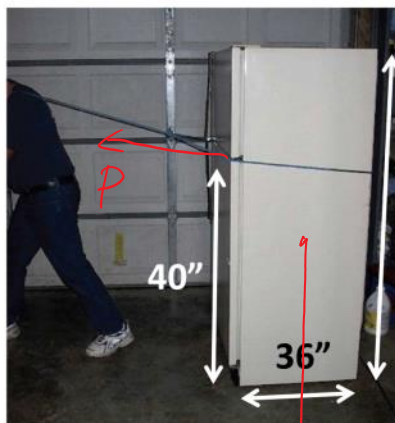


# Lecture Objectives



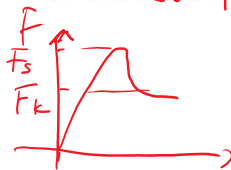
## Friction

1

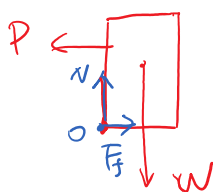


**Given:** Fridge weight = 250 lb and  $\mu_s = 0.4$

**Find:** The maximum horizontal force P that can be applied at without causing movement of the ~~crate~~ fridge



$$\downarrow 250 \text{ lb} \quad P = F_s = \mu_s \cdot W = 0.4 \times 250 = \boxed{100 \text{ lb}}$$

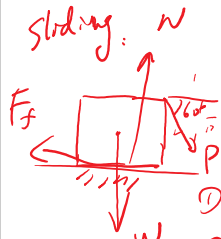


$$\Sigma M_o = P \cdot 40'' - W \cdot 18'' = 0$$

$$P = \frac{18'' \cdot 250}{40} = \frac{9 \times 25}{2} = 112.5 \text{ lb}$$

A 4'x2' 100 lb box with a wide base is pulled by a force  $P = 80$  lb and  $\mu_s = 0.4$ . Will the block remain stationary?

Yes



$$\textcircled{1} \sum F_x = F_f - P \cos 60^\circ = 0$$

$$\textcircled{2} \sum F_y = N - P \sin 60^\circ - W = 0$$

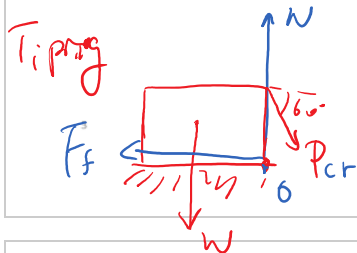
$$N = 80 \cdot \frac{\sqrt{3}}{2} + 100 \text{ lb}$$

$$F_f = 80 \cdot \frac{1}{2} = 40 \text{ lb}$$

$$F_s = \mu_s N =$$

$$0.4(40\sqrt{3} + 100) = 67.7 \text{ lb}$$

$$F_f < F_s \text{ , No sliding}$$



$$\sum M_o = W \cdot 2 - P_{cr} \cdot \frac{1}{2} \cdot 2 = 0$$

$$P_{cr} = 200 \text{ lb} > P = 80 \text{ lb}$$

no tiping

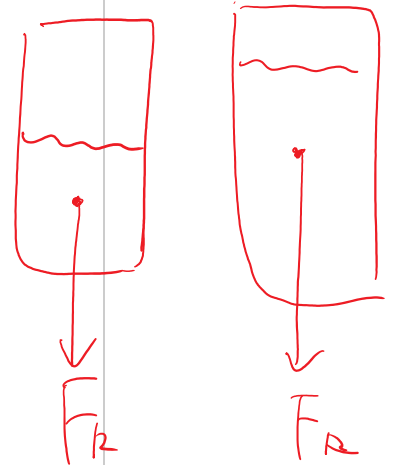
## Center of gravity



To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

$$F_R = W_w + W_t$$



## Center of gravity

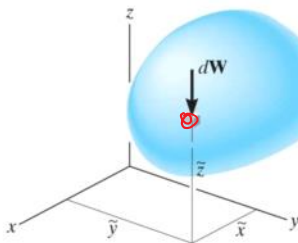


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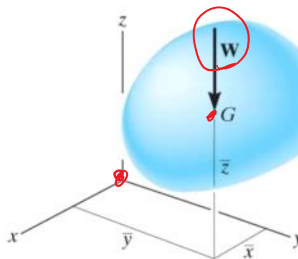


© Paramount Pictures

## Center of gravity



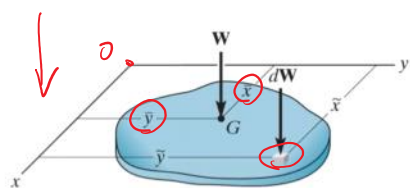
A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight  $dW$ .  $w = \int dW$



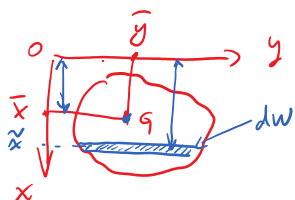
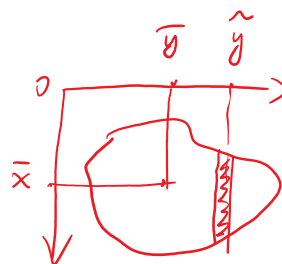
The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

## Center of gravity



$$W = \int dW$$



$$W \cdot \bar{x} = \int \tilde{x} dW$$

$$\bar{x} = \frac{\int \tilde{x} dW}{\int dW}$$

$$\bar{y} = \frac{\int \tilde{y} dW}{\int dW}$$

## Center of Mass

$$\bar{x} = \frac{\int \tilde{x} dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y} dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z} dm}{\int dm}$$

## Center of Volume

$$\bar{x} = \frac{\int \tilde{x} dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y} dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z} dV}{\int dV}$$

## Center of Area →

$$\bar{x} = \frac{\int \tilde{x} dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y} dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z} dA}{\int dA}$$

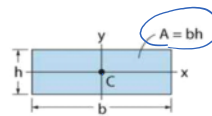
## Centroid

The centroid,  $C$ , is a point defining the geometric center of an object.

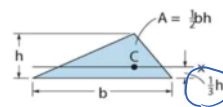
The centroid coincides with the center of mass or the center of gravity **only** if the material of the body is **homogeneous** (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

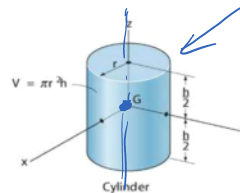
In some cases, the centroid may not be located on the object.



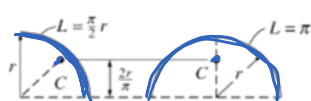
Rectangular area



Triangular area



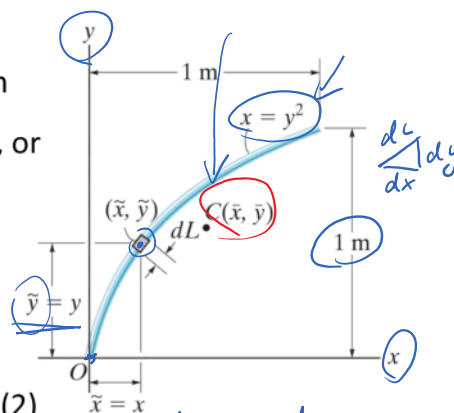
Cylinder



Quarter and semicircle arcs

## Centroid – Analysis Procedure

1. Select an appropriate coordinate system
2. Define the appropriate element ( $dL$ ,  $dA$ , or  $dV$ )
3. Express (2) in terms of the coordinate system
4. Identify any symmetry
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



$$y = \sqrt{x}$$

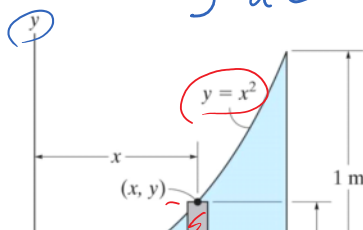
$$dy = \frac{dx}{2\sqrt{x}}$$

$$\bar{y} = \frac{\int \tilde{y} dL}{\int dL} = \frac{\int_0^1 y \sqrt{1+4y^2} \cdot dy}{\int_0^1 \sqrt{1+4y^2} \cdot dy}$$

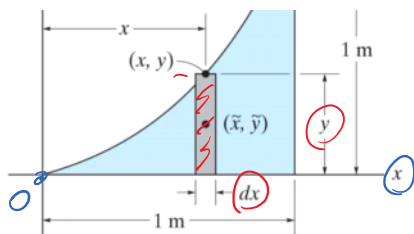
$$dL = \sqrt{dx^2 + dy^2} = \sqrt{(1+4y^2)} dy = \sqrt{1+4y^2} dy$$

$$dx = 2y dy$$

$$\bar{x} = \frac{\int \tilde{x} dL}{\int dL} =$$



Locate the centroid of the area.



$$\underline{dA} = y dx = \underline{x^2 \cdot dx}$$

$$\bar{x} = \frac{\int_0^1 x x^2 dx}{\int_0^1 x^2 dx} = \frac{\frac{1}{4} x^4 \big|_0^1}{\frac{1}{3} x^3 \big|_0^1} = \frac{3}{4} \text{ m}$$