

# Moments of Inertia

# Applications



Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

What primary property of these members influences design decisions?



Terminology: the term **moment** in this module refers to the mathematical sense of different “measures” of an area or volume.

- The zeroth moment is the total mass.
- The first moment (a single power of position) gave us the centroid.
- The second moment will allow us to describe the “width.”
- An analogy that may help: in *probability* the first moment gives you the mean (the center of the distribution), and the second is the standard deviation (the width of the distribution).

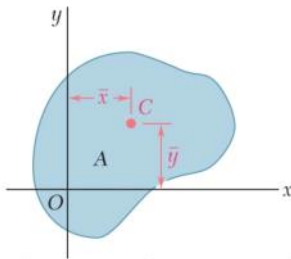
$$\int dm = m$$

$$\bar{x} = \frac{\int x dm}{m}$$

$$\frac{\int x^2 dm}{m}$$

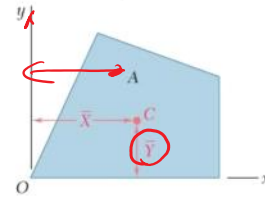
# Recap: First moment of an area (centroid of an area)

- The first moment of the area  $A$  with respect to the  $x$ -axis is given by  $Q_x = \int_A y dA$
- The first moment of the area  $A$  with respect to the  $y$ -axis is given by  $Q_y = \int_A x dA$
- The centroid of the area  $A$  is defined as the point  $C$  of coordinates  $\bar{x}$  and  $\bar{y}$ , which satisfies the relation



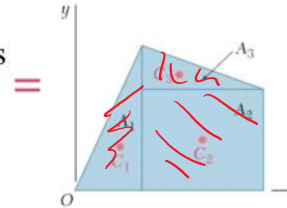
$$\int_A x dA = A \bar{x}$$

$$\int_A y dA = A \bar{y}$$



- In the case of a composite area, we divide the area  $A$  into parts

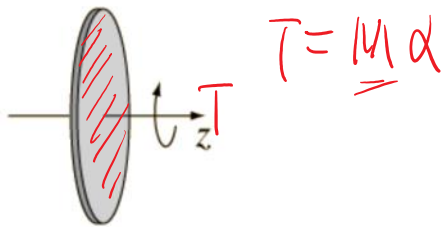
$$A_{total} \bar{X} = \sum_i A_i \bar{x}_i \quad A_{total} \bar{Y} = \sum_i A_i \bar{y}_i$$



# Mass Moment of Inertia

- **Mass moment of inertia** is the mass property of a rigid body that determines the torque needed for a desired angular acceleration about an axis of rotation.
- A larger mass moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis
- Mass moment of inertia depends on the shape and density of the body and is different around different axes of rotation.

Torque-acceleration relation:



# Second moment of area

**Moment of inertia** is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis.

Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

- The moment of inertia of the area A with respect to the x-axis is given by

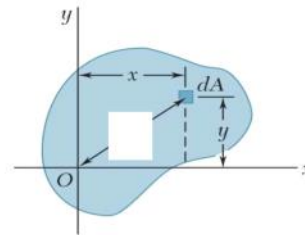
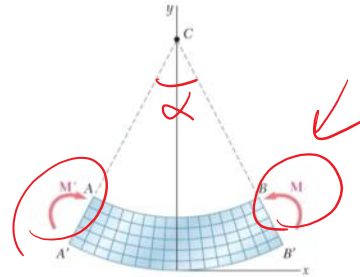
$$I_x = \int y^2 dA$$

- The moment of inertia of the area A with respect to the y-axis is given by

$$I_y = \int x^2 dA$$

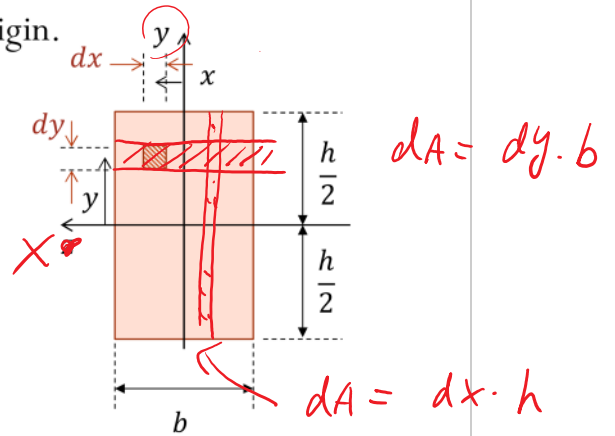
- The moment of inertia of the area A with respect to the origin is given by (Polar Mol)

$$J_o = \int (x^2 + y^2) dA = I_x + I_y$$



Moment of inertia of a rectangular area about the origin.

$$\begin{aligned}
 J_o &= \int (x^2 + y^2) dA \\
 &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} (x^2 + y^2) dx dy \\
 &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \int_{-\frac{b}{2}}^{\frac{b}{2}} (x^2 + y^2) dy \right) dx
 \end{aligned}$$



$$J_o = I_x + I_y$$

$$\begin{aligned}
 I_x &= \int y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} b \cdot y^2 \cdot dy \\
 I_y &= \int x^2 dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} h \cdot x^2 \cdot dx
 \end{aligned}$$