

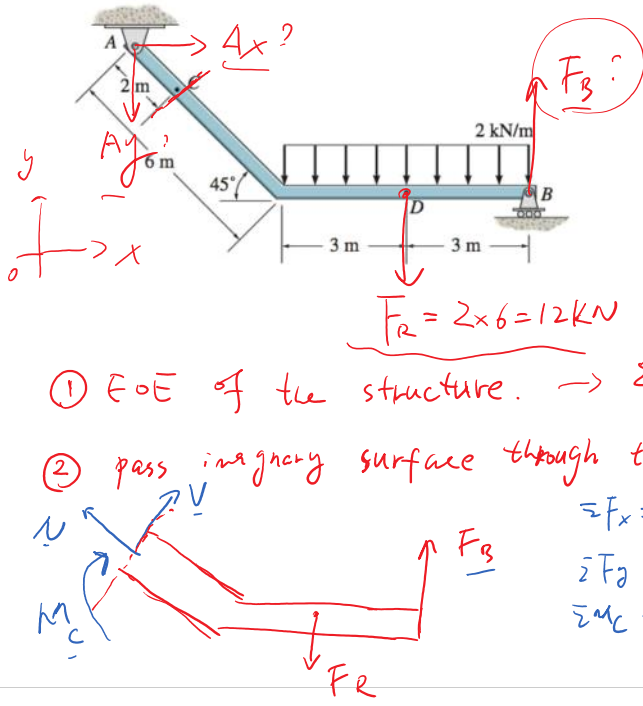
# Lecture Objectives



## Internal Forces

1

Determine the normal force, shear force, and bending moment at C.

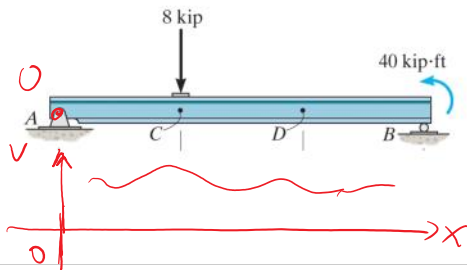


## Shear and Moment Diagram

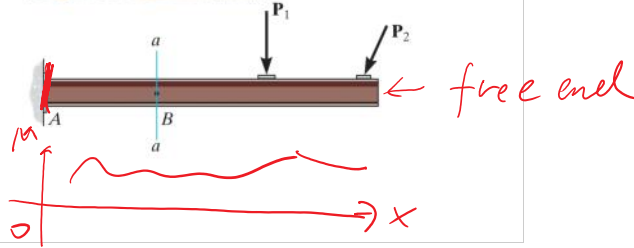
# Shear and Moment Diagram

Beams: structural members designed to support loadings applied perpendicular to their axes.

Simply supported beam



Cantilever beam

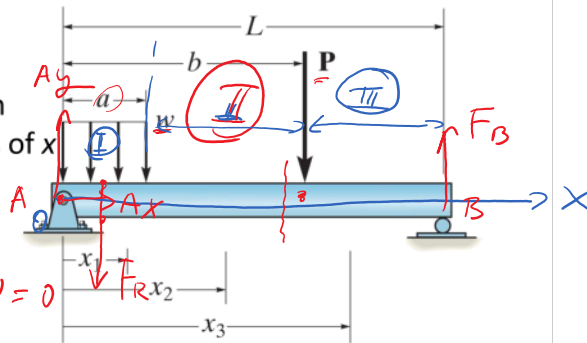


## Shear and Moment Diagram

Goal: provide detailed knowledge of the variations of internal loadings (V and M) throughout the beam

### Procedure

1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinates x
3. Divide the beam into regions
4. Draw FBD of a segment
5. Apply equations of equilibrium to derive V and M as functions of x



$$\textcircled{1} \sum F_x = A_x = 0$$

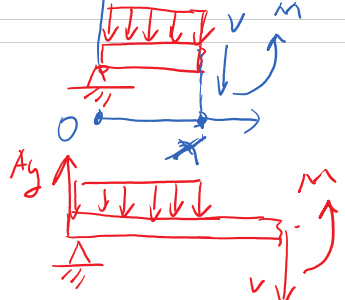
$$\sum F_y = A_y + F_b - P - aw = 0$$

$$\sum M_A = F_b L - P \cdot b - aw \cdot \frac{a}{2} = 0$$

$\textcircled{2}$

$$A_y - w \cdot x - V = 0$$

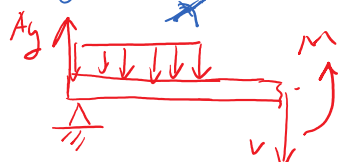
$\textcircled{3}$  Region I:



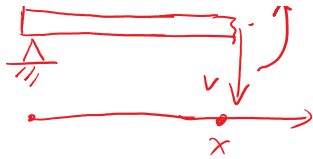
$$\sum M_A = M - V \cdot x - w \cdot x \cdot \frac{x}{2} = 0$$

$$\begin{cases} V = A_y - wx \\ M = \frac{w}{2} x^2 + Vx \end{cases}$$

Region II:



Region II:



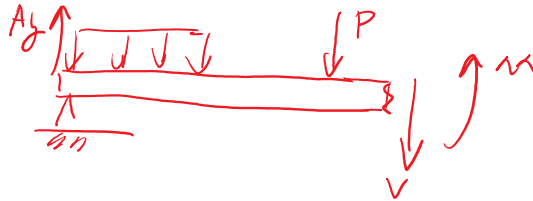
$$\sum F_y = A_y - w \cdot a - V = 0$$

$$\sum M_A = M - V \cdot x - \frac{a}{2} w \cdot a = 0$$

$$V = A_y - wa$$

$$M = Vx + \frac{w}{2} a^2$$

Region III:



$$\sum F_y = A_y - P - w \cdot a - V = 0$$

$$\sum M_A = M - V \cdot x - P \cdot b - \frac{a}{2} w \cdot a = 0$$

$$V = A_y - P - wa$$

$$M = Vx + Pb + \frac{w}{2} a^2$$

L2 - Gen Principles & Force Vectors

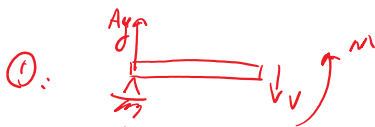
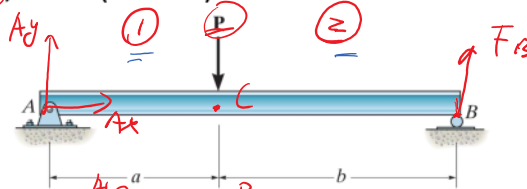
5

### Example

Draw the V diagrams for:  $a = b$ ; Let  $L (= a + b)$  be the total length of the beam.

$$A_x = 0$$

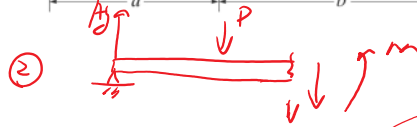
$$A_y = F_B = \frac{P}{2}$$



$$V = A_y = \frac{P}{2}$$

$$\sum M_A = M - Vx = 0$$

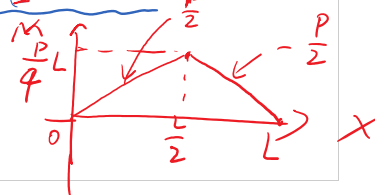
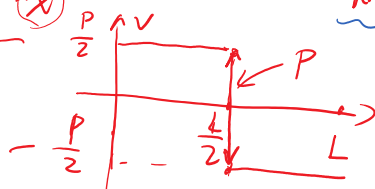
$$M = \frac{P}{2} x$$



$$A_y - P - V = 0 \quad V = -\frac{P}{2}$$

$$M - P \cdot \frac{L}{2} - Vx = 0$$

$$M = Vx + \frac{P}{2} L = -\frac{P}{2} x + \frac{P}{2} L$$



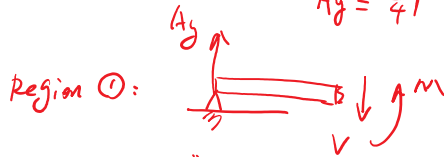
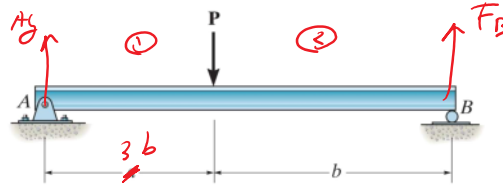
### Example

Draw the V diagrams for:  $a = 3b$ . Let  $L (= a + b)$  be the total length of the beam.

$$A_y + F_B = P$$

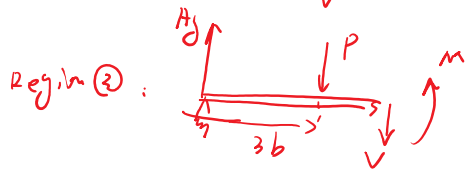
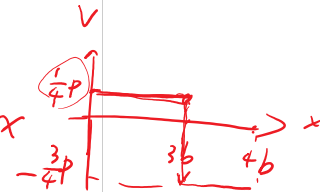
$$P \cdot 3b = F_B \cdot 4b \quad F_B = \frac{3}{4}P$$

$$A_y = \frac{1}{4}P$$



$$A_y - V = 0 \quad V = \frac{1}{4}P$$

$$M - Vx = 0 \quad M = \frac{1}{4}P \cdot x$$



$$A_y - V - P = 0$$

$$M - Vx - P \cdot 3b = 0$$

$$V = A_y - P = -\frac{3}{4}P$$

$$M = Vx + 3Pb = -\frac{3}{4}P \cdot x + 3Pb$$

