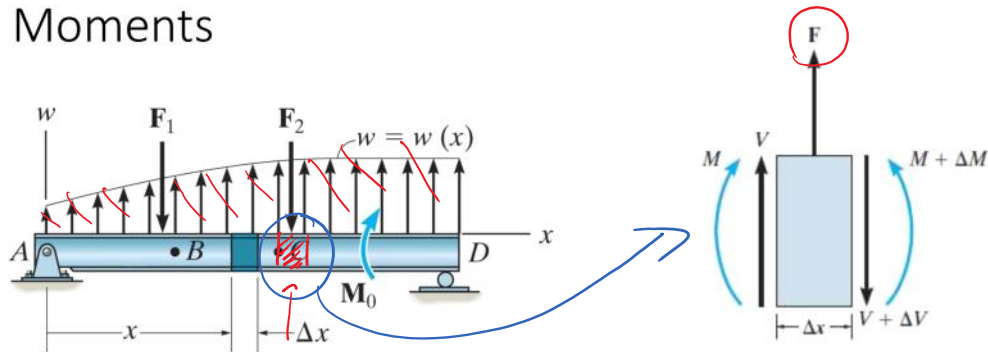


Lecture  
Objectives



Internal Forces

## Relations Among Load, Shear and Bending Moments



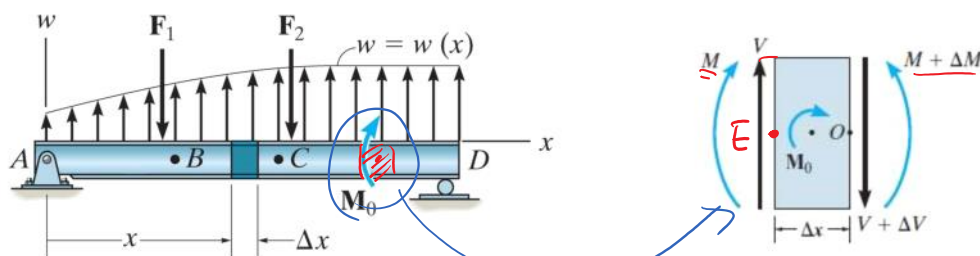
$$\sum F_y = V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

Wherever there is an external concentrated force, there will be a change (jump) in internal shear force.

2

## Relations Among Load, Shear and Bending Moments



$$\sum M_E = -\cancel{M} - M_0 + \cancel{M + \Delta M} - (V + \Delta V) \cdot \Delta x = 0$$

$$\Delta M = M_0 + (V + \Delta V) \Delta x \quad \Delta x \rightarrow 0$$

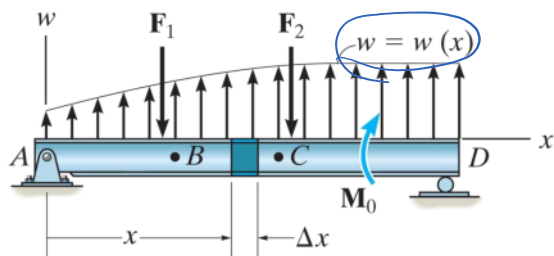
$$\Delta M = M_0$$

clockwise couple moment ↑

anticlockwise couple moment ↓

3

# Relations Among Load, Shear and Bending Moments

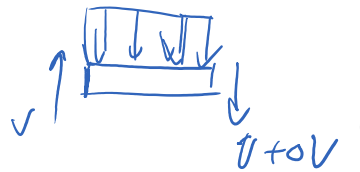
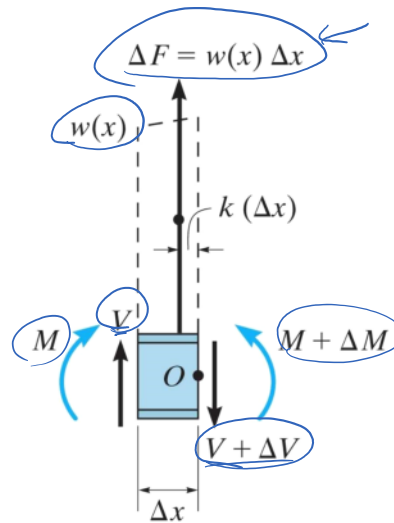


Relationship between load and shear:

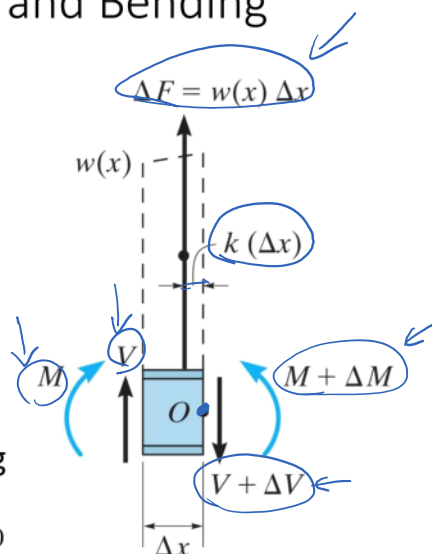
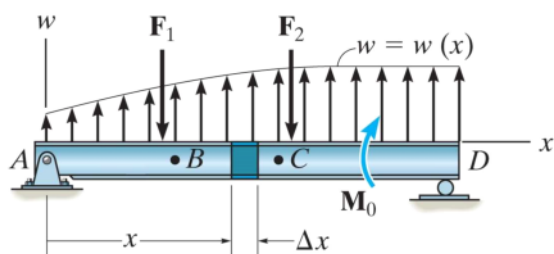
$$\sum F_y = 0: V - (V + \Delta V) + w \Delta x = 0$$

Dividing by  $\Delta x$  and letting  $\Delta x \rightarrow 0$ , we get:

$$\frac{dV}{dx} = -w \quad \Delta V = \int w \, dx$$



## Relations Among Load, Shear and Bending Moments



Relationship between shear and bending moment:

$$\sum M_O = 0: (M + \Delta M) - M - V \Delta x - w \Delta x \left( \frac{\Delta x}{2} \right) = 0$$

$$\Delta M = V \Delta x + w k(\Delta x)^2$$

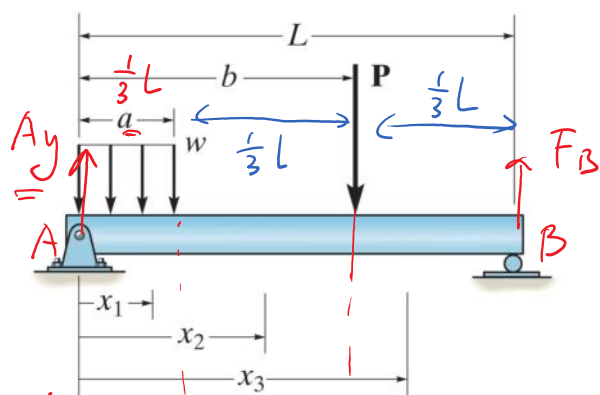
Dividing by  $\Delta x$  and letting  $\Delta x \rightarrow 0$ , we get:

$$\frac{dM}{dx} = V \quad \Delta M = \int V dx$$

$w k(\Delta x)^2 \ll V \Delta x$

## Relationships between w, V, M

Draw the shear and moment diagrams for the beam.



$$a = \frac{1}{2} b = \frac{1}{3} L$$

$$\sum F_y = A_y + F_B - P - w \cdot \frac{1}{3} L$$

$$\sum M_A = - \frac{1}{3} L \cdot w \cdot \frac{1}{6} L - P \cdot \frac{2}{3} L + F_B \cdot L = 0$$

$$F_B = \frac{2}{3} P + \frac{1}{18} w \cdot L$$

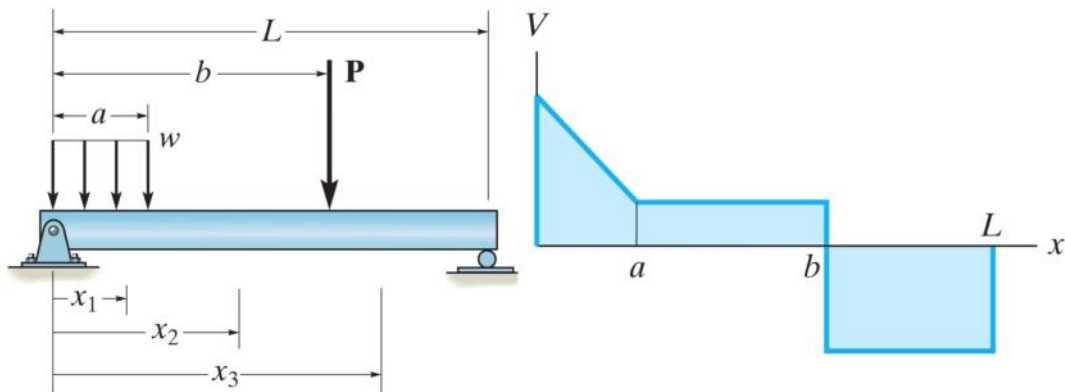
$$A_y = P + \frac{1}{3} w L - F_B = \frac{1}{3} P + \frac{5}{18} w L$$



$$-\frac{2}{3}P - \frac{wL}{18} \quad - \quad \downarrow \downarrow \downarrow$$

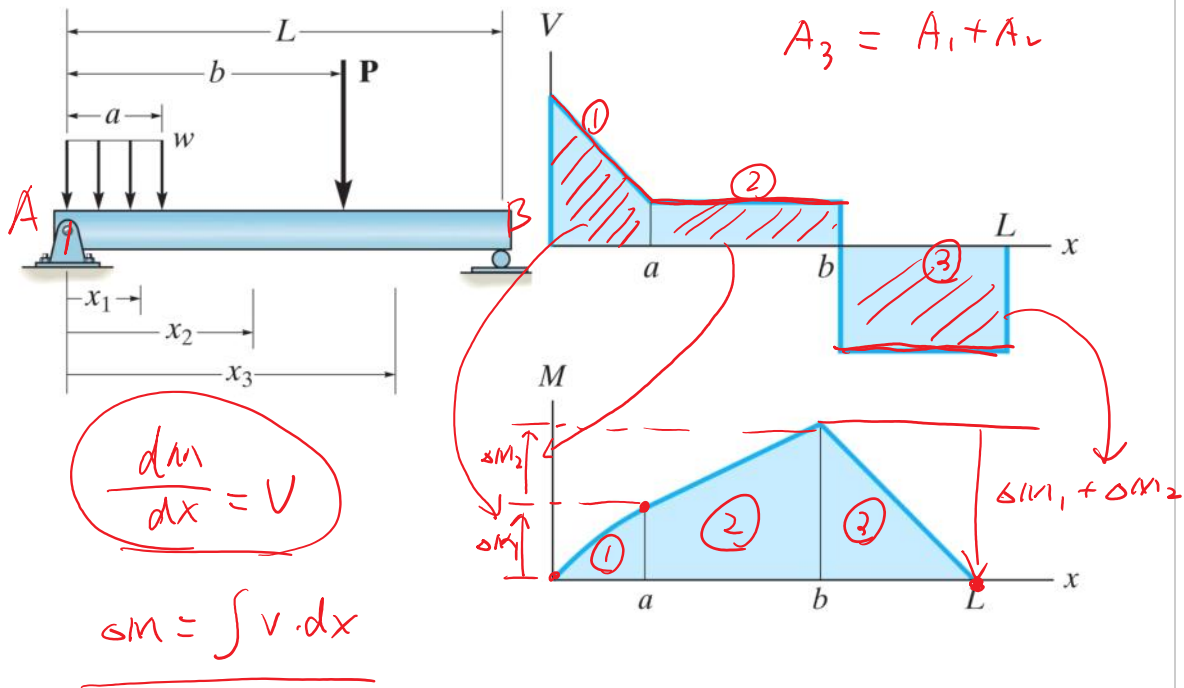
# Relationships between w, V, M

Draw the shear and moment diagrams for the beam.



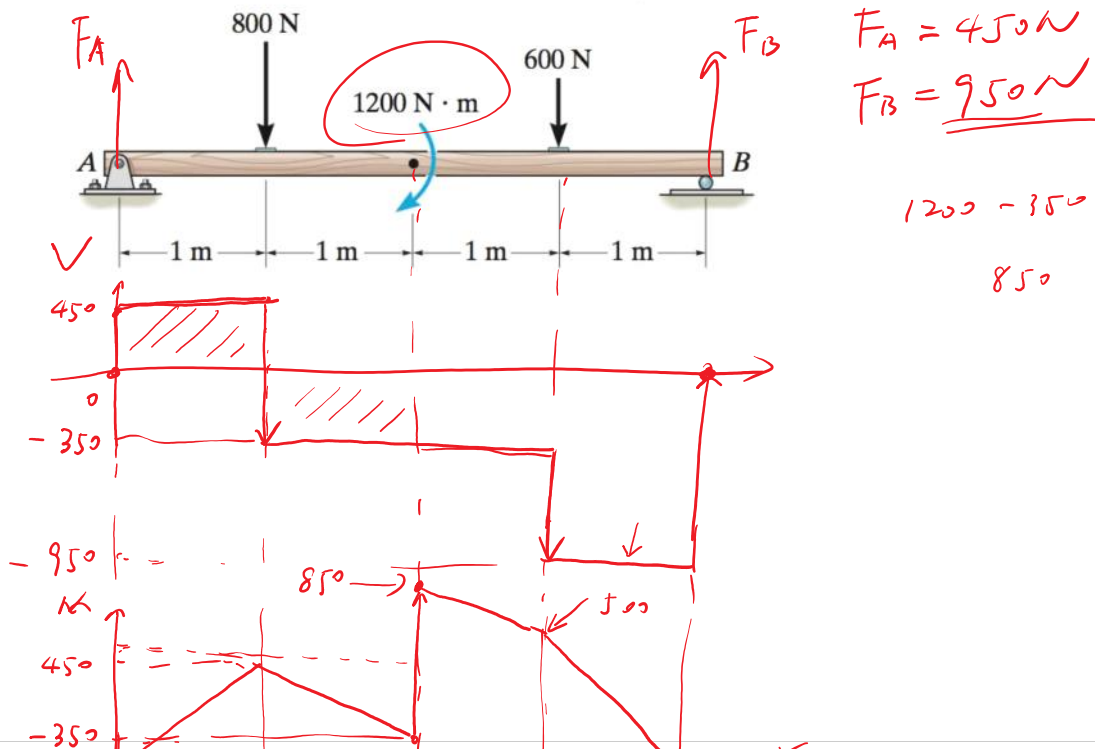
## Relationships between $w$ , $V$ , $M$

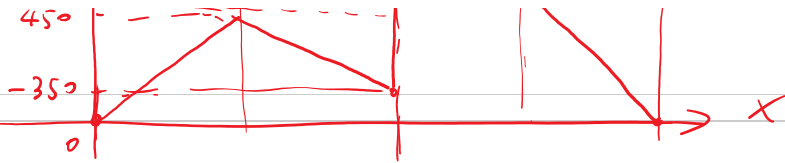
Draw the shear and moment diagrams for the beam.



## Example

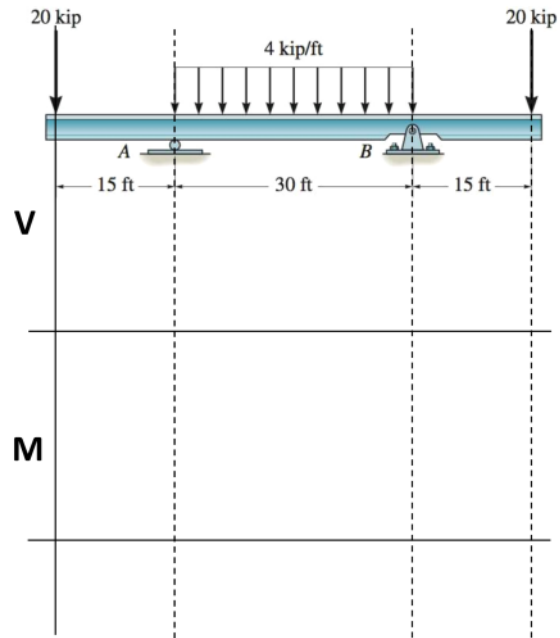
Draw the shear and moment diagrams for the beam.





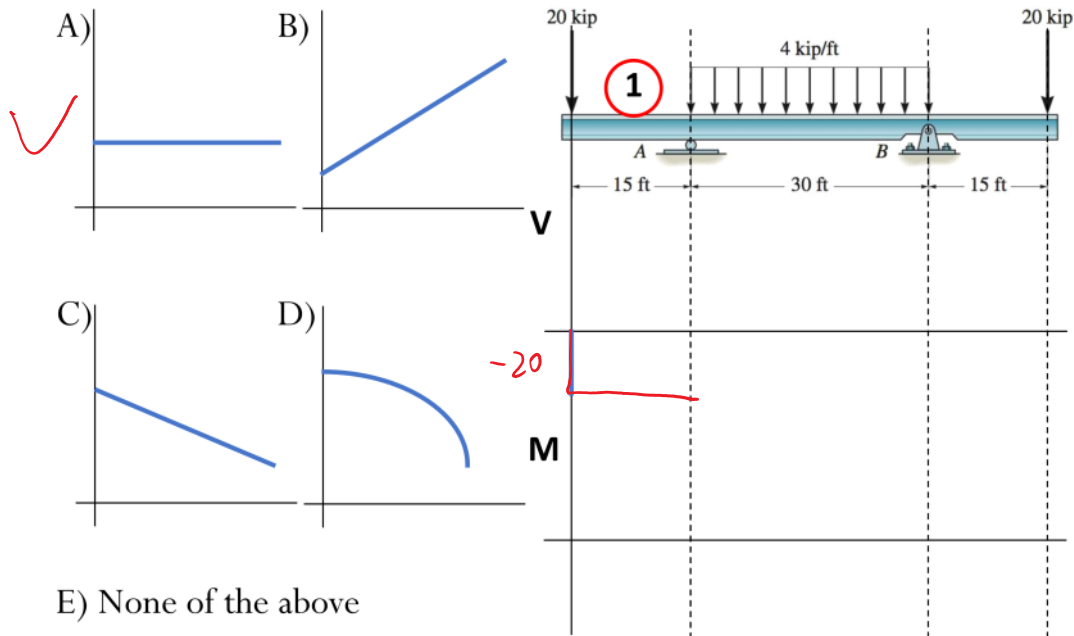
## Example

Draw the shear force and bending moment diagrams for the beam.



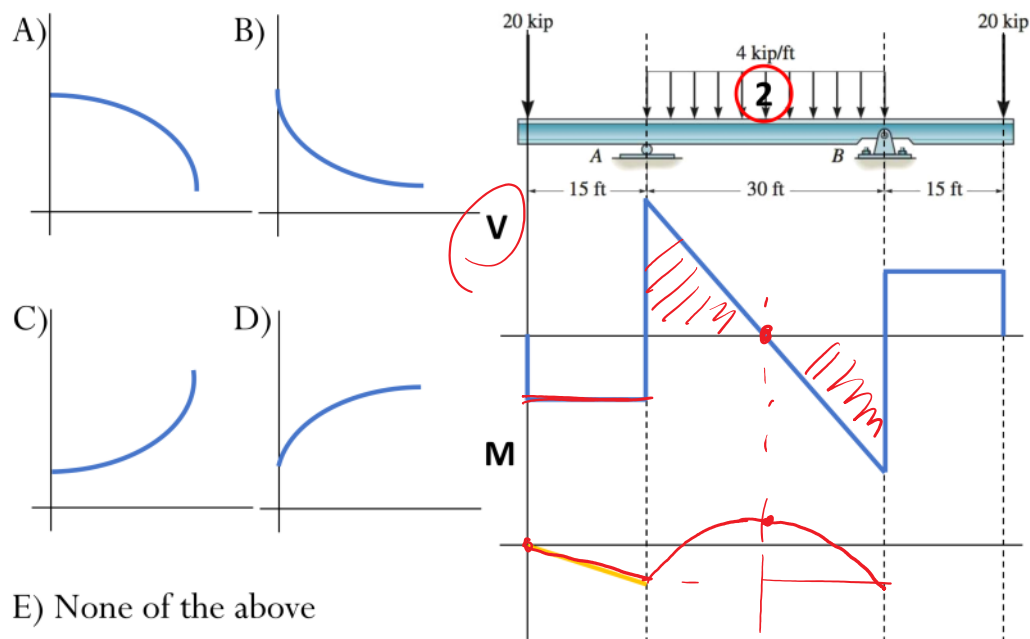
## i-Clicker Time

What is the shape of  $V(x)$  for region 1?



## i-Clicker Time

What is the shape of  $M(x)$  for region 2?

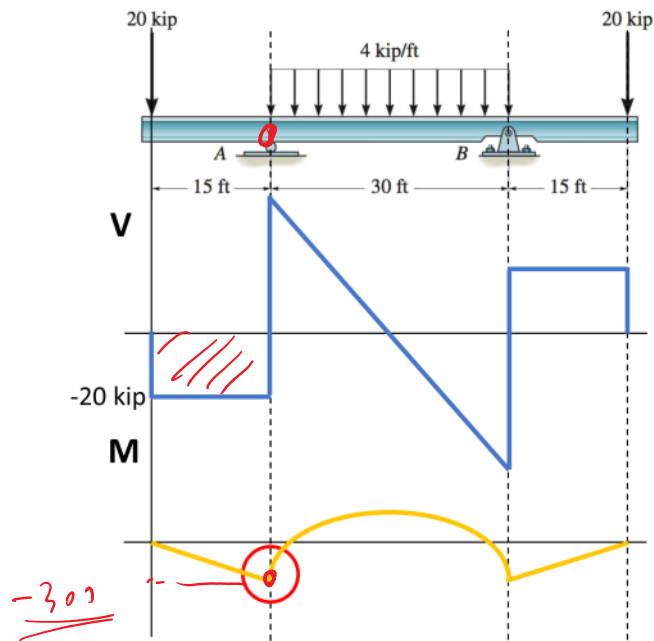




## i-Clicker Time

What is the value of  $M$  at  $A$ ?

- A) 200 kip-ft
- B) 300 kip-ft
- C) 400 kip-ft
- D) 500 kip-ft
- E) None of the above



## Example

Draw the shear and moment diagrams for the beam.

