

R/LHS - Normal Equation

```
t = np.array([2, 4, 6])
b = np.array([0.74, 0.79, 1])
A = np.vstack([np.cos(t), np.sin(t)]).T

n = A.T @ b
M = A.T @ A
print('n', repr(n))
print('M', repr(M))

n array([ 0.13584317, -0.20440937])
M array([[ 1.52235515, -0.15200858],
        [-0.15200858,  1.47764485]])
```

minimal norm  $\|x\|_2$

```
U = np.array([[ -1,  0],
              [ 0,  1]])
S = np.array([2])
VT = np.array([[np.sqrt(3),  1],
               [-1, np.sqrt(3)]])/2
b = np.array([7, 0])
temp1 = np.divide(np.dot(U.T, b), S)
weights_linear = np.dot(VT.T, temp1)
weights_linear

array([-3.03108891, -1.75      ])
```

拟合

```
t = np.array([4, 0, -3.2, 3.1])
y = np.array([9.3, 0.9, -5, 7.1])
A = np.vstack([t, np.ones(len(t))]).T

m, b = la.lstsq(A, y)[0]
m, b
```

solu

```
Sigma_p = 1 / Sigma.T
Sigma_p[Sigma_p == np.inf] = 0
x = VT.T @ Sigma_p @ U.T @ b
x
```

quad-fit

```
t = np.array([-3.0, 1.8, -4.9])
y = np.array([3.3, 7.9, 15])
A = np.vstack([t, t**2]).T
coeffs = la.lstsq(A, y)[0]
coeffs
```

```
full_matrices = False
a=0
sv= np.diag(s)@V
for i in range(k):
    a += np.power(U[:,i],sv[i])
```

A:  $m \times n$  rank = r  
rank(m+n+1) size.  
min(m,n)

$\|A\|_2 = \sigma_{\max}$   
 $\|A - A_n\|_2 = \sigma_n$   
 $\|A^+\|_2 = \frac{1}{\sigma_{\min}}$

$$\begin{bmatrix} 1, 2, 3 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$
  
$$\begin{matrix} 1 \\ 2 \end{matrix} \quad 1 \quad 2 \quad 3.$$
  
$$\begin{pmatrix} 4, 1 \end{pmatrix}$$
  
$$\begin{pmatrix} 4, 1 \end{pmatrix}$$

$$A = U \Sigma V^T$$

The left sv	$AA^T$	✓
Diagonal	$A^T A$	×
Diagonal	$AA^T$	✓
The right	$A^T A$	✓

$$\text{The rank of } A = k \quad \checkmark$$

$$B = \sigma, U, V,^T \quad \checkmark$$

$$A^+ = V \Sigma^+ U^T$$

$$\text{If } A \text{ or } , A^{-1} = A^+ \quad \checkmark$$

$$\text{If } A^{-1} \text{ exist, } A^+ = A^{-1} \quad \checkmark$$

$$\text{Possible } A^+ \text{ exist, when } A^{-1} \text{ not } \checkmark$$

$$\text{For any mat. P-inv exist } \checkmark$$

$$x^3, \text{ SVD, mm, LU}$$