Problema 1

$$L = L(\ddot{q}, \dot{q}, q, t)$$

Del principio de Mínima acción

$$ds[x^A] = \left\{ \delta \int_t L(\dot{q}, \ddot{q}, q, t)\, dt \right\}$$

$$\Rightarrow \int_t dL(\ddot{q}, \dot{q}, q, t)\, dt = \int_t \left[\frac{\partial L}{\partial \ddot{x}^A}(\delta \ddot{x}^A) + \frac{\partial L}{\partial \dot{x}^A}(\partial \dot{x}^A) - \frac{\partial L}{\partial x^A}(\delta x^A) \right] dt$$

$$= \int_t \left[\frac{\partial L}{\partial \ddot{x}^A}\left(\frac{d}{dt}\delta \dot{x}^A\right) + \frac{\partial L}{\partial \dot{x}^A}\left(\frac{d}{dt}\delta x^A\right) + \frac{\partial L}{\partial x^A}(\delta x^A) \right] dt$$

Hacemos cambio de variable.

$$u = \frac{\partial L}{\partial \ddot{x}^A} \to \partial u' = \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{x}^A}\right)$$

$$dV = \frac{d}{dt} dx^A \Rightarrow V = \delta \dot{x}^A$$

$$S = \frac{\partial L}{\partial x^A} \to dS = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^A}\right)$$

$$dt = \frac{d}{dt} dx^A \to t = \delta x^A$$

Sustituyendo en la integral

$$= \int_t \left[-\frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{x}^A}\right)(dx^A) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^A}\right)(d\dot{x}^A) + \frac{\partial L}{\partial x^A}(\delta x^A) \right] dt$$

$$+ \frac{\partial L}{\partial \ddot{x}^A} d\ddot{x}^A \Big|_t^0 + \frac{\partial L}{\partial \dot{x}^A} d\dot{x}^A \Big|_0^t = 0$$

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$$P = -\frac{d}{dt}\frac{\partial L}{\partial x^A} \rightarrow dp = -\frac{d}{dt^2}\left(\frac{\partial L}{\partial \dot{x}^A}\right)$$

$$dq \doteq \frac{d}{dt}\delta x^A \qquad q = \delta x^A$$

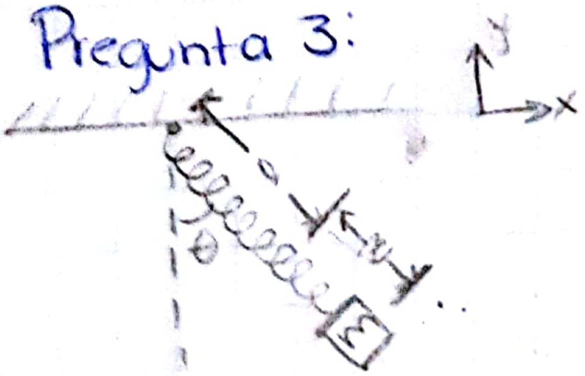
Integramos

$$dS[x^A] = \int_t \left[\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial x^A}\right)(\delta x^A) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^A}\right)(\delta \dot{x}^A)\left(\frac{\partial L}{\partial \dot{x}^A}\right) + \frac{\partial L}{\partial x^A}(\delta x^A)\right] dt - \frac{d}{dt}\frac{\partial L}{\partial x^A}(\partial x^A)\Big|_t^0 = 0$$

$$\left[\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{x}^A}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^A}\right) + \frac{\partial L}{\partial x^A}\right] dx^A = 0$$

$$\boxed{\therefore \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{x}^A}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^A}\right) + \frac{\partial L}{\partial x^A} = 0}$$

Pregunta 3:



Coordenadas generalizados

$$x = (l+z)\sin\theta$$
$$y = (l+z)\cos\theta$$

Energía Cinética

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) = \frac{1}{2}m[(\dot{z}\sin\theta + (l+z)\dot{\theta}\cos\theta)^2 + (\dot{z}\cos\theta - (l+z)\dot{\theta}\sin\theta)^2]$$

$$T = \frac{1}{2}m[\dot{z}^2\sin^2\theta + 2(l+z)\dot{z}\dot{\theta}\sin\theta\cos\theta + (l+z)^2\dot{\theta}^2\cos^2\theta + \dot{z}^2\cos^2\theta - 2(l+z)\dot{z}\dot{\theta}\sin\theta\cos\theta + (l+z)^2\dot{\theta}^2\sin^2\theta]$$

$$T = \frac{1}{2}m[\dot{z}^2 + (l+z)^2\dot{\theta}^2]$$

$$U = mgy + \frac{1}{2}kz^2 = mg(l+z)\cos\theta) + \frac{1}{2}kz^2$$

$$L = T - U = \frac{1}{2}m[\dot{z}^2 + (l+z)^2\dot{\theta}^2] + mg(l+z)\cos\theta - \frac{1}{2}kz^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = 0 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial z} = \frac{1}{\cancel{2}}m\dot{\theta}^2\cancel{2}(l+z) + mg\cos\theta - kz$$

$$\frac{\partial L}{\partial \dot{z}} = m\dot{z} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = m\ddot{z}$$

$$\boxed{\ddot{z} - (l+z)\dot{\theta}^2 - g\cos\theta + \frac{k}{m}z = 0}$$

$$\frac{\partial L}{\partial \dot{\theta}} = m(l+z)^2\dot{\theta} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \cancel{m}(l+z)^2\ddot{\theta} + 2\cancel{m}(l+z)\dot{\theta}\dot{z}$$

$$\frac{\partial L}{\partial \theta} = -\cancel{m}g(\cancel{l+z})\operatorname{sen}\theta$$

$$\boxed{(l+z)\ddot{\theta} + 2\dot{z}\dot{\theta} + g\operatorname{sen}\theta = 0}$$

Puntos de equilibrio y estabilidad

$$\frac{K}{m}z - g\cos\theta = 0$$

$$\frac{K}{m}z = g\cos\theta$$

como θ toma valores $n\pi \Rightarrow \cos\theta = (-1)^n$

$$z = \pm\frac{mg}{K}$$

$$g\,\text{sen}\,\theta = 0$$

para que $g\,\text{sen}\,\theta$ sea cero $\Rightarrow \text{sen}\,\theta = 0$

$\therefore \theta = n\pi \quad n = 0, 1, \ldots$

Pregunta 4

$$L = e^{bt}\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}k^2q^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$\frac{\partial L}{\partial q} = -e^{bt}k^2q$$

$$\frac{\partial L}{\partial \dot{q}} = e^{bt}m\dot{q} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = be^{bt}m\dot{q} + e^{bt}m\ddot{q}$$

$$be^{bt}m\dot{q} + e^{bt}m\ddot{q} + e^{bt}k^2q = 0$$

$$\ddot{q} + b\dot{q} + \frac{k^2}{m}q = 0$$

Se parece al oscilador amortiguado.

$$Q = e^{bt/2}q$$

$$q = \frac{Q}{e^{bt/2}} \qquad \dot{q} = \frac{\dot{Q}}{e^{bt/2}} + \left[-\frac{bQ}{2e^{bt/2}}\right]$$

$$\dot{q} = \frac{2\dot{Q} - bQ}{2e^{bt/2}}$$

$$L = e^{bt}\left(\frac{1}{2}m\left(\frac{\dot{Q} - \frac{b}{2}Q}{e^{bt/2}}\right)^2 - \frac{1}{2}k^2\left(\frac{Q}{e^{bt/2}}\right)^2\right)$$

$$L = \cancel{e^{bt}}\left[\frac{m(\dot{Q}^2 - b\dot{Q}Q + \frac{b^2}{4}Q^2)}{2\cancel{e^{bt}}} - \frac{k^2Q^2}{2\cancel{e^{bt}}}\right]$$

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$$\frac{m\left(\dot{Q}^2 - b\dot{Q}Q + \frac{b^2}{4}Q^2\right)}{2} - \frac{k^2Q^2}{2}$$

$$L = \frac{m\dot{Q}^2}{2} - \frac{mb\dot{Q}Q}{2} + \frac{mb^2Q^2}{8} - \frac{k^2Q^2}{2}$$

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