$\boxed{1}$ $L = L(\ddot{q}_i, \dot{q}_i, q_i, t)$

$$S = \int_{t_{ini}}^{t_{fin}} L(\ddot{q}_i, \dot{q}_i, q_i)$$

$$\delta S = \int_{t_{ini}}^{t_{fin}} \delta L(\ddot{q}_i, \dot{q}_i, q_i)$$

$$= \int_{t_{ini}}^{t_{fin}} \left(\frac{\partial L}{\partial \ddot{q}_i} \delta \ddot{q}_i + \frac{\partial L}{\partial \dot{q}_i} \delta \dot{q}_i + \frac{\partial L}{\partial q_i} \delta q_i \right) dt$$

$$\delta \dot{q}_i = \frac{d}{dt} \delta q_i$$

$$\delta \ddot{q}_i = \frac{d^2}{dt^2} \delta q_i$$

$$\delta S = \int_{t_{ini}}^{t_{fin}} \Bigg(\underbrace{\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) \delta \ddot{q}_i}_{1} + \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) \delta \dot{q}_i}_{2} + \underbrace{\frac{\partial L}{\partial q_i} \delta q_i}_{3} \Bigg) dt$$

1 $$\int_{t_{ini}}^{t_{fin}} \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) \delta \ddot{q}_i \, dt = \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}_i} \delta \dot{q}_i \Big|_{t_{ini}}^{t_{fin}} - \int \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) \delta \dot{q}_i \, dt \right)$$

$$u = \frac{\partial L}{\partial \ddot{q}_i} \qquad dv = \frac{d}{dt} \delta \ddot{q}_i \, dt$$

$$\frac{du}{dt} = \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) \qquad v = \delta \dot{q}_i$$

$$\frac{d}{dt}\left(\cancelto{0}{\frac{\partial L}{\partial \ddot{q}_i}\delta q_i}\Big|_{t_{ini}}^{t_{fin}} - \int_{t_{ini}}^{t_{fin}} \left(\frac{\partial L}{\partial \ddot{q}_i}\right)\frac{d}{dt}\delta q_i \, dt\right)$$

$$u = \frac{\partial L}{\partial \ddot{q}_i} \qquad dv = \frac{d}{dt}\delta q_i$$

$$\frac{du}{dt} = \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}_i} \qquad v = \delta q_i$$

$$= -\cancelto{0}{\frac{\partial L}{\partial \ddot{q}_i}\delta q_i}\Big|_{t_{ini}}^{t_{fin}} + \int_{t_{ini}}^{t_{fin}} \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\delta q_i \, dt\right)$$

De modo que: $\boxed{\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) = 0}$

(2) $$\int_{t_{ini}}^{t_{fin}} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\delta \dot{q}_i \, dt$$

$$u = \frac{\partial L}{\partial \dot{q}_i} \qquad dv = \frac{d}{dt}\delta \dot{q}_i \, dt$$

$$\frac{du}{dt} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) \qquad v = \delta q_i$$

$$= \cancelto{0}{\frac{\partial L}{\partial \dot{q}_i}\delta q_i}\Big|_{t_{ini}}^{t_{fin}} - \int_{t_{ini}}^{t_{fin}} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\delta q_i \, dt$$

$$-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$$

(3) $$\int_{t_{ini}}^{t_{fin}} \frac{\partial L}{\partial q_i}\delta q_i\, dt = \frac{\partial L}{\partial q_i} = 0$$

Entonces EL:

$$\boxed{\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0}$$

2.- $$L(\dot{q}, q, t) = \frac{1}{2} g_{ab}(q^c)\dot{q}^a\dot{q}^b$$

Mediante EL:

$$\frac{\partial L}{\partial \dot{q}^c} = \frac{1}{2} g_{cb}\dot{q}^b + \frac{1}{2} g_{ac}\dot{q}^a = g_{cb}\dot{q}^b$$

simétrico $\rightarrow = \frac{1}{2} g_{cb}\dot{q}^b$

$$\frac{\partial L}{\partial q^c} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^c} = \frac{1}{2}\frac{\partial g_{ab}}{\partial q^c}\dot{q}^a\dot{q}^b - \frac{d}{dt}\left(g_{cb}\dot{q}^b\right)$$

Desarrollando:

$$\underbrace{\frac{1}{2}\frac{\partial g_{ab}}{\partial q^c}\dot{q}^a\dot{q}^b - \frac{\partial g_{cb}}{\partial q^a}\dot{q}^a\dot{q}^b}_{\text{Símbolo de christoffel}} - g_{cb}\ddot{q}^b = 0$$

Definición:

$$\Gamma^c_{ab} = \frac{1}{2} g^{cm}\left(\frac{\partial g_{ma}}{\partial x^b} + \frac{\partial g_{mb}}{\partial x^a} - \frac{\partial g_{ab}}{\partial x^m}\right)$$

$$= \frac{1}{2}\frac{\partial g_{ca}}{\partial x^b} + \frac{1}{2}\frac{\partial g_{cb}}{\partial x^a} - \frac{1}{2}\frac{\partial g_{ab}}{\partial x^m}$$

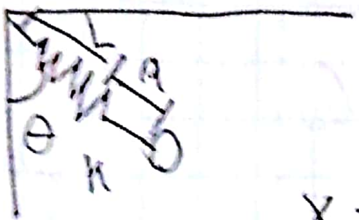
$$\frac{\partial g_{ca}}{\partial x^b} = \frac{\partial g_{cb}}{\partial x^a} \longrightarrow \text{simétrico}$$

$$= \frac{\partial g_{cb}}{\partial x^a} - \frac{1}{2}\frac{\partial g_{ab}}{\partial x^c} = \frac{\partial g_{cb}}{\partial q^a} - \frac{1}{2}\frac{\partial g_{ab}}{\partial q^c}$$

De modo que:

$$\boxed{\ddot{q}^{b}+\Gamma^{c}_{ab}\dot{q}^{a}\dot{q}^{b}=0}$$

3.-

a) 

$$L=T-V$$

$$x=\operatorname{sen}\theta(L+a)$$
$$y=-\cos\theta(L+a)$$

$$\dot{x}=\dot{a}\operatorname{sen}\theta+(L+a)\cos\theta\,\dot{\theta}$$
$$\dot{y}=-\dot{a}\cos\theta+(L+a)\operatorname{sen}\theta\,\dot{\theta}$$

$$T=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)=\frac{1}{2}m\Big[(\dot{a}\operatorname{sen}\theta+(L+a)\cos\theta\,\dot{\theta})^2$$
$$+(-\dot{a}\cos\theta+(L+a)\operatorname{sen}\theta\,\dot{\theta})^2\Big]$$
$$=\frac{1}{2}m\Big[\dot{a}^2\operatorname{sen}^2\theta+2\dot{a}\dot{\theta}(L+a)\cos\theta\operatorname{sen}\theta+(L+a)^2\cos^2\theta\,\dot{\theta}^2$$
$$+\dot{a}^2\cos^2\theta-2\dot{a}\dot{\theta}(L+a)\cos\theta\operatorname{sen}\theta+(L+a)^2\operatorname{sen}^2\theta\,\dot{\theta}^2\Big]$$
$$=\frac{1}{2}m(\dot{a}^2+(L+a)^2\dot{\theta}^2)$$

$$V=mgy+\frac{1}{2}ka^2$$
$$=-mg\cos\theta(L+a)+\frac{1}{2}ka^2$$

$L = T - V$

$= \frac{1}{2} m(\dot{q}^2 + (L+q)^2\dot{\theta}^2) + mg\cos\theta\,(L+q) - \frac{1}{2}kq^2$

EL:

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0$$

$$-mg\,\text{sen}\theta(L+q) - \frac{d}{dt}\left(m(L+q)^2\dot{\theta}\right)$$

$$= -mg\,\text{sen}\theta\,(L+q) - 2m\dot{q}\dot{\theta} - m(L+q)^2\ddot{\theta} = 0$$

$$\boxed{\ddot{\theta}(L+q) - 2\dot{\theta}\dot{q} - g\,\text{sen}\theta = 0}$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$= m(L+q)\dot{\theta}^2 + mg\cos\theta - kq + m\ddot{q} = 0$$

$$\boxed{(L+q)\dot{\theta}^2 + g\cos\theta - \frac{kq}{m} + \ddot{q} = 0}$$

b) $\ddot{\theta} = 0, \dot{\theta} = 0$

$$-g\,\text{sen}\theta = 0$$

$$\theta =$$

$$(L+q)\dot{\theta}^2 + g\cos\theta - \frac{kq}{m} + \ddot{q} = 0$$

$$\ddot{q} = -(L+q)\dot{\theta}^2 - g\cos\theta + \frac{kq}{m}$$

$L+a \neq 0$, para que $\dot{\theta}^2=0$

Entonces:

$a=0$ y $\theta=(2n+1)\frac{\pi}{2}$, $n=0,1,\ldots$

Los puntos de equilibrio son entonces:

$\ddot{\theta}=0$	$\ddot{a}=0$
$\dot{\theta}=0$	$\dot{a}=0$
$\theta=n\pi$	$a=0$
$\theta=(2n+1)\frac{\pi}{2}$	