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a) Para probar que son transformaciones canónicas, podemos verificar mediante corchetes de Poisson.

$$\{q_1, q_2\} = \frac{\partial q_1}{\partial x}\frac{\partial q_2}{\partial p_x} - \frac{\partial q_1}{\partial p_x}\frac{\partial q_2}{\partial x} + \frac{\partial q_1}{\partial y}\frac{\partial q_2}{\partial p_y} - \frac{\partial q_1}{\partial y}\frac{\partial q_2}{\partial p_y}$$

$$= \cos(\mu)\sin(\mu) - \sin(\mu)\cos(\mu) = 0$$

$$\{P_1, P_2\} = \frac{\partial P_1}{\partial x}\frac{\partial P_2}{\partial p_x} - \frac{\partial P_1}{\partial p_x}\frac{\partial P_2}{\partial x} + \frac{\partial P_1}{\partial y}\frac{\partial P_2}{\partial p_y} - \frac{\partial P_1}{\partial p_y}\frac{\partial P_2}{\partial y}$$

$$= \cos\mu\sin(\mu) - \cos(\mu)\sin(\mu) = 0$$

$$\{q_1, P_1\} = \frac{\partial q_1}{\partial x}\frac{\partial P_1}{\partial p_x} - \frac{\partial q_1}{\partial p_x}\frac{\partial P_1}{\partial x} + \frac{\partial q_1}{\partial y}\frac{\partial P_1}{\partial p_y} - \frac{\partial q_1}{\partial p_y}\frac{\partial P_1}{\partial y}$$

$$= \cos^2\mu + \sin^2\mu = 1$$ $\Rightarrow$ las transformaciones son canónicas

b) Tomando las cantidades

$$q_1^2 = x^2\cos^2\mu + 2xp_y\cos\mu\sin\mu + p_y^2\sin^2\mu$$

$$q_2^2 = y^2\cos^2\mu + 2yp_x\cos\mu\sin\mu + p_x^2\sin^2\mu$$

$$P_1^2 = p_x^2\cos^2\mu - 2p_xy\cos\mu\sin\mu + y^2\sin^2\mu$$

$$P_2^2 = p_y^2\cos^2\mu - 2p_yx\cos\mu\sin\mu + x^2\sin^2\mu$$

$$q_1^2 + q_2^2 + P_1^2 + P_2^2 = x^2 + y^2 + p_x^2 + p_y^2$$

El Hamiltoniano $\mathcal{H} = \dfrac{x^2 + y^2 + p_x^2 + p_y^2}{2}$

c) las ecs de Hamilton $\dot{P}_i = -\dfrac{\partial \mathcal{H}}{\partial q_i}$, $\dot{q}_i = \dfrac{\partial \mathcal{H}}{\partial P_i}$ Para $y=0$, $p_y=0$

$$\dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x} = \frac{-2x}{2} = -x$$

$\dot{p}_x = m\ddot{x}$ Así $m\ddot{x} + x = 0$

2.- a) Para determinar los momentos de inercia

Tomando el tensor de inercia con simetría $I = \begin{pmatrix} y^2+z^2 & 0 & 0 \\ 0 & x^2+z^2 & 0 \\ 0 & 0 & y^2+x^2 \end{pmatrix}$

$I_{11} = \int \rho y^2 dxdy$, $I_{22} = I_{11}$

$I_{33} = I_{22} + I_{11}$ Para calcular I_{33} en $I_{33} = \int \rho (x^2+y^2) dxdy$

coordenadas polares

$$I_{33} = \rho \int_0^{2\pi} \int_0^r r^2 r dr d\theta = \rho \frac{1}{4} A^4 (2\pi)$$ tomando la densidad $\rho = \frac{M}{\pi A^2}$

$$I_{33} = \frac{1}{2} MA^2$$ lo que implica $I_{11} = I_{22} = \frac{1}{4} MA^2$

Así $$I = \begin{pmatrix} \frac{1}{4}MA^2 & 0 & 0 \\ 0 & \frac{1}{4}MA^2 & 0 \\ 0 & 0 & \frac{1}{2}MA^2 \end{pmatrix}$$

Para los ejes principales (autovectores)

$$\begin{pmatrix} \frac{1}{4}MA^2 - \lambda & 0 & 0 \\ 0 & \frac{1}{4}MA^2 - \lambda & 0 \\ 0 & 0 & \frac{1}{2}MA^2 - \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Del polinomio característico

$$(\tfrac{1}{4}MA^2 - \lambda)^2(\tfrac{1}{2}MA^2 - \lambda) = 0$$ $\lambda_1 = \lambda_2 = \frac{1}{4}MA^2$

$\lambda_3 = \frac{1}{2}MA^2$

Sustituimos

$$\Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}MA^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ Así parametrizando

$$\bar{x}_i = s\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Para el otro eigenvector

$$\begin{pmatrix} \frac{1}{2}MA^2 & 0 & 0 \\ 0 & \frac{1}{2}MA^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ $\bar{x}_i = u\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

b) Para el vector de momento angular

$L_i = I_{ij} w_j$ en este caso $i=j$

$L_1 = I_{11} w_1$

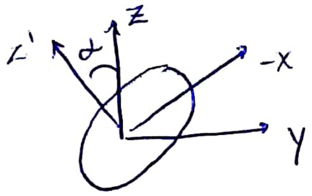
$L_2 = I_{22} w_2$

$L_3 = I_{33} w_3$

Debemos encontrar las proyecciones w

Y es ortogonal a z

por lo tanto $w_2 = 0$



$L_1 = \frac{1}{4} M A^2 w \operatorname{sen}(\alpha)$

$L_2 = 0$

$L_3 = \frac{1}{2} M A^2 w \cos(\alpha)$

$\Rightarrow \vec{L} = \left(\frac{1}{4} M A^2 w \operatorname{sen}(\alpha), 0, \frac{1}{2} M A^2 \cos(\alpha) w\right)$

la Magnitud

$$\|L\| = \sqrt{\left(\tfrac{1}{2} M A^2 w\right)^2 \left(\tfrac{1}{4} \operatorname{sen}^2\alpha + \cos^2\alpha\right)}$$

$$= \frac{1}{2} M A^2 w \sqrt{\tfrac{1}{4} \operatorname{sen}^2\alpha + \cos^2\alpha}$$

la dirección

de L $\theta = \tan^{-1}\left(\frac{L_1}{L_3}\right) = \tan^{-1}\left(\frac{1}{2}\tan\alpha\right)$

c) la torca es $\vec{\tau} = \frac{d\vec{L}}{dt}$ pero L no depende de t, ni su ángulo α

Así pues el momento se conserva (momento angular)

$\vec{\tau} = 0$

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