a) $q_1 = x\cos\mu + P_y \sin\mu$ $\quad q_2 = y\cos\mu + P_x \sin\mu$

$P_1 = P_x \cos\mu - y\sin\mu$ $\quad P_2 = P_y \cos\mu - x\sin\mu$

Para demostrar que son canonicas los corchetes de Poisson deben ser 1 y 0 para P_i y q_i y la transformacion es de la manera

$$(q, p) \longrightarrow (\bar{X}, \bar{P})$$

Comenzaremos demostrando

$$\{q_1, p_1\}_{(\bar{x},\bar{p})} = \frac{\partial q_1}{\partial x}\frac{\partial P_1}{\partial P_x} - \frac{\partial q_1}{\partial P_x}\frac{\partial P_1}{\partial x} + \frac{\partial q_1}{\partial y}\frac{\partial P_1}{\partial P_y} - \frac{\partial q_1}{\partial P_y}\frac{\partial P_1}{\partial y}$$

$$+ \cancel{\frac{\partial q_1}{\partial z}\frac{\partial P_1}{\partial P_z}}^{\,0} \cancel{- \frac{\partial q_1}{\partial P_z}\frac{\partial P_1}{\partial z}}$$

$$\{q_1, p_1\} = \frac{\partial q_1}{\partial x}\frac{\partial P_1}{\partial P_x} - \frac{\partial q_1}{\partial P_x}\frac{\partial P_1}{\partial x} + \frac{\partial q_1}{\partial y}\frac{\partial P_1}{\partial P_y} - \frac{\partial q_1}{\partial P_y}\frac{\partial P_1}{\partial y}$$

$$\{q_1, p_1\} = \frac{\partial}{\partial x}(x\cos\mu + P_y\sin\mu)\frac{\partial}{\partial P_x}(P_x\cos\mu - y\sin\mu) - \cancel{\frac{\partial}{\partial P_x}(x\cos\mu + P_y\sin\mu)}^{\,0}$$

$$\cancel{\frac{\partial}{\partial x}(P_x\cos\mu - y\sin\mu)}^{\,0} + \cancel{\frac{\partial}{\partial y}(x\cos\mu + P_y\sin\mu)}^{\,0}\cancel{\frac{\partial}{\partial P_y}(P_x\cos\mu - y\sin\mu)}^{\,0}$$

$$- \frac{\partial}{\partial P_y}(x\cos\mu + P_y\sin\mu)\frac{\partial}{\partial y}(P_x\cos\mu - y\sin\mu)$$

$\{q_1, q_2\} = (\cos\mu)(\cos\mu) - (\sin\mu)(-\sin\mu)$

$\{q_1, q_2\} = \cos^2\mu + \sin^2\mu = \underline{\underline{1}}$

ahora para q_2 y P_2

$$\{q_2, P_2\} = \frac{\partial q_2}{\partial x}\frac{\partial P_2}{\partial P_x} - \frac{\partial q_1}{\partial P_x}\frac{\partial P_1}{\partial x} + \frac{\partial q_2}{\partial y}\frac{\partial P_2}{\partial P_x} - \frac{\partial q_2}{\partial P_y}\frac{\partial P_2}{\partial y}$$

$$\{q_2, P_2\} = \frac{\partial}{\partial x}\cancel{(y\cos\mu + P_x\sin\mu)}\frac{\partial}{P_y}\cancel{(P_y\cos\mu - x\sin\mu)}^{0} -$$

$$\frac{\partial}{\partial P_x}(y\cos\mu + P_x\sin\mu)\frac{\partial}{\partial x}(P_y\cos\mu - x\sin\mu)$$

$$+ \frac{\partial}{\partial y}(y\cos\mu + P_x\sin\mu)\frac{\partial}{P_y}(P_y\cos\mu - x\sin\mu)$$

$$- \frac{\partial}{P_y}\cancel{(y\cos\mu + P_x\sin\mu)}\frac{\partial}{\partial y}(P_y\cos\mu - x\sin\mu)^{\to 0}$$

$$\Rightarrow \{q_2, P_2\} = -(\sin\mu)(-\sin\mu) + (\cos\mu)(\cos\mu)$$

$$= \cos^2\mu + \sin^2\mu = \underline{\underline{1}}$$

Seguimos con los cruzados

$$\{q_1, P_2\} = \frac{\partial}{\partial x}(x\cos\mu + P_y\sin\mu)\frac{\partial}{\partial P_x}\cancel{(P_y\cos\mu - x\sin\mu)}^{\to 0}$$

$$- \frac{\partial}{P_x}\cancel{(x\cos\mu + P_y\sin\mu)}^{\to 0}\frac{\partial}{\partial x}(P_y\cos\mu - \sin\mu)$$

$$+\frac{\partial}{\partial y}(x\cos\mu+P_y\sin\mu)\frac{\partial}{\partial P_y}(P_y\cos\mu-x\sin\mu)$$

$$-\frac{\partial}{\partial P_y}(x\cos\mu+P_y\sin\mu)\frac{\partial}{\partial y}(P_y\cos\mu-x\sin\mu)$$

$\therefore \{q_1, P_2\}=0$

$$\{q_2, P_1\}=\frac{\partial}{\partial x}(y\cos\mu+P_x\sin\mu)\frac{\partial}{P_x}(P_x\cos\mu-y\sin\mu)$$

$$-\frac{\partial}{P_x}(y\cos\mu+P_x\sin\mu)\frac{\partial}{\partial x}(P_x\cos\mu-y\sin\mu)+\frac{\partial}{\partial y}(y\cos\mu+P_x\sin\mu)$$

$$\frac{\partial}{\partial P_y}(P_x\cos\mu-y\sin\mu)-\frac{\partial}{P_y}(y\cos\mu+P_x\sin\mu)\frac{\partial}{\partial y}(P_x\cos\mu-y\sin\mu)$$

$\therefore \{q_2, P_1\}=0$

$\therefore$ Cumple que
$\{q_i, P_j\}=\delta_{ij}$

Ahora probamos que $\{P_i, P_i\}=0$

$$\{P_1, P_1\}=\frac{\partial P_1}{\partial q_1}\frac{\partial P}{\partial P_1}-\frac{\partial P_1}{\partial q_1}\frac{\partial P_1}{\partial P_1}+\frac{\partial P_1}{\partial q_2}\frac{\partial P_1}{\partial q_2}-\frac{\partial P_1}{\partial q_2}\frac{\partial P_1}{\partial P_2}$$

$\therefore \{P_1, P_1\}=0$, Analogamente con $\{q_1, q_1\}=0$

b) $\mathcal{H} = (q_1^2 + q_2^2 + P_1^2 + P_2^2)/2$

Sustituimos

$$\bar{\mathcal{H}} = [(x\cos\mu + P_y \sin\mu)^2 + (y\cos\mu + P_x \sin\mu)^2 + (P_x\cos\mu - y\sin\mu)^2 + (P_y\cos\mu - x\sin\mu)^2]/2$$

$$\bar{\mathcal{H}} = [x^2\cos^2\mu + 2P_y x\cos\mu\sin\mu + P_y^2\sin^2\mu + y^2\cos^2\mu + 2P_x y\cos\mu\sin\mu + P_x^2\sin^2\mu + P_x^2\cos^2\mu - 2P_x y\cos\mu\sin\mu + y^2\sin^2\mu + P_y^2\cos^2\mu - 2P_y x\cos\mu\sin\mu + x^2\sin^2\mu]/2$$

$$\Rightarrow \bar{\mathcal{H}} = [x^2(\cos^2\mu + \sin^2\mu) + P_y^2(\sin^2\mu + \cos^2\mu) + y^2(\cos^2\mu + \sin^2\mu) + P_x^2(\cos^2\mu + \sin^2\mu) + 2\cos\mu\sin\mu(\cancel{P_y x} + \cancel{P_x y} - \cancel{P_x y} - \cancel{P_y x})]/2$$

$$\Rightarrow \bar{\mathcal{H}} = [x^2 + P_y^2 + y^2 + P_x^2]/2$$

$\therefore$

$$\boxed{\bar{\mathcal{H}} = [x_i + P_j]/2} \quad \text{para } i, j = 1, 2$$

c) $y = P_y = 0$

tenemos el nuevo Hamiltoniano con las restricciones

$$\bar{\mathcal{H}} = [x^2 + P_x^2]/2$$

tenemos las ecuaciones de Hamilton

$$\dot{q}_i = \frac{\partial H}{\partial P_i}, \quad \dot{P}_i = -\frac{\partial H}{\partial q}, \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$\dot{q}_1 = \frac{\partial}{\partial P_x}\left([x^2 + P_x^2]/2\right) = \underline{P_x},$$

$$\dot{q}_2 = \frac{\partial}{\partial P_y}(H) = 0 \quad , \quad \dot{P}_x = \frac{-\partial H}{\partial x} = \underline{-x},$$

$$\dot{P}_y = \frac{-\partial H}{\partial y} = 0$$

ahora $\frac{\partial H}{\partial t} = \frac{-\partial L}{\partial t} = \underline{0},$

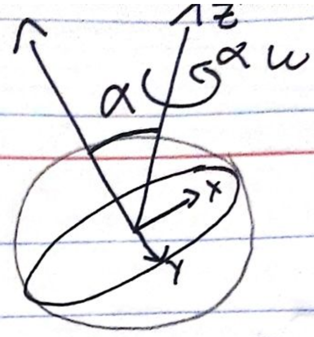
Rescribiendo

$$\dot{q} = P_x, \quad , \quad \dot{P}_x = -x,$$

$$\Rightarrow \quad \dot{q}_1 - P_x = 0 \quad \Rightarrow \quad \dot{x} - m\dot{x} = 0$$

$$\dot{P}_x = -x \Rightarrow m\ddot{x} + x = 0$$

2)



a) tenemos que el Tensor de Inercia continua dado por la expresión

$$I_{ab} = \int d^3\bar{r}\,\rho(r)\begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix}$$

donde $\rho(r)$ para un disco es

$$\rho = M/\pi A^2$$

como pasa por su centro de masa y es simetrico con respecto a x y y podemos reescribir

$$I_{ab} = \int_0^A \left(\frac{M}{\pi A^2}\right) d^3\bar{r} \begin{pmatrix} y^2+z^2 & 0 & 0 \\ 0 & x^2+z^2 & 0 \\ 0 & 0 & x^2+y^2 \end{pmatrix}$$

como $z=0$ por ser unicamente un area tenemos

$$I_{11} = \int_0^A \rho\, y\, d^2\bar{x} = \frac{M}{\pi A^2}\int_0^A y\, d^2\bar{x}$$

Igualmente para z

$$I_{22} = \int_0^A \rho\, x^2 d^2\bar{x} = \frac{M}{\pi A^2}\int_0^A x^2 d^2\bar{x}$$

para $I_{33} = \int_0^A \rho (x^2+y^2) dA$

Sabiendo el diferencial de area de polares
y viendo que $I_1 + I_2 = I_3$
tenemos

$$I_{33} = \frac{M}{\pi A^2} \int_0^{2\pi} \int_0^A r \, r \, dr \, d\theta$$

$$I_{33} = \frac{2M\pi}{\pi A^2} \int_0^A r^3 dr = \frac{2M\cancel{\pi}}{\cancel{\pi} A^2} r^4 \Big|_0^A$$

$$= \frac{MA^2}{2} = I_1 + I_2 \therefore I_1 = \frac{MA^2}{4} + \frac{MA^2}{4}$$

Rescribimos el tensor de la sig manera

$$I_{ab} = \begin{pmatrix} \frac{MA^2}{4} & 0 & 0 \\ 0 & \frac{MA^2}{4} & 0 \\ 0 & 0 & MA^2 \end{pmatrix}$$

Con esta matriz determinaremos los ejes principales dado por la ec. caracteristica por simetria

$$X_i^{(2)} = \begin{pmatrix} l \\ h \\ 0 \end{pmatrix} = \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ h \\ 0 \end{pmatrix} \qquad l, h \in \mathbb{R}$$

Para $\lambda = \frac{MA^2}{2}$

$$\Rightarrow \begin{pmatrix} -\frac{MA^2}{4} & 0 & 0 \\ 0 & -\frac{MA^2}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix} = \bar{0}_{3\times 1}$$

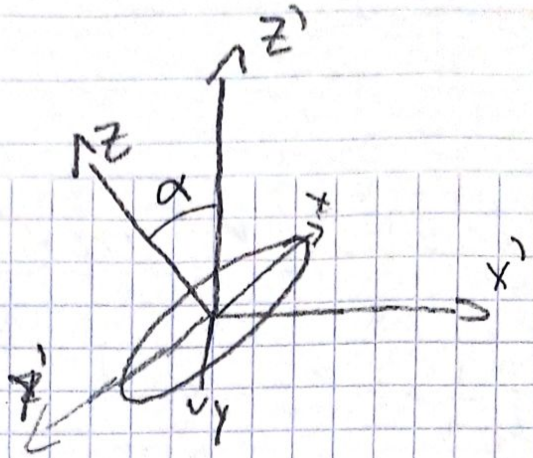
$$\left(-\frac{MA^2}{4}\right)(x^1) = 0 \quad \therefore \quad x^1 = 0$$

$$\left(-\frac{MA^2}{4}\right)(x^2) = 0 \quad \therefore \quad x^2 = 0$$

$$0(x^3) = 0 \qquad x^3 = s$$

$$\therefore \quad \boxed{X_i^{(3)} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix}} \qquad s \in \mathbb{R}$$

b) $L = ?$
tenemos que



$$L_a = I_{ab}\,\omega_b$$

$$L_a = \begin{pmatrix} \frac{MA^2}{4} & 0 & 0 \\ 0 & \frac{MA^2}{4} & 0 \\ 0 & 0 & \frac{MA^2}{4} \end{pmatrix} \begin{pmatrix} \omega_1 \cos\alpha \\ \omega_2 \operatorname{sen}\alpha \\ 0 \end{pmatrix}$$

$$L_1 = \frac{MA^2}{4}\omega_1 \operatorname{sen}\alpha$$

$$L_2 = 0$$

$$L_3 = \frac{MA^2}{2}\cos\alpha \quad \therefore \quad \underline{L_i = \left(\frac{MA^2}{4}\omega_1 \operatorname{sen}\alpha,\ 0,\ \frac{MA^2}{4}\cos\alpha\right)}$$

c) $\tau = ?$

$$\tau = \frac{dL}{dt} = \dot{L}$$

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{MA^2}{4}\omega_1 \operatorname{sen}\alpha,\ 0,\ \frac{MA^2}{4}\cos\alpha,\right)$$

como α no depende de t es decir $\alpha \neq \alpha(t)$

$$\therefore \ \underline{\tau = 0}$$

$$\|\bar{L}\| = \sqrt{\left(\frac{MA^2}{4}\omega_1 \operatorname{sen}\alpha\right)^2 + \left(\frac{MA^2}{4}\cos\alpha\right)^2}$$

$$= \frac{MA^2\omega}{2}\sqrt{\frac{1}{4}\operatorname{sen}^2\alpha + \cos\alpha}$$

$=$