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Pregunta 1)

a) Prueba que las siguientes transformaciones son canónicas para cualquier μ.

$$q_1 = x\cos\mu + P_y\sin\mu \qquad q_2 = y\cos\mu + P_x\sin\mu$$

$$P_1 = P_x\cos\mu - y\sin\mu \qquad P_2 = P_y\cos\mu - x\sin\mu$$

1- Debemos asegurarnos que los corchetes de Poisson son invariantes

Es decir, $\{x_i, p_j\} = \delta_{ij}$ o alternativamente $\{q_i, P_j\} = \delta_{ij}$

Así,

$$\{q_1, P_1\} = \frac{\partial q_1}{\partial x_i}\frac{\partial P_1}{\partial p_i} - \frac{\partial q_1}{\partial p_i}\frac{\partial P_1}{\partial x_i}$$

$$= \frac{\partial q_1}{\partial x}\frac{\partial P_1}{\partial P_x} + \cancel{\frac{\partial q_1}{\partial y}}^{0}\frac{\partial P_1}{\partial P_y} - \cancel{\frac{\partial q_1}{\partial P_x}}^{0}\frac{\partial P_1}{\partial x} - \frac{\partial q_1}{\partial P_y}\frac{\partial P_1}{\partial y}$$

$$= \cos^2\mu + \mathrm{sen}^2\mu = 1$$

$$\{q_1, P_2\} = \frac{\partial q_1}{\partial x}\cancel{\frac{\partial P_2}{\partial P_x}}^{0} + \cancel{\frac{\partial q_1}{\partial y}}^{0}\frac{\partial P_2}{\partial P_y} - \cancel{\frac{\partial q_1}{\partial P_x}}^{0}\frac{\partial P_2}{\partial x} - \frac{\partial q_1}{\partial P_y}\cancel{\frac{\partial P_2}{\partial y}}^{0}$$

$$= 0$$

$$\{q_2, P_1\} = \cancel{\frac{\partial q_2}{\partial x}}^{0}\frac{\partial P_1}{\partial P_x} + \frac{\partial q_2}{\partial y}\cancel{\frac{\partial P_1}{\partial P_y}}^{0} - \cancel{\frac{\partial q_2}{\partial P_x}}^{0}\frac{\partial P_1}{\partial x} - \cancel{\frac{\partial q_2}{\partial P_y}}^{0}\frac{\partial P_2}{\partial y}$$

$$= 0$$

$$\{q_2, P_2\} = \frac{\partial q_2}{\partial x}\frac{\partial P_2}{\partial P_x} + \frac{\partial q_2}{\partial y}\frac{\partial P_2}{\partial P_y} - \frac{\partial q_2}{\partial P_x}\frac{\partial P_2}{\partial x} - \frac{\partial q_2}{\partial P_y}\frac{\partial P_2}{\partial y}$$

$$= \cos^2\mu + \mathrm{sen}^2\mu$$

$$= 1.$$

Por lo tanto, $\{q_i, P_j\} = \delta_{ij}$ //

Además, se debe probar que $\{q_i, q_j\} = 0$ y $\{P_i, P_j\} = 0$

Por propiedades de los corchetes de Poisson, $\{q_1, q_1\} = \{q_2, q_2\} = 0$ y $\{P_1, P_1\} = \{P_2, P_2\} = 0$

Sólo queda demostrar los restantes:

$$\{q_1, q_2\} = \frac{\partial q_1}{\partial x}\frac{\partial q_2}{\partial P_x} + \overset{0}{\frac{\partial q_1}{\partial y}}\frac{\partial q_2}{\partial P_y} - \overset{0}{\frac{\partial q_1}{\partial P_x}}\frac{\partial q_2}{\partial x} - \frac{\partial q_1}{\partial P_y}\frac{\partial q_2}{\partial y}$$

$$= \cos\mu \sin\mu - \sin\mu \cos\mu$$

$$= 0$$

$$\{P_1, P_2\} = \overset{0}{\frac{\partial P_1}{\partial x}}\frac{\partial P_2}{\partial P_x} + \frac{\partial P_1}{\partial y}\frac{\partial P_2}{\partial P_y} - \frac{\partial P_1}{\partial P_x}\frac{\partial P_2}{\partial x} - \frac{\partial P_1}{\partial P_y}\frac{\partial P_2}{\partial y}$$

$$= -\sin\mu\cos\mu + \cos\mu\sin\mu$$

$$= 0$$

Por lo tanto, $\{q_i, q_j\} = 0$ y $\{P_i, P_j\} = 0$ //

Así, las transformaciones son canónicas para cualquier μ.

2- La segunda forma de probar que la transformación es canónica, es probar que el jacobiano es simpléctico.

En este caso $J = \begin{pmatrix} \frac{\partial x_i}{\partial q_j} & \frac{\partial x_i}{\partial p_j} \\ \frac{\partial P_i}{\partial q_j} & \frac{\partial P_i}{\partial p_j} \end{pmatrix}$ donde $x_i = (x, y)$
$P_i = (P_x, P_y)$
$q_i = (q_1, q_2)$
$p_i = (P_1, P_2)$

Primero encontramos las expresiones para $x = x(q_i, p_i)$, $y = y(q_i, p_i)$ $P_x = P_x(q_i, p_i)$ y $P_y = P_y(q_i, p_i)$

De $q_1 = x\cos\mu + P_y \sin\mu$ y $P_2 = P_y\cos\mu - x\sin\mu$ despejamos x.

$\Rightarrow x = \dfrac{q_1 - P_y \sin\mu}{\cos\mu}$ y $x = \dfrac{P_y\cos\mu - P_2}{\sin\mu}$.

Igualando

$$\frac{q_1 - P_y \sin\mu}{\cos\mu} = \frac{P_y\cos\mu - P_z}{\sin\mu} \Leftrightarrow q_1 - P_y\sin\mu = \frac{\cos^2\mu\, P_y}{\sin\mu} - \frac{P_z\cos\mu}{\sin\mu}$$

$$\Leftrightarrow P_y\left(\frac{\cos^2\mu}{\sin\mu} + \sin\mu\right) = q_1 + \frac{P_z\cos\mu}{\sin\mu} \Rightarrow \boxed{P_y = q_1\sin\mu + P_z\cos\mu}$$

Sustituyendo en x, $x = \dfrac{q_1 - q_1\sin^2\mu - P_z\cos\mu\sin\mu}{\cos\mu} = q_1\left(\dfrac{1}{\cos\mu} - \dfrac{\sin^2\mu}{\cos\mu}\right) - P_z\sin\mu$

$$\Rightarrow \boxed{x = q_1\cos\mu - P_z\sin\mu}$$

Luego, despejamos y de $q_2 = y\cos\mu + P_x\sin\mu$ y $P_1 = P_x\cos\mu - y\sin\mu$

$$\Rightarrow y = \frac{q_2 - P_x\sin\mu}{\cos\mu}, \quad y = \frac{P_x\cos\mu - P_1}{\sin\mu}$$

Igualamos:

$$\frac{P_x\cos\mu - P_1}{\sin\mu} = \frac{q_2 - P_x\sin\mu}{\cos\mu} \Leftrightarrow \frac{P_x\cos\mu - P_1}{\sin\mu} = \frac{q_2 - P_x\sin\mu}{\cos\mu}$$

$$\Rightarrow P_x\cos\mu - P_1 = q_2\frac{\sin\mu}{\cos\mu} - P_x\frac{\sin^2\mu}{\cos\mu}$$

$$\Rightarrow P_x\left(\cos\mu + \frac{\sin^2\mu}{\cos\mu}\right) = q_2\frac{\sin\mu}{\cos\mu} + P_1 \Rightarrow \boxed{P_x = q_2\sin\mu + P_1\cos\mu}$$

Sustituimos P_x en y:

$$y = \frac{q_2 - q_2\sin^2\mu - P_1\cos\mu\sin\mu}{\cos\mu} \Rightarrow \boxed{y = q_2\cos\mu - P_1\sin\mu}$$

Hacemos las matrices de 2x2 del jacobiano:

$$\frac{\partial x_i}{\partial q_j} = \begin{pmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} \end{pmatrix} = \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix}$$

$$\frac{\partial x_i}{\partial P_j} = \begin{pmatrix} \frac{\partial x}{\partial P_1} & \frac{\partial x}{\partial P_2} \\ \frac{\partial y}{\partial P_1} & \frac{\partial y}{\partial P_2} \end{pmatrix} = \begin{pmatrix} 0 & -\sin\mu \\ -\sin\mu & 0 \end{pmatrix}$$

$$\frac{\partial P_i}{\partial q_j} = \begin{pmatrix} \frac{\partial P_x}{\partial q_1} & \frac{\partial P_x}{\partial q_2} \\ \frac{\partial P_y}{\partial q_1} & \frac{\partial P_y}{\partial q_2} \end{pmatrix} = \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix}$$

$$\frac{\partial P_i}{\partial p_j} = \begin{pmatrix} \frac{\partial P_x}{\partial p_1} & \frac{\partial P_x}{\partial p_2} \\ \frac{\partial P_y}{\partial p_1} & \frac{\partial P_y}{\partial p_2} \end{pmatrix} = \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix}$$

Hacemos JJ^T donde $J^T = \begin{pmatrix} \left(\frac{\partial x_i}{\partial q_j}\right)^T & \left(\frac{\partial P_i}{\partial q_j}\right)^T \\ \left(\frac{\partial x_i}{\partial p_j}\right)^T & \left(\frac{\partial P_i}{\partial p_j}\right)^T \end{pmatrix}$

Así,

$$J J^T = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} & \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -\sin\mu \\ -\sin\mu & 0 \end{pmatrix} & \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & -\sin\mu \\ -\sin\mu & 0 \end{pmatrix} & \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} \\ \begin{pmatrix} -\cos\mu & 0 \\ 0 & -\cos\mu \end{pmatrix} & \begin{pmatrix} 0 & -\sin\mu \\ -\sin\mu & 0 \end{pmatrix} \end{pmatrix}$$

Además, $J = \begin{pmatrix} \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} & \begin{pmatrix} 0 & -\sin\mu \\ -\sin\mu & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix} & \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} \end{pmatrix}$

Llamamos $A = \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix}$ y $B = \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix}$

Así,

(5)

$$J(J J^T) = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}\begin{pmatrix} -B & A \\ -A & -B \end{pmatrix}$$

$$= \begin{pmatrix} -AB + BA & A^2 + B^2 \\ -B^2 - A^2 & BA - AB \end{pmatrix}$$

donde $AB = \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix}\begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix} = \begin{pmatrix} 0 & \cos\mu\sin\mu \\ \cos\mu\sin\mu & 0 \end{pmatrix}$

$$BA = \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix}\begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} = \begin{pmatrix} 0 & \cos\mu\sin\mu \\ \cos\mu\sin\mu & 0 \end{pmatrix} = AB$$

$$AA = \begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix}\begin{pmatrix} \cos\mu & 0 \\ 0 & \cos\mu \end{pmatrix} = \begin{pmatrix} \cos^2\mu & 0 \\ 0 & \cos^2\mu \end{pmatrix}$$

$$BB = \begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix}\begin{pmatrix} 0 & \sin\mu \\ \sin\mu & 0 \end{pmatrix} = \begin{pmatrix} \sin^2\mu & 0 \\ 0 & \sin^2\mu \end{pmatrix}$$

Por lo tanto,

$$J J J^T = \begin{pmatrix} 0 & \begin{pmatrix} \cos^2\mu & 0 \\ 0 & \cos^2\mu \end{pmatrix} + \begin{pmatrix} \sin^2\mu & 0 \\ 0 & \sin^2\mu \end{pmatrix} \\ -\begin{pmatrix} \cos^2\mu & 0 \\ 0 & \cos^2\mu \end{pmatrix} - \begin{pmatrix} \sin^2\mu & 0 \\ 0 & \sin^2\mu \end{pmatrix} & 0 \end{pmatrix}$$

$$J J J^T = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

El Jacobiano es simpléctico

$\therefore$ las transformaciones son canónicas

b) Si el Hamiltoniano original es $H = (q_1^2 + q_2^2 + p_1^2 + p_2^2)/2$ encuentra en nuevo Hamiltoniano como función de x y y y sus momentos conjugados.

Sustituimos q_1, q_2, P_1 y P_2.

$$H = \frac{1}{2}(q_1^2 + q_2^2 + P_1^2 + P_2^2)$$

$$= \frac{1}{2}\big(x^2\cos^2\mu + \cancel{2xP_y\sin\mu\cos\mu} + P_y^2\sin^2\mu + y^2\cos^2\mu + \cancel{2yP_x\sin\mu\cos\mu} + P_x^2\sin^2\mu + P_x^2\cos^2\mu - \cancel{2yP_x\sin\mu\cos\mu} + y^2\sin^2\mu + P_y^2\cos^2\mu - \cancel{2xP_y\sin\mu\cos\mu} + x^2\sin^2\mu\big)$$

$$= \frac{1}{2}\big(x^2\cos^2\mu + x^2\sin^2\mu + P_y^2\sin^2\mu + P_y^2\cos^2\mu + P_x^2\sin^2\mu + P_x^2\cos^2\mu + y^2\cos^2\mu + y^2\sin^2\mu\big)$$

$$= \frac{1}{2}(x^2 + y^2 + P_x^2 + P_y^2)$$

c) Usa el nuevo Hamiltoniano para resolver la dinámica con la restricción $y = P_y = 0$

Si $y = P_y = 0 \Rightarrow H = \frac{1}{2}(x^2 + P_x^2)$

Así, las ecuaciones de Hamilton son

$$\dot{x} = \frac{\partial H}{\partial P_x} = P_x \quad \Rightarrow \quad \dot{x} = P_x \qquad (1)$$

$$\dot{P}_x = -\frac{\partial H}{\partial x} = -x \quad \Rightarrow \quad \dot{P}_x = -x \qquad (2)$$

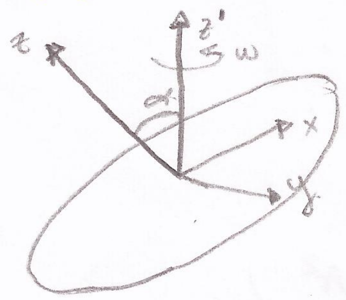
derivamos (1): $(\dot{x} = P_x)^{\cdot} \Rightarrow \ddot{x} = \dot{P}_x$, y sustituimos $\dot{P}_x = -x$.

$\therefore \ddot{x} = -x \Rightarrow x(t) = A\cos(t+\delta)$ // donde $A, \delta \in \mathbb{R}$.

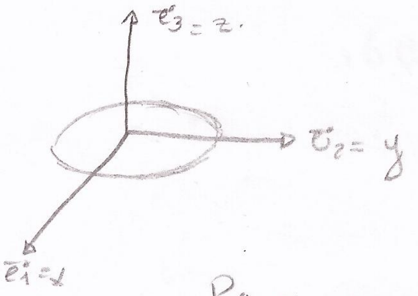
Y de (1), $P_x(t) = -A\,\mathrm{sen}(t+\delta)$ //

Pregunta 2.

Un disco delgado uniforme de masa M y radio A, rota sin fricción con una velocidad angular uniforme w sobre un eje vertical fijo que pasa sobre su centro y tiene un ángulo alfa con el eje de simetría del disco.



a) Determina los momentos de inercia y los ejes principales.



ρ uniforme $\Rightarrow \rho = \frac{M}{\pi A^2}$

radio A.

masa M

Para cuerpos continuos

$$I = \int d^3r\, \rho(r) \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix}$$

Y como no hay variación en z y hay simetría en x y y los únicos valores diferentes de cero en el tensor de inercia son

$$I_{iu} = \begin{pmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{pmatrix}$$

Así, calculamos I_{11}

$$I_{11} = \int \rho y^2\, dx dy = \underbrace{\int \rho r^2 \sin^2\theta\, r\, d\theta\, dr}_{\text{coordenadas polares}} = \int_0^A \int_0^{2\pi} \rho r^3 \sin^2\theta\, d\theta\, dr$$

$$I_{11} = \rho \int_0^{2\pi} \operatorname{sen}^2\theta \, d\theta \left(\frac{1}{4} r^4 \right) \Bigg|_0^A = \frac{1}{4} A^4 \left(\frac{M}{\pi A^2} \right) \int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta$$

$$= \frac{1}{4} \frac{A^2 M}{\pi} \left(\frac{1}{2}\theta \Big|_0^{2\pi} - \cancel{\frac{1}{4} \operatorname{sen} 2\theta \Big|_0^{2\pi}} \right)$$

$$= \frac{1}{4} \frac{A^2 M}{\pi} \frac{1}{\cancel{2}} (\cancel{2}\pi)$$

$$= \frac{1}{4} M A^2$$

Y por simetría en x y y, $I_{22} = I_{11} = \frac{1}{4} M A^2$

Luego

coordenadas polares

$$I_{33} = \int \rho (x^2+y^2)\, dx\, dy = \rho \int r^2 r\, dr\, d\theta = \rho \int_0^A \int_0^{2\pi} r^3 \, d\theta \, dr$$

$$= \frac{M}{\pi A^2} (2\pi) \frac{1}{4} r^4 \Big|_0^A$$

$$= \frac{1}{4} \frac{M}{\pi A^2} (2\pi) A^4$$

$$= \frac{1}{2} M A^2$$

Por lo tanto, el tensor de inercia es:

$$I_{ik} = \begin{pmatrix} \frac{1}{4} M A^2 & 0 & 0 \\ 0 & \frac{1}{4} M A^2 & 0 \\ 0 & 0 & \frac{1}{2} M A^2 \end{pmatrix}$$

Los ejes principales son los autovectores del tensor de inercia.

Primero, obtenemos la ecuación característica:

$$\begin{vmatrix} \frac{1}{4}MA^2-\lambda & 0 & 0 \\ 0 & \frac{1}{4}MA^2-\lambda & 0 \\ 0 & 0 & \frac{1}{2}MA^2-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda_1 = \frac{1}{4}MA^2$, $\lambda_2 = \frac{1}{4}MA^2$ y $\lambda_3 = \frac{1}{2}MA^2$

Así, los autovectores correspondientes a cada λ son:

- Para $\lambda_1 = \lambda_2 = \frac{1}{4}MA^2$

$$\begin{pmatrix} \frac{1}{4}MA^2-\lambda_1 & 0 & 0 \\ 0 & \frac{1}{4}MA^2-\lambda_2 & 0 \\ 0 & 0 & \frac{1}{2}MA^2-\lambda_2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}MA^2 \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$A_1 =$ libre

$A_2 =$ libre

$A_3 = 0$

dos ejes principales

$$\therefore \vec{A} = \begin{pmatrix} s \\ t \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix}} + \underbrace{\begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix}} \quad \text{donde } s, t \in \mathbb{R}$$

- Para $\lambda_3 = \frac{1}{2}MA^2$:

$$\begin{pmatrix} \frac{1}{4}MA^2-\frac{1}{2}MA^2 & 0 & 0 \\ 0 & \frac{1}{4}MA^2-\frac{1}{2}MA^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{1}{2}MA^2 & 0 & 0 \\ 0 & \frac{1}{2}MA^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow B_1 = 0$

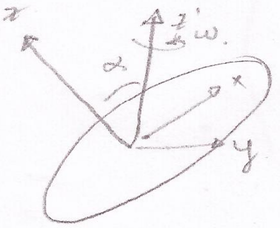
$B_2 = 0$

$B_3 = \text{libre}$ — 1 eje principal

$\therefore \vec{B} = \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$ donde $k \in \mathbb{R}$.

Por lo tanto, los ejes principales son $(s,0,0)$, $(0,t,0)$ y $(0,0,k)$ donde $s,t,k \in \mathbb{R}$, los cuales son x, y y z.

b) Encuentra el vector de momento angular (magnitud y dirección)



Si proyectamos $\vec{\omega}$ en los ejes principales:

$\omega_1 = \omega \cos(90-\alpha) = \omega \operatorname{sen}\alpha$

$\omega_2 = 0$

$\omega_3 = \omega \cos\alpha$

Así, de $L_a = I_{ab}\omega_b$, las componentes del momento angular son

$L_1 = I_{11}\omega_1 = \frac{1}{4} M A^2 \omega \operatorname{sen}\alpha$

$L_2 = I_{22}\omega_2 = 0$

$L_3 = I_{33}\omega_3 = \frac{1}{2} M A^2 \omega \cos\alpha$

Por lo tanto, $\vec{L} = \frac{1}{2} M A^2 \omega \left(\frac{1}{2}\operatorname{sen}\alpha, 0, \cos\alpha\right)$

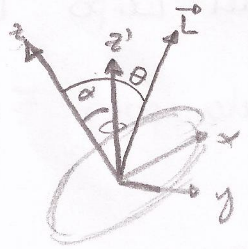
Y su magnitud es

$$\|\vec{L}\| = \sqrt{\frac{1}{4} M^2 A^4 \omega^2 \left(\frac{1}{4}\operatorname{sen}^2\alpha + 0 + \cos^2\alpha\right)}$$

$$= \frac{1}{2} M A^2 \omega \sqrt{\frac{1}{4}\operatorname{sen}^2\alpha + \cos^2\alpha}$$

$$= \frac{1}{2} M A^2 \omega \sqrt{1 - \frac{3}{4}\operatorname{sen}^2\alpha}$$

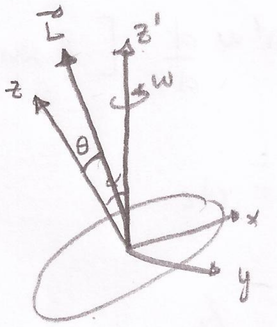
La dirección de $\vec{L}$ está dada por el ángulo θ que hace con respecto a z, donde



$$\theta = \tan^{-1}\left(\frac{L_1}{L_3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{4} M A^2 \omega \operatorname{sen}\alpha}{\frac{1}{2} M A^2 \omega \cos\alpha}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\tan\alpha\right)$$



Por lo tanto, $\theta = \tan^{-1}\left(\frac{1}{2}\tan\alpha\right)$

continúa en la siguiente hoja.

c) ¿Cuál es la magnitud y dirección de la torca relativa al sistema de referencia del cuerpo (x, y, z)?

El torque está dado por $\vec{\tau} = \frac{d\vec{L}}{dt}$

Así,

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d\left[\frac{1}{2}MA^2w\left(\frac{1}{2}\operatorname{sen}\alpha,\ 0,\ \cos\alpha\right)\right]}{dt}$$

$$\vec{\tau} = \frac{1}{2}MA^2w\frac{d}{dt}\left[\frac{1}{2}\operatorname{sen}\alpha\,\hat{\imath} + \cos\alpha\,\hat{k}\right]$$

Pero $\alpha = cte$

$\therefore\ \vec{\tau} = 0$ //