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Mecanica Analitica tarea #4

1.- a) $q_1 = x\cos\mu + p_y \operatorname{sen}\mu$ $\qquad q_2 = y\cos\mu + p_x \operatorname{sen}\mu$

$p_1 = p_x\cos\mu - y\operatorname{sen}\mu$ $\qquad p_2 = p_y\cos\mu - x\operatorname{sen}\mu$

Si son canonicas, deben cumplir con las propiedades de los corchetes de Poisson,

$$\{q_i, p_j\} = \delta_{ij} \qquad \{p_i, p_j\} = 0 \qquad \{q_i, q_j\} = 0$$

$$\{q_1, q_2\}(x_i, y_i) = \frac{\partial q_1}{\partial x}\frac{\partial q_2}{\partial p_x} - \frac{\partial q_2}{\partial x}\frac{\partial q_1}{\partial p_x} + \frac{\partial q_1}{\partial y}\frac{\partial q_2}{\partial p_y} - \frac{\partial q_2}{\partial y}\frac{\partial q_1}{\partial p_y}$$

$x_1 = x \quad y_1 = p_x$

$x_2 = y \quad y_2 = p_y$

$$= \cos\mu \cdot \operatorname{sen}\mu - 0\cdot 0 + 0\cdot 0 - \cos\mu \cdot \operatorname{sen}\mu = 0$$

$$\Longrightarrow \{q_1, q_2\}(x_i, y_i) = 0$$

$$\{p_1, p_2\}(x_i, y_i) = \frac{\partial p_1}{\partial x}\frac{\partial p_2}{\partial p_x} - \frac{\partial p_2}{\partial x}\frac{\partial p_1}{\partial p_x} + \frac{\partial p_1}{\partial y}\frac{\partial p_2}{\partial p_y} - \frac{\partial p_2}{\partial y}\frac{\partial p_1}{\partial p_y}$$

$$= 0\cdot 0 - (-\operatorname{sen}\mu)\cos\mu + (-\operatorname{sen}\mu)\cos\mu - 0\cdot 0 = 0$$

$$\Longrightarrow \{p_1, p_2\}(x_i, y_i) = 0$$

$$\{q_1, p_1\}(x_i, y_i) = \frac{\partial q_1}{\partial x}\frac{\partial p_1}{\partial p_x} - \frac{\partial p_1}{\partial x}\frac{\partial q_1}{\partial p_x} + \frac{\partial q_1}{\partial y}\frac{\partial p_1}{\partial p_y} - \frac{\partial q_1}{\partial y}\frac{\partial q_1}{\partial p_y}$$

$$= \cos\mu\cos\mu - 0\cdot 0 + 0\cdot 0 - (-\operatorname{sen}\mu)\operatorname{sen}\mu = 1$$

$$\Longrightarrow \{q_1, p_1\}(x_i, y_i) = 1$$

$$\{q_1, p_2\}(x_i, y_i) = \frac{\partial q_1}{\partial x}\frac{\partial p_2}{\partial p_x} - \frac{\partial p_2}{\partial x}\frac{\partial q_1}{\partial p_x} + \frac{\partial q_1}{\partial y}\frac{\partial p_2}{\partial p_y} - \frac{\partial p_2}{\partial y}\frac{\partial q_1}{\partial p_y}$$

$$= \cos\mu\cdot 0 - (-\operatorname{sen}\mu)\cdot 0 + 0\cdot\cos\mu - 0\cdot\operatorname{sen}\mu = 0$$

$$\Longrightarrow \{q_1, p_2\}(x_i, y_i) = 0$$

$$\{q_2, p_1\}(x_i, y_i) = \frac{\partial q_2}{\partial x}\frac{\partial p_1}{\partial p_x} - \frac{\partial p_1}{\partial x}\frac{\partial q_2}{\partial p_x} + \frac{\partial q_2}{\partial y}\frac{\partial p_1}{\partial p_y} - \frac{\partial p_1}{\partial y}\frac{\partial q_2}{\partial p_y}$$

$$= 0 \cdot 0 - 0 \cdot \operatorname{sen}\mu + \cos\mu \cdot 0 - (-\operatorname{sen}\mu) \cdot 0 = 0$$

$$\Rightarrow \{q_2, p_1\}(x_i, y_i) = 0$$

$$\{q_2, p_2\}(x_i, y_i) = \frac{\partial q_2}{\partial x}\frac{\partial p_2}{\partial p_x} - \frac{\partial p_2}{\partial x}\frac{\partial q_2}{\partial p_x} + \frac{\partial q_2}{\partial y}\frac{\partial p_2}{\partial p_y} - \frac{\partial p_2}{\partial y}\frac{\partial q_2}{\partial p_y}$$

$$= 0 \cdot 0 - (-\operatorname{sen}\mu) \cdot \operatorname{sen}\mu + \cos\mu \cdot \cos\mu - 0 \cdot 0 = 1$$

$$\Rightarrow \{q_2, p_2\}(x_i, y_i) = 1$$

$\boxed{\therefore \text{ son canonicas.}}$

b) $H = \frac{(p_1^2 + p_2^2 + q_1^2 + q_2^2)}{2}$

$H = \frac{1}{2}\left(p_x^2\cos^2\mu - 2p_x y\,\mathrm{sen}\,\mu\cos\mu + y^2\mathrm{sen}^2\mu + p_y^2\cos^2\mu - 2p_y x\cos\mu\,\mathrm{sen}\,\mu + x^2\mathrm{sen}^2\mu\right.$

$\left. + x^2\cos^2\mu + 2x p_y\,\mathrm{sen}\,\mu\cos\mu + p_y^2\mathrm{sen}^2\mu + y^2\cos^2\mu + 2p_x y\,\mathrm{sen}\,\mu\cos\mu + p_x^2\mathrm{sen}^2\mu\right)$

$H = \frac{1}{2}\left[\cos^2\mu\,(p_x^2 + p_y^2 + x^2 + y^2) + \mathrm{sen}^2(p_x^2 + p_y^2 + x^2 + y^2)\right]$

$$\boxed{H = \frac{p_x^2 + p_y^2 + x^2 + y^2}{2}}$$

c) con $y = p_y = 0$ $\quad H = \frac{x^2 + p_x^2}{2}$

$\dot{q}_i = \frac{\partial H}{\partial p_i}$

$\dot{p}_i = -\frac{\partial H}{\partial q_i}$

$\dot{x} = \frac{\partial H}{\partial p_x} = p_x$

$\dot{p}_x = -\frac{\partial H}{\partial x} = -x \longrightarrow \frac{\partial \dot{p}_x}{\partial t} = \ddot{p}_x = -\dot{x}$

$x = -\dot{p}_x$

$\dot{x} = -p_x$

$-p_x = \ddot{p}_x$

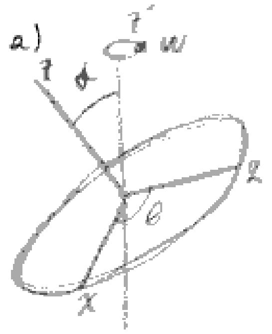
$$\boxed{\ddot{p}_x + p_x = 0}$$

para x, $\dot{x} = -p_x$, $x = \dot{p}_x$

$\frac{\partial \dot{x}}{\partial t} = \ddot{x} = -\dot{p}_x$

$x = -\ddot{x} \Rightarrow \boxed{\ddot{x} + x = 0}$

2.- a)



momentos de inercia y los ejes principales.

$$I_{ij} = \int_V \rho(r) \left(\delta_{ij} x_k^2 - x_i x_j \right) dx\,dy\,dz$$

por simetria en ejes x y y.

$$I_{11} = I_{22} \qquad I_{11} \neq I_{33}$$

con $\rho = \frac{M}{\pi A^2}$

considerando la matriz $(\delta_{ij} x_k^2 - x_i x_j)$

$$\begin{pmatrix} y^2+z^2 & & \\ & x^2+z^2 & \\ & & y^2+x^2 \end{pmatrix}$$ considerando $z = 0$.

$$\Longrightarrow \quad I_{11} = \frac{M}{\pi A^2} \int_0^r\!\!\int_0^r y^2\,dx\,dy = I_{22} = \frac{M}{\pi A^2} \int_0^r\!\!\int_0^r x^2\,dx\,dy$$

$$I_{33} = I_{11} + I_{22} = \frac{M}{\pi A^2} \int_0^r\!\!\int_0^r (x^2+y^2)\,dx\,dy$$

cambiando es sistema de coordenadas a cilindricas.

$$I_{33} = I_{11} + I_{22} = \frac{M}{\pi A^2} \int_0^{2\pi}\!\!\int_0^A r^3\,dr\,d\theta$$

$$= \frac{M}{\pi A^2} \frac{A^4}{4} \cdot 2\pi$$

$$I_{33} = \frac{M}{2} A^2$$

$$\Longrightarrow \quad I_{11} = I_{22} = \frac{M}{4} A^2$$

$$I_{ij} = \begin{pmatrix} \frac{M}{4}A^2 & 0 & 0 \\ 0 & \frac{M}{4}A^2 & 0 \\ 0 & 0 & \frac{M}{2}A^2 \end{pmatrix}$$

b) Vector momento angular

$$\underline{L} = I_{ab}\, w_b = \begin{pmatrix} I_{11} & & \\ & I_{22} & \\ & & I_{33} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$L_1 = I_{11} w_1 = \frac{MA^2}{4} w_1$$

$$L_2 = I_{22} w_2 = \frac{MA^2}{4} w_2$$

$$L_3 = I_{33} w_3 = \frac{MA^2}{2} w_3$$

$$\left.\begin{aligned} I_2 \dot{w}_2 &= -w_2 w_3 (I_2 - I_3) \\ I_2 \dot{w}_1 &= w_3 w_1 (I_1 - I_3) \end{aligned}\right\} \quad \begin{aligned} \dot{w}_1 &= \Omega w_2 \\ \dot{w}_2 &= -\Omega w_1 \end{aligned}$$

donde $\Omega = w_3 (I_1 - I_3)$

$$(w_1, w_2) = w_0 (\operatorname{sen} \Omega t, \cos \Omega t)$$

c) Dirección de la torca.

$$\underline{T} = \frac{d\underline{L}}{\partial t}$$