FORMA CANÓNICA CONTROLABLE (FCC)

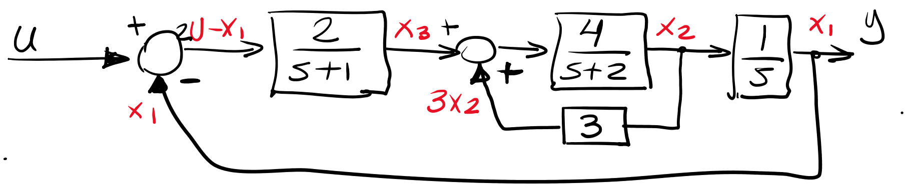
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$\overline{\mathbf{A}} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} \qquad \mathbf{P} = \mathbf{S}\mathbf{M}$$

$$\overline{\mathbf{B}} = \mathbf{P}^{-1}\mathbf{B} \qquad \mathbf{S} = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Ejm:



$$x_2\left(\frac{1}{s}\right) = x_1 \qquad (x_3 + 3x_2)\left(\frac{4}{s+2}\right) = x_2 \qquad (u - x_1)\left(\frac{2}{s+1}\right) = x_3$$

$$\dot{x}_1 = x_2 \qquad 4x_3 + 12x_2 = sx_2 + 2x_2 \qquad \dot{x}_3 = -2x_1 - x_3 + 2u$$

$$\dot{x}_2 = 4x_3 + 10x_2$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B} u \qquad y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

F.C.C.

$$S = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 8 & 72 \\ 2 & -2 & 2 \end{bmatrix}$$

borrador

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 10 & 4 \\ -8 & 100 & 36 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} (+) & (-) & (+) \end{array} \\ \Delta c = |sI - A| = \left|\begin{bmatrix} s & -1 & 0 \\ 0 & s-10 & -4 \\ 2 & 0 & s+1 \end{bmatrix}\right| \end{array}$$

$$\Delta c = s(s-10)(s+1) + 8 = s^3 - 9s^2 - 10s + 8$$

$\therefore$ $P = SM$

$$P = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 8 & 72 \end{bmatrix}\begin{bmatrix} -10 & -9 & 1 \\ -9 & 1 & 0 \end{bmatrix}$$

$$\Delta c = s(s-10)(s+1) + 8 = s^3 \underbrace{-9}_{a_2} s^2 \underbrace{-10}_{a_1} s \underbrace{+8}_{a_0}$$

$$P = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 8 & 72 \\ 2 & -2 & 2 \end{bmatrix} \begin{bmatrix} -10 & -9 & 1 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -10 & -9 & 1 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & -20 & 2 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0.125 & 0 & 0 \\ 0 & 0.125 & 0 \\ 0 & 1.25 & 0.5 \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} 0.125 & 0 & 0 \\ 0 & 0.125 & 0 \\ 0 & 1.25 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\bar{x}_1 = 0.125\, x_1$

$\bar{x}_2 = 0.125\, x_2$

$\bar{x}_3 = 1.25\, x_2 + 0.5\, x_3$

$\therefore$

$$\bar{A} = P^{-1} A P$$

$$\bar{A} = \underbrace{\begin{bmatrix} 0.125 & 0 & 0 \\ 0 & 0.125 & 0 \\ 0 & 1.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix}} \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & -20 & 2 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0 & 0.125 & 0 \\ 0 & 1.25 & 0.5 \\ -1 & 12.5 & 4.5 \end{bmatrix} \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & -20 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & 10 & 9 \end{bmatrix}$$

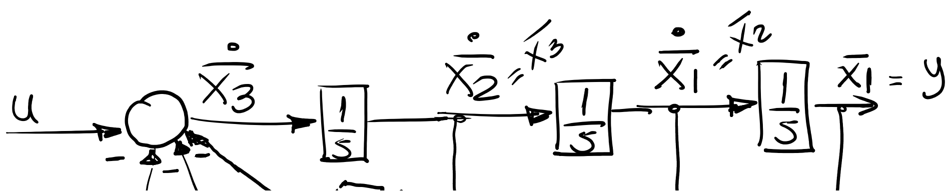
$$\bar{B} = \begin{bmatrix} 0.125 & 0 & 0 \\ 0 & 0.125 & 0 \\ 0 & 1.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

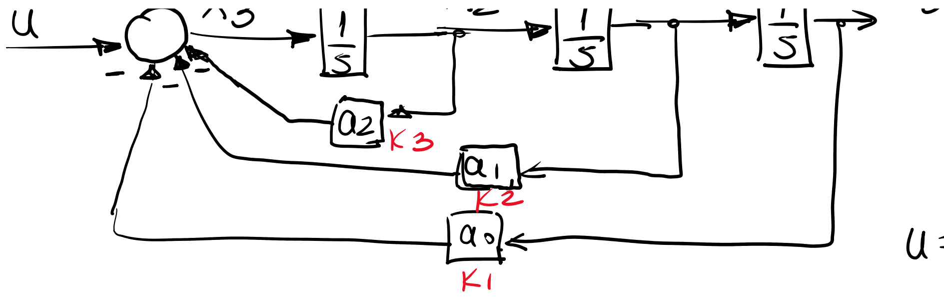
$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & 10 & 9 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$\dot{\bar{x}}_1 = \bar{x}_2$

$\dot{\bar{x}}_2 = \bar{x}_3$

$\dot{\bar{x}}_3 = -8\bar{x}_1 + 10\bar{x}_2 + 9\bar{x}_3 + u$





$u = -Kx$

$$u = -\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

FORMA CANÓNICA OBSERVABLE (FCO)

La matriz **Q** de la tranformación a la FCO, viene dada por:

$$\mathbf{Q} = (\mathbf{MV})^{-1}$$

en donde **M** viene dada por la ec. (15) y

$$\mathbf{V} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}$$

De donde se obtiene

$$\overline{\mathbf{A}} = \mathbf{Q}^{-1}\mathbf{AQ} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & & & \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix}$$

y

$$\overline{\mathbf{C}} = \mathbf{CQ} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

además

$$\overline{\mathbf{B}} = \mathbf{Q}^{-1}\mathbf{B} \quad \text{y} \quad \overline{\mathbf{D}} = \mathbf{D}$$

$$\Delta c = s(s-10)(s+1) + 8 = s^3 \underbrace{-9}_{a_2} s^2 \underbrace{-10}_{a_1} s + \underbrace{8}_{a_0}$$

$$M = \begin{bmatrix} -10 & -9 & 1 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix}}_{A} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 10 & 4 \\ -8 & 100 & 36 \\ 2 & -2 & 1 \end{bmatrix}$$

$$Q = (MV)^{-1} = \left(\begin{bmatrix} -10 & -9 & 1 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 4 \end{bmatrix} \right)^{-1} = \Bigg(\underbrace{\begin{bmatrix} -10 & 1 & 4 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{Q^{-1}} \Bigg)^{-1}$$

$$Q = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 9 \\ 1/4 & -1/4 & 1/4 \end{bmatrix}$$

$$\bar{A} = Q^{-1} A Q$$

$$= \underbrace{\begin{bmatrix} -10 & 1 & 4 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 10 & 4 \\ -2 & 0 & -1 \end{bmatrix}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 9 \\ 0.25 & -0.25 & 0.25 \end{bmatrix}$$

$$\begin{bmatrix} -8 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 9 \\ 0.25 & -0.25 & 0.25 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & 10 \\ 0 & 1 & 9 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} -10 & 1 & 4 \\ -9 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 9 \\ 0.25 & -0.25 & 0.25 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 9 \end{bmatrix} ; \quad \begin{bmatrix} 0 \end{bmatrix}$$

$$\bar{C} = CQ = [1\ 0\ 0]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 9 \\ 1/4 & -1/4 & 1/4 \end{bmatrix} = [0\ \ 0\ \ 1]$$

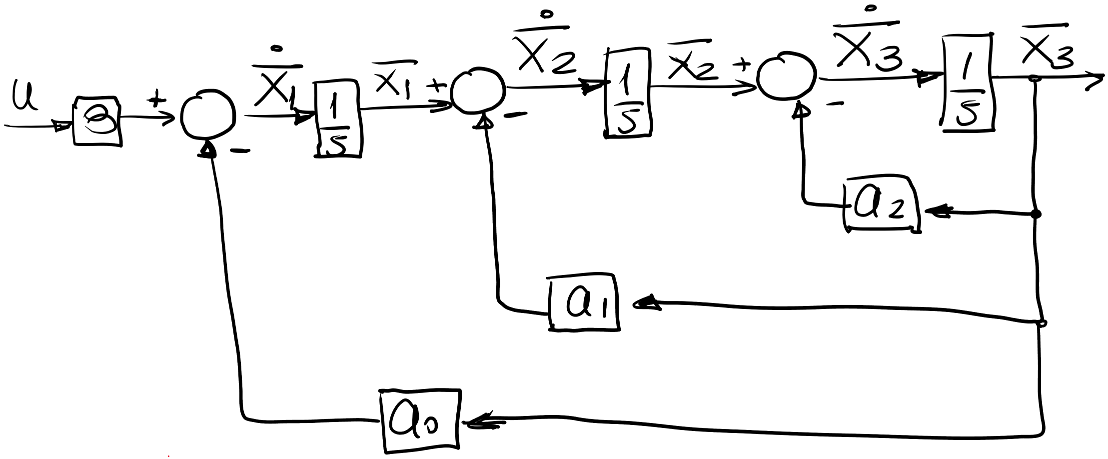
$$\dot{\bar{X}}_o = \bar{A}_o \bar{X}_o + \bar{B}_o u \quad ; \quad y = \bar{C}_o \bar{X}_o$$

$$\dot{\bar{X}}_o = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & 10 \\ 0 & 1 & 9 \end{bmatrix}\begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \end{bmatrix} + \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} u \qquad y = [0\ \ 0\ \ 1]\begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_3 \end{bmatrix}$$

$$\dot{\bar{X}}_1 = -8\bar{X}_3 + 8u \qquad y = \bar{X}_3$$

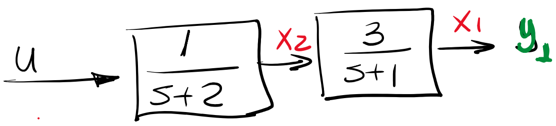
$$\dot{\bar{X}}_2 = \bar{X}_1 + 10\bar{X}_3$$

$$\dot{\bar{X}}_3 = \bar{X}_2 + 9\bar{X}_3$$



DISEÑO DE CONTROLADORES

Ejemplo:



- Diseñar un controlador
  - $Mp\% < 10\%$
  - $Ts_{2\%} < 4s$

$$U\left(\frac{1}{s+2}\right) = X_2 \quad ; \quad X_2\left(\frac{3}{s+1}\right) = X_1$$

$$\dot{x}_2 = -2x_2 + u \quad ; \quad \dot{x}_1 = -x_1 + 3x_2$$

$$\dot{x} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$10 = 100 \cdot e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\xi = \sqrt{\frac{5.3}{9.87+5.3}} = 0.6$$

$$\frac{4}{\xi\, w_n} = 4$$

$$\dot{x} = \underbrace{\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{[1 \ 0]}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

• controlador:

$$S = [B \ AB] = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$$

$$|S| = -3 \neq 0 \ \checkmark$$

$$\text{rango}(S) = 2 \ \checkmark$$

$$\Delta c = |sI - A| = \left| \begin{bmatrix} s+1 & -3 \\ 0 & s+2 \end{bmatrix} \right| = (s+1)(s+2)$$

$$= s^2 + \underbrace{3}_{a_1}s + \underbrace{2}_{a_0}$$

$$M = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore P = SM = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \rightarrow P^{-1} = \frac{\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}}{3}$$

$$P^{-1} = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix}$$

$$\frac{4}{\xi . w_n} = 4$$

$$w_n = 1.67$$

$$\lambda_1 ; \lambda_2 = -\xi . w_n \pm w_n \sqrt{1-\xi^2} \, j$$

$$\lambda_1 ; \lambda_2 = -1 \pm 1.34 j$$

$$\Delta d = (s - \lambda_1)(s - \lambda_2)$$

$$= (s+1-1.34j)(s+1+1.34j)$$

$$\Delta d = s^2 + \underbrace{2}_{\alpha_1}s + \underbrace{2.8}_{\alpha_0}$$

$$K^* = [(\alpha_0 - a_0) \ (\alpha_1 - a_1)]$$

$$K^* = [0.8 \ -1]$$

$$K = K^* P^{-1}$$

$$K = [0.8 \ -1] \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix} = [\underbrace{0.6}_{K_1} \ \underbrace{-1}_{K_2}]$$