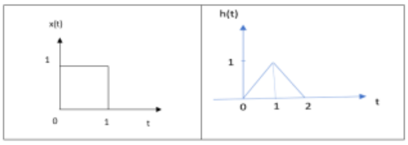
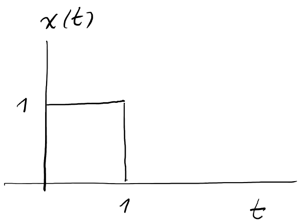
PREGUNTA 1 (5 PTOS)

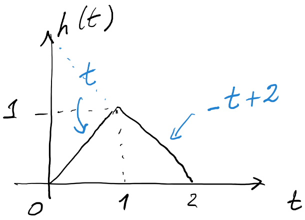
06:39

Hallar la salida $y(t)$ de un sistema lineal invariante en el tiempo con respuesta al impulso $h(t)$ y entrada $x(t)$ mediante la convolución. Trabajar la convolución gráficamente, paso por paso.

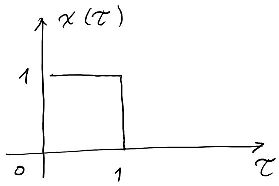
a) Hallar el resultado analitico (5 ptos.: 1 pto. por cada intervalo de t y su correspondiente función analitica correctamente hallados).

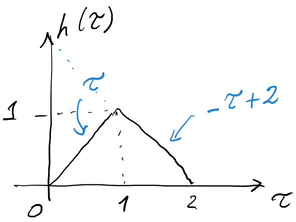


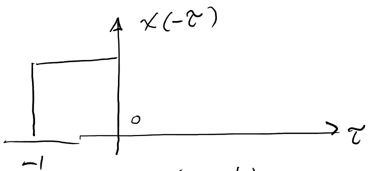


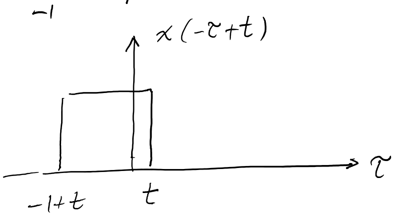


$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(-\tau+t)\, d\tau$$



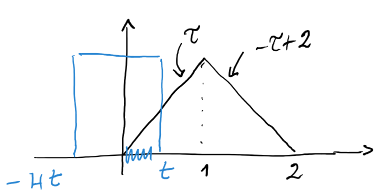






① Si $t<0 \Rightarrow y(t)=0$ ... (CASO 1) 1 pto.

② Si $0 \leq t < 1$

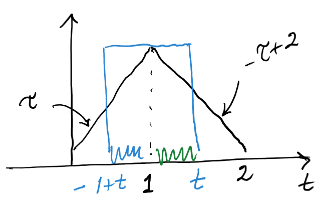


$$y(t) = \int_0^t (1)(\tau)\, d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}$$

(CASO 2) $0 \leq t < 1$

1 pto.

③ Si $1 \leq t < 2$

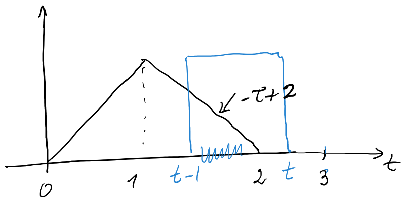


$$y(t) = \int_{-1+t}^{1} (\tau)(1)\, d\tau + \int_1^t (-\tau+2)(1)\, d\tau$$

$$= -t^2 + 3t - \frac{3}{2}$$ ... (1 p)

(CASO 3) $1 \leq t < 2$

(4) Si $2 \leq t < 3$



$$y(t) = \int_{t-1}^{2} (-\tau+2)(1)\, d\tau$$

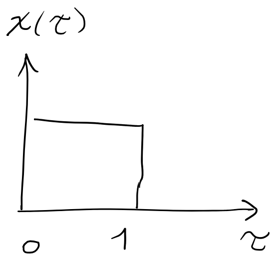
CASO 4. $(2 \leq t < 3)$

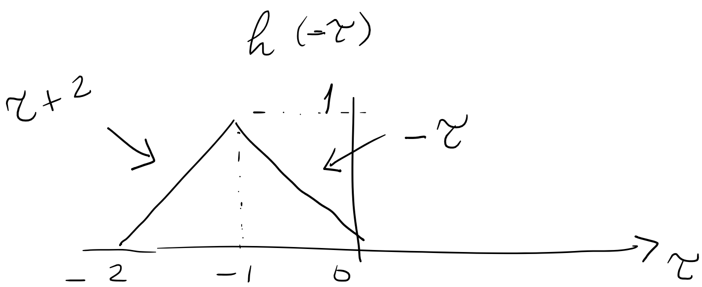
$\dfrac{(t-3)^2}{2}$ ... $(1p)$.

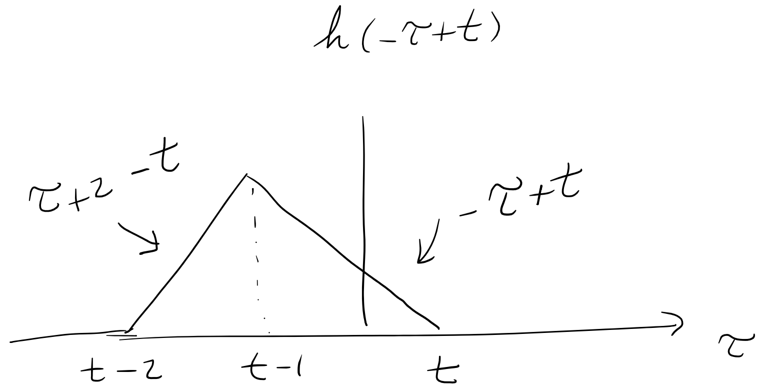
CASO 5: $t \geq 3$

(5) Si $3 \leq t \Rightarrow y(t) = 0$ $(1p.)$

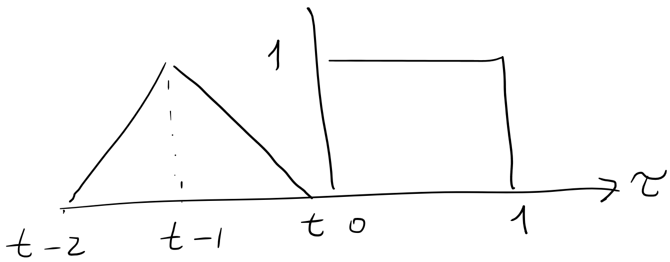
Otra manera:



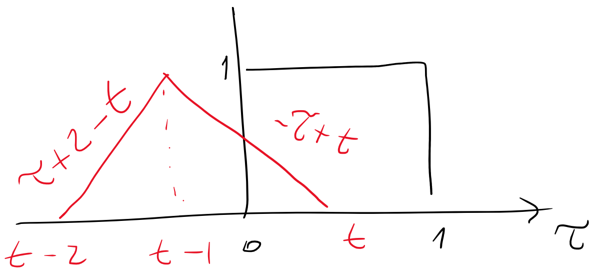




CASO 1: $t < 0$ $\Rightarrow$ $y(t) = 0$



Caso 2: $0 < t \le 1$

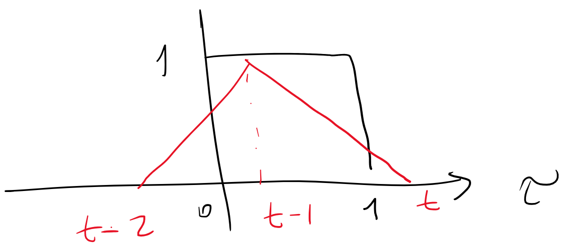


$$y(t) = \int_0^t -\tau + t \, d\tau =$$

$$= -\frac{\tau^2}{2} + t\tau \Big|_0^t$$

$$= -\frac{t^2}{2} + t^2 = \frac{t^2}{2}$$

Caso 3: $0 < t-1 \le 1 \Rightarrow 1 < t \le 2$



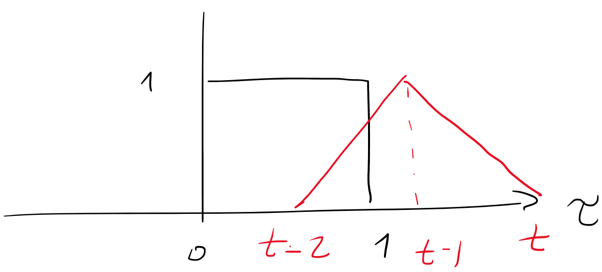
$$y(t) = \int_0^{t-1} \tau + 2 - t \, d\tau +$$

$$\int_{t-1}^{1} -\tau + t \, d\tau$$

$$y(t) = -t^2 + 3t - \frac{3}{2}$$

Caso 4: $2 < t \le 3$

$$y(t) = \int_{t-2}^{1} \tau + 2 - t \, d\tau$$



$$y(t) = \frac{1}{2}(t-3)^2$$

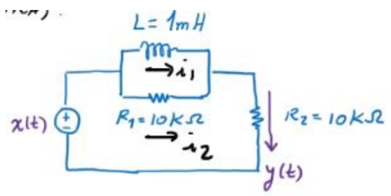
Caso 5: $t > 3$; $y(t) = 0$.

PREGUNTA 2 (5 PTOS)

06:42

Dado el circuito, donde la entrada es el voltaje x(t) y la salida es la corriente y(t), y el sistema es causal.
Hallar:

a) Ecuación diferencial en forma estándar y con valores numéricos (2 ptos)
b) Solución homogénea (1 pto.)
c) Solución particular (1 pto.)
d) Solución total (1 pto.)



$$y(t) = i_1(t) + i_2(t)$$

$$y(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt + \frac{v_L}{R_1}$$

$$x(t) = v_L(t) + y(t)R_2$$

$$v_L(t) = x(t) - y(t)R_2$$

$$y(t) = \frac{1}{L}\int_{-\infty}^{t}\left(x(t) - y(t)R_2\right)dt + \frac{1}{R_1}\left(x(t) - y(t)R_2\right)$$

$$\frac{dy(t)}{dt} = \frac{1}{L}\left(x(t) - y(t)R_2\right) + \frac{1}{R_1}\left[\frac{dx(t)}{dt} - R_2\frac{dy(t)}{dt}\right]$$

$$\frac{dy(t)}{dt} = \frac{1}{L}x(t) - \frac{R_2}{L}y(t) + \frac{1}{R_1}\frac{dx(t)}{dt} - \frac{R_2}{R_1}\frac{dy(t)}{dt}$$

$$\left(1 + \frac{R_2}{R_1}\right)\frac{dy}{dt} + \frac{R_2}{L}y(t) = \frac{1}{L}x(t) + \frac{1}{R_1}\frac{dx(t)}{dt}.$$

Reemplazando: $L = 10^{-3}$; $R_1 = R_2 = 10^4$

(2p.) $\boxed{2y'(t) + 10^7 y(t) = 10^3 x(t) + 10^{-4} x'(t)}$ ... ec. dif. canónica.

Sol. homogénea

$$y_h(t) = Ke^{st} \Rightarrow$$

$$2Kse^{st} + 10^7 Ke^{st} = 0$$

$$Ke^{st}(2s + 10^7) = 0$$

$$s = -0.5\times 10^7$$

$$y_h(t) = Ke^{-0.5\times 10^7 t} \quad (1p.)$$

Sol. particular para escalón unitario

$$y_p(t) = M$$

$$10^7 M = 10^3$$

$$M = 10^{-4} \quad (1p.)$$

$$y_T(t) = Ke^{-0.5\times10^7 t} + 10^{-4} \quad \dots(\alpha)$$

De la ec. diferencial:

$$\underbrace{\int_{-\infty}^{t=0} 2\frac{dy(\tau)}{d\tau}d\tau}_{2y(0)} + \underbrace{10^7\int_{-\infty}^{t=0} y(\tau)\,d\tau}_{0} = \underbrace{10^3\int_{-\infty}^{t=0} x(\tau)\,d\tau}_{0} + \underbrace{10^{-4}\int_{-\infty}^{t=0}\frac{dx(\tau)}{d\tau}d\tau}_{10^{-4}x(0)}$$

$$2y(0) + 0 = 0 + 10^{-4}x(0)$$

$$y(0) = 0.5\times10^{-4}x(0)$$

$$y(0) = 0.5\times10^{-4}(1) = 0.5\times10^{-4} \quad \dots(\beta)$$

(β) en (α):

$$K + 10^{-4} = 0.5\times10^{-4}$$

$$K = 0.5\times10^{-4} - 10^{-4} = 10^{-4}(0.5-1) = -0.5\times10^{-4}$$

$$y_T(t) = -0.5\times10^{-4}e^{-0.5\times10^7 t} + 10^{-4}$$

$$y_T(t) = -0.5\times 10^{-4}\, e^{-0.5\times 10\, t} + 10^{-4}$$

$$y_T(t) = 10^{-4}\left(1 - 0.5\, e^{-0.5\times 10^{7} t}\right) := s(t).$$ (1p.)

PREGUNTA 3 (5 PTOS)

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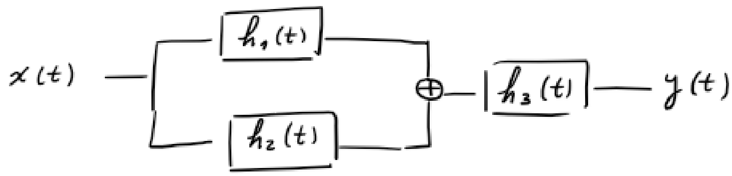
PARTE 1 (2 PTOS)

Dado el siguiente sistema, donde se muestran respuestas al impulso $h_1(t)$, $h_2(t)$, $h_3(t)$.



Donde
$$h_1(t) = \delta(t-2)$$
$$h_2(t) = \delta(t-1)$$
$$h_3(t) = e^{-2t}u(t)$$

Hallar $h_{eq}(t)$. (h equivalente del sistema).

PARTE 2 (3 PTOS)

II) Si la respuesta al impulso de un sistema lineal invariante en el tiempo es:

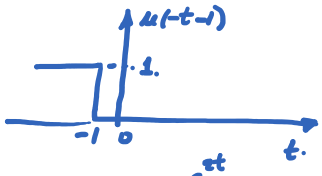
$$h(t) = e^{2t}u(-1-t)$$

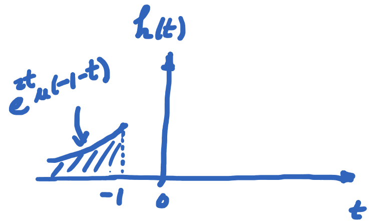
a) Indicar y justificar si el sistema tiene o no memoria (1 pto.)
b) Indicar y justificar si el sistema es o no causal (1 pto.)
c) Indicar y justificar si el sistema es o no es estable (1 pto.)

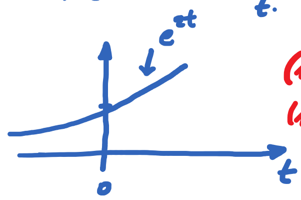
Solución:

$$h_{eq}(t) = e^{-2t}u(t) * [\delta(t-1) + \delta(t-2)]$$
$$= e^{-2(t-1)}u(t-1) + e^{-2(t-2)}u(t-2) \quad (2p)$$

$h(t) = e^{2t}u(-1-t)$







(1p.) a) Sí tiene memoria porque $h(t) \neq k\delta(t)$

(1p.) b) No es causal porque $h(t) \neq 0$ para $t < 0$.

(1p.) c) $\int_{-\infty}^{-1} h(t)\,dt = \,?$

$$\int_{-\infty}^{-1} e^{2t}\,dt = \frac{1}{2}e^{2t}\Big|_{-\infty}^{-1} = \frac{1}{2}\left[e^{-2} - \underbrace{e^{-\infty}}_{0}\right]$$

$$= \frac{1}{2}e^{-2} \text{ (converge)}$$

$\Rightarrow$ es estable.