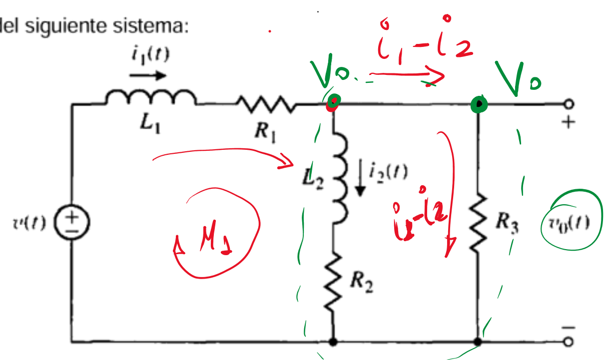
1) A partir del siguiente sistema:



$x_1 = i_1$

$x_2 = i_2$

$U = V(t)$

$y = V_0(t)$

Considerando que las variables están en sus unidades internacionales y los valores siguiente: L1=2 ; R1=10; L2= 4; R2=2; R3=5

$$V(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + L_2 \frac{di_2}{dt} + R_2 i_2$$

Las variables de estado son las corrientes i1, i2 y la salida Vo(t). además la señal de control U es V(t)

$$U = 2\dot{x}_1 + 10x_1 + \underbrace{4\dot{x}_2 + 2x_2}$$

a) Hallar el modelo espacio estados de todo el proceso. (3 ptos)
b) Hallar la F.T. del sistema Vo(t)/V(t) a partir del modelo espacio estados. (2 ptos)

a)

$$y = V_0 = \underbrace{L_2 \frac{di_2}{dt} + R_2 i_2} = R_3(i_1 - i_2) \quad ..(1)$$

$$y = 5(x_1 - x_2)$$

$$\dot{X} = \underbrace{\begin{bmatrix} -7.5 & 2.5 \\ 1.25 & -1.75 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}}_{B} u$$

$$U = 2\dot{x}_1 + 10x_1 + 5(x_1 - x_2)$$

$$\dot{x}_1 = -7.5x_1 + 2.5x_2 + 0.5u$$

$$y = \underbrace{[5 \; -5]}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$4\dot{x}_2 + 2x_2 = 5(x_1 - x_2)$$

$$\dot{x}_2 = 1.25x_1 - 1.75x_2$$

b) $G(s) = C \cdot (sI - A)^{-1} \cdot B$

$$= [5 \; -5] \begin{bmatrix} s+7.5 & -2.5 \\ -1.25 & s+1.75 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

$$[5 \; -5] \frac{\begin{bmatrix} s+1.75 & 2.5 \\ 1.25 & s+7.5 \end{bmatrix}}{s^2 + 9.25s + 10} \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{5s + 2.5}{s^2 + 9.25s + 10} & \dfrac{-5s - 25}{s^2 + 9.25s + 10} \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

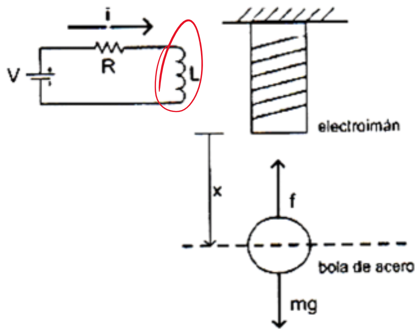
$$G(s) = \frac{2.5s + 1.25}{s^2 + 9.25s + 10}$$

1) Considerar que el levitador magnético tiene las siguientes variables.

$$\underline{V_0 = 24V}, L = 0.5H, \underline{R = 200\Omega}, K = \underline{15}\left(\frac{Nm^3}{A^2}\right), m = 0.1Kg$$

Si la fuerza f esta dado por: $f = \underline{K}\frac{i^2}{x^3}$

Hallar:



2° ley Kirchoff

$$V = i.R + \cancel{L\frac{di}{dt}} \rightarrow$$

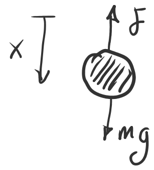
equilibrio

$$V_0 = i_0 . R$$

$$24 = i_0 . 200$$

$$i_0 = 0.12$$

DCL 2° ley Newton



$$mg - f = m\ddot{x}$$

$$mg - 15\frac{i^2}{x^3} = m\ddot{x}$$

$$\Sigma F = 0$$

$$mg = f$$

$$0.1(9.81) = 15\frac{i_0^2}{X_0^3}$$

$$X_0 = \sqrt[3]{\frac{15(0.12)^2}{0.981}} = 0.6$$

Linealizar

1. $f = 0$

$$\rightarrow f = m\ddot{x} + 15\frac{i^2}{x^3} - mg$$

2. variables

$$x \rightarrow x_0 = 0.6$$

$$\ddot{x} \rightarrow \ddot{x}_0 = 0$$

$$i \rightarrow i_0 = 0.12$$

3. $$\left.\frac{df}{dx}\right|_{p.o.}(x - x_0) + \left.\frac{df}{d\ddot{x}}\right|_{p.o}(\ddot{x} - \cancel{\ddot{x}_0}^{\,0}) + \left.\frac{df}{di}\right|_{p.o}(i - i_0) = 0$$

$$-\frac{45 i_0^2}{X_0^4}(x - x_0) + 0.1\ddot{x} + \frac{30 i_0}{X_0^3}(i - i_0) = 0$$

$$-5x \cancel{+3} + 0.1\ddot{x} + 16.67 i \cancel{-2} = 0$$

$$\ddot{x} = 50x - 166.7 i - 10$$

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$x_3 = i$$

$$u = V$$

$$\dot{x}_1 = x_2$$

$$u = 200x_3 + 0.5\dot{x}_3$$

$$V = 200 i + 0.5\frac{di}{dt}$$

$$\dot{x}_2 = 50x_1 - 166.7x_3 - 10$$

$$\dot{x}_3 = -400x_3 + 2u$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 50 & 0 & -166.7 \\ 0 & 0 & -400 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix}}_{w}$$

$$\underbrace{\begin{bmatrix}0 & 0 & -400\end{bmatrix}}_{A}\begin{bmatrix}x_3\end{bmatrix} \quad \underbrace{\begin{bmatrix}2\end{bmatrix}}_{B} \quad \underbrace{\begin{bmatrix}\ \end{bmatrix}}_{w}$$

$$y = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}\cdot B = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}s & -1 & 0\\ -50 & s & 166.7\\ 0 & 0 & s+400\end{bmatrix}^{-1}\begin{bmatrix}0\\0\\2\end{bmatrix}$$

Reparo

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\2 & 1 & 3\end{bmatrix}^{-1} = \frac{\operatorname{adj}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\2 & 1 & 3\end{bmatrix}}{\left|\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\2 & 1 & 3\end{bmatrix}\right|} = \frac{\begin{bmatrix}1 & -3 & 1\\4 & -3 & -2\\-2 & 3 & 1\end{bmatrix}}{3} = \begin{bmatrix}0.33 & -1 & 0.33\\1.33 & -1 & -0.66\\-0.66 & 1 & 0.33\end{bmatrix}$$

$$1-(-8)+(-6) = 3$$

$$\begin{bmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{bmatrix}$$

$$\operatorname{adj}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 2\\2 & 1 & 3\end{bmatrix} = \begin{matrix}\begin{vmatrix}1 & 2\\1 & 3\end{vmatrix} & -\begin{vmatrix}0 & 2\\2 & 3\end{vmatrix} & \begin{vmatrix}0 & 1\\2 & 1\end{vmatrix}\\ -\begin{vmatrix}2 & 3\\1 & 3\end{vmatrix} & \begin{vmatrix}1 & 3\\2 & 3\end{vmatrix} & -\begin{vmatrix}1 & 2\\2 & 1\end{vmatrix}\\ \begin{vmatrix}2 & 3\\1 & 2\end{vmatrix} & -\begin{vmatrix}1 & 3\\0 & 2\end{vmatrix} & \begin{vmatrix}1 & 2\\0 & 1\end{vmatrix}\end{matrix} = \begin{matrix}1 & 4 & -2\\-3 & -3 & 3\\1 & -2 & 1\end{matrix} = \begin{matrix}1 & -3 & 1\\4 & -3 & -2\\-2 & 3 & 1\end{matrix}$$

$$G(s) = C(sI-A)^{-1}\cdot B = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{array}{c}\begin{matrix}+ & - & +\end{matrix}\\ \begin{bmatrix}s & -1 & 0\\ -50 & s & 166.7\\ 0 & 0 & s+400\end{bmatrix}^{-1}\end{array}\begin{bmatrix}0\\0\\2\end{bmatrix}$$

$$\to \det(s^3+400s^2-50s-2000)$$

$$\begin{bmatrix}+\begin{pmatrix}s & 166.7\\0 & s+400\end{pmatrix} & -\begin{pmatrix}-50 & 166.7\\0 & s+400\end{pmatrix} & +0\\ -\begin{pmatrix}-1 & 0\\0 & s+400\end{pmatrix} & +\begin{pmatrix}s & 0\\0 & s+400\end{pmatrix} & -0\\ +\begin{pmatrix}-1 & 0\\s & 166.7\end{pmatrix} & -\begin{pmatrix}s & 0\\-50 & 166.7\end{pmatrix} & +\begin{pmatrix}s & -1\\-50 & s\end{pmatrix}\end{bmatrix} = \begin{bmatrix}s^2+400s & 50s+20000 & 0\\ s+400 & s^2+400s & 0\\ -166.7 & -166.7s & s^2-50\end{bmatrix}$$

$$[1\ 0\ 0]\ \frac{\begin{bmatrix} s^2+400s & s+400 & -166.7 \\ 50s+\cancel{20000} & s^2+\cancel{400}s & -\cancel{166}.7s \\ \cancel{0} & \cancel{0} & \cancel{s^2}-50 \end{bmatrix}}{s^3+400s^2-50s-20000}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$G(s) = \frac{-333.4}{s^3+400s^2-50s-20000}$$