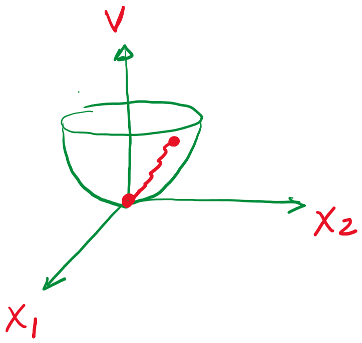
Criterio de Estabilidad de Lyapunov

$$\left.\begin{aligned}\dot{x} &= Ax+Bu\\ u &= -Kx\end{aligned}\right\}\quad \begin{aligned}\dot{x} &= Ax + B(-Kx)\\ \dot{x} &= \underbrace{(A-BK)}_{A_c}x\end{aligned}$$

si los autovalores de A_c son positivos < 0
$\therefore$ El sistema es estable.



$$\left.\begin{aligned}V(x) &\geq 0\\ \dot{V}(x) &< 0\end{aligned}\right\}\ \text{sistema estable}$$

$$V(x) = x^T P x \geq 0$$
$\hookrightarrow$ P positiva

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} \quad ; \quad \dot{x} = A_c x$$

$$\dot{V}(x) = x^T A_c^T P x + x^T P A_c x$$

$$\dot{V}(x) = x^T \underbrace{\left[A_c^T P + P A_c\right]}_{\text{negativo}} x$$

de Ricatti

P; Q; R son matrices simétricas.

$$P^T = P$$
$$Q^T = Q$$
$$R^T = R$$

$$K = R^{-1}B^TP \rightarrow K^T = P^T B R^{-1} = PBR^{-1}$$

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \; - PBR^{-1}B^TP$$
$$+ PBR^{-1}B^TP$$

$$(A^T - PBR^{-1}B^T)P + P(A - BR^{-1}B^TP) + PBR^{-1}B^TP + Q = 0$$

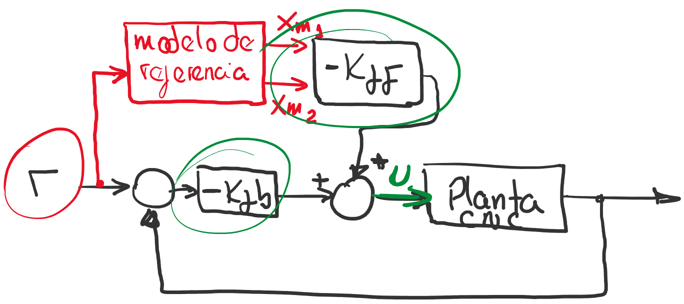
$$(A^T - PBR^{-1}B^T)P + P(A - BR^{-1}B^TP) + PBR^{-1}B^TP + Q = 0$$

$$(\underbrace{A^T - K^TB^T}_{})P + P(\underbrace{A - BK}_{}) + PBR^{-1}B^TP + Q = 0$$

$$\underbrace{A_c^TP + PA_c + \underbrace{PBR^{-1}B^TP}_{>0} + \underbrace{Q}_{>0}}_{\geq 0} = 0$$

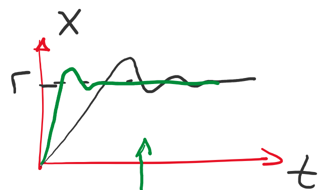
$$\therefore \quad A_c^TP + PA_c < 0$$

Controlador Feedback + Feedforward



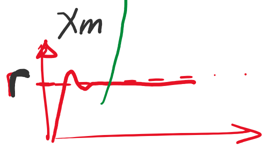
Planta CNC

$$x = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}$$



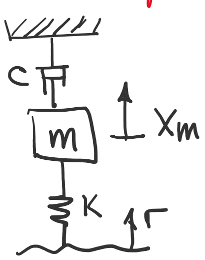
modelo de referencia

$$x_m = \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}$$

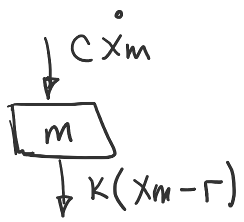


Modelo de referencia

Sky-hook-damper



DCL



$$\Sigma F = m \cdot a$$

$$-k(x_m - r) - c\dot{x}_m = m\ddot{x}_m$$

$$\underline{x_m} = \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}$$

$$\dot{x}_m = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k}{m} \end{bmatrix} r \qquad y_m = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_m} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}$$

$$\dot{x}_m = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}}_{A_m} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{k}{m} \end{bmatrix}}_{B_m} r \qquad y_m = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_m} \begin{bmatrix} x_m \\ \dot{x}_m \end{bmatrix}$$

$$\mathbb{X} = \begin{bmatrix} X_{3\times1} \\ X_{m\,2\times1} \end{bmatrix}$$

$$\dot{\mathbb{X}} = \begin{bmatrix} \dot{X}_{3\times1} \\ \dot{X}_{m\,2\times1} \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} A & 0 \\ \hline 0 & A_m \end{array}\right]}_{\mathbb{A}} \begin{bmatrix} X_{3\times1} \\ X_{m\,2\times1} \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\mathbb{B}} u + \underbrace{\begin{bmatrix} 0 \\ B_m \end{bmatrix}}_{\mathbb{W}} r$$

Función de Costo

$$J = \int_0^\infty \left((y - y_m)^T q (y - y_m) \right) + u^T r u \,)\, dt$$

$$J = \int_0^\infty \left((Cx - C_m X_m)^T q (Cx - C_m X_m) \right) + u^T r u \,)\, dt$$

$$J = \int_0^\infty \Big(\underbrace{\begin{bmatrix} X^T & X_m^T \end{bmatrix}}_{\mathbb{X}^T} \underbrace{\begin{bmatrix} C^T \\ -C_m^T \end{bmatrix} q \begin{bmatrix} C & -C_m \end{bmatrix}} \underbrace{\begin{bmatrix} X \\ X_m \end{bmatrix}}_{\mathbb{X}} + u^T r u \Big) dt$$

$$J = \int_0^\infty \Big(\mathbb{X}^T \underbrace{\begin{bmatrix} C^T q C & -C^T q C_m \\ -C_m^T q C & C_m^T q C_m \end{bmatrix}}_{\mathbb{Q}} \mathbb{X} + u^T r u \Big) dt$$

$$J = \int_0^\infty (\mathbb{X}^T \mathbb{Q} \mathbb{X} + u^T r u) dt$$

Ricatti

$$\mathbb{A}^T P + P \mathbb{A} - P \mathbb{B} r^{-1} \mathbb{B}^T P + \mathbb{Q} = 0 \qquad ; \quad u = -\mathbb{K}\mathbb{X}$$

$$\mathbb{A}^T\mathbb{P} + \mathbb{P}\mathbb{A} - \mathbb{P}\mathbb{B}\mathbb{R}^{-1}\mathbb{B}^T\mathbb{P} + \mathbb{Q} = 0 \quad ; \quad u = -\mathbb{K}\mathbb{X}$$

$$\begin{bmatrix} A^T & 0 \\ 0 & A_m^T \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix}$$

$$-\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} B \\ 0 \end{bmatrix} r^{-1}\begin{bmatrix} B^T & 0 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} C^T q C & -C^T q C_m \\ -C_m^T q C & C_m^T q C_m \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbb{K} = \mathbb{R}^{-1}\mathbb{B}^T\mathbb{P}$$

$$\mathbb{K} = r^{-1}\begin{bmatrix} B^T & 0 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

$$\mathbb{K} = \begin{bmatrix} \underbrace{r^{-1}B^T P_{11}}_{K_{fb}} & \underbrace{r^{-1}B^T P_{12}}_{K_{ff}} \end{bmatrix}$$

$$u = -K_{fb}X - K_{ff}X_m$$

$$\begin{bmatrix} * & \\ & \end{bmatrix} \quad A^T P_{11} + P_{11}A - P_{11}Br^{-1}B^T P_{11} + C^T q C = 0$$

$$q \to \begin{bmatrix} q_1 & & 0 \\ & q_2 & \\ 0 & & q_3 \end{bmatrix}$$

$$\begin{bmatrix} & * \\ & \end{bmatrix} \quad A^T P_{12} + P_{12}A_m - P_{11}Br^{-1}B^T P_{12} + (-C^T q C_m) = 0$$

ecuación de Lyapunov:

$$Ax + xB + Q = 0$$

$$P_{12} = X = \mathrm{lyap}(A, B, Q)$$

$$\underbrace{(A^T - P_{11}Br^{-1}B^T)}_{A_{PR}}\underbrace{P_{12}}_{X} + \underbrace{P_{12}}_{X}\underbrace{A_m}_{B} + \underbrace{(-C^T q C_m)}_{Q} = 0$$