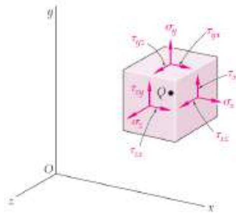
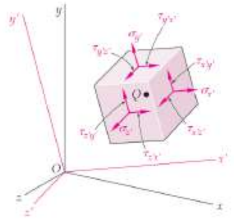
Transformaciones de esfuerzos y deformaciones

Suponga que existe un estado de esfuerzo plano en el punto Q

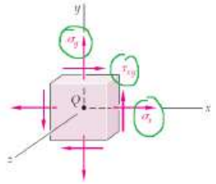


rotando con respecto a z

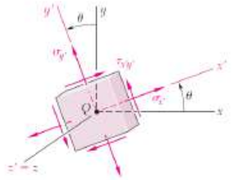


b)

Lo que se busca es una transformación de esfuerzos

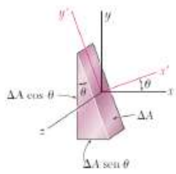


a)

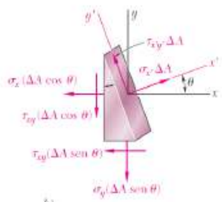


b)

Transformación de esfuerzo plano



a)



b)

$$\Sigma F_{x'} = 0: \quad \sigma_{x'}\,\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\operatorname{sen}\theta - \sigma_y(\Delta A\operatorname{sen}\theta)\operatorname{sen}\theta - \tau_{xy}(\Delta A\operatorname{sen}\theta)\cos\theta = 0$$

$$\Sigma F_{y'} = 0: \quad \tau_{x'y'}\,\Delta A + \sigma_x(\Delta A\cos\theta)\operatorname{sen}\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta - \sigma_y(\Delta A\operatorname{sen}\theta)\cos\theta + \tau_{xy}(\Delta A\operatorname{sen}\theta)\operatorname{sen}\theta = 0$$

Agrupando y despejando:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\operatorname{sen} 2\theta$$

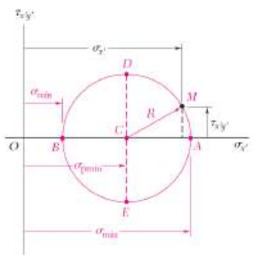
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\operatorname{sen} 2\theta + \tau_{xy}\cos 2\theta$$

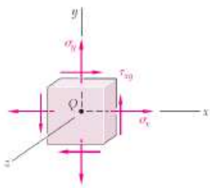
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\operatorname{sen} 2\theta$$

Esfuerzos principales. Esfuerzo cortante máximo

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

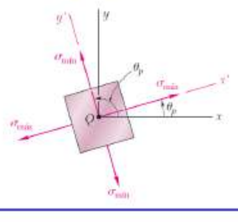
$$\sigma_{\text{prom}} = \frac{\sigma_x + \sigma_y}{2} \qquad \text{y} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$





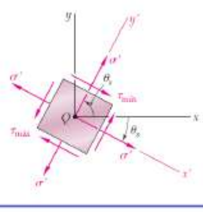
Circulo de Mohr

Esfuerzos principales



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \Rightarrow \quad \sigma_{\text{máx, mín}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Esfuerzo cortante máximo

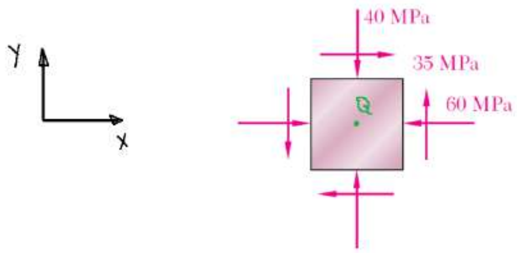


$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\text{máx}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Problema 02

Para el estado de esfuerzo dado, determine a) los planos principales, b) los esfuerzos principales.



$\sigma_x = -60\,MPa$

$\sigma_y = -40\,MPa$

$\tau_{xy} = 35\,MPa$

a) En el sentido de las agujas del reloj ⟶ Arriba *b)* En el sentido contrario a las agujas del reloj ⟶ Abajo

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2 \times 35}{-60-(-40)} = -3{,}5 \qquad 2\theta_p = -74{,}05^\circ$$

$$\theta_p = -37^\circ$$

$\sigma_x = -60\,MPa$

$\sigma_y = -40\,MPa$

$\tau_{xy} = 35\,MPa$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\,\text{sen}\,2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\,\text{sen}\,2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\,\text{sen}\,2\theta + \tau_{xy}\cos 2\theta$$

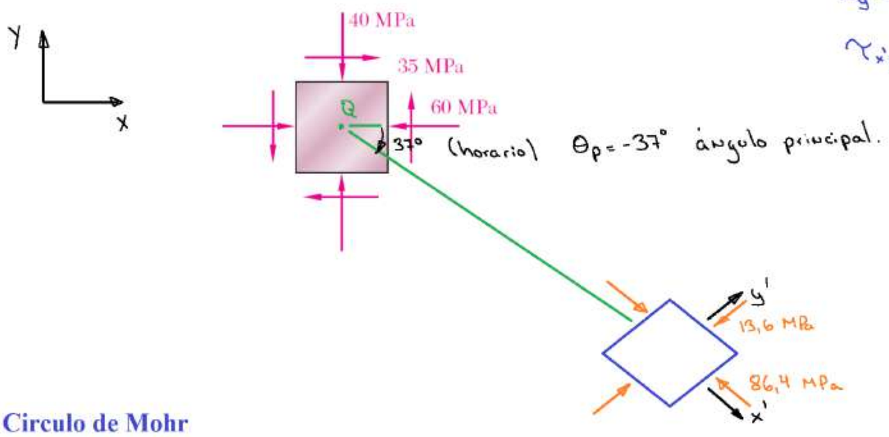
$$\sigma_{x'} = \frac{-60-40}{2} + \frac{-60+40}{2} \times \cos(-74{,}05) + 35.\text{sen}(-74{,}05)$$

$$\sigma_{x'} = -86{,}4\,MPa$$

$$\sigma_{y'} = \frac{-60-40}{2} - \frac{-60+40}{2} \times \cos(-74{,}05) - 35.\text{sen}(-74{,}05)$$

$$\sigma_{y'} = -13{,}6\,MPa$$

$$\tau_{x'y'} = 0$$



Circulo de Mohr

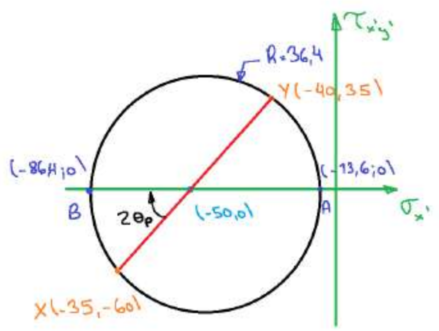
$\sigma_x = -60\,MPa$

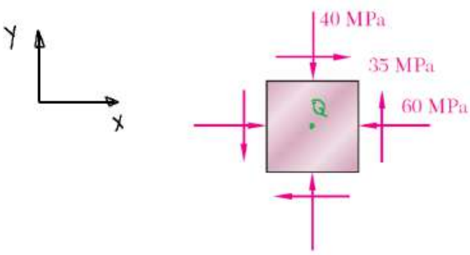
$\sigma_y = -40\,MPa$

$\tau_{xy} = 35\,MPa$

$$\sigma_{prom} = \frac{\sigma_x + \sigma_y}{2} \quad \text{y} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

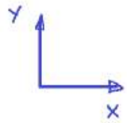
$$\sigma_{prom} = \frac{-60-40}{2} = -50 \qquad R = \sqrt{\left(\frac{-60+40}{2}\right)^2 + 35^2} = 36{,}4$$

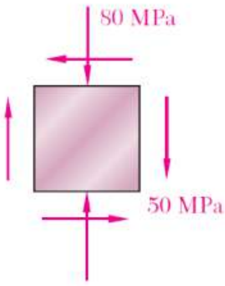




Problema 03

Para el estado de esfuerzo dado, determine los esfuerzos normal y cortante después de girar el elemento mostrado a) 25° en el sentido de las manecillas del reloj, b) 10° en el sentido contrario a las manecillas del reloj.





$$\boxed{\begin{aligned}\sigma_x &= 0\\ \sigma_y &= -80\,MPa\\ \tau_{xy} &= -50\,MPa\end{aligned}}$$

a) $\theta = -25°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\,\text{sen}\,2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\,\text{sen}\,2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\,\text{sen}\,2\theta$$

$$\sigma_{x}' = \frac{0-80}{2} + \frac{0+80}{2}\cdot \text{Cos}(-50) - 50\cdot\text{Sen}(-50)$$

$$\sigma_{x}' = 24{,}01\ MPa$$

$$\tau_{x'y'} = -\frac{0+80}{2}\cdot\text{Sen}(-50) + (-50)\cdot\text{Cos}(-50)$$

$$\tau_{x'y'} = -1{,}5\,MPa$$

$$\sigma_{y}' = \frac{0-80}{2} - \frac{0+80}{2}\cdot\text{Cos}(-50) + 50\cdot\text{Sen}(-50)$$

$$\sigma_{y'} = -104\,MPa$$

b) $\theta = 10°$

$$\sigma_{x'} = \frac{0-80}{2} + \frac{0+80}{2}\cdot\text{Cos}(20) - 50\cdot\text{Sen}(20) = -19{,}51\,MPa$$

$$\tau_{x'y'} = -\frac{0+80}{2}\cdot\text{Sen}(20) + (-50)\cdot\text{Cos}(20) = -60{,}7\,MPa$$

$$\sigma_{y'} = \frac{0-80}{2} - \frac{0+80}{2}\cdot\text{Cos}(20) + 50\cdot\text{Sen}(20) = -60{,}5\,MPa$$

Circulo de Mohr

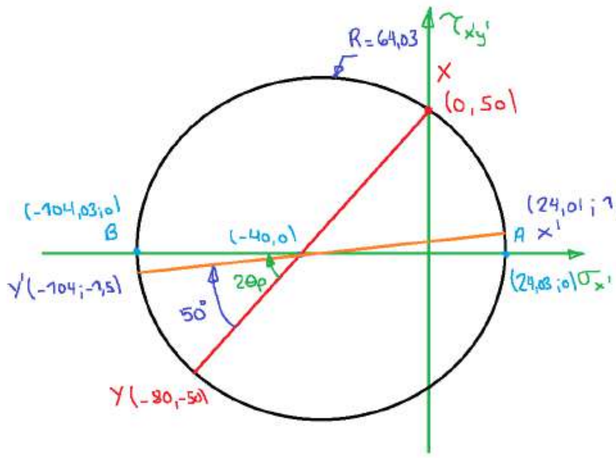
$$\sigma_{\text{prom}} = \frac{\sigma_x + \sigma_y}{2}$$

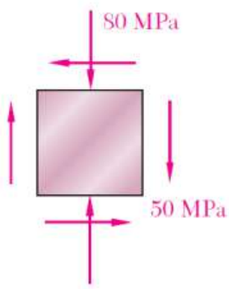
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\begin{aligned}\sigma_x &= 0\\ \sigma_y &= -80\,MPa\\ \tau_{xy} &= -50\,MPa\end{aligned}}$$

$$\sigma_{prom} = \frac{0-80}{2} = -40$$

$$R = \sqrt{\left(\frac{0+80}{2}\right)^2 + (-50)^2} = 64{,}03$$





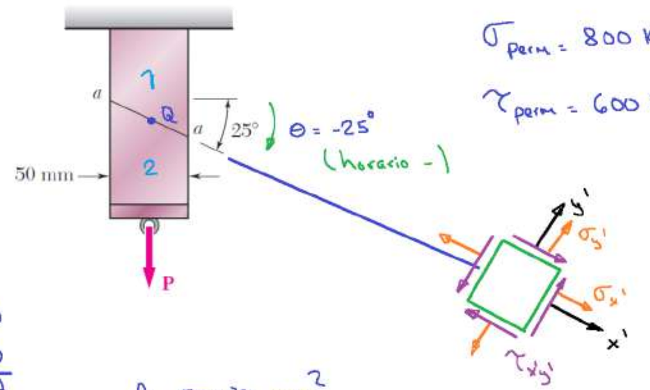
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{Tan}\ 2\theta_p = \frac{2\times(-50)}{0+80}$$

$$2\theta_p = -51{,}34°$$

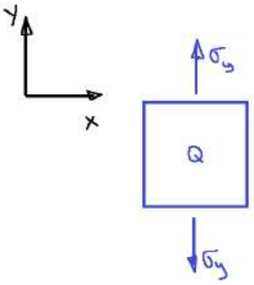
Problema 04

Dos elementos de sección transversal uniforme de 50 x 80 mm se pegan a lo largo del plano $a - a$ que forma un ángulo de 25° con la horizontal. Si se sabe que los esfuerzos permisibles para la junta pegada son σ = 800 kPa y τ = 600 kPa, determine la carga axial máxima **P** que puede aplicarse.



$\sigma_{perm} = 800$ KPa

$\tau_{perm} = 600$ KPa



$\sigma_x = 0$

$\sigma_y = \frac{P}{A}$

$\tau_{xy} = 0$

$A = 50 \times 80 \ mm^2$

$A = 4000 \times 10^{-6} \ m^2$

$A = 4 \times 10^{-3} \ m^2$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\,\mathrm{sen}\, 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\,\mathrm{sen}\, 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\,\mathrm{sen}\, 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{x'} = \frac{0 + \frac{P}{A}}{2} + \frac{0 - \frac{P}{A}}{2} \times \cos(-50) + 0$$

$$\sigma_{x'} = \frac{P}{2A} - \frac{P}{2A} \cdot 0{,}642787 = 0{,}1786 \cdot \frac{P}{A}$$

$$\sigma_{y'} = \frac{0 + \frac{P}{A}}{2} - \frac{0 - \frac{P}{A}}{2} \times \cos(-50) - 0$$

$$\sigma_{y'} = 0{,}82139 \ \frac{P}{A}$$

$$\tau_{x'y'} = -\frac{0 - \frac{P}{A}}{2} \cdot \mathrm{Sen}(-50) + 0$$

$$\tau_{x'y'} = -0{,}383022 \ \frac{P}{A}$$

Evaluando la carga máxima permisible

$\sigma_{perm} = 800$ KPa

$\tau_{perm} = 600$ KPa

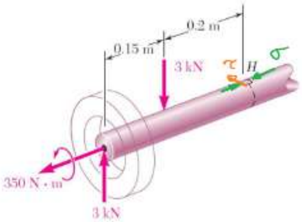
$$\sigma = 800 \times 10^3 \ Pa = 0{,}82139 \cdot \frac{P}{4 \times 10^{-3} \ m^2} \Rightarrow P = 3895{,}84 \ N \approx 3{,}9 \ KN$$

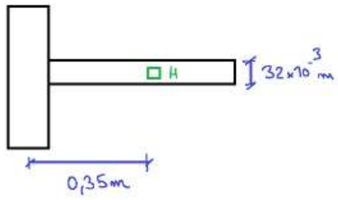
$$\tau = 600 \times 10^3 \ Pa = 0{,}383022 \ \frac{P}{4 \times 10^{-3} \ m^2} \Rightarrow P = 6265{,}96 \ N \approx 6{,}3 \ KN$$

Se escoge el menor: P = 3.9 kN

Problema 05

El eje de un automóvil está sometido a las fuerzas y al par que se muestran en la figura. Si se sabe que el diámetro del eje sólido es de 32 mm, determine a) los planos principales y los esfuerzos principales en el punto H localizado en la parte superior del eje, b) el esfuerzo cortante máximo en el mismo punto.





Para la Torsión:

$$T = 350\,N.m$$

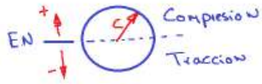
$$\tau = \frac{T.c}{J} \qquad J = \frac{\pi . c^4}{2}$$

$$\tau = \frac{T.c}{\frac{\pi.c^4}{2}} = \frac{2.T}{\pi.c^3} = \frac{2 \times 350\,N.m}{\pi.(16\times10^{-3}\,m)^3} = 54{,}40\,MPa$$

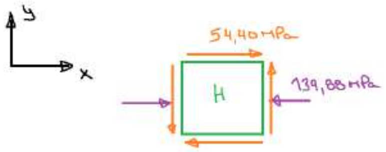
Para la flexión

$$M = 3\,KN \times 0{,}15\,m = 0{,}45\,KN.m$$

Se refiere al signo de c



$$\sigma = -\frac{M.c}{I} = -\frac{0{,}45\times10^3\,N.m \times (16\times10^{-3}\,m)}{\frac{\pi}{4}\times(16\times10^{-3}\,m)^4} = -139{,}88\,MPa$$



$\sigma_x = -139{,}88\,MPa$

$\sigma_y = 0$

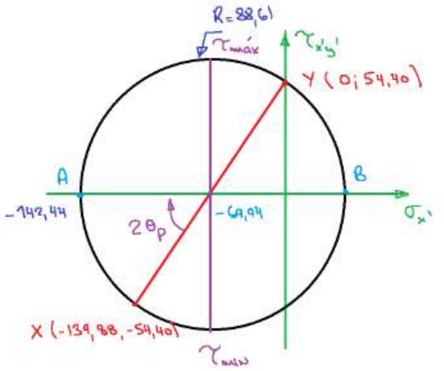
$\tau_{xy} = 54{,}40\,MPa$

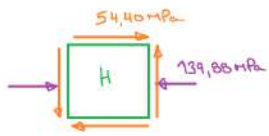
Circulo de Mohr

$$\sigma_{prom} = \frac{\sigma_x + \sigma_y}{2} \qquad \sigma_{prom} = \frac{-139{,}88 + 0}{2} = -69{,}94$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad R = \sqrt{\left(\frac{0 + 139{,}88}{2}\right)^2 + 54{,}40^2} = 88{,}61$$

de la figura $\tau_{máx} = 88{,}61\,MPa$





$\sigma_A = -69{,}94 - 88{,}61 = -142{,}44\,MPa$

$\sigma_B = 88{,}61 - 69{,}94 = 18{,}67\,MPa$

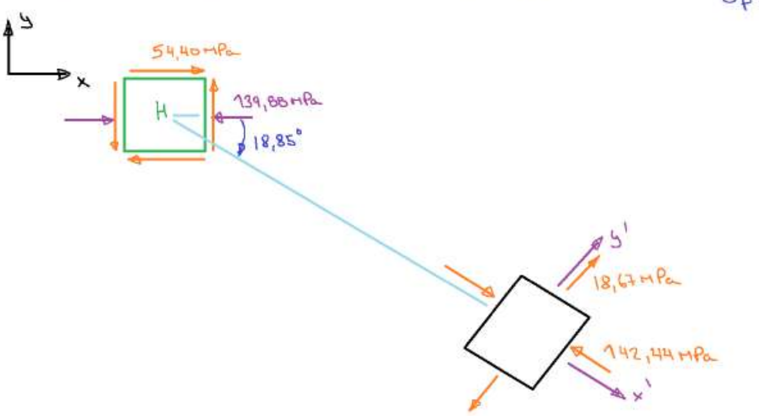
esfuerzos principales

Para el ángulo principal

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \mathrm{Tan}\ 2\theta_p = \frac{2 \times 54{,}40}{-139{,}88 - 0}$$

$2\theta_p = -37{,}7°$

$\theta_p = -18{,}85°$



esfuerzos principales.