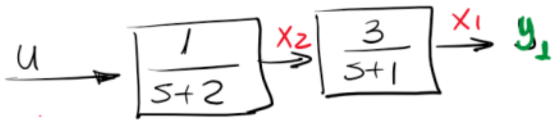
Diseño de Observadores

Ejemplo:



- Diseñar un controlador
  - $M_p\% < 10\%$
  - $T_{s2\%} < 4s$

$U\left(\frac{1}{s+2}\right) = X_2$ | $X_2\left(\frac{3}{s+1}\right) = X_1$

$\dot{x}_2 = -2x_2 + u$ | $\dot{x}_1 = -x_1 + 3x_2$

$$\dot{x} = \underbrace{\begin{bmatrix}-1 & 3\\ 0 & -2\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{B}u$$

$$y = \underbrace{[1\ \ 0]}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$10 = 100\cdot e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\xi = \sqrt{\frac{5.3}{9.87+5.3}} = 0.6$$

$$\frac{4}{\xi\cdot\omega_n} = 4$$

$$\omega_n = 1.67$$

$$\underbrace{\lambda_1;\lambda_2}_{\text{raíces del controlador}} = -\xi\cdot\omega_n \pm \omega_n\sqrt{1-\xi^2}\,j = -1 \pm 1.34j$$

$$\underbrace{\mu_1;\mu_2}_{\text{raíces del observador}} = -5\xi\omega_n \pm \sqrt{1-\xi^2}\,j = -5 \pm 1.34j$$

$$V = \begin{bmatrix}C\\ CA\end{bmatrix} = \begin{bmatrix}1 & 0\\ -1 & 3\end{bmatrix}$$

$|V| = 3 \neq 0$ ✓

rango(v) = 2 ✓

$$\Delta c = |sI - A| = \left|\begin{bmatrix}s+1 & -3\\ 0 & s+2\end{bmatrix}\right| = (s+1)(s+2) = s^2 + \underbrace{3}_{a_1}s + \underbrace{2}_{a_0}$$

$$\therefore M = \begin{bmatrix}3 & 1\\ 1 & 0\end{bmatrix}$$

$$Q = (MV)^{-1} = \left(\begin{bmatrix}3 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 3\end{bmatrix}\right)^{-1} = \begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}^{-1}$$

$$\Delta d_0 = (s+5+1.34j)(s+5-1.34j)$$

$$\Delta d_0 = s^2 + \underbrace{10}_{\alpha_1}s + \underbrace{26.8}_{\alpha_0}$$

$$Q = \frac{\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}}{-3} = \begin{bmatrix}0 & 1\\ 1/3 & -2/3\end{bmatrix}$$

$$K_0' = \begin{bmatrix}\alpha_0 - a_0\\ \alpha_1 - a_1\end{bmatrix} = \begin{bmatrix}24.8\\ 7\end{bmatrix}$$

$$\therefore K_0 = Q K_0' = \begin{bmatrix}0 & 1\\ 1/3 & -2/3\end{bmatrix}\begin{bmatrix}24.8\\ 7\end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 - a_1 \end{bmatrix} \quad \begin{bmatrix} 7 \end{bmatrix} \qquad \qquad \begin{bmatrix} 13 & -43 \end{bmatrix} \begin{bmatrix} + \end{bmatrix}$$

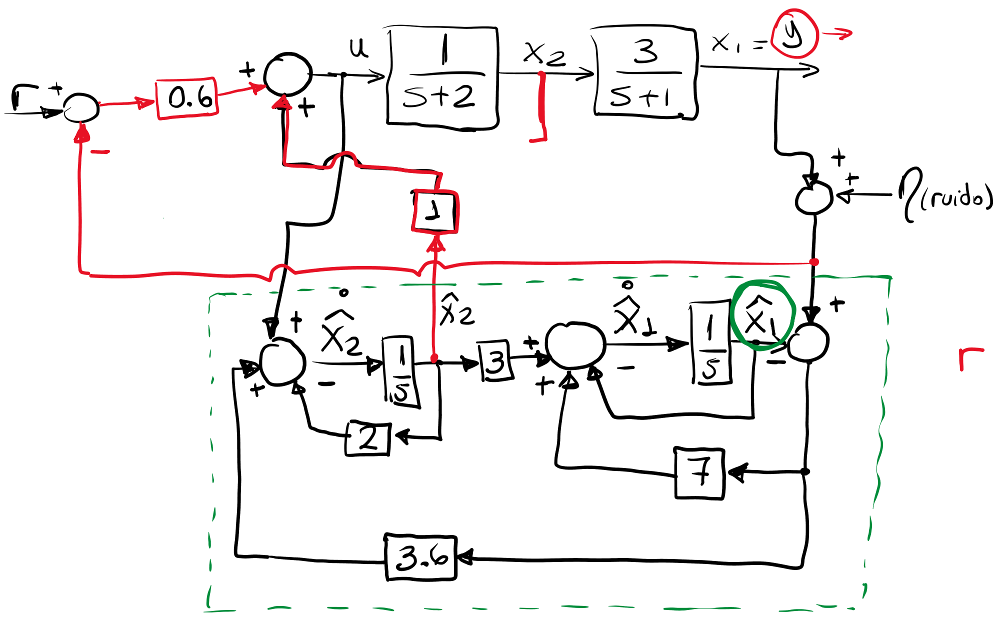
$$K_o = \begin{bmatrix} 7 \\ 3.6 \end{bmatrix} \qquad K_c = \begin{bmatrix} 0.6 & -1 \end{bmatrix}$$

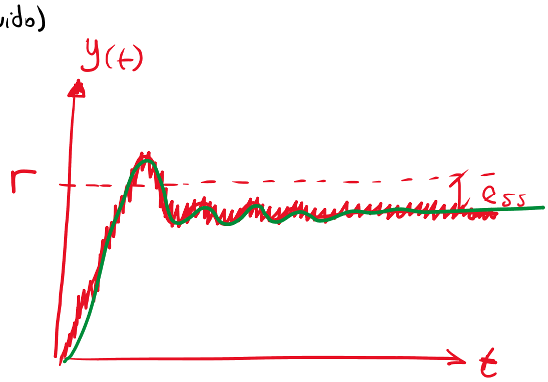
$$\dot{x} = \underbrace{\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K_o(y - \hat{y})$$

$$\dot{\hat{x}}_1 = -\hat{x}_1 + 3\hat{x}_2 + 7(\underline{x_1 - \hat{x}_1})$$

$$\dot{\hat{x}}_2 = -2\hat{x}_2 + u + 3.6(x_1 - \hat{x}_1)$$





OBSERVADOR

SISTEMAS DE SEGUIMIENTO

$$\dot{x} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$x_1 = ✓$

$x_2 = ✓$

$$\dot{x}_3 = r - x_1$$

para garantizar $e_{ss} = 0$ el sistema debe tener un integrador puro.

$$x_3 = \int r - y = \int r - x_1$$

$$\dot{x}_i = \underbrace{\begin{bmatrix} -1 & 3 & 0 \\ 0 & -2 & 0 \\ \underbrace{-1}_{-C} & 0 & 0 \end{bmatrix}}_{A_i} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{B_i} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$S_i = \begin{bmatrix} B_i & A_iB_i & A_i^2B_i \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 & 0 \\ 0 & -2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 & 0 \\ 0 & -2 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} * & -9 & * \\ * & 4 & * \\ * & -3 & * \end{bmatrix}$$

$$S_i = \begin{bmatrix} 0 & 3 & -9 \\ & & \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} * & 4 & * \\ * & -3 & * \end{bmatrix}$$

$$S_i = \begin{bmatrix} 0 & 3 & -9 \\ 1 & -2 & 4 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\Delta c = |sI - A_i| = \left| \begin{bmatrix} s+1 & -3 & 0 \\ 0 & s+2 & 0 \\ 1 & 0 & s \end{bmatrix} \right| = s^3 + \underbrace{3}_{a_2}s^2 + \underbrace{2}_{a_1}s + \underbrace{0}_{a_0}$$

$$M = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad P = S_i M = \begin{bmatrix} 0 & 3 & -9 \\ 1 & -2 & 4 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 1 & 1 \\ -3 & 0 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 0 & -1/3 \\ 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \end{bmatrix}$$

$\lambda_1; \lambda_2 = -1 \pm 1.34j$

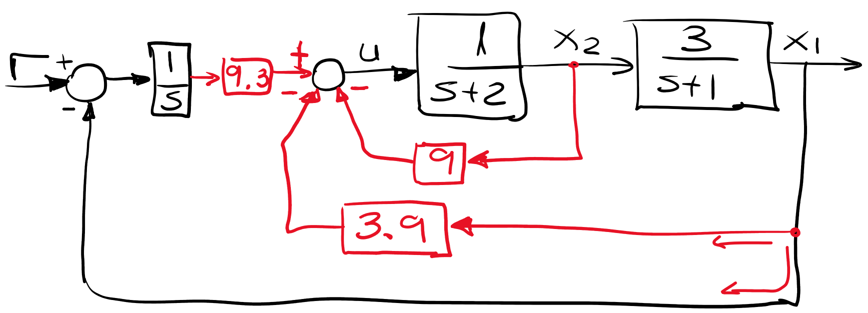
$\lambda_3 = -10$

$$\Delta d_c = (s+1+1.34j)(s+1-1.34j)(s+10)$$
$$= (s^2 + 2s + 2.8)(s+10) = s^3 + \underbrace{12}_{\alpha_2}s^2 + \underbrace{22.8}_{\alpha_1}s + \underbrace{28}_{\alpha_0}$$

$$K^* = [\,28 \quad 20.8 \quad 9\,]$$

$$\therefore K_c = K^* P^{-1} = [28 \;\; 20.8 \;\; 9] \begin{bmatrix} 0 & 0 & -1/3 \\ 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \end{bmatrix}$$

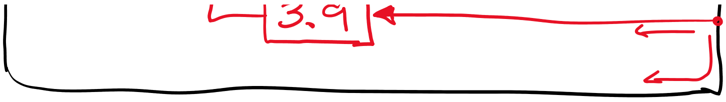
$$K_c = [\underbrace{3.9}_{K_1} \;\; \underbrace{9}_{K_2} \;\; \underbrace{-9.3}_{K_I}]$$



$Mp\% = 10\%$

$Ts_{2\%} = 4s$

$e_{ss} = 0$ ✓


3.9