**1.** Sea la señal $f(t)$ de periodo T=2π dada por $f(t) = e^{-t}$ , $-\pi \le t \le \pi$.

a. Determine el coeficiente $c_n$ de la serie compleja de Fourier.

b. Calcular la suma $S = \sum_{n=-\infty}^{\infty} |c_n|^2$

$$c_0 = \frac{(e^{\pi} - e^{-\pi})}{2\pi}$$

**a)** $\mathbf{f(t) = e^{-t},\ \ -\pi \le t \le \pi,\ \ T = 2\pi,\ \ w_o = \frac{2\pi}{T} = 1}$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_o t}$$

Siendo:

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_o t} dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-t}.\, e^{-jn(1)t} dt$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-(jn+1)t}\, dt = \frac{1}{2\pi}\left[\frac{e^{-(jn+1)t}}{-(jn+1)}\right]\Big|_{-\pi}^{\pi}$$

$$* \int e^{\alpha t} dt = \frac{e^{\alpha t}}{\alpha}$$

$$c_n = \frac{e^{(jn+1)\pi} - e^{-(jn+1)\pi}}{2\pi(jn+1)} = \frac{e^{jn\pi}.e^{\pi} - e^{-jn\pi}.e^{-\pi}}{2\pi(jn+1)}$$

$$c_n = \frac{(-1)^n(e^{\pi} - e^{-\pi})}{2\pi(jn+1)} \quad (Rpta)$$

$\rightsquigarrow -\infty < n < \infty$

$* \; e^{jn\pi} = \underbrace{\cos n\pi}_{(-1)^n} + j\,\cancel{\operatorname{sen} n\pi}^{\,0}$ $\qquad * \; e^{-jn\pi} = \underbrace{\cos n\pi}_{(-1)^n} - j\,\cancel{\operatorname{sen} n\pi}^{\,0}$

$* \; e^{jn\pi} = \underbrace{\cos n\pi} + j\underbrace{\operatorname{sen} n\pi}_{0}$

$e^{jn\pi} = (-1)^n.$

$* \; e^{-jn\pi} = (-1)^n$

**1.** Sea la señal $f(t)$ de periodo T=2π dada por $f(t)=e^{-t}\quad,-\pi\leq t\leq\pi$.

a. Determine el coeficiente $c_n$ de la serie compleja de Fourier.

b. Calcular la suma $S=\sum_{n=-\infty}^{\infty}|c_n|^2$

b) Se tiene: $c_n=\frac{(-1)^n(e^{\pi}-e^{-\pi})}{2\pi(jn+1)}=\frac{(-1)^n(e^{\pi}-e^{-\pi})}{2\pi}.\frac{1}{jn+1},\quad -\infty<n<\infty$

Usando el **teorema de Párseval:**

$$\boxed{S=\sum_{-\infty}^{\infty}|c_n|^2=\frac{1}{T}\int_{-T/2}^{T/2}\{f(t)\}^2dt}$$

$$S=\sum_{-\infty}^{\infty}|c_n|^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\{e^{-t}\}^2dt=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-2t}.dt$$

$$S=\sum_{-\infty}^{\infty}|c_n|^2=\frac{1}{2\pi}.\frac{e^{-2t}}{-2}\Big|_{-\pi}^{\pi}=\frac{1}{2\pi}.\frac{e^{2\pi}-e^{-2\pi}}{2}$$

$$\boxed{S=\sum_{-\infty}^{\infty}|c_n|^2=\frac{e^{2\pi}-e^{-2\pi}}{4\pi}}$$ Rpta.

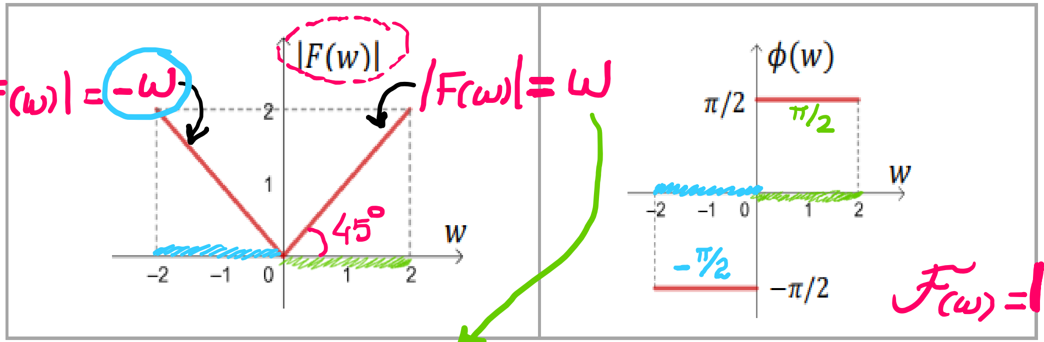
Otra respuesta puede ser, si se tiene en cuenta que:

$$\boldsymbol{senh(U)=\frac{e^{U}-e^{-U}}{2}}$$

$$\boxed{S=\sum_{-\infty}^{\infty}|c_n|^2=\frac{senh(2\pi)}{2\pi}}$$

**2.** Halle la señal cuya transformada de Fourier tiene por módulo $|F(w)|$ y fase $\phi(w)$ que están dadas en la siguiente figura.

$\underbrace{\text{la señal}}_{x(t)}$ $\quad \mathcal{F}(x(t))$ $\quad \underbrace{\phi(w)}_{\text{ARGUMENTO.}}$




$\mathcal{F}(\omega) = |F(\omega)| . e^{j\phi(\omega)}$

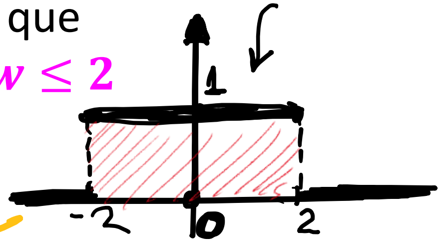
$$F(w) = wj, \qquad -2 \le w \le 2$$

Debe hallarse una señal $x(t)$ tal que

$$\Im(x(t)) = wj . 1 \qquad -2 \le w \le 2$$

O equivalentemente:

$$\Im(x(t)) = wj . \underbrace{P_4(w)}$$




**SOLUCIÓN:**

$$|F(w)| = \begin{cases} w, & 0 \le w \le 2 \\ -w, & -2 \le w < 0 \end{cases}$$

$$\phi(w) = \begin{cases} \frac{\pi}{2}, & 0 \le w \le 2 \\ -\frac{\pi}{2}, & -2 \le w < 0 \end{cases}$$

$F(\omega) = |F(\omega)| e^{j\phi(\omega)}$

$$F(w) = \begin{cases} w e^{\frac{\pi}{2}j} \to j, & 0 \le w \le 2 \\ -w e^{-\frac{\pi}{2}j} \to -j, & -2 \le w < 0 \end{cases}$$

$* \; e^{\frac{\pi}{2}j} = \cos\frac{\pi}{2} + j\,\text{sen}\frac{\pi}{2}$ $\quad 0 \quad 1$

$e^{\frac{\pi}{2}j} = j$

$* \; e^{-\frac{\pi}{2}j} = -j$

$$F(w) = \begin{cases} wj, & 0 \le w \le 2 \\ -w(-j), & -2 \le w < 0 \end{cases}$$

**Por formula 18:** $\Im\left(\frac{sen(2t)}{\pi t}\right) = P_4(w)$

**Por fórmula 9:** $\Im\left(\left(\frac{sen(2t)}{\pi t}\right)'\right) = (jw). P_4(w) \to \mathcal{F}(x(t))$

Luego $x(t) = \frac{1}{\pi} . \left(\frac{sen(2t)}{t}\right)' = \frac{2\cos(2t)t - sen(2t)}{\pi t^2}$

| | | | |
|---|---|---|---|
| 9 | $\mathcal{F}\left(f^{(n)}(t)\right)$ | $=$ | $(jw)^n F(w)$ |
| 18 | $\mathcal{F}\left(\frac{\text{sen}(at)}{\pi t}\right)$ | $=$ | $P_{2a}(w)$ |

**3.** Halle la señal cuya transformada de Fourier es:

a. $X(w) = \dfrac{jw+2}{-w^2+jw+1}$ b. $X(w) = \dfrac{jw+3}{-jw^3-6w^2+8jw}$

$(jw)^2 = -w^2$

| | | | |
|---|---|---|---|
| 21 | $\mathcal{F}\left(e^{-at}\text{sen}(bt)\,u(t)\right)$ | $=$ | $\dfrac{b}{(jw+a)^2+b^2}\,,\ a>0$ |
| 22 | $\mathcal{F}\left(e^{-at}\cos(bt)\,u(t)\right)$ | $=$ | $\dfrac{jw+a}{(jw+a)^2+b^2}\,,\ a>0$ |

a)

$$\Rightarrow \Im(x(t)) = X(w) = \frac{jw+2}{(jw)^2+jw+1} = \frac{jw+2}{(jw+\frac{1}{2})^2+\frac{3}{4}}$$

$2 \to \frac{1}{2}+\frac{3}{2}$

$$\Im(x(t)) = \frac{jw+\frac{1}{2}+\frac{3}{2}}{(jw+\frac{1}{2})^2+\frac{3}{4}} = \frac{jw+\frac{1}{2}}{(jw+\frac{1}{2})^2+\frac{3}{4}} + \frac{\frac{3}{2}}{(jw+\frac{1}{2})^2+\frac{3}{4}}$$

$3 \to \sqrt{3}.\sqrt{3}$

$$\Im(x(t)) = \frac{jw+\frac{1}{2}}{(jw+\frac{1}{2})^2+\frac{3}{4}} + \sqrt{3}.\frac{\frac{\sqrt{3}}{2}}{(jw+\frac{1}{2})^2+\frac{3}{4}}$$

#22 #21

$$\cancel{\Im}(x(t)) = \cancel{\Im}\left(e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)u(t)\right) + \sqrt{3}.\cancel{\Im}\left(e^{-\frac{1}{2}t}\text{sen}\left(\frac{\sqrt{3}}{2}t\right)u(t)\right)$$

$$x(t) = e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)u(t) + \sqrt{3}.e^{-\frac{1}{2}t}\text{sen}\left(\frac{\sqrt{3}}{2}t\right)u(t).$$

**3.** Halle la señal cuya transformada de Fourier es:

a. $X(w) = \dfrac{jw+2}{-w^2+jw+1}$  b. $X(w) = \dfrac{jw+3}{-jw^3 - 6w^2 + 8jw}$

$(jw)^3 + 6(jw)^2$

**b)**

$$\Im(x(t)) = X(w) = \boxed{\frac{jw+3}{(jw)^3+6(jw)^2+8(jw)}} = \frac{jw+3}{(jw)(jw+2)(jw+4)} = \frac{A}{jw} + \frac{B}{jw+2} + \frac{C}{jw+4}$$

Usando fracciones parciales (deben mostrar el procedimiento)

$$\Im(x(t)) = \frac{3/8}{jw} - \frac{1/4}{jw+2} - \frac{1/8}{jw+4}$$

$$\cancel{\Im}(x(t)) = \frac{3}{16}.\cancel{\Im}(\mathrm{sgn}(t)) - \frac{1}{4}.\cancel{\Im}(e^{-2t}u(t)) - \frac{1}{8}.\cancel{\Im}(e^{-4t}u(t))$$

$$x(t) = \frac{3}{16}(\mathrm{sgn}(t)) - \frac{1}{4}(e^{-2t}u(t)) - \frac{1}{8}(e^{-4t}u(t))$$

\#43

$* \; \frac{3}{8} \cdot \frac{1}{jw} = \frac{3}{16} \cdot \frac{2}{jw} = \frac{3}{16} \mathcal{F}(\mathrm{sgn}(t))$

$* \; \frac{1}{4} \cdot \frac{1}{jw+2} = \frac{1}{4} \mathcal{F}(e^{-2t} \cdot u(t))$

$* \; \frac{1}{8} \cdot \frac{1}{jw+4} = \frac{1}{8} \mathcal{F}(e^{-4t} \cdot u(t))$

\#14

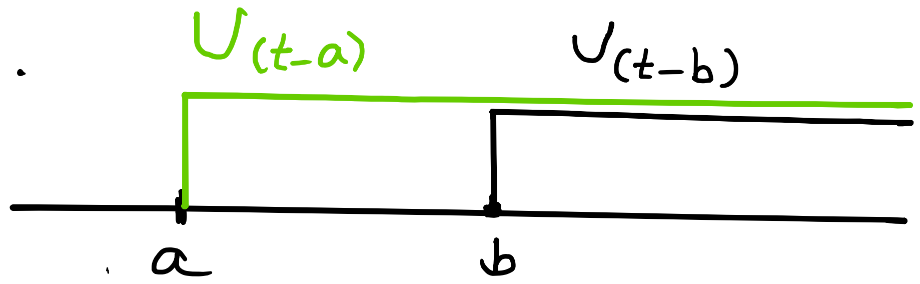
| | | | |
|---|---|---|---|
| 14 | $\mathcal{F}(e^{-at}u(t))$ | = | $\dfrac{1}{jw+a}$, $a>0$ |

| | | | |
|---|---|---|---|
| 43 | $\mathcal{F}(\mathrm{sgn}(t))$ | = | $2/(jw)$ |

$$\frac{\mathcal{F}(\mathrm{sgn}(t))}{2} = \frac{1}{jw} \Leftarrow$$

**4.** Halle la transformada de Fourier de:

$$f(t) = -2[u(t+3) - u(t+1)] + 2[u(t-1) - u(t-3)]$$




$$\mathfrak{I}(f(t)) = -2\mathfrak{I}(P_2(t+2)) + \mathfrak{I}(P_2(t-2))$$

$$U_{(t-a)} - U_{(t-b)} = P_{(b-a)}\left(t - \frac{a+b}{2}\right)$$

$t=a$ $t=b$

Se sabe que:

$$\mathfrak{I}(P_2(t)) = \frac{2sen(w)}{w} \rightarrow \mathfrak{I}(P_2(t+2)) = e^{+2jw}.\frac{2sen(w)}{w}$$

$$\mathfrak{I}(P_2(t)) = \frac{2sen(w)}{w} \rightarrow \mathfrak{I}(P_2(t-2)) = e^{-2jw}.\frac{2sen(w)}{w}$$

$$\mathfrak{I}(f(t)) = -2e^{+2jw}.\frac{2sen(w)}{w} + e^{-2jw}.\frac{2sen(w)}{w}$$

traslación en $t$

| | | |
|---|---|---|
| 4 | $f(t-a)$ | $e^{-jwa}F(w)$ |
| 17 | $P_a(t)$ | $\frac{asen(wa/2)}{wa/2}$ , $a > 0$ |

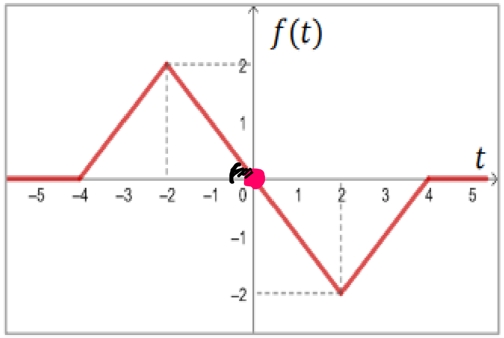
**5.** Dada la señal $f(t)$ en la figura mostrada:

a. Halle $\int_{-\infty}^{+\infty} F(w)dw$

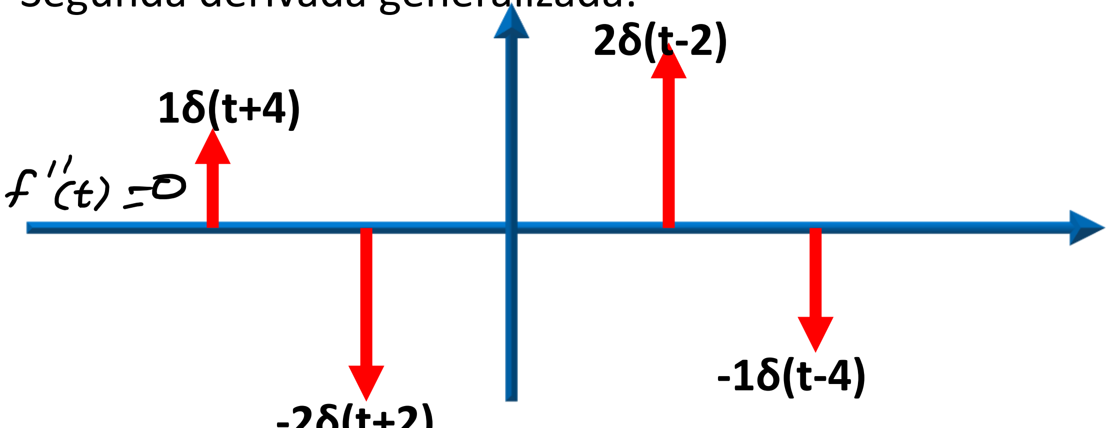
b. Halle la 1ra y 2da derivada generalizada.

c. Determine la transformada de Fourier de la 2da derivada generalizada.

d. Deduzca la transformada de Fourier a partir de lo hallado en el ítem c.




Segunda derivada generalizada:




a) Por **definición de transformada inversa**:

$$\boldsymbol{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{jwt}dw}$$

$$\rightarrow f(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{jw0}dw$$

$$\int_{-\infty}^{\infty} F(w)\, dw = 2\pi f(0) = 2\pi(0) = 0$$

C) $f''(t) = 1\delta(t+4) - 2\delta(t+2) + 2\delta(t-2) - 1\delta(t-4)$

$$\Im(f''(t)) = \Im(1\delta(t+4) - 2\delta(t+2) + 2\delta(t-2) - 1\delta(t-4))$$

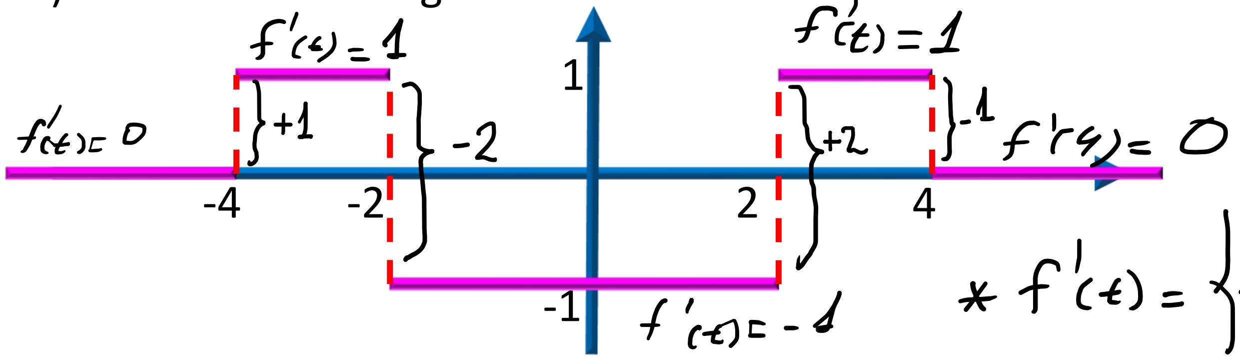
$$\Im(f''(t)) = e^{+4jw} - 2e^{+2jw} + e^{-2jw} - e^{-4jw}$$

| | | |
|---|---|---|
| 27 | $\delta(t-a)$ | $e^{-jwa}$ |

$$\mathcal{F}[f(t)] = F(w) = \int_{-\infty}^{+\infty} f(t)e^{-jwt}dt$$

$$f(t) = \mathcal{F}^{-1}[F(w)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(w)e^{jwt}dw$$

b) Primera derivada generalizada:




$$* f'(t) = \begin{cases} 1, & t\in[-4,-2]\cup[2,4] \\ -1, & -2\le t\le 2 \\ 0, & \text{en otros casos.} \end{cases}$$

d) La transformada de Fourier de f(t) es

$\mathcal{F}(f(t))$

$$\boldsymbol{\Im(f(t)) = F(w) \rightarrow \Im(f''(t)) = (jw)^2 F(w)}$$

$$e^{j\theta} - e^{-j\theta} = 2j\,\text{sen}\,\theta$$

Luego, se puede igualar con lo hallado en ( c ):

$$\underbrace{\Im(f''(t))} = e^{+4jw} - 2e^{+2jw} + e^{-2jw} - e^{-4jw}$$

$$(jw)^2 F(w) = e^{4jw} - 2e^{2jw} + e^{-2jw} - e^{-4jw}$$

$$F(w) = \frac{1}{(jw)^2}\left[e^{4jw} - 2e^{2jw} + e^{-2jw} - e^{-4jw}\right]$$

$\mathcal{F}(f(t))$ .

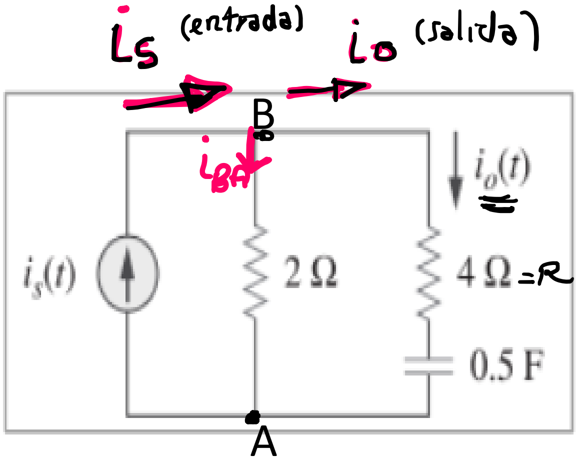
**6.** El siguiente circuito es un sistema LTI.

a. Halle la función de transferencia del sistema si la entrada es la corriente $i_s(t)$ y la salida es corriente $i_0(t)$.

b. Encuentre la ecuación diferencial del sistema asociado.

c. Determine la respuesta al impulso unitario.

d. Determine la respuesta al escalón unitario.




$En\ B$: $\boldsymbol{i_s = i_o + i_{BA}} \rightarrow \boldsymbol{i_{BA} = i_s - i_o}$

$$\boldsymbol{V_{BA} = R.io + \frac{1}{C}\int_0^t i(x)dx = R_{BA}.i_{BA}} \rightarrow \boldsymbol{4io + 2\int_0^t i_o(x)dx = 2(i_s - i_o)}$$

Ahora se deriva: $4i'_o + 2i_o = 2i'_s - 2i'_0$

**(b)** La ecuación diferencial es: $\boldsymbol{6i'_o + 2i_o = 2i'_s} \rightarrow \boxed{\boldsymbol{3i'_o + i_o = i'_s}}$

(a) Tomamos Transformada de Fourier:

$$\boldsymbol{3\Im(i'_o) + \Im(i_o) = \Im(i'_s)} \rightarrow \boldsymbol{3(jw)I_o(w) + I_o(w) = (jw)I_s(w)}$$

Función de transferencia: $\mathrm{H(w)} = \dfrac{\boldsymbol{I_o(w)}}{\boldsymbol{I_s(w)}} = \dfrac{(jw)}{3(jw)+1}$ Rpta.

salida entrada

$\delta(t) \rightarrow \mathcal{F}(\delta(t)) = 1$

( c ) Respuesta al impulso unitario $\boldsymbol{I_s(w) = 1}$ :

$$\mathrm{H(w)} = \frac{\boldsymbol{I_o(w)}}{\boldsymbol{1}} = \frac{(jw)}{3(jw)+1}$$

$$\boldsymbol{I_o(w)} = \frac{(jw)}{3(jw)+1} = \frac{1}{3}\left[\frac{(jw)}{(jw)+\frac{1}{3}}\right]$$

$$\boldsymbol{I_o(w)} = \frac{1}{3}\left[\frac{(jw)+\frac{1}{3}-\frac{1}{3}}{(jw)+\frac{1}{3}}\right] = \frac{1}{3}\left[1 - \frac{1}{3}\cdot\frac{1}{\mathrm{jw}+\frac{1}{3}}\right]$$

$$\boldsymbol{I_o(w)} = \frac{\boldsymbol{1}}{\boldsymbol{3}}[\Im(\delta(\boldsymbol{t})) - \frac{\boldsymbol{1}}{\boldsymbol{3}}\Im(e^{-\frac{1}{3}t}u(t))]$$

$\mathcal{F}(i_o(t))$

$$\boldsymbol{io(t)} = \frac{\boldsymbol{1}}{\boldsymbol{3}}[\delta(\boldsymbol{t}) - \frac{\boldsymbol{1}}{\boldsymbol{3}}e^{-\frac{1}{3}t}u(t)]$$

(d) Respuesta al escalón unitario.

Se tiene que $\mathrm{H}(\mathrm{w}) = \dfrac{\boldsymbol{I_o(w)}}{\boldsymbol{I_s(w)}} = \dfrac{(jw)}{3(jw)+1}$

Siendo ahora: $\boldsymbol{I_s(w)} = \Im\big(\boldsymbol{u(t)}\big) = \pi\delta(w) + \dfrac{1}{jw}$

$$\boldsymbol{I_o(w)} = \frac{(jw)}{3(jw)+1}.\left(\pi\delta(w) + \frac{1}{jw}\right)$$

$$\boldsymbol{I_o(w)} = \frac{(jw)}{3(jw)+1}.\pi\delta(w) + \frac{(jw)}{3(jw)+1}.\frac{1}{jw}$$

$$\boldsymbol{I_o(w)} = \frac{(0)}{3(0)+1}.\pi\delta(w) + \frac{(jw)}{3(jw)+1}.\frac{1}{jw}$$

$$\boldsymbol{I_o(w)} = 0 + \frac{1}{3(jw)+1}$$

$$\boldsymbol{I_o(w)} = 0 + \frac{1}{3}.\frac{1}{(jw)+\frac{1}{3}}$$

$$\boldsymbol{I_o(w) = 0 + \frac{1}{3}}.\Im\left(e^{-\frac{1}{3}t}u(t)\right)$$

| | | |
|---|---|---|
| 30 | $u(t)$ | $\pi\delta(w) + \frac{1}{jw}$ |

| | | |
|---|---|---|
| 32 | $1$ | $2\pi\delta(w)$ |

| | | |
|---|---|---|
| 14 | $e^{-at}u(t)$ | $\frac{1}{jw+a}$ , $a > 0$ |

$$\boldsymbol{io(t) = \frac{1}{3}}.e^{-\frac{1}{3}t}u(t)$$