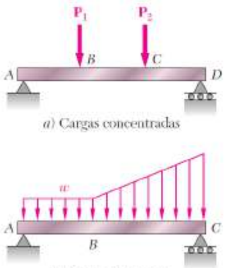
Análisis y diseño de vigas para flexión

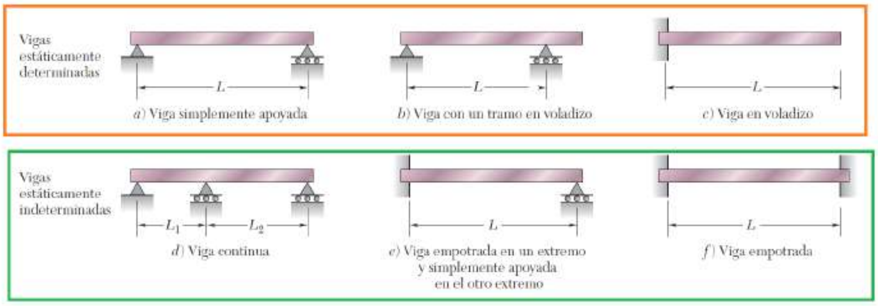
Vigas cargas transversalmente



a) Cargas concentradas

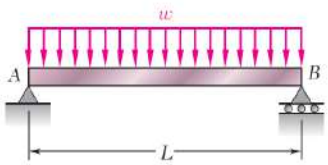
b) Carga distribuida

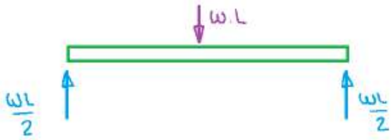
Configuraciones de apoyo en vigas

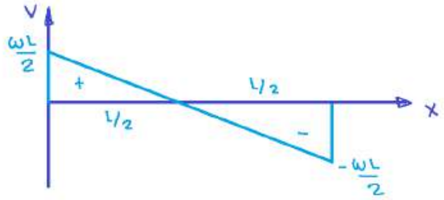


Problema 01

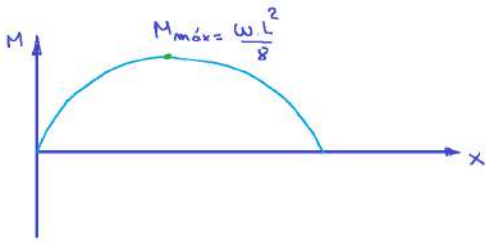
Para la viga y las cargas que se muestran en la figura, a) dibuje los diagramas de cortante y de momento flector, b) determine las ecuaciones de las curvas de cortante y de momento flector.

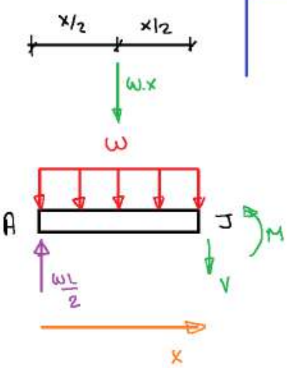






$$A_+ = \frac{L}{2} \cdot \frac{wL}{2} \cdot \frac{1}{2} = \frac{w.L^2}{8}$$





$$\sum F_y = 0 \qquad \frac{wL}{2} - w.x - V = 0$$

$$V = w\left(\frac{L}{2} - x\right)$$

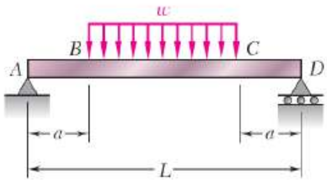
$$+\circlearrowleft \sum M_J = 0$$

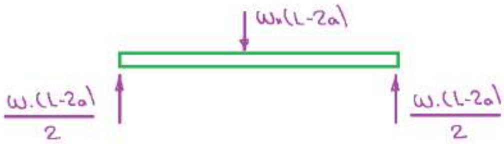
$$-\frac{wL}{2} \cdot x + wx \cdot \frac{x}{2} + M = 0$$

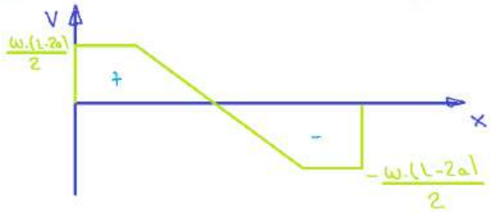
$$M = \frac{w}{2}\left(Lx - x^2\right)$$

Problema 02

Para la viga y las cargas que se muestran en la figura, a) dibuje los diagramas de cortante y de momento flector, b) determine las ecuaciones de las curvas de cortante y de momento flector.

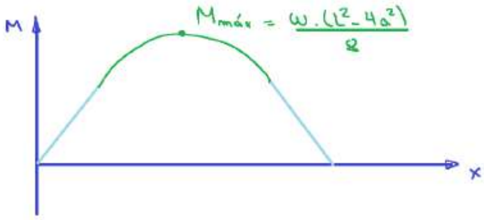




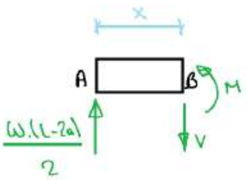


$$A_+ = \frac{\omega.(L-2a)}{2}.a + \frac{\omega.(L-2a)}{2}.\frac{(L-2a)}{2}.\frac{1}{2}$$

$$A_+ = \frac{\omega.(L^2-4a^2)}{8}$$



Para la sección A - B

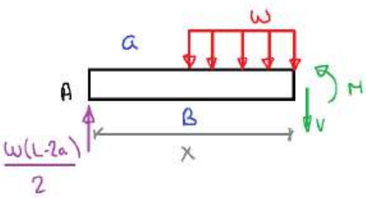


$0 < x < a$

$$\Sigma F_y = 0 \qquad \frac{\omega.(L-2a)}{2} - V = 0 \Rightarrow V = \frac{\omega.(L-2a)}{2}$$

$$+\circlearrowleft \Sigma M = 0 \qquad -\frac{\omega.(L-2a)}{2}.x + M = 0 \Rightarrow M = \frac{\omega.(L-2a)}{2}.x$$

Para la sección B - C

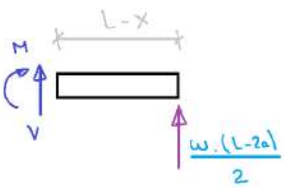


$a < x < L-a$

$$\Sigma F_y = 0 \qquad \frac{\omega.(L-2a)}{2} - \omega.(x-a) - V = 0 \Rightarrow V = \omega.\left(\frac{L}{2} - x\right)$$

$$\Sigma M = 0 \qquad \frac{\omega.(L-2a)}{2}.x + \omega(x-a).\frac{(x-a)}{2} + M = 0 \Rightarrow M = \frac{1}{2}\omega\left[(L-2a)x - (x-a)^2\right]$$

Para la sección C - D



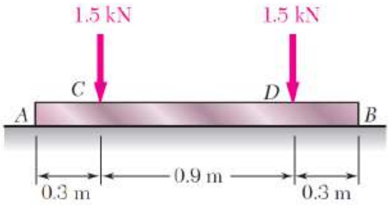
$L-a < x < L$

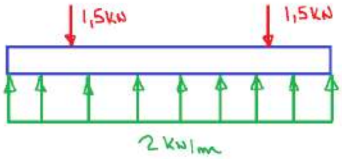
$$\Sigma F_y = 0 \qquad V + \frac{\omega(L-2a)}{2} = 0 \Rightarrow V = -\frac{1}{2}\omega.(L-2a)$$

$$+\circlearrowleft \Sigma M = 0 \qquad -M + \frac{\omega.(L-2a)}{2}.(L-x) = 0 \Rightarrow M = \frac{1}{2}\omega(L-2a).(L-x)$$

Problema 04

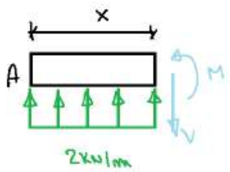
Si se supone que la reacción del suelo está uniformemente distribuida, dibuje los diagramas de cortante y de momento flector para la viga AB y determine el máximo valor absoluto a) del esfuerzo cortante, b) del momento flector.





Para el sector A - C

$0 < x < 0,3$

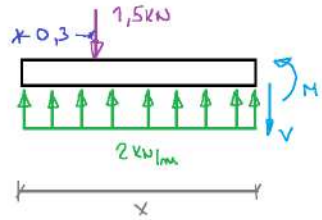


$\Sigma F_y = 0 \qquad 2x - V = 0 \Rightarrow V = 2x$

$\Sigma M = 0 \qquad -2x \cdot \left(\frac{x}{2}\right) + M = 0 \Rightarrow M = x^2$

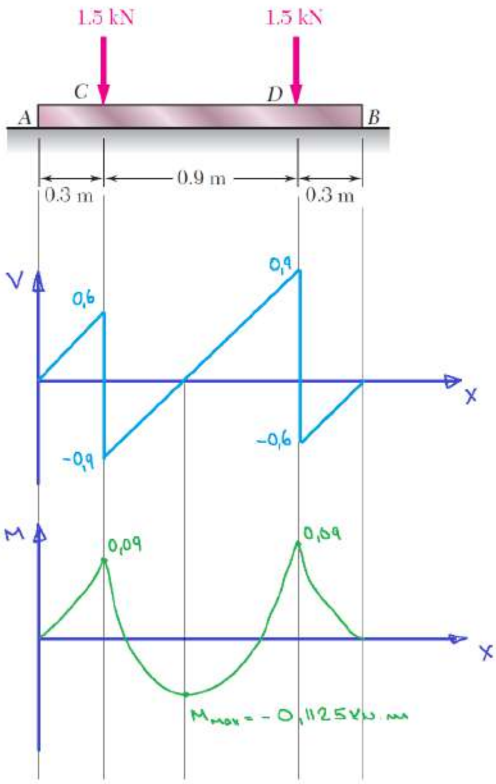
Sector C - D

$0,3 < x < 1,2$



$\Sigma F_y = 0 \qquad -1,5 + 2x - V = 0 \Rightarrow V = 2x - 1,5$

$\Sigma M = 0 \qquad 1,5 \cdot (x - 0,3) - 2x \cdot \left(\frac{x}{2}\right) + M = 0 \Rightarrow M = (x^2 - 1,5x + 0,45)$



$M = (x^2 - 1,5x + 0,45) \qquad x = 0,75 \qquad M = -0,1125 \text{ kN.m}$

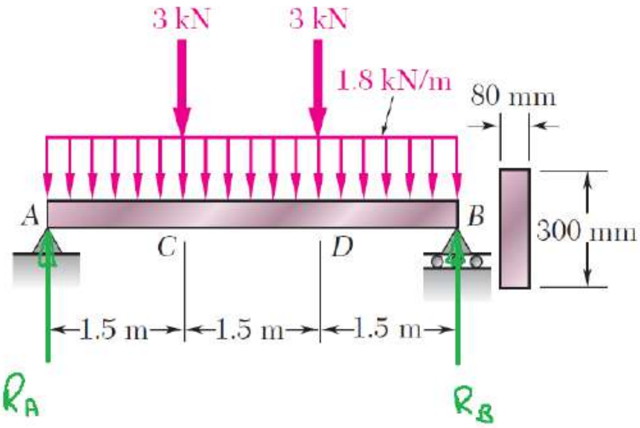
$M = (x^2 - 1,5x + 0,45)$ para $x = 1,2 \text{ m} \qquad M = 0,09$

a) $|V_{máx}| = 0,9 \text{ kN} = 900 \text{ N}$

$|M_{máx}| = 0,1125 \text{ kN.m} = 112,5 \text{ N.m}$

Problema 05

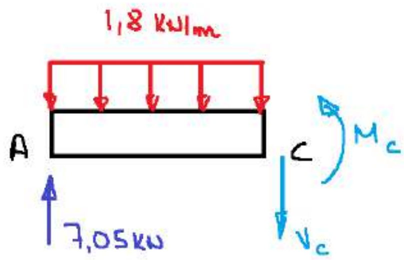
Para la viga y las cargas mostradas en la figura, determine el esfuerzo normal máximo debido a la flexión sobre un corte transversal en C.



$\sum M_B = 0$

$$-R_A \cdot 4{,}5 + 3\times 3 + 3\times 1{,}5 + (1{,}8\times 4{,}5)\times 2{,}25 = 0$$

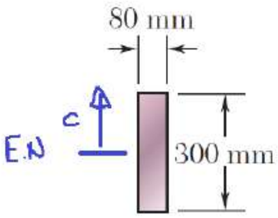
$$R_A = 7{,}05\ kN$$



$\sum M_c = 0$

$$M_c - 7{,}05\,kN \cdot 1{,}5\,m + \left(1{,}8\,\frac{kN}{m}\times 1{,}5\,m\right)\cdot 0{,}75\,m = 0$$

$$M_c = 8{,}55\ kN\cdot m$$

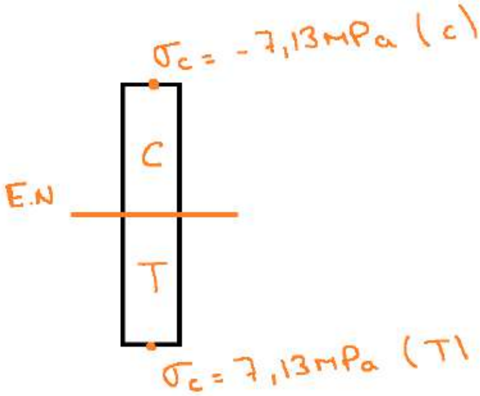


Cálculo del momento de inercia

$$I = \frac{1}{12}\cdot(300)^3\cdot 80 = 180\times10^{6}\ mm^4 = 180\times10^{-6}\ m^4$$

$C = 150\ mm$

$$\sigma_c = -\frac{M_c\cdot C}{I} = -\frac{8{,}55\times10^{3}\ N\cdot m\cdot(0{,}15\,m)}{180\times10^{-6}\ m^4}$$

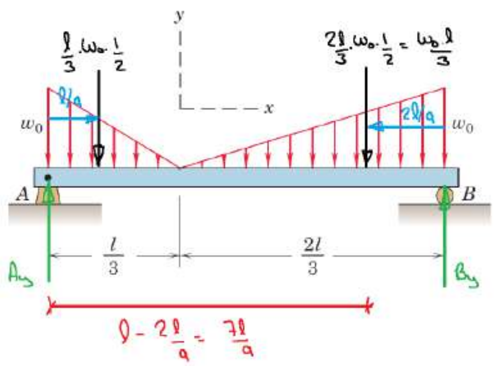


$$\sigma_c = -7{,}125\times10^{6}\ Pa$$

$$\sigma_c = -7{,}13\ MPa \quad (\text{compresión})$$

Problema 17

Calculate the support reactions at A and B for the loaded beam.



$\circlearrowleft^{+}\ \Sigma M_A = 0$

$$-\frac{w_0 l}{6}\cdot\frac{l}{3} - \frac{w_0 l}{3}\cdot\frac{7l}{9} + B_y\cdot l = 0$$

$$B_y = \frac{5}{18}\, w_0 l$$

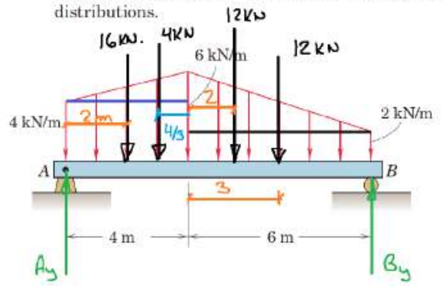
$$\Sigma F_y = 0$$

$$A_y - \frac{w_0 l}{6} - \frac{w_0 l}{3} + B_y = 0$$

$$A_y = \frac{2}{9}\cdot w_0 l$$

Problema 18

Calculate the support reactions at A and B for the beam subjected to the two linearly varying load distributions.



$\circlearrowleft^{+}\ \Sigma M_A = 0$

$$-16\,KN\cdot 2m - 4KN\cdot 2{,}67m - 12KN\cdot 6m - 12KN\cdot 7m + B_y\cdot 10 = 0$$

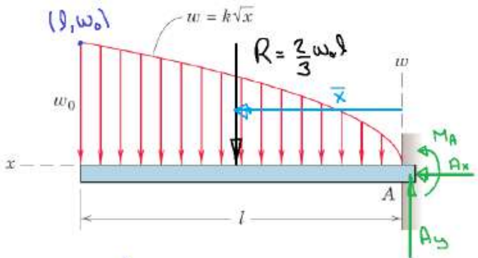
$$B_y = 19{,}87\ KN$$

$$\Sigma F_y = 0$$

$$A_y - 44\,KN + B_y = 0 \qquad A_y = 24{,}13\ KN$$

Problema 21

Determine the force and moment reactions at the support A of the cantilever beam subjected to the load distribution shown.



$$w = k\cdot\sqrt{x}$$

$$w_0 = k\cdot\sqrt{l} \qquad k = \frac{w_0}{\sqrt{l}}$$

$$w = \frac{w_0}{\sqrt{l}}\cdot\sqrt{x}$$

$$R = \int w\,dx = \int_0^l \frac{w_0}{\sqrt{l}}\cdot\sqrt{x}\,dx = \frac{w_0}{\sqrt{l}}\cdot\left(\frac{2}{3}x^{3/2}\right)\Big|_0^l = \frac{2}{3}\cdot w_0\cdot l$$

$$\bar{x} = \frac{\int x\cdot w\cdot dx}{\int w\,dx} \qquad \int x\cdot w\cdot dx = \int_0^l x\cdot\frac{w_0}{\sqrt{l}}\cdot\sqrt{x}\cdot dx$$

$$\int_0^l x\cdot\frac{w_0}{\sqrt{l}}\cdot\sqrt{x}\cdot dx = \frac{w_0}{\sqrt{l}}\cdot\int_0^l x^{3/2}\cdot dx = \frac{2}{5}\frac{w_0}{\sqrt{l}}\cdot x^{5/2}\Big|_0^l = \frac{2}{5} w_0\cdot l^{5/2-1/2} = \frac{2}{5} w_0\cdot l^2$$

$$\bar{x} = \frac{\int x\cdot w\cdot dx}{\int w\,dx} = \frac{\frac{2}{5} w_0\cdot l^{\cancel{2}}}{\frac{2}{3}\cdot w_0\cdot \cancel{l}} = \frac{3}{5}\cdot l$$

Por condicion de equilibrio:

$$\Sigma M_A = 0$$

$$\frac{2}{3} w_0 l\cdot\frac{3}{5}l + M_A = 0 \qquad M_A = -\frac{2}{5} w_0\cdot l^2$$

$$A_y = \frac{2}{3} w_0\cdot l$$

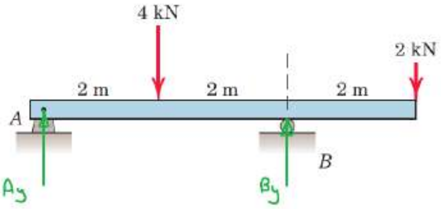
$$A_x = 0$$

Problema 23

Draw the shear and moment diagrams for the loaded beam and determine the distance d to the right of A where the moment is zero.

$$w = -\frac{dV}{dx}$$

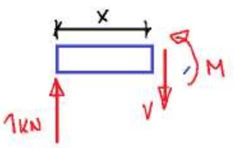
$$V = \frac{dM}{dx}$$



$\circlearrowleft + \; M_A = 0 \qquad -4\,kN \cdot 2\,m + B_y \cdot 4\,m - 2\,kN \cdot 6\,m = 0 \qquad B_y = 5\,kN$

$\Sigma F_y = 0 \qquad A_y + B_y - 6\,kN = 0 \qquad A_y = 1\,kN$

Para 0 ≤ x < 2 m



$\Sigma F_y = 0$

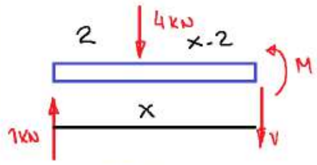
$1\,KN - V = 0 \qquad V = 1\,KN$

$\circlearrowleft + \; \Sigma M_A = 0$

$-V \cdot x + M = 0$

$M = 1 \cdot x$

Para 2 < x < 4



$\Sigma F_y = 0$

$1\,KN - 4\,KN - V = 0$

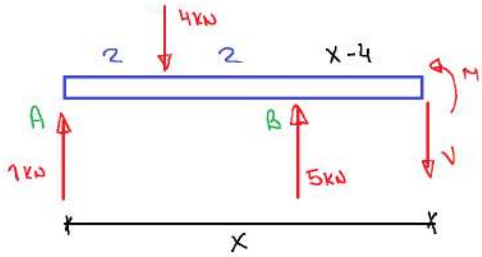
$V = -3\,KN$

$\circlearrowleft + \; \Sigma M_A = 0$

$-4\,KN \cdot 2\,m - Vx + M = 0$

$M = -3x + 8$

Para 4 < x < 6 m



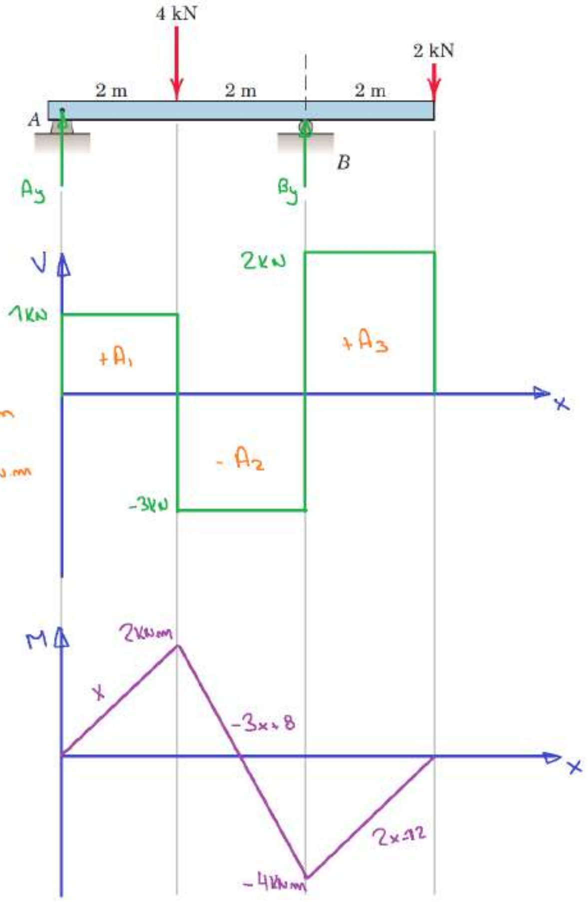
$\Sigma F_y = 0$

$1 - 4 + 5 - V = 0 \qquad V = 2\,KN$

$\circlearrowleft + \; \Sigma M_A = 0$

$-4 \times 2 + 5 \cdot 4 - V \cdot x + M = 0$

$M = 2x - 12$



$A_1 = 2\,KNm$

$A_2 = -6\,KN\,m$

diagrama de fuerza cortante

$$w = -\frac{dV}{dx}$$

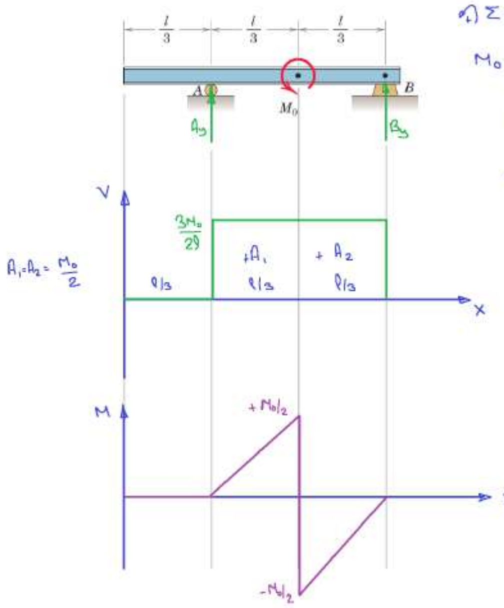
$$V = \frac{dM}{dx}$$

$\int C \, dx = \int dM$

diagrama momento flector

Problema 24

Draw the shear and moment diagrams for the loaded beam. What are the values of the shear and moment at midbeam?



$\circlearrowleft^{+}\ \Sigma M_A = 0$

$$M_0 + B_y \cdot \frac{2l}{3} = 0$$

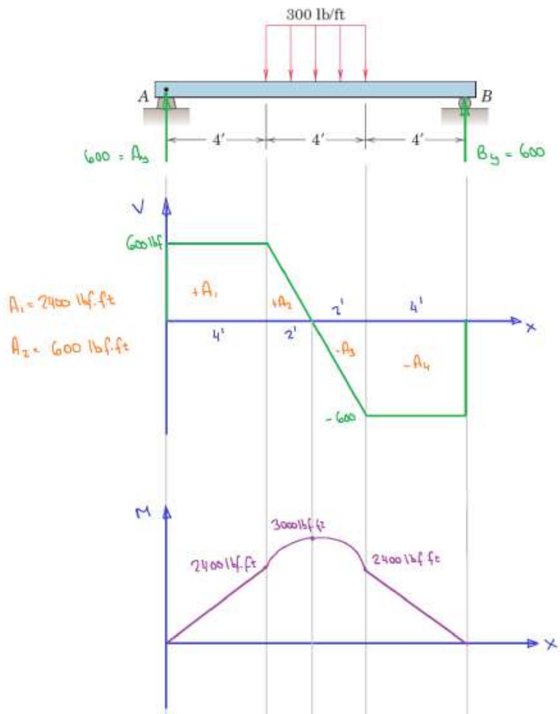
$$B_y = -\frac{3M_0}{2l}$$

$\Sigma F_y = 0$

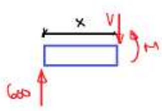
$$A_y = \frac{3M_0}{2l}$$

Problema 27

Draw the shear and moment diagrams for the loaded beam and determine the maximum value M_{max} of the moment.



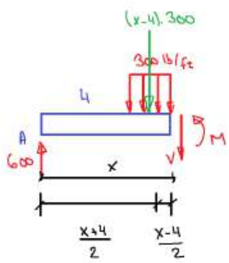
0 < x < 4 ft



$V = 600$

$V \cdot x = M$

$M = 600x$



4 < x < 8 ft

$\Sigma F_y = 0$

$$600 - 300 \cdot (x-4) - V = 0$$

$$V = 1800 - 300x$$

$\circlearrowleft^{+}\ \Sigma M_A = 0$

$$-(x-4) \cdot 300 \cdot \left(\frac{x+4}{2}\right) + (1800 - 300x) \cdot x + M = 0$$

$$-150(x^2 - 16) + 1800x - 300x^2 + M = 0$$

$$M = 150(x^2 - 16) - 1800x + 300x^2$$