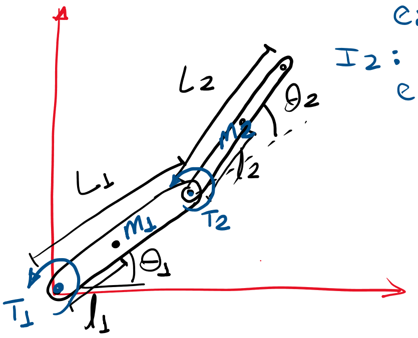


I_1: inercia eslabón 1
I_2: " eslabón 2

$$\rightarrow \quad M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

$$M = \begin{bmatrix} m_1\bar{l}_1^{\,2} + m_2\left(l_1^2 + 2l_1\bar{l}_2c_2 + \bar{l}_2^{\,2}\right) + I_1 + I_2 & m_2\left(l_1\bar{l}_2c_2 + \bar{l}_2^{\,2}\right) + I_2 \\ m_2\left(l_1\bar{l}_2c_2 + \bar{l}_2^{\,2}\right) + I_2 & m_2\bar{l}_2^{\,2} + I_2 \end{bmatrix}$$

$$C = \begin{bmatrix} -2m_2l_1\bar{l}_2s_2\dot{q}_2 & -m_2l_1\bar{l}_2s_2\dot{q}_2 \\ m_2l_1\bar{l}_2s_2\dot{q}_1 & 0 \end{bmatrix}$$

$$G = g\begin{bmatrix} (m_1\bar{l}_1 + m_2l_1)c_1 + m_2\bar{l}_2c_{12} \\ m_2\bar{l}_2c_{12} \end{bmatrix}$$

$$\ddot{q} = \begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}, \dot{q} = \begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}, \tau = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

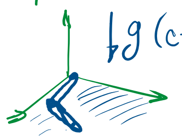
$$\begin{bmatrix} m_1l_1^2 + m_2\left(L_1^2 + 2L_1l_2\cos(\theta_2) + l_2^2\right) + I_1 + I_2 & m_2\left(L_1l_2\cos(\theta_2) + l_2^2\right) + I_2 \\ m_2\left(L_1l_2\cos(\theta_2) + l_2^2\right) + I_2 & m_2l_2^2 + I_2 \end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}$$

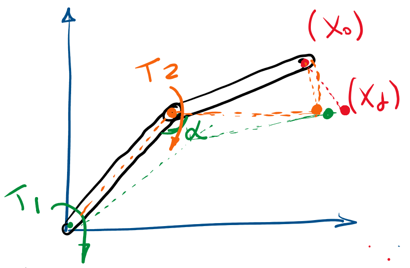
$$+ \begin{bmatrix} -2m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_2 & -m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_2 \\ m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_1 & 0 \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} \cancel{+ g\begin{bmatrix} (m_1l_1 + m_2L_1)\cos(\theta_1) + m_2l_2\cos(\theta_1+\theta_2) \\ m_2l_2\cos(\theta_1+\theta_2) \end{bmatrix}}$$

$$= \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

* Por trabajar sobre un plano horizontal

g (cte)





$$T_1 = \left[m_1l_1^2 + m_2\left(L_1^2 + 2L_1l_2\cos(\theta_2) + l_2^2\right) + I_1 + I_2\right]\ddot{\theta}_1 + \left[m_2\left(L_1l_2\cos(\theta_2) + l_2^2\right) + I_2\right]\ddot{\theta}_2$$
$$\underbrace{- 2m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_2\dot{\theta}_1} - \underbrace{m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_2^2}$$

$$T_2 = \left[m_2\left(L_1l_2\cos(\theta_2) + l_2^2\right) + I_2\right]\ddot{\theta}_1 + \left[m_2l_2^2 + I_2\right]\ddot{\theta}_2 + \underbrace{m_2L_1l_2\,\text{sen}(\theta_2)\dot{\theta}_1^2}$$

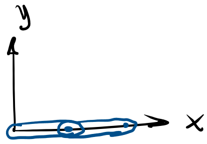
$$T_1 - T_2 = \left[m_1l_1^2 + m_2\left(L_1^2 + L_1l_2\cos(\theta_2)\right) + I_1\right]\ddot{\theta}_1 + \left[m_2\left(L_1l_2\cos(\theta_2)\right)\right]\ddot{\theta}_2$$
$$- m_2L_1l_2\,\text{sen}(\theta_2)\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2$$

$$\underbrace{\begin{bmatrix} m_1l_1^2 + m_2\left(L_1^2 + L_1l_2\overset{1}{\cancel{\cos(\theta_2)}}\right) + I_1 & m_2L_1l_2\overset{1}{\cancel{\cos(\theta_2)}} \\ m_2\left(L_1l_2\overset{1}{\cancel{\cos(\theta_2)}} + l_2^2\right) + I_2 & m_2l_2^2 + I_2 \end{bmatrix}}_{M}\underbrace{\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}}_{\ddot{\theta}} + \underbrace{\begin{bmatrix} -m_2L_1l_2\,\text{sen}(\overset{\theta_0}{\cancel{\theta_2}})\overset{0}{\cancel{(\dot{\theta}_1 + \dot{\theta}_2)^2}} \\ m_2L_1l_2\,\text{sen}(\overset{\theta_0}{\cancel{\theta_2}})\overset{0}{\cancel{\dot{\theta}_1}} \end{bmatrix}}_{\overset{0}{\cancel{C}}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{S}\underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}}_{T}$$

ecuación : $M\ddot{\theta} + C = S.T.$

* Para llevar a espacio estados

<u>Linealizar</u>



$\theta << 1$

$\operatorname{sen}(\theta) \approx \theta$

$\cos(\theta) \approx 1$

$\theta_0 = 0$

$\dot{\theta}_0 = 0$

$$\underbrace{\begin{bmatrix} m_1 l_1^2 + m_2(L_1^2 + L_1 l_2) + I_1 & m_2 L_1 l_2 \\ m_2(L_1 l_2 + l_2^2) + I_2 & m_2 l_2^2 + I_2 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}_{\ddot{\theta}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}}_{T}$$

$$\longrightarrow \ddot{\theta} = M^{-1} S T$$

$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \longrightarrow \dot{x} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \multicolumn{2}{c}{M^{-1}S} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}}_{u}$$

$u = -Kx$

$K = R^{-1} B^T P$

Paso 1: Resuelva la ecuación algebraica de Riccati, hallando **P**.

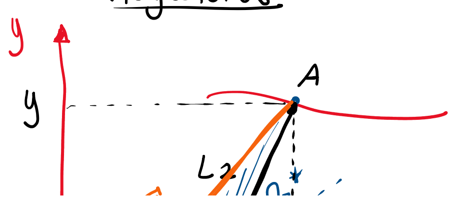
$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

Paso 2: Calcule K usando la ecuación:

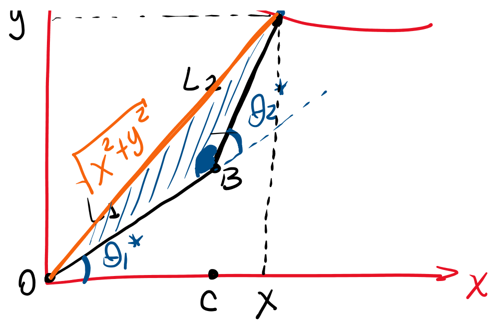
$$K = R^{-1} B^T P$$

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}$$

<u>Trayectorias:</u>



$$\theta_1^* = \underbrace{\measuredangle AOC}_{\operatorname{acos}\left(\frac{x}{\sqrt{x^2+y^2}}\right)} - \measuredangle AOB$$



$$\operatorname{acos}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$\sphericalangle AOB : \left(\sqrt{x^2+y^2}\right)^2 + L_1^2 = L_2^2 + 2\left(\sqrt{x^2+y^2}\,L_1\right)\cos\sphericalangle AOB$$

$$\sphericalangle AOB = \frac{x^2+y^2+L_1^2-L_2^2}{2L_1\left(\sqrt{x^2+y^2}\right)}$$

$$\theta_1^* = \operatorname{acos}\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \operatorname{acos}\left(\frac{x^2+y^2+L_1^2-L_2^2}{2L_1\sqrt{x^2+y^2}}\right)$$

$$\sphericalangle ABO + \theta_2^* = 180^\circ$$

$$\theta_2^* = 180^\circ - \sphericalangle ABO$$

$\sphericalangle ABO$

$$L_1^2+L_2^2 = x^2+y^2+2L_1L_2\underbrace{\cos\sphericalangle ABO}_{-\cos(\theta_2^*)}$$

$$\theta_2^* = \operatorname{acos}\left(\frac{L_1^2+L_2^2-x^2-y^2}{-2L_1L_2}\right)$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = U = -KX = -\underbrace{\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix}}_{K} \begin{bmatrix} \theta_1 - \theta_1^* \\ \theta_2 - \theta_2^* \\ \dot\theta_1 - 0 \\ \dot\theta_2 - 0 \end{bmatrix}$$