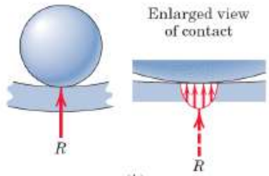
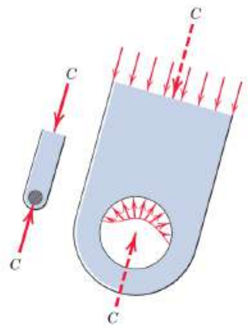
Fuerzas Distribuidas

(a)

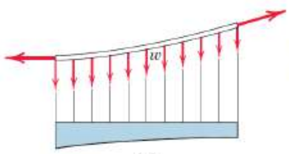


(b)



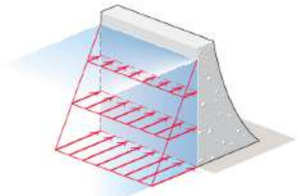
elemento Tipo barra

(c)



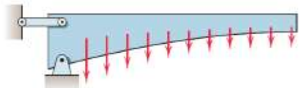
Catenaria

(a)



(b)

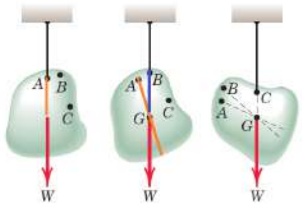
$P = \rho . g . h$



(c)

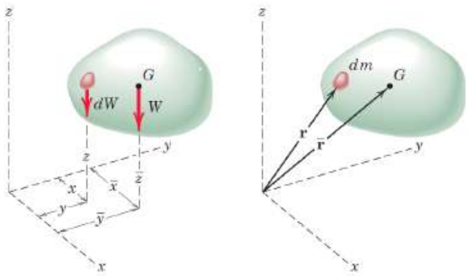
Para este ejemplo la distribución del peso es variable.

Centro de masa



(a) (b) (c)

$\bar{x}, \bar{y}$: las coordenadas del centro de gravedad.



(a) (b)

Por condición de equilibrio:

ΣM con respecto al eje y es 0

$$\int x \, dw = W . \bar{x}$$

$$\boxed{\bar{x} = \frac{\int x \, dw}{W}}$$

$$\bar{x} = \frac{\int x \, dW}{W} \qquad \bar{y} = \frac{\int y \, dW}{W} \qquad \bar{z} = \frac{\int z \, dW}{W}$$

With the substitution of $W = mg$ and $dW = g \, dm$, the expressions for the coordinates of the center of gravity become

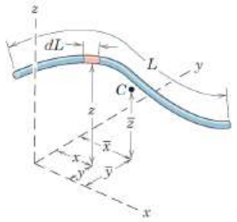
$$\bar{x} = \frac{\int x \, dm}{m} \qquad \bar{y} = \frac{\int y \, dm}{m} \qquad \bar{z} = \frac{\int z \, dm}{m} \qquad \textbf{(5/1b)}$$

Notación vectorial

$$\bar{\mathbf{r}} = \frac{\int \mathbf{r} \, dm}{m}$$

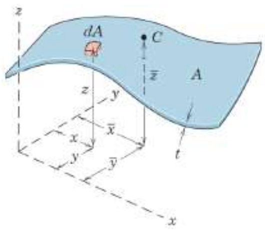
Centroide líneas áreas y volúmenes

Lineas:



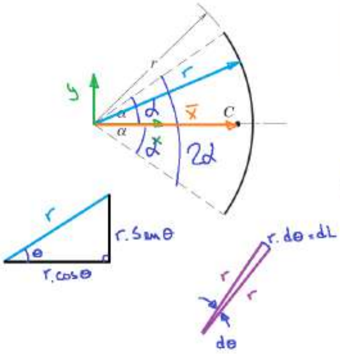
$$\bar{x} = \frac{\int x\, dL}{L} \qquad \bar{y} = \frac{\int y\, dL}{L} \qquad \bar{z} = \frac{\int z\, dL}{L}$$

Areas



$$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A} \qquad \bar{z} = \frac{\int z\, dA}{A}$$

Ejemplo: Localizar el centroide



$$\bar{x} = \frac{\int x\, dL}{L}$$

$$L.\bar{x} = \int x\, dL$$

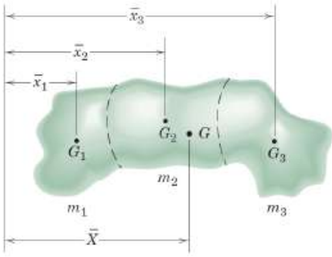
$$r.2\alpha.\bar{x} = \int_{-\alpha}^{\alpha} r.\cos\theta.r.d\theta$$

$$r.2\alpha.\bar{x} = r^2.(-\mathrm{Sen}\,\theta)\Big|_{-\alpha}^{\alpha}$$

$$2.r.\alpha.\bar{x} = 2r^2.\mathrm{Sen}\,\alpha$$

$$\bar{x} = \frac{r.\mathrm{Sen}\,\alpha}{\alpha}$$

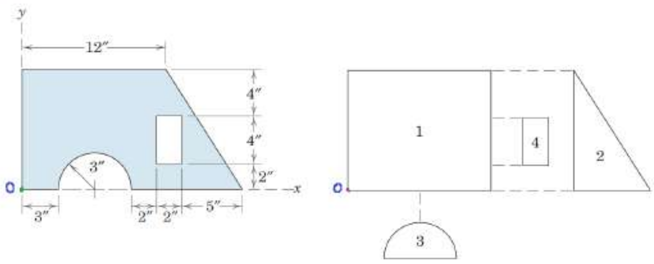
Figuras y cuerpos compuestos



$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} \qquad \bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} \qquad \bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m}$$

$$\bar{X} = \frac{m_1.\bar{X}_1 + m_2.\bar{X}_2 + m_3.\bar{X}_3}{m_1 + m_2 + m_3}$$

Ejemplo



PART	A in.^{2}	$\bar{x}$ in.	$\bar{y}$ in.	$\bar{x}A$ in.^{3}	$\bar{y}A$ in.^{3}
1	120	6	5	720	600
2	30	14	10/3	420	100
3	−14.14	6	1.273	−84.8	−18
4	−8	12	4	−96	−32
TOTALS	127.9			959	650

$$\bar{x} = \frac{\Sigma A x_c}{\Sigma A} \qquad \bar{y} = \frac{\Sigma A y_c}{\Sigma A}$$

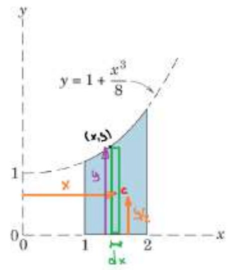
$$\bar{X} = \frac{959}{127{,}9} = 7{,}5 \text{ in}$$

$$\bar{y} = \frac{650}{127{,}9} = 5{,}08 \text{ in}$$

Seminario 2

Problema 02

Determine the x- and y-coordinates of the centroid of the shaded area.



$$\bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\int y\,dA}{A}$$

$$dA = y.dx = \left(1+\frac{x^3}{8}\right)dx$$

$$A = \int_1^2 \left(1+\frac{x^3}{8}\right)dx = \left(x+\frac{x^4}{32}\right)\Big|_1^2$$

$$A = \left(2+\frac{16}{32}-1-\frac{1}{32}\right) = \frac{47}{32}$$

$$\int x\,dA = x.\left(1+\frac{x^3}{8}\right)dx = \left(\frac{x^2}{2}+\frac{x^5}{40}\right)\Big|_1^2 = \frac{91}{40}$$

$$\int \frac{y}{2}.y\,dx = \int \frac{y^2}{2}dx = \frac{1}{2}\int_1^2 \left(1+\frac{x^3}{8}\right)^2 dx = \frac{1}{2}\int_1^2 \left(1+\frac{x^3}{4}+\frac{x^6}{64}\right)dx$$

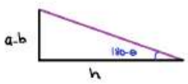
$$\frac{1}{2}\int_1^2 \left(1+\frac{x^3}{4}+\frac{x^6}{64}\right)dx = \frac{1}{2}\left(x+\frac{x^4}{16}+\frac{x^7}{448}\right)\Big|_1^2 = \frac{995}{89}$$

$$\bar{X} = \frac{\int x\,dA}{A} = \frac{\frac{91}{40}}{\frac{47}{32}} = 1{,}549$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\frac{995}{89}}{\frac{47}{32}} = 0{,}756$$

Problema 03

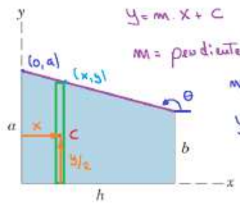
Determine the x- and y-coordinates of the centroid of the trapezoidal area.



$$\mathrm{Tan}(180-\theta) = \frac{a-b}{h}$$

$$-\mathrm{Tan}\,\theta = \frac{a-b}{h}$$

$$\mathrm{Tan}\,\theta = \frac{b-a}{h}$$



$y = m.x + C$

m = pendiente de la recta

$m = \mathrm{Tan}\,\theta$

$$y = \left(\frac{b-a}{h}\right)x + C$$

pasa por $(0,a)$

$$a = \left(\frac{b-a}{h}\right)0 + C$$

$$C = a$$

$$y = \left(\frac{b-a}{h}\right)x + a$$

$$A = \int y\,dx = \int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right]dx = \left[\frac{(b-a).x^2}{2h} + ax\right]\Big|_0^h = \frac{h}{2}.(a+b)$$

$$\int x.\,dA = \int_0^h x.\left[\left(\frac{b-a}{h}\right)x + a\right]dx = \left[\frac{(b-a).x^3}{3} + \frac{ax^2}{2}\right]\Big|_0^h = h^2\left(\frac{b}{3}+\frac{a}{6}\right)$$

$$\int y\,dA = \int_0^h \frac{y}{2}.y\,dx = \frac{1}{2}\int_0^h y^2\,dx = \frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right]^2 dx$$

$$\frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right]^2 dx = \frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)^2 x^2 + 2ax\left(\frac{b-a}{h}\right) + a^2\right]dx$$

$$\frac{1}{2}\int_0^h \left[\left(\frac{b-a}{h}\right)^2 x^2 + 2ax\left(\frac{b-a}{h}\right) + a^2\right]dx = \frac{1}{2}\left[\left(\frac{b-a}{h}\right)^2.\frac{x^3}{3} + ax^2\left(\frac{b-a}{h}\right) + a^2x\right]\Big|_0^h$$

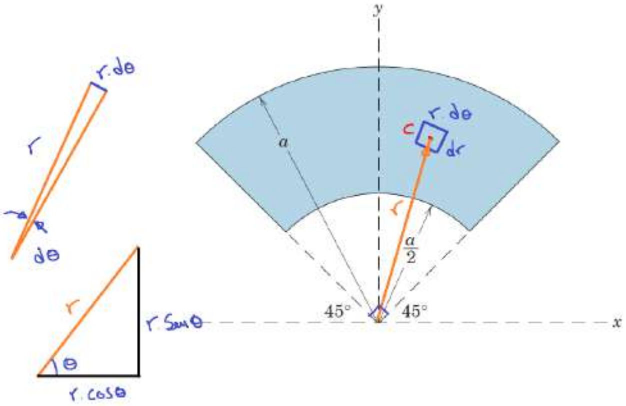
$$\frac{1}{2}\left[\left(\frac{b-a}{h}\right)^2.\frac{x^3}{3} + ax^2\left(\frac{b-a}{h}\right) + a^2x\right]\Big|_0^h = \frac{h}{6}\left[a^2+ab+b^2\right]$$

$$\bar{X}_c = \frac{\int x.\,dA}{A} = \frac{h^2\left(\frac{b}{3}+\frac{a}{6}\right)}{\frac{h}{2}.(a+b)} = \frac{h.(a+2b)}{3(a+b)}$$

$$\bar{y}_c = \frac{\int y\,dA}{A} = \frac{\frac{h}{6}\left[a^2+ab+b^2\right]}{\frac{h}{2}.(a+b)} = \frac{(a^2+ab+b^2)}{3(a+b)}$$

Problema 06

Determine the y-coordinate of the centroid of the shaded area.



$$dA = r\,dr.d\theta$$

$$A = \frac{\pi.a^2}{4} - \frac{\pi.(a/2)^2}{4}$$

$$A = \frac{3}{16}\pi.a^2$$

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$\bar{y} = \frac{\int r.\mathrm{Sen}\theta.r\,dr.d\theta}{A}$$

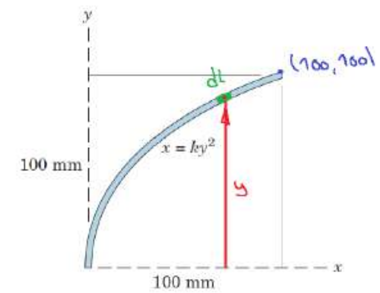
$$\int_{\pi/4}^{3\pi/4}\int_{a/2}^{a} r.\mathrm{Sen}\theta.r\,dr.d\theta = \int_{\pi/4}^{3\pi/4} \frac{r^3}{3}.\mathrm{Sen}\theta\,d\theta \Bigg|_{r=a/2}^{r=a} = \frac{7}{24}a^3\int_{\pi/4}^{3\pi/4}\mathrm{Sen}\theta\,d\theta$$

$$\frac{7}{24}.a^3.(-\cos\theta)\Bigg|_{\pi/4}^{3\pi/4} = \frac{7.\sqrt{2}}{24}.a^3$$

$$\bar{y} = \frac{\int r.\mathrm{Sen}\theta.r\,dr.d\theta}{A} = \frac{\frac{7.\sqrt{2}}{24}.a^3}{\frac{3}{16}\pi.a^2} = \frac{14.\sqrt{2}.a}{9.\pi}$$

Problema 08

The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y-coordinate of the mass center of the rod. (*Reminder:* A differential arc length is $dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (dx/dy)^2}\,dy$.)



$$100 = K.100^2$$

$$K = \frac{1}{100}$$

$$x = \frac{1}{100}.y^2$$

$$dx = \frac{1}{100}.2y\,dy = \frac{y\,dy}{50}$$

$$\frac{dx}{dy} = \frac{y}{50}$$

$$\bar{y} = \frac{\int y\,dL}{L}$$

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{(dy)^2\left[\left(\frac{dx}{dy}\right)^2+1\right]} = dy.\sqrt{\left(\frac{dx}{dy}\right)^2+1}$$

$$L = \int dL = \int dy.\sqrt{\left(\frac{dx}{dy}\right)^2+1} = \int\sqrt{\left(\frac{y}{50}\right)^2+1}\,dy = \int\sqrt{\frac{y^2+50^2}{50^2}}\,dy = \frac{1}{50}\int_0^{100}\sqrt{y^2+50^2}\,dy$$

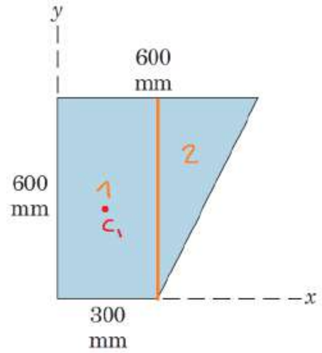
$$\frac{1}{50}\times\frac{1}{2}\left[y\sqrt{50^2+y^2} + 50^2.\mathrm{Ln}\left(y+\sqrt{50^2+y^2}\right)\right]\Bigg|_0^{100} = 147{,}89\,mm$$

$$\int y.dL = \int_0^{100} y.dy.\sqrt{\left(\frac{dx}{dy}\right)^2+1} = \frac{1}{50}.\frac{1}{3}\left(50^2+y^2\right)^{3/2}\Bigg|_0^{100} = 8483{,}62\,mm^2$$

$$\bar{y} = \frac{\int y\,dL}{L} = \frac{8483{,}62\,mm^2}{147{,}89\,mm} = 57{,}36\,mm$$

Problema 09

Determine the coordinates of the centroid of the trapezoidal area shown.



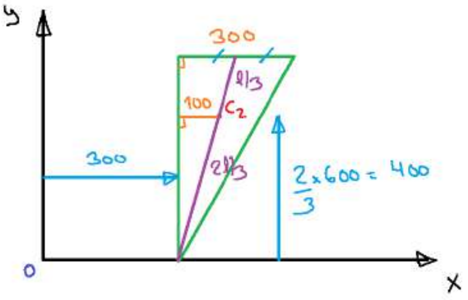
$A_1 = 18\times10^4\ mm^2$

$A_2 = 9\times10^4\ mm^2$

Para el sector 1, las coordenadas de su centroide con respecto al eje x,y

$\bar{x}_1 = 150\ mm$

$\bar{y}_1 = 300\ mm$



$\bar{x}_2 = 400\ mm$

$\bar{y}_2 = 400\ mm$

Aplicando la Teoria

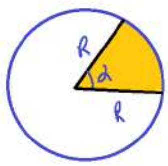
$$\bar{x} = \frac{18\times10^4 \cdot 150 + 9\times10^4 \cdot 400}{18\times10^4 + 9\cdot10^4} = 233\ mm$$

$$\bar{y} = \frac{18\times10^4 \cdot 300 + 9\cdot10^4 \cdot 400}{18\times10^4 + 9\cdot10^4} = 333\ mm$$

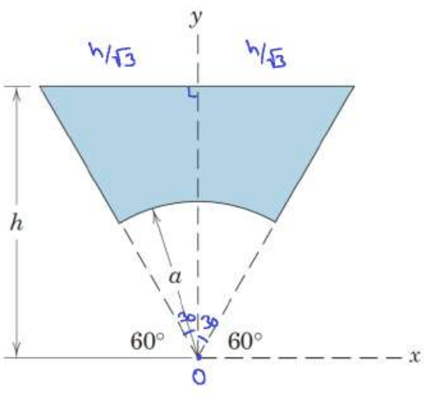
$$(\bar{x},\bar{y}) = (233, 333)$$

Problema 10

Determine the y-coordinate of the centroid of the shaded area.



$$Area = \frac{R^2 \cdot \alpha}{2}$$



1: Triangulo

$$A_1 = \frac{2h}{\sqrt{3}} \times h \cdot \frac{1}{2} = \frac{h^2}{\sqrt{3}}$$

$$y_1 = \frac{2h}{3}$$

2: Sector circular

$$A_2 = a^2 \cdot \frac{\pi}{3} \cdot \frac{1}{2} = \frac{a^2 \pi}{6}$$

$$y_2 = \frac{2}{3} r \cdot \frac{\operatorname{Sen} \alpha}{\alpha}$$

$2\alpha = 60$

$\alpha = 30$

$\alpha = \pi/6$

$$y_2 = \frac{2}{3} \cdot a \cdot \frac{1}{2} \cdot \frac{6}{\pi}$$

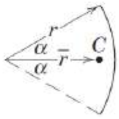
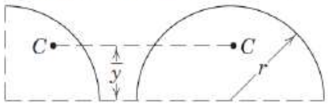
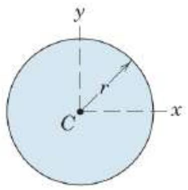
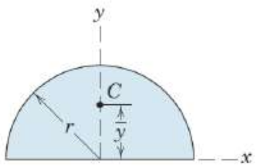
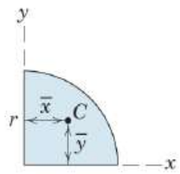
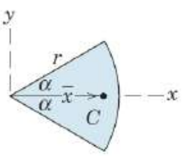
$$y_2 = 2a/\pi$$

$$A = A_1 - A_2 = \frac{h^2}{\sqrt{3}} - \frac{a^2 \pi}{6}$$

$$\bar{y} = \frac{\sum A \cdot \bar{y}}{A} = \frac{\frac{h^2}{\sqrt{3}} \cdot \frac{2h}{3} - \frac{a^2\pi}{6} \cdot \frac{2a}{\pi}}{\frac{h^2}{\sqrt{3}} - \frac{a^2\pi}{6}}$$

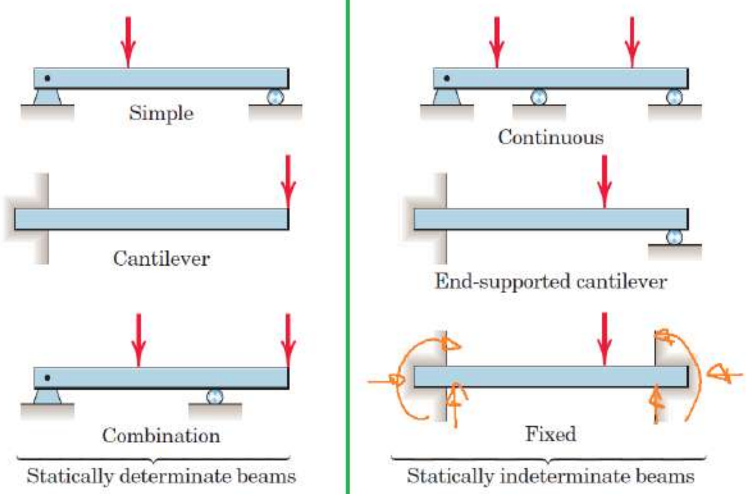
$$\bar{y} = \frac{4h^3 - 2\sqrt{3}\,a^3}{6h^2 - \sqrt{3}\pi \cdot a^2}$$

TABLE D/3 PROPERTIES OF PLANE FIGURES

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \dfrac{r \sin\alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3}\dfrac{r \sin\alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}\left(\alpha - \dfrac{1}{2}\sin 2\alpha\right)$ $I_y = \dfrac{r^4}{4}\left(\alpha + \dfrac{1}{2}\sin 2\alpha\right)$ $I_z = \dfrac{1}{2} r^4 \alpha$

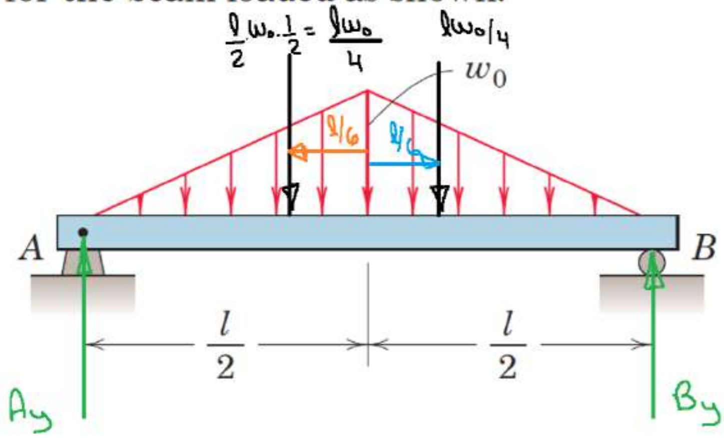
Vigas - Efectos externos

Tipos de vigas



Problema 16

Determine the reactions at the supports A and B for the beam loaded as shown.



Por simetria $A_y = B_y$

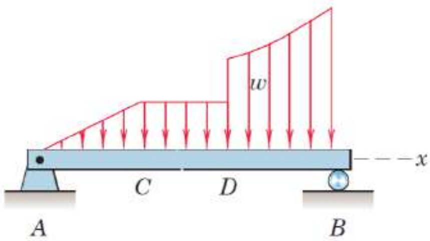
$$\Sigma F_y = 0$$

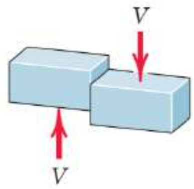
$$A_y - \frac{l w_0}{4} - \frac{l w_0}{4} + B_y = 0$$

$$A_y + B_y = \frac{l w_0}{2}$$

$$A_y = B_y = \frac{l w_0}{4}$$

Vigas - Efectos internos



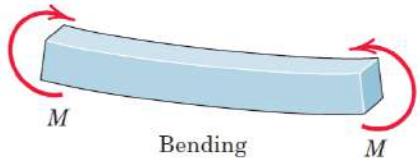


Shear

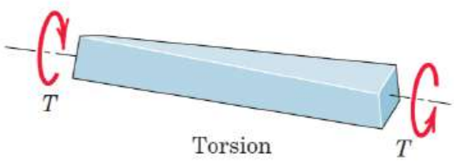
V: fuerza cortante

M: momento flector

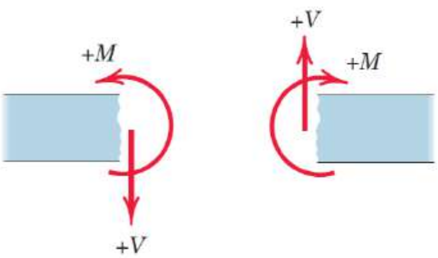
T: momento torsor

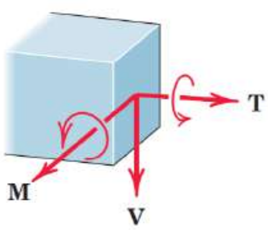


Bending

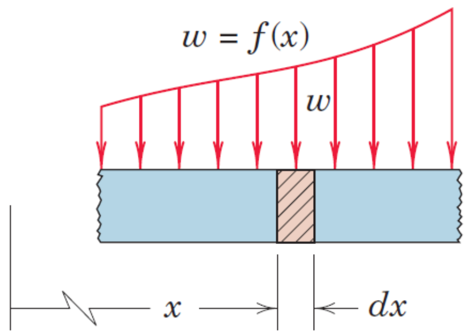


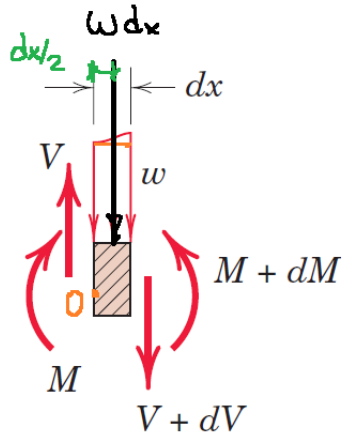
Torsion





Combined loading





Por equilibrio en el elemento diferencial

$$\sum F_y = 0 \Rightarrow \cancel{V} - w\,dx - \cancel{V} - dV = 0$$

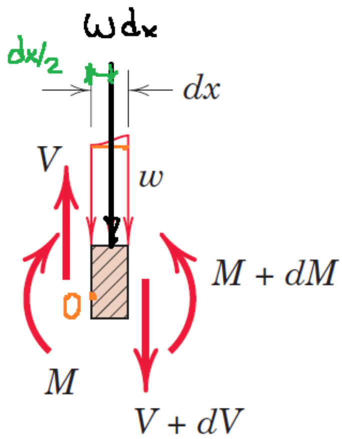
$$dV = -w\,dx$$

$$\frac{dV}{dx} = -w$$

Lo que desea obtener es la funcion de la fuerza cortante V

$$\curvearrowright + \quad \sum M_o = 0$$

$$\cancel{M} + w\,dx \cdot \overset{\approx 0}{\frac{dx}{2}} + (V + dV) \cdot dx - \cancel{M} - dM = 0$$



$$V = \frac{dM}{dx}$$

$$\int V \cdot dx = \int dM$$

Lo que se desea es obtener la función M

$$\left.\begin{array}{l} V = \dfrac{dM}{dx} \\ \\ w = -\dfrac{dV}{dx} \end{array}\right\} \qquad w = -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$w = -\frac{d^2M}{dx^2}$$