Control 2

entrada

salida

A, B y C son matrices

A cuadrada C fila

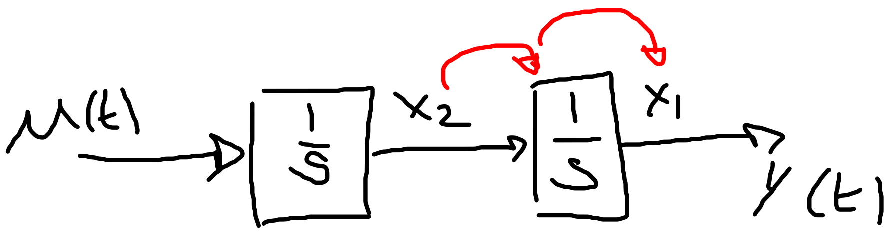
B columna

$$\dot{x} = Ax + Bu \qquad y = Cx$$

variable derivada

variables

$A_{n\times n}$ n= orden del SIS



$y = x_1$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$
$$y = x_1$$

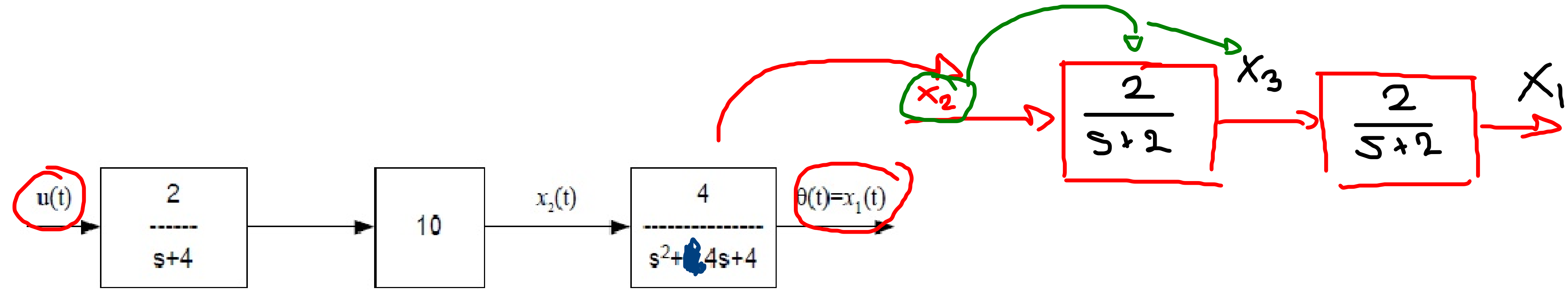
$$x_2\left(\frac{1}{s}\right) = x_1 \qquad u\left(\frac{1}{s}\right) = x_2 \to u = s x_2$$

$\mathcal{L}^{-1}$ $u = \dot{x}_2$

$$\mathcal{L}^{-1}\left(x_2 = s x_1\right| \to x_2 = \dot{x}_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$u\left(\frac{20}{s+4}\right) = x_2 \rightarrow 20u = s x_2 + 4x_2 \rightarrow \dot{x}_2 = -4x_2 + 20u$$

$$x_2\left(\frac{2}{s+2}\right) = x_3 \rightarrow \dot{x}_3 = -2x_3 + 2x_2$$

$$x_3\left(\frac{2}{s+2}\right) = x_1 \rightarrow \dot{x}_1 = -2x_1 + 2x_3$$

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \overset{A}{\begin{bmatrix} -2 & 0 & 2 \\ 0 & -4 & 0 \\ 0 & 2 & -2 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 20 \\ 0 \end{bmatrix}} u$$

$$y = \overset{C}{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

FUNCIÓN TANSFERENCIA (F.T)

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Matriz identidad orden n

$$F.T = G_{(s)} = C\,(sI - A)^{-1} B$$

$$G_{(s)} = \frac{C \cdot \operatorname{adj}(s.I - A)^{-1} B}{|sI - A|}$$

Ecuación
De característica

Nota

$$(Z)^{-1} = \frac{\operatorname{adj}(Z)}{|Z|}$$

$$Z = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\operatorname{adj}(Z) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$|Z| = a.d - bc$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$G_{(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \Bigg(\underbrace{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\left(\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}\right)^{-1}} \Bigg)^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$G(s) = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2} = \frac{\begin{bmatrix} s & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2} = \frac{1}{s^2}$$

En función del tiempo

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$\underset{C.I}{x_{(t)}} = \mathcal{L}^{-1}\left((sI - A)^{-1}\right)x_0 \qquad \underset{u}{x_{(t)}} = \mathcal{L}^{-1}\left((sI - A)^{-1}Bu\right)$$

REPASITO

Ec. Variables de estado

$\dot{x} = AX + Bu$ / $y = Cx$

Función transferencia

$F.T = C(s.I - A)^{-1} B$

$\Delta_{ec} = |s.I - A|$

Variables en el tiempo por Laplace

$X_{c.i}(t) = \mathcal{L}^{-1}\left((s.I-A)^{-1}\right).x_0$

$X_{u}(t) = \mathcal{L}^{-1}\left((s.I-A)^{-1} B u_{(s)}\right)$

$X(t) = X_{cc}(t) + X_{u}(t)$

Forma canónica controlable

$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$

$\bar{y} = \bar{c}\bar{x}$

$\bar{A} = P^{-1}AP$

$\bar{B} = P^{-1}B$

$\bar{C} = CP$

$S = [B \quad AB \quad A^2B \ldots A^{n-1}B]$

$$M = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$P = SM$

Forma canónica observable

$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$

$\bar{y} = \bar{c}\bar{x}$

$\bar{A} = Q^{-1}AQ$

$\bar{B} = Q^{-1}B$

$\bar{C} = CQ$

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$Q = (MV)^{-1}$

Controlador por ub. de polos

Matriz de ganancia

$u = -Kx(t)$

$\dot{x} = Ax - BKx(t)$

$\dot{x} = (A - BK)x$

$\Delta_{ec} = |sI - A + BK|$

$\Delta_d = (s+\lambda_1)(s+\lambda_2)\ldots$

$\Delta_d = s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1 s + \alpha_0$

$K^* = \alpha_{i-1} - a_{i-1}$ (fila)

Para la Planta original

$K = K^* P^{-1}$

Diseño de observador

$\dot{\tilde{x}} = A\tilde{x} + Bu + Ke(y - \tilde{y})$

Ec. error

$\dot{e} = (A - KeC)e$

$\Delta_e = |sI - A + KeC|$

$\Delta_d = (s - u_1)(s - u_2)\ldots$

$\Delta_d = s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1 s + \alpha$

$K_e' = \alpha_{i-1} - a_{i-1}$ (columna)

$K_e = Q K_e'$

$Mp\% = 100\, e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$

$T_{s2\%} = \frac{4}{\xi . w_n}$

$\lambda_1, \lambda_2 = -\xi w_n \pm w_n\sqrt{1-\xi^2}\, j$