Quantum Information Theory

Pawel Wocjan

University of Central Florida

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Aristotle's law of motion

- Aristotle lived in a world dominated by friction.
- To make anything move – a heavy cart with wooden wheels, for instance – you had to push it, you had to apply a force to it.
- The harder you pushed it, the faster it moved; but if you stopped pushing, the cart very quickly came to a rest.
- Aristotle came to some wrong conclusions because he didn't understand that friction is a force.
- But still, it's worth exploring his ideas in modern language.

Aristotle's law of motion

- If he had known calculus, Aristotle might have proposed the following law of motion:

  The velocity of any object is proportional to the total applied force.

Aristotle's law of motion

- Had he known how to write vector equations, his law would have looked like this:
$$\vec{F} = m\vec{v}$$
- $\vec{F}$ is the applied force, and the response according to Aristotle would be velocity vector $\vec{v}$.
- The factor m relating the two is some characteristic quantity describing the resistance of the body to being moved; for a given force, the bigger the m of the object, the smaller the velocity.
- With little reflection, the philosopher might have identified m as the mass of the object.

Aristotle's law of motion

- Consider one-dimensional motion of a particle along the x axis under the influence of a given force.
- Using the fact that the velocity is the time derivative of position, x, we find that Aristotle's equation takes the form

$$\frac{dx(t)}{dt} = \frac{F(t)}{m}$$

Aristotle's law of motion

- One obvious difference to the deterministic laws that we studied earlier it that Aristotle's equation is not stroboscopic.
- Nevertheless, we can see the similarity if we assume that time is broken up into intervals of size Δt and replace the derivative by $\frac{\Delta x}{\Delta t}$

$$x(t + \Delta t) = x(t) + \Delta t \frac{F(t)}{m} \tag{1}$$

Aristotle's law of motion

- Let's go back to the exact equation of motion:

$$\frac{dx(t)}{dt} = \frac{F(t)}{m}$$

- Equations for unknown functions that involve derivatives are called differential equations.
- This is a first-order differential equation because it contains only first derivatives.
- Equations like this are easy to solve.

Aristotle's law of motion

- The trick is to integrate both sides of the equation

$$\int \frac{dx(t)}{dt} dt = \int \frac{F(t)}{m} dt$$

- Assuming that $F(t)$ is constant, the solution is given by

$$x(t) = \frac{F}{m} t + c$$

- The constant c is fixed by the initial conditions.

Aristotle's law of motion

- Aristotle's equations of motion are deterministic, but are they reversible?
- The analogous procedure to reversing all the arrows when time is continuous is very simple.
- Everywhere you see time in the equations, replace it with minus time.
- That will have the effect of interchanging the future and the past.

Aristotle's law of motion

- Let's go back to Aristotle's equation

$$F(t) = m\frac{dx}{dt}$$

  and change the sign of time.
- The result is

$$F(-t) = -m\frac{dx}{dt}$$

- Note that reversing the arrows means changing the differential dt to $-dt$.

Aristotle's law of motion

- We obtain
$$-F(-t) = m\frac{dx}{dt}$$
- The implication is simple: The reversed equation of motion is exactly like the original, but with a different rule for the force as a function of time.
- The conclusion is clear: If Aristotle's equations of motion are deterministic into the future, they are also deterministic into the fact.
- The problem with Aristotle's equation is not that they are inconsistent, they are just the wrong equations.

Mass, acceleration, and force

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