Quantum Information Theory

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Particle motion

- The concept of a point particle is an idealization.
- No object is so small that it is a point.
- But in many situations we can ignore the extended structure of objects and treat them as points.
- The position of a particle is specified by giving a value for each of the three spatial coordinates, and the motion of the particle is defined by it position at every time.
- Mathematically, we can specify a position by giving the three spatial coordinates $x(t), y(t), y(t)$.

Particle motion

- The position can also be thought of as a vector $\vec{r}(t)$ whose components are $x(t), y(t), z(t)$ at time t.
- The path of the particle – its trajectory – is specified by $\vec{r}(t)$.
- The job of classical mechanics is to figure out $\vec{r}(t)$ from some initial condition and some dynamical law.

Particle motion

- Next, to its position, the most important thing about a particle is its velocity.
- Velocity is also a vector.
- Consider the displacement of the particle between time t and a little bit later at time $t + \Delta t$.
- During that time interval the particle moves from $x(t), y(t), z(t)$ to $x(t + \Delta t), y(t + \Delta t), z(t + \Delta t)$.
- Expressed in vector notation, from $\vec{r}(t)$ to $\vec{r}(t + \Delta t)$.

Particle motion

- The displacement is defined as

$$\begin{aligned}\Delta x &= x(t+\Delta t) - x(t)\\ \Delta y &= y(t+\Delta t) - y(t)\\ \Delta z &= z(t+\Delta t) - z(t)\end{aligned}$$

or

$$\Delta \vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t).$$

Particle motion

- To get the velocity, we divide the displacement by Δt and take the limit as Δt shrinks to zero. For example,

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- This, of course, is the definition of the derivative of x with respect to t

$$v_x = \frac{dx}{dt} = \dot{x}$$

- Placing a dot over a quantity is standard shorthand for taking the time derivative.

Particle motion

- It would be cumbersome to keep writing x, y, z so we condense the notation.
- The three coordinates x, y, z are denoted collectively by x_i and the velocity components by v_i

$$v_i = \frac{dx_i}{dt} = \dot{x}_i$$

where i takes the values x, y, z, or, in vector notation

$$\vec{v} = \frac{\vec{r}}{dt} = \dot{\vec{r}}$$

Particle motion

- Acceleration is the quantity that tells you how the velocity is changing.
- If an object is moving with constant velocity vector, it experiences no acceleration.
- A constant velocity vector implies not only a constant speed but also constant direction.
- You feel acceleration only when your velocity vector changes in magnitude or direction.

Particle motion

- In fact, acceleration is the time derivative of the velocity

$$a_i = \frac{dv_i}{dt} = \dot{v}_i$$

or, in vector notation,

$$\vec{a} = \dot{\vec{v}}$$

Particle motion

- Because v_i is the time derivative of x_i and a_i is the time derivative of v_i, it follows that acceleration is the second time derivative of x_i

$$a_i = \frac{d^2 x_i}{dt^2} = \ddot{x}_i$$

where the double-dot notation means the second time derivative.

Examples of motion: uniform acceleration

- Suppose a particle starts to move at time $t = 0$ according to the equations

$$v_x(t) = 0$$
$$v_y(t) = 0$$
$$v_z(t) = v(0) - gt$$

- Let's calculate the acceleration

$$a_x(t) = 0$$
$$a_y(t) = 0$$
$$a_z(t) = -g$$

- The acceleration along the z axis is constant and negative. If the z axis were to represent altitude, the particle would accelerate downward in just a way a falling object would.

Examples of motion: oscillation

- Let's consider an oscillating particle that moves back and forth along the x axis.
- Because there is no motion in the other two directions, we will ignore them.
- A simple oscillatory motion uses trigonometric functions:

$$x(t) = \sin \omega t$$

- The larger the constant ω, the more rapid the oscillation.
- This kind of motion is called simple harmonic motion.

Examples of motion: oscillation

- The velocity and acceleration are

$$v_x = \omega \cos \omega t$$
$$a_x = -\omega^2 \sin \omega t$$

- Whenever the position x is at its maximum or minimum, the velocity is zero.
- The opposite is also true: When the position is at $x = 0$, then the velocity is either a maximum or a minimum (90° out of phase).

Examples of motion: oscillation

- The position and acceleration are also related, both being proportional to $\sin \omega t$.
- Notice the minus sign in the acceleration.
- Whenever x is positive (negative), the acceleration is negative (positive).
- In other words, wherever the particle is, it is being accelerated back to the origin (180° out of phase).

Examples of motion: circular motion

- Let's consider a particle moving with uniform circular motion about the origin.
- The most general (counterclockwise) uniform circular motion about the origin has the mathematical form

  $$x(t) = R\cos\omega t$$
  $$y(t) = R\sin\omega t$$

  where ω is the angular frequency.

Examples of motion: circular motion

- The components of velocity and acceleration are:

$$
\begin{aligned}
v_x &= -R\omega \sin \omega t \\
v_y &= R\omega \cos \omega t \\
a_x &= -R\omega^2 \cos \omega t \\
a_y &= -R\omega^2 \sin \omega t
\end{aligned}
$$

- This shows an interesting property of circular motion that Newton used in analyzing the motion of the moon:
  The acceleration of a circular motion is parallel to the position vector, but it is oppositely directed.
- In other words, the acceleration vector points radially inward toward the origin.