COT6602 Quantum Information Theory
Fall 2019

Homework 2

Due: will be posted on Webcourses

Type up your solutions using a computer. Do not turn in handwritten notes.

Problem 1 (Double pendulum)

A pendulum swings in the x, y plan supported at the origin. The rod of the pendulum is massless, and the bob (weight at the end is) m. To make it simple, let the rod be 1 meter in length and let the bob be 1 kilogram in mass. Next, take another identical pendulum, but suspend it from the bob of the first pendulum.

Let θ be the angle of the first pendulum from the vertical. Let α be the angle of the second pendulum to be measured relative to the first rod rather than the vertical.

The easiest way to compute the kinetic energy is to temporarily refer to the cartesian coordinates of both bobs. Those of the first bob are:

$$x_1 = \sin\theta$$
$$y_1 = \cos\theta$$

Those of the second bob are:

$$y_1 = \sin\theta + \sin(\alpha + \theta)$$
$$y_2 = \cos\theta + \cos(\alpha + \theta)$$

The kinetic energy of the first bob is:

$$T_1 = \frac{\dot{\theta}^2}{2}$$

The kinetic energy of the second bob is:

$$T_2 = \frac{\dot{\theta}^2 + (\dot{\theta} + \dot{\alpha})^2}{2} + \dot{\theta}(\dot{\theta} + \dot{\alpha})\cos\alpha$$

Assume that there is no gravitational field so that the potential energy is always 0.

- Work out the Euler-Lagrange equations for θ and α.
- Determine the symmetries of the Lagrangian and the corresponding preserved quantities.

Problem 2 (Harmonic oscillator)

The Lagrangian of a harmonic oscillator is

$$L = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$$

- Show that if you make the change in variables $q = (km)^{1/4}x$, the Langrangian has the form

$$L = \frac{1}{2\omega}\dot{q}^2 - \frac{\omega}{2}q^2$$

- What is the connection among k, m, and ω?
- Starting with the above form of the Lagrangian, calculate the Hamiltonian in terms of p and q.
- What are the corresponding equations of motion?