Quantum Information Theory

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Aristotle's law of motion

- Aristotle lived in a world dominated by friction.
- To make anything move – a heavy cart with wooden wheels, for instance – you had to push it, you had to apply a force to it.
- The harder you pushed it, the faster it moved; but if you stopped pushing, the cart very quickly came to a rest.
- Aristotle came to some wrong conclusions because he didn't understand that friction is a force.
- But still, it's worth exploring his ideas in modern language.

Aristotle's law of motion

- If he had known calculus, Aristotle might have proposed the following law of motion:

  The velocity of any object is proportional to the total applied force.

Aristotle's law of motion

- Had he known how to write vector equations, his law would have looked like this:
$$\vec{F} = m\vec{v}$$
- $\vec{F}$ is the applied force, and the response according to Aristotle would be velocity vector $\vec{v}$.
- The factor m relating the two is some characteristic quantity describing the resistance of the body to being moved; for a given force, the bigger the m of the object, the smaller the velocity.
- With little reflection, the philosopher might have identified m as the mass of the object.

Aristotle's law of motion

- Consider one-dimensional motion of a particle along the x axis under the influence of a given force.
- Using the fact that the velocity is the time derivative of position, x, we find that Aristotle's equation takes the form

$$\frac{dx(t)}{dt} = \frac{F(t)}{m}$$

Aristotle's law of motion

- One obvious difference to the deterministic laws that we studied earlier it that Aristotle's equation is not stroboscopic.
- Nevertheless, we can see the similarity if we assume that time is broken up into intervals of size Δt and replace the derivative by $\frac{\Delta x}{\Delta t}$

$$x(t + \Delta t) = x(t) + \Delta t \frac{F(t)}{m} \tag{1}$$

Aristotle's law of motion

- Let's go back to the exact equation of motion:

$$\frac{dx(t)}{dt} = \frac{F(t)}{m}$$

- Equations for unknown functions that involve derivatives are called differential equations.
- This is a first-order differential equation because it contains only first derivatives.
- Equations like this are easy to solve.

Aristotle's law of motion

- The trick is to integrate both sides of the equation

$$\int \frac{dx(t)}{dt} dt = \int \frac{F(t)}{m} dt$$

- Assuming that $F(t)$ is constant, the solution is given by

$$x(t) = \frac{F}{m} t + c$$

- The constant c is fixed by the initial conditions.

Aristotle's law of motion

- Aristotle's equations of motion are deterministic, but are they reversible?
- The analogous procedure to reversing all the arrows when time is continuous is very simple.
- Everywhere you see time in the equations, replace it with minus time.
- That will have the effect of interchanging the future and the past.

Aristotle's law of motion

- Let's go back to Aristotle's equation

$$F(t) = m\frac{dx}{dt}$$

  and change the sign of time.
- The result is

$$F(-t) = -m\frac{dx}{dt}$$

- Note that reversing the arrows means changing the differential dt to $-dt$.

Aristotle's law of motion

- We obtain
$$-F(-t) = m\frac{dx}{dt}$$
- The implication is simple: The reversed equation of motion is exactly like the original, but with a different rule for the force as a function of time.
- The conclusion is clear: If Aristotle's equations of motion are deterministic into the future, they are also deterministic into the fact.
- The problem with Aristotle's equation is not that they are inconsistent, they are just the wrong equations.

Mass, acceleration, and force

- Aristotle's mistake was to think that a net applied force is needed to keep an object moving.
- The right idea is to that one force – the applied force – is needed to overcome another force – the force of friction.
- An isolated object moving in free space, with no forces acting on it, requires nothing to keep it moving.
- In fact it needs a force to stop it.
- This is the law of inertia.
- What forces do is change the state of motion of a body.

Mass, acceleration, and force

- If the body is initially at rest, it takes a force to start it moving.
- If it's moving, it takes a force to stop it.
- If it is moving in a particular direction, it takes a force to change the direction of motion.
- All of these examples involve a change in the velocity of an object, and therefore an acceleration.

Mass, acceleration, and force

- From experience we know that some object have more inertia than others; it requires a larger force to change their velocities.
- The quantitative measure of an object's inertia is its mass.
- Newton's law of motion involves three quantities acceleration, mass, and force.
- Acceleration we studies earlier.
- By monitoring the position of an object as it moves, a clever observer can determine its acceleration.

Mass, acceleration, and force

- Mass is a new concept that is usually defined in terms of force and acceleration.
- But so far we haven't defined force.
- It sounds like we are in a logical circle in which force is defined by the ability to change the motion of a given mass, and mass is defined by the resistance to that change.
- To break that circle, let's take a closer look at how force is defined and measured in practice.

Mass, acceleration, and force

- Use white board to explain how force is measured with a spring balance.

Mass, acceleration, and force

- Newton's second law of motion tells us that force equals mass times acceleration,
$$\vec{F} = m\vec{a}$$
- This equation can also be written in the form
$$\vec{F} = m\frac{d\vec{v}}{dt}$$
In other words, force equals mass time the rate of change of velocity: no force $\Rightarrow$ no change in velocity.

Units

length

$[x] = [\text{length}] = \text{meters} = \text{m}$

time

$[t] = [\text{time}] = \text{seconds} = \text{s}$

Units

velocity

$[v] = [\text{length } / \text{ time}] = \text{m/s}$

acceleration

$[a] = [\text{length/time}]\ [1/\text{time}] = [\text{length } / \text{ time}^2] = \text{m/s}^2$

Units

mass

$[m] = [\text{mass}] = \text{kilogram} = \text{kg}$

force

$[F] = [\text{force}] = [ma] = [\text{mass} \times \text{length} \;/\; \text{time}^2] = [\text{kg m/s}^2]$

Simple examples of solving Newton's equations

- Consider a particle with no forces acting on it.
- The equation of motion is then

$$m\frac{d\vec{v}}{dt} = 0$$

or, using the dot equation for time derivative,

$$m\dot{\vec{v}} = 0$$

- We can drop the factor of mass and write the equation in component form as

$$\begin{aligned} \dot{v}_x &= 0 \\ \dot{v}_y &= 0 \\ \dot{v}_z &= 0 \end{aligned}$$

Simple examples of solving Newton's equations

- The solution is simple: the components of the velocity are constant and can just be set equal to their initial values

$$v_x(t) = v_x(0)$$
$$v_y(t) = v_y(0)$$
$$v_z(t) = v_z(0)$$

- This is often referred to as Newton's first law of motion:

  Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

Simple examples of Newton's equations

- The equations are called Newton's second law of motion:

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

- The first law is simply a special case of the second law when the force is zero.

Simple examples of Newton's equations

- The equations

$$
\begin{aligned}
v_x(t) &= v_x(0) \\
v_y(t) &= v_y(0) \\
v_z(t) &= v_z(0)
\end{aligned}
$$

can be expressed in the form

$$
\begin{aligned}
x(t) &= x_0 + v_x(0)t \\
y(t) &= y_0 + v_y(0)t \\
z(t) &= z_0 + v_z(0)t
\end{aligned}
$$

or, in vector notation

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t$$

Simple examples of Newton's equations

- Discuss the two cases (i) constant force in z direction and (ii) harmonic oscillator on white board.