Quantum Information Theory

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Particle motion

- The concept of a point particle is an idealization.
- No object is so small that it is a point.
- But in many situations we can ignore the extended structure of objects and treat them as points.
- The position of a particle is specified by giving a value for each of the three spatial coordinates, and the motion of the particle is defined by it position at every time.
- Mathematically, we can specify a position by giving the three spatial coordinates $x(t), y(t), y(t)$.

Particle motion

- The position can also be thought of as a vector $\vec{r}(t)$ whose components are $x(t), y(t), z(t)$ at time t.
- The path of the particle – its trajectory – is specified by $\vec{r}(t)$.
- The job of classical mechanics is to figure out $\vec{r}(t)$ from some initial condition and some dynamical law.

Particle motion

- Next, to its position, the most important thing about a particle is its velocity.
- Velocity is also a vector.
- Consider the displacement of the particle between time t and a little bit later at time $t + \Delta t$.
- During that time interval the particle moves from $x(t), y(t), z(t)$ to $x(t + \Delta t), y(t + \Delta t), z(t + \Delta t)$.
- Expressed in vector notation, from $\vec{r}(t)$ to $\vec{r}(t + \Delta t)$.

Particle motion

- The displacement is defined as

$$
\begin{aligned}
\Delta x &= x(t+\Delta t) - x(t) \\
\Delta y &= y(t+\Delta t) - y(t) \\
\Delta z &= z(t+\Delta t) - z(t)
\end{aligned}
$$

or

$$\Delta \vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t).$$

Particle motion

- To get the velocity, we divide the displacement by Δt and take the limit as Δt shrinks to zero. For example,

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- This, of course, is the definition of the derivative of x with respect to t

$$v_x = \frac{dx}{dt} = \dot{x}$$

- Placing a dot over a quantity is standard shorthand for taking the time derivative.

Particle motion

- It would be cumbersome to keep writing x, y, z so we condense the notation.
- The three coordinates x, y, z are denoted collectively by x_i and the velocity components by v_i

$$v_i = \frac{dx_i}{dt} = \dot{x}_i$$

where i takes the values x, y, z, or, in vector notation

$$\vec{v} = \frac{\vec{r}}{dt} = \dot{\vec{r}}$$

Particle motion

- Acceleration is the quantity that tells you how the velocity is changing.
- If an object is moving with constant velocity vector, it experiences no acceleration.
- A constant velocity vector implies not only a constant speed but also constant direction.
- You feel acceleration only when your velocity vector changes in magnitude or direction.

Particle motion

- In fact, acceleration is the time derivative of the velocity

$$a_i = \frac{dv_i}{dt} = \dot{v}_i$$

or, in vector notation,

$$\vec{a} = \dot{\vec{v}}$$

Particle motion

- Because v_i is the time derivative of x_i and a_i is the time derivative of v_i, it follows that acceleration is the second time derivative of x_i

$$a_i = \frac{d^2 x_i}{dt^2} = \ddot{x}_i$$

where the double-dot notation means the second time derivative.