Energy

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Force and potential energy

- One often learns that there are many forms of energy (kinetic, potential, heat, chemical, nuclear, ...) and that the sum total of all of them is conserved.
- But when reduced to the motion of particles, classical physics really has only two forms of energy: kinetic and potential.
- The best way to derive the conservation of energy is to jump right into the formal mathematical principles and then step back and see what we have.

Force and potential energy

- The basic principle – call it the potential energy principle – asserts that all forces derive from a potential energy function, denoted $V(\{x\})$.
- Here $\{x\}$ represents the entire set of $3N$ coordinates – the configuration space – of all particles in the system.
- Let's begin with the simplest case of a single particle moving along the x axis under the influence of a force $F(x)$.
- According to the potential energy principle, the force on the particle is related to the derivative of the potential energy $V(x)$:

$$F(x) = -\frac{dV(x)}{dx}$$

Force and potential energy

- In the one-dimensional case, the potential energy principle is really just the definition of $V(x)$.
- The potential energy can be reconstructed from the force by integrating the above equation

$$V(x) = -\int F(x)dx$$

- We can think of the potential energy principle as follows: The force is always directed in such as way that pushes the particle toward lower potential energy (note the minus sign).
- Moreover, the steeper $V(x)$, the stronger the force.
- *Force pushes you down the hill.*

Force and potential energy

- Potential energy by itself is not conserved.
- As the particle moves, $V(x)$ varies.
- What is conserved is the sum of potential energy and kinetic energy.
- Roughly speaking, as the particle rolls down the hill, it picks up speed.
- As it rolls up the hill, it loses speed.

Force and potential energy

- Kinetic energy is defined in terms of the velocity v and m of the particle

$$T = \frac{1}{2}mv^2$$

- The total energy E of the particle is the sum of the kinetic and potential energies:

$$E = \frac{1}{2}mv^2 + V(x)$$

- As the particle rolls along the x axis, the two types of energy individually vary, but always in such a way that the sum is preserved.
- Present derivation on whiteboard.