Quantum Information Theory

Pawel Wocjan

University of Central Florida

Fall 2019

The nature of classical physics

What is classical physics?

- The term classical physics refers to physics before the advent of quantum mechanics.
- Classical physics includes Newton's equations of motion, the Maxwell-Faraday theory of electromagnetic fields, and Einstein's general theory of relativity.
- But it is more than just specific theories of specific phenomena.
- It is a set of principles and rules – an underlying logic – that governs all phenomena for which quantum uncertainty is not important.
- Those general rules are called classical mechanics.

What is classical physics?

- The job of classical mechanics is to predict the future.
- The great eighteenth-century physicist Pierre-Simon Laplace said it out in a famous quote:

  > *We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.*

  – Pierre Simon Laplace, *A Philosophical Essay on Probabilities*

What is classical physics?

- In classical physics, if you know everything about a system at some instant of time, and you also know the equations that govern how the system changes, then you can predict the future.
- That is what we mean when we say that the classical laws are deterministic.
- If we can say the same thing, but with the past and future reversed, then the same equations tell you everything about the past.
- Such a system is called reversible.

Simple dynamical systems and the space of states

- A collection of objects – particles, fields, waves, or whatever – is called a system.
- A system that is either the entire universe or is so isolated from everything else that it behaves as if nothing else exists is a closed system.

Simple dynamical systems and the space of states

- To get an idea of what deterministic and reversible mean, we are going to begin with some extremely simple closed systems.

Simple dynamical systems and the space of states

- Imagine an abstract object that has only one state.
- We could think of it as a coin glued to the table – forever showing heads.
- In physics jargon, the collection of all states occupied by a system is its space of states, or more simply, its state-space.
- The state-space is not ordinary space; it's a mathematical set whose elements label the possible states of the system.
- Here the state-space consists of a single point – namely Heads (or just H) – because the system has only one state.
- Predicting the future is extremely simple: nothing ever happens and the outcome of any observation is always H.

Simple dynamical systems and the space of states

- The next simplest system has state-space consisting of two points; in this case we have one abstract object and two possible states.
- Imagine a coin that can be either Heads or Tails (H or T).

Simple dynamical systems and the space of states

- In classical mechanics we assume that systems evolve smoothly, without any jumps or interruptions.
- Such behaviour is said to be continuous.
- Obviously, we cannot move between Heads and Tails smoothly.
- So let's assume that time comes in discrete jumps labeled by integers.
- A world whose evolution is discrete could be called stroboscopic.

Simple dynamical systems and the space of states

- A system that changes with time is called a dynamical system.
- A dynamical system consists of more than a space of states.
- It also entails a law of motion or dynamical law.
- The dynamical law is a rule that tells us the next state given the current state.

Simple dynamical systems and the space of states

- One simple dynamical law is that whatever the state at some instant, the next state is the same.
- In this case, it has two possible histories: H H H H H H ... and T T T T T T ...
- Another dynamical law dictates that whatever the current state, the next state is the opposite.

Simple dynamical systems and the space of states

- We can even write these dynamical laws in equation form.
- The variables describing a system are called its degrees of freedom.
- Our coin has one degree of freedom, which we denote by σ.
- The two possible states are:
  $\sigma = +1$ for H
  $\sigma = -1$ for T
- The state at time n is described by $\sigma(n)$.

Simple dynamical systems and the space of states

- The first law in equation form is:

$$\sigma(n+1) = \sigma(n)$$

- The second law in equation form is:

$$\sigma(n+1) = -\sigma(n)$$

- These two laws are deterministic because in each case the future behaviour is completely determined by the initial state.
- All the basic laws of classical mechanics are deterministic.

Simple dynamical systems and the space of states

- To make things more interesting, let's generalize the system by increasing the number of states.
- Draw diagrams representing different dynamical laws on white board.

The minus-first law

- According to the rules of classical physics, not all laws are legal.
- It's not enough for a dynamical law to be deterministic; it must also be reversible.
- The meaning of reversible – in the context of physics – can be described in a few different ways.
- If you reverse all the arrows, the resulting law is still deterministic.
- Another way is to say that the laws are deterministic into the past as well as the future.
- Recall Laplace's remark, "for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

The minus-first law

- Can one conceive laws that are deterministic into the future, but not into the past?
- In other words, can we formulate irreversible laws?
- Draw diagrams of irreversible laws on white board.

Dynamical systems with an infinite number of states

- There is no reason why you can't have a dynamical system with an infinite number of states.
- For example, image a line with an infinite number of discrete points along it – like a train track with an infinite sequence of stations in both directions.
- In this case, the state-space is equal to the set $\mathbb{Z}$ of integers.
- Suppose that a marker of some sort can jump from one point to another according to some rule.
- The history of the marker would consist of a function $\sigma(n)$ telling you the position of the marker along the track at every time $n \in \mathbb{Z}$.

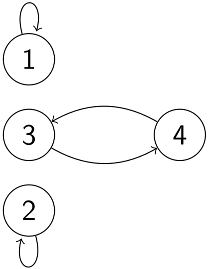
Dynamical systems with an infinite number of states

- Here are some example of possible dynamical laws:

$$\sigma(n+1) = \sigma(n) + 1$$
$$\sigma(n+1) = \sigma(n) + 2$$
$$\sigma(n+1) = \sigma(n)^2$$
$$\sigma(n+1) = (-1)^{\sigma(n)}\sigma(n)$$

Cycles and conservation laws

- When the state-space is separated into several cycles, the system remains in whatever cycles it started in.
- Each cycle has its own dynamical rule, but they are all part of the same state-space because they describe the same dynamical system.

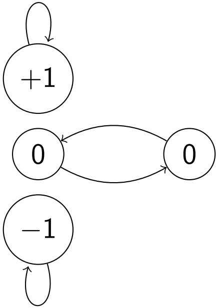


Cycles and conservation laws

- Whenever a dynamical law divides the state-space into separate cycles, there is a memory of which cycle they started in.
- Such memory is called a conservation law; it tells us something is kept intact for all the time.
- To make the conservation law quantitative, we give each cycle a numerical value $\mathcal{Q}$.
- The three cycles in the previous example are $\mathcal{Q} = +1$, $\mathcal{Q} = -1$, and $\mathcal{Q} = 0$.

Cycles and conservation laws

- Whatever the value of $\mathcal{Q}$, it remains the same for all time because the dynamical law does not allow jumping from one cycle to another cycle.
- Simply stated, $\mathcal{Q}$ is conserved.



Cycles and conservation laws

- We will take up the problem of continuous motion in which both time and the state-space are continuous.
- All the things we have discussed for simple discrete systems have their analogs for the more realistic system.

The limits of precision

- Laplace may have been overly optimistic about how predictable the world is, even in classical physics.
- ...