Binomial Distribution

→ Bernoulli trial.

The Binomial Experiment.

→ X (Random Variable) that equals the # of trials that result in a success (x) from among n Bernoulli trials is called a Binomial RV with parameters

$$0 < p < 1 \quad \text{and} \quad n = 1, 2, 3, \ldots$$

Suppose,

- n is the total # of trials,
- x is the # of trials that result in a success,
- $n-x$ is the # of trials that result in a failure,
- p = Prob (success) $= \frac{x}{n}$
- q = P(failure) $= \frac{n-x}{n}$ or $1-p$.

$$p + q = 1$$

P (Success)

P(failure)

To develop the PD for a Binomial Random Experiment:

(i) Must first determine the prob. of any one way the event of interest can occur.

$\#$ of success $= x$

$P(\text{success}) = p$

(ii) Then multiply this prob. by the total $\#$ of ways that this event of interest can occur.

$$C(n,x) = {}^nC_x = \frac{n!}{x!(n-x)!}$$

and $0! = 1$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

The Prob. of one of the ways the event of interest can occur $= p^x \cdot q^{n-x}$

Then,

Binomial RV X's PMF :

$$\boxed{f(x) = P(X=x) = {}^nC_x \, p^x \cdot q^{n-x}}$$

flavours = {Vanilla, Choc, Strawberry, Black Current, Pistachio}

Q:) Assuming every transaction involves the purchase of a single ice cream only, find the probability that exactly 1 customer purchases one 'Strawberry' ice cream, in the next 5 customers.

Solⁿ: We can use Binomial Dist, to determine its probability.

Experiment: determining the Prob of next customer purchasing a strawberry flavoured ice-cream.

Event of interest = "A customer purchasing one strawberry ice-cream."

$$\text{Prob(success)} = p = \frac{1}{5} = 0.2$$

$$\text{Prob (failure)} = q = 1-p = 1-0.2 = 0.8 \left(\frac{4}{5}\right)$$

# of trials: $n = 5$

# of successes x	# of failure $n-x$	${}^{n}C_{x}$	Total Possible ways	
0	5	${}^{5}C_{0}$	1	
1	4	${}^{5}C_{1}$	5	
2	3	${}^{5}C_{2}$	10	
3	2	${}^{5}C_{3}$	10	
4	1	${}^{5}C_{4}$	5	
5	0	${}^{5}C_{5}$	1	
			$\Sigma = 32$	

Event of interest:

No. of successes, $x = 1$

$X = \{0, 1, 2, 3, 4, 5\}$

# of outcomes associated with our event of interest:

$${}^{5}C_{1} = \underline{5}$$

No. of trials (n)

No. of successes (x)

success is → "A customer purchasing ONE strawberry ice-cream."

This table shows the different ways in which the event of interest can occur:

Trials	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	$p^x q^{n-x} = (0.2)^1 \times (0.8)^4$
Outcome	S 0.2	F 0.8	F	F	F	0.08192
	F 0.8	S 0.2	F	F	F	0.08192
	F 0.8	F 0.8	S 0.2	F	F	"
	F 0.8	F ·	F	S 0.2	F	"
	F 0.8	F ·	F	F	S 0.2	"

$p = 0.2$
$q = 0.8$

$$P(-) = {}^5C_1 \times 0.08192 = 5 \times 0.0819$$
$$= 0.409599$$
$$= 40.9599\%$$

Poisson Distribution

The Poisson Process:

In an interval of Real Numbers, let's assume their counts (ie success or an outcome of interest → x) occurring randomly throughout the interval

If you say that the interval could be further subdivided into sub-intervals of length small enough such that it possesses the following characteristics for each sub-interval:

(i) P(success>1) is almost 0.

(ii) P(success=1) would be the same for every subinterval & its value would be proportional to the sub-interval length.

(iii) n(success) in each sub-interval would be independent of other sub intervals.

↳ Poisson Process.

The Poisson Dist:

The RV (X) that equals the # of counts in the interval of a Poisson process is called a Poisson RV with parameter $0 < \lambda$, and the PMF:

$$f(x) = P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

where $x = 0, 1, 2, 3, \ldots$ = # of counts ie. the total # of successes in the provided interval.

λ = successes count we are expecting in the provided interval.

$e = 2.718$ (Euler's Number).

Q) $\mu = 10$ customers / hr

She wants to plan out to close the parlour for the day in another one hour.

Calculate the Prob of 12 or more customers visiting the parlour.

Sol^n: Experiment: determine ≥ 12 customers.

Interval: 1 hour

$\mu = \lambda = 10$

Event of interest:

$P(X \geq 12) = P(X=12) + P(X=13) + P(X=14) + \ldots$

$= 1 - P(X \leq 11)$

Random Variable (X):

X = # of customers visiting the parlour in the next one hour

$X = \{0, 1, 2, 3, \dots\}$