Probability Mass Function (PMF)

→ aka the mass of a discrete RV, defines that the probability of a RV with discrete value is exactly equal to some value.

$$X = \{x_1, x_2, x_3, \dots\}$$

or $X = x_k$ where $k = 1, 2, 3, \dots$

The PMF (f), of a discrete RV (X):

$$f(x_k) = P(X = x_k)$$

The PMF will have 2 properties:

1. $f(x_k) \geq 0 \quad \forall \; x_k \in \mathbb{R}$

2. Summation of $f(x_k) = 1$

# of ice creams Purchased at a time	Total # of Customers	Calculation for PMF ($f x_k$) $P(X = x_k)$	PMF	CDF
1	235	235/540	0.435	0.435
2	175	175/540	0.324	0.759
3	70	70/540	0.13	0.889
4	25	25/540	0.046	0.935
5	25	25/540	0.046	0.981
6	10	10/540	0.019	1.00
	$\Sigma = 540$		$\Sigma = 1.00$	

Table: Sales details for March

The Prob. of a customer buying exactly 1 ice-cream = 43.5%

Q.) The Prob of the next customer purchasing at most 2 ice-creams?

A: $\underbrace{P(X \leq 2)}$ = P(X=1) + P(X=2)

$\hookrightarrow$ Cumulative Distribution Function.

Q.) How many of 200 customers of a random sample are expected to buy at most 2 ice-creams?

A: $P(x \leq 2) = P(x=1) + P(x=2) = 0.435 + 0.324$

$= 0.759 \sim \underline{76\%}$

Hence, in the selected random sample,

76% of 200 = 0.76 × 200 = 152 customers

are expected to buy at most 2 ice-creams.

Q.) In the same random sample, how many

are expected to buy at least 2 ice-creams?
(inclusive of 2)

A: $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) + \cdots + P(X=6)$

$= 0.324 + 0.131 \ldots + 0.019$

$= 0.565 \approx 56.5\%$

OR.

$1 - P(X=1) = 1 - 0.435 = 0.565$
or
56.5%

Discrete Uniform Distribution

→ Fixed # of outcomes
→ Constant Prob. for each outcome.

∴ X has a discrete Uniform dist., if every m values in its range, say,

$a_1, a_2, a_3, \ldots, a_m$ has an equal likeliness to occur.

$$P(X=a_1) = P(X=a_2) = \cdots = P(X=a_m) = \frac{1}{m}$$

Let's say you're rolling a fair die.

RV (X) = Outcome of a die roll.

$X = \{1, 2, 3, 4, 5, 6\}$

$f(x) = P(X=1) = P(X=2) = \dots = P(X=6) = \frac{1}{6} = 0.1\bar{6}\bar{6}$