Computer Vision - 2025

A Tiny Lecture #2. Room for Doubt

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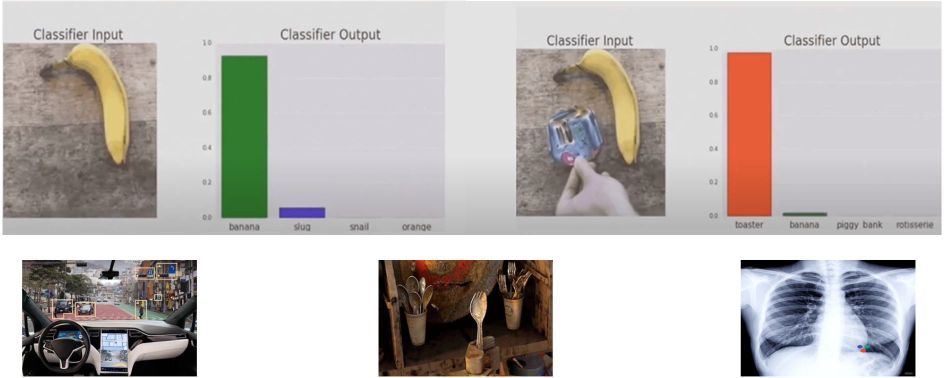
Agenda

Section 1. On uncertainty in AI

All models are wrong, but models that know when they are wrong, are useful /George Box + some corrections/



Overconfidence effect (first line) and some uncertainty domains (second line).

Goals

The goal of this lecture is:

1. To teach students how to incorporate uncertainty into account in an ML model

Section 2. Recap

Binary Cross-Entropy (BCE) Loss Intuition

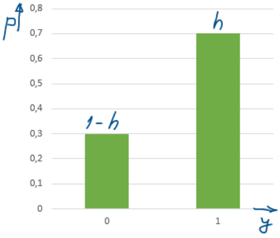


Figure: The Bernoulli distribution Bishop and Nasrabadi (2006); Prince (2023).

Derivation of BCE Loss from MLE

For a pair (x_i, y_i), the Bernoulli distribution takes the form:

$$p(y_i \mid h_i) = h_i^{y_i}(1 - h_i)^{1-y_i}.$$

This represents the probability of observing y_i given the predicted probability h_i.
For a dataset of N i.i.d. pairs, the joint probability is:

$$P(y_1, y_2, \dots, y_N \mid h_1, h_2, \dots, h_N) = \prod_{i=1}^{N} h_i^{y_i}(1 - h_i)^{1-y_i} \to \max .$$

Then we take the negative logarithm of the joint probability:

$$-\log P = -\sum_{i=1}^{N} [y_i \log h_i + (1 - y_i) \log(1 - h_i)] \to \min .$$

Binary Cross-Entropy (BCE) Loss Intuition

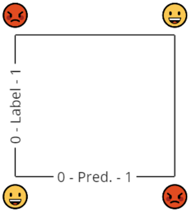


Figure: The BCE loss values intuition concerning the predictions and labels Prince (2023); Goodfellow et al. (2016).

Derivation of BCE Loss from MLE

The Binary Cross-Entropy (BCE) loss can be derived from Maximum Likelihood Estimation (MLE) for binary classification problems. Given a set of predictions $\hat{y}_i$ and true labels y_i, the BCE loss is defined as:

$$\mathcal{L}_{\text{BCE}} = -\frac{1}{N}\sum_{i=1}^{N}\left[y_i \log h_i + (1 - y_i)\log(1 - h_i)\right],$$

where:

- N is the number of samples,
- $y_i \in \{0, 1\}$ is the true label,
- $h_i \in (0, 1)$ is the predicted probability.

Section 3. Uncertainty-aware Loss Intuition

Uncertainty-aware Loss Intuition

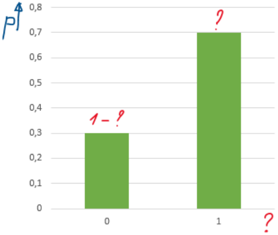


Figure: The Bernoulli distribution Bishop and Nasrabadi (2006); Prince (2023).

Derivation of UA Loss from MLE

For a pair (x_i, y_i), the Bernoulli distribution takes the form:

$$p(\quad|\quad) = \qquad\qquad .$$

This represents the probability of observing . . . given the predicted probability . . .
For a dataset of N i.i.d. pairs, the joint probability P is:

$$P = \prod_{i=1}^{N} \qquad (1- \quad)^{1-} \qquad \to \max .$$

Then we take the negative logarithm of the joint probability:

$$-\log P = -\sum_{i=1}^{N} [\quad \log \quad + (1- \quad) \log(1- \quad)] \to \min .$$

Uncertainty-Aware Loss Intuition

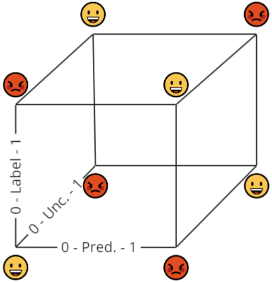


Figure: The UA loss values intuition concerning the predictions, the labels, and the uncertainties .

Derivation of UA Loss from MLE

The UA loss can be derived from Maximum Likelihood Estimation (MLE) for binary classification problems. Given a set of predictions $\hat{y}_i$ and true labels y_i, the BCE loss is defined as:

$$\mathcal{L}_{\text{UA}} = -\frac{1}{N}\sum_{i=1}^{N}[\delta_i \log u_i + (1-\delta_i)\log(1-u_i)] \to \min,$$

where:

- N is the number of samples,
- $\delta_i = (y_i - h_i)^2 \in (0,1)$ is the pseudo label,
- $y_i \in \{0,1\}$ is the true label,
- $h_i \in (0,1)$ is the prediction,
- $u_i \in (0,1)$ is the uncertainty of the prediction.

Section 4. Conclusion

Conclusion

Taking the uncertainty into account makes a model slightly more complicated but also more flexible for real-world data. A more precise test of the research domain helps determine whether the application of uncertainty-aware models is necessary.

Bibliography

Bishop, C. M. and Nasrabadi, N. M. (2006). Pattern recognition and machine learning, volume 4. Springer.

Goodfellow, I., Bengio, Y., and Courville, A. (2016). Deep Learning. MIT Press. http://www.deeplearningbook.org.

Prince, S. J. (2023). Understanding Deep Learning. The MIT Press.