Genetic Algorithm Approach for Spur Gears Design Optimization

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Abstract— **Gears are used in most types of machinery and vehicles for the power transmission system. The design of gears is highly complicated involving the satisfaction of many constraints such as strength, pitting resistance, bending stress, scoring wear, and interference in involute gears. In addition, using conventional or traditional optimization techniques to solve this problem could not give optimum results. A stochastic approach as a Genetic Algorithm (GA) is applied in this paper to find the optimal combination of design parameters for minimum weight of spur gears. The purpose of this study is minimizing the weight and the center distance of one pair of spur gears. This objective was accomplished by the means of the GA under some constraint such as bending strength, a contact stress and each dimension conditions of gears, which must be satisfied. The results are calculated by using *Matlab* tools of Genetic algorithm with four type of materials, which are standard steel, stainless steel, gray cast iron, and Alloy copper.**

Keywords- Spur gear; Weight; Center distance; Constraints; Genetic algorithm; Materials

I. INTRODUCTION

Gear trains are used to transmit power in various industrial and defense applications. The weight is the important criteria in the mobile applications affecting cost. Therefore, many researches in design optimization are focused on gears train systems. Yokota et al. [1] described genetic algorithm for the optimization of gear weight. Savsani et al. [2] described the design of minimum weight gear trains using particle swarm optimization and simulated annealing algorithms. Gologlu and Zeyveli [3] presented an automated preliminary design of gear drives by minimizing volume of gear trains using a genetic algorithm. Bonori et al. [4] performed the optimization of gear pairs aiming to reduce vibration and noise by using a genetic algorithm. Mendi et al. [5] presented a dimensional optimization using a genetic algorithm. Thompson et al. [6] presented their work on the optimization of multi-stage spur gear reduction units taking into account minimum volume and surface fatigue life, as objective functions, employing quasi-Newton method. Zarefar and Muthukrishnan [7] used random search algorithm for the weight optimization of helical gear. Four design variables, module, helix angle, number of teeth on pinion, and face width were taken along with constraints on contact stress and bending stress. Wang and Wang [8] considered center distance, weight, tooth deflection, and gear life as objective functions. The authors had used modified iterative weighted Tchebycheff (MIWT) method for the solution. Chong et al. [9] described a method for reduction of geometrical volume and meshing vibration of cylindrical gear pairs while satisfying strength and geometric constraints using a goal programming formulation. Mao [10] described optimization of gear surface micro-geometry for the fatigue wear reduction. Wang [11] presented an optimization of tooth profile of gears for the reduction of noise. The rotational movement of gears is treated as the input of the gear system and the acoustical noise signal as the output. Daoudi and Boudi [12] proposed a methodology for the shape optimization of the epicyclical gear train system based on the parametric shape optimization. The objective is a lighter gearbox with respect of kinematic and dynamic behavior. Daoudi et Boudi [13] have illustrated a comparison between two advanced optimization algorithms known as Genetic algorithm (GA) and particle swarm optimization (PSO) in order to find the optimal combination of design parameters for minimum volume of planetary gear train. Marjanovic et al. [14] have presented one practical approach to gear train optimization. It provides a description for selection of the optimal concept, based on selection matrix, selection of optimal materials, optimal gear ratio and optimal positions of shaft axes. Also, Original software for gear train optimization named GTO was developed. B. Harinath Reddy et al. [15] have used the advanced optimization technique, Genetic Algorithm (GA) to find the optimal combination of design parameters for minimum weight of a gear train. P.Rai et al. [16]Perforùmed the optimization of the volume of helical gear with profile shift by the means of real-coded genetic algorithm.

Optimization design of gear reducer includes the continuous variables and discrete variables. The traditional method is to round the optimal design to the adjacent discrete points. Thus, design point might run out of the feasible region, besides traditional optimization method is mostly based on gradient algorithm, which is likely trapped into local minimum search. Many studies indicate that the genetic algorithm has strong ability of general optimization, which is very effective

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in treating optimization problem containing continuous and discrete variables [17, 18].

II. THE PRINCIPE OF GENETIC ALGORITHM

Originally developed by Holland, a genetic algorithm (GA) is a robust technique, based on the natural selection and genetic production mechanism. The genetic algorithm works with a group of possible solutions within a search space instead of working with a single solution as is seen in gradient optimization methods [19].

Genetic Algorithm (GA) is referred to as a search method of optimal solution to simulating Darwin's genetic selection and biological evolution process. Genetic algorithm is a series of random iterations and evolutionary computations simulating the process of selection, crossover and mutation occurred in natural selection and population genetics, in according to the survival of the fittest, through crossover and mutation, good quality gradually maintained and combined, while continually producing better individuals and out of bad individuals. Through the generational produce and optimizing the individual, the whole group evolves forward and constantly approaches to the optimal solution.

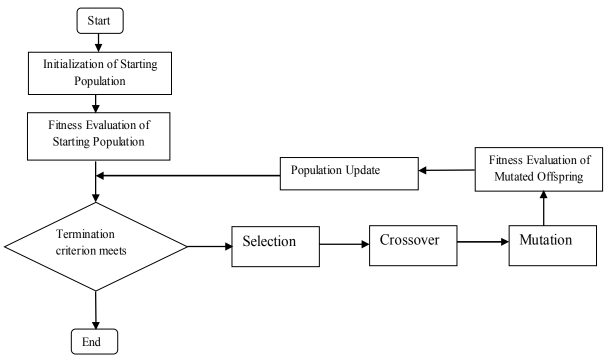

Figure 1. Flowchart of GA for solving problem.

This flowchart present the principal operators of genetic algorithm. Selection is the first genetic operator. Fitness function is evaluated for each individual in the population and at least two individuals with low fitness function are selected to form next generation. Crossover is the second genetic operator that allows producing offspring by recombining the chromosomes of two individuals. Mutation is the third genetic operator allowing creating a new individual by changing 0 to 1 or changing 1 to 0. Each bit of each individual is possible subject to mutation [21].

Genetic algorithm, not requiring gradient information and continuous function, optimization results being global, applied to mechanical design optimization problems, can effectively avoid local optimal solutions, and get the global optimal solution [20].

III. DEFINITION PROBLEM

Single gear pair optimization have concentrated on investigating the optimal bending and contact stresses on the gear tooth with regard weight, center distance, and material capabilities.

The model studied in this paper is one stage of spur gears train as shown in figure 2. The Optimization of weight and center distance is carried out by using GA with help of a program developed on *Matlab* platform. Many Factors are considered in this optimization as such as torque, material, width of tooth, gear ratio and input power as defined in the table below (Table 1).

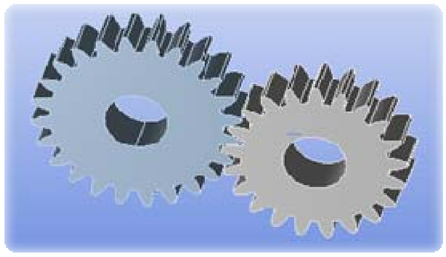

Figure 2. Pair of spur gears, model of application *ANSYS 16.0*.

A. *Objective Functions*

In our cases, multi-objective optimization is presented which are the minimization of the center distance and the weight of spur gear pair.

- *Center distance*

The user of gears or products, which contain gears often, demands smaller gear sets. In general, the most desirable gear set is the smallest one that will perform the required job. Smaller gears are easier to make, run more smoothly due to small inertial loads and pitch line velocities and also less expensive. Smaller gears would require less material to make and less space to operate it. The following formula define the equation govern the center distance of gear set [10].

$$a = \frac{m}{2}(Z_1 + Z_2) \tag{1}$$

Thus, the objective function for minimizing center distance of spur gear train is expressed in (2).

$$F_{1\text{-objective}} = F(Z_1\,,\, Z_2,\, m) \tag{2}$$

Where a is the center distance, m is the module, Z_1 and Z_2 is the number of teeth on pinion and on gear respectively.

- *Gears Weight*

A weight reduction of gears improves performance of non-stationary systems and also it saves the material, which leads to cost reduction and easy assembling. The following is the equation of the weight of the gear set [22].

$$W = \frac{\pi}{4} * \rho * m^2 * b * (Z_1^2 + Z_2^2) \tag{3}$$

So, the objective function for minimizing weight is presented as shown in (4).

$$F_{2\text{-objective}} = F\,(Z_1\,, m,\, b) \tag{4}$$

Where b represent the tooth width and ρ the density of the material.

B. Constraints

The geometry of gears train is very specific. Therefore, they are many constraints of assembly, dimension, contact stress, and bending stress must be satisfied and introduced in the objective function for the best selection of optimum results. All the constraint are formulated in equations (5),…, (10).

- The maximum and the minimum of tooth width

$$g_1(x) = \frac{b}{m} - 12 \leq 0 \quad (5)$$
$$g_2(x) = 6 - \frac{b}{m} \leq 0 \quad (6)$$

- The maximum and the minimum of tooth number

$$g_3(x) = 18 - Z_1 \leq 0 \quad (7)$$
$$g_4(x) = Z_1 - 30 \leq 0 \quad (8)$$

- The constraint of bending stress

$$g_5(x) = \frac{(i+1)}{a*m*b*y}[M_t] - [\sigma_b] \leq 0 \quad (9)$$

- The constraint of contact stress

$$g_6(x) = 0.74 * \frac{(i+1)}{a}\sqrt{\frac{i+1}{i*b} E * [M_t]} - [\sigma_c] \leq 0 \quad (10)$$

C. Gear design parameters

The gear design parameters and materials are taken from the mechanical references [24]. They are listed below in the table1. The choice of this material was based on the more useful materials for gears.

TABLE I. DESCRIPTION OF PARAMETERS AND DESIGN VARIABLES

Parameters	Symbol	Unit	Value
Power delivered	P	Kw	15
Input speed	N	Rpm	1500
Gear ratio	I	-	3.5
Density material			
Standard steel	ρ	N/mm2	7850
Stainless steel	-	-	7750
Gray Cast Iron	-	-	7200
Alloy copper	-	-	8300
Young's modulus			
Standard /Stainless steel	E	N/mm2	2.1e+5
Gray cast Iron		-	1.4e+5
Alloy copper		-	1.2e+5
Module	M	mm	[1.5 6]
Number of pinion teeth	Z_1		[18 30]
Tooth width	b	mm	[9 72]

IV. IMPLEMENTATION OF THE GENETIC ALGORITHM

The program code of GA is implemented in *MATLAB* software. For solving the problem with GA optimization, several step must be followed.

a) Step 1

The variables of our problem must be converted to the useful form of *Matlab* as expressed in the vector x. The vector x present all the variable of optimization.

$$X=[X_1\ X_2\ X_3]=[Z_1\ m\ b]$$

Then, some input parameter need to be initialized in the first time for the execution of code. There are proper to the GA, for example, the choice of 200 for the ngen it wasn't arbitrary, because the number of generation exceed the product of number of variable and npop , for the crossover and mutation probability were be changed according to the fineness of the desirable result, which are named as:

```
ngen=200 ;        %Number of generation
nPop =50;         % Population size
nvar =3;          %Number of the variables
stringlength=8;          % Length string
x_bound = [18,30;1.5,6;9,72];
                  % Lower and Upper bound for each parameter.
Pc =0.9           %Crossover probability
Pm =0.05          %Mutation probability
```

b) Step 2:

Definition of the objective functions as defined below.

```
function y = object_fitness(x)
y(1)=(pi/4)*(x(2)^2)*(x(1)^2+((x(1)^2)/i))*x(3
)*rho/1e+9
y(2)=x(1)*x(2)*(i+1)/(2*i)
end
```

c) Step3

Every optimization need some constraint to find an optimal points. Thus, all the constraints are formulated in the constraint file.

```
function [c, ceq] = object_constraint(x)
   c = [(x(3)/x(1))-12,
        6-(x(3)/x(1)),
              .
              .
         ((2*i/((x(1)^2)*x(2)*x(3)))*Mt)-
sigmab,

(2*i/(x(1)*x(2))*sqrt(((i+1)/i*x(3))*E*Mt))-
sigmac];
   ceq = [];
end
```

V. GENETIC ALGORITHM RESULTS

The optimum design parameters values obtained by using the Genetic algorithm are given in the table 2.

TABLE II. OPTIMIZATION RESULTS OF SPUR GEARS SET

Materials	Gen	Z1	m	b	Center distance	Weight
Standard steel	102	29	6	14.507	111.237	3.443
Stainless steel	112	28	5.75	13.714	109.565	3.118
Gray cast iron	104	27	6	15.495	105.715	3.046
Alloy copper	122	28	5.5	13.564	107.028	3.151

The table show that the gray cast iron present the best fitness of the weight and the center distance with a value of 3.046 kg and 105.715 mm respectively. The value of width tooth was maximal and was found to be equal to 15.495 mm in the case of Gray cast iron. The model of gears train with the standard steel represent the high weight and center distance with an average in the tooth width value. The results of Stainless steel and gray cast iron are nearly the same with the difference in module and width tooth value. The optimal result of the module was found to be equal to 5.5 mm with the Alloy copper material. The variation of the best fitness of the weight and center distance are presented in the figures below.

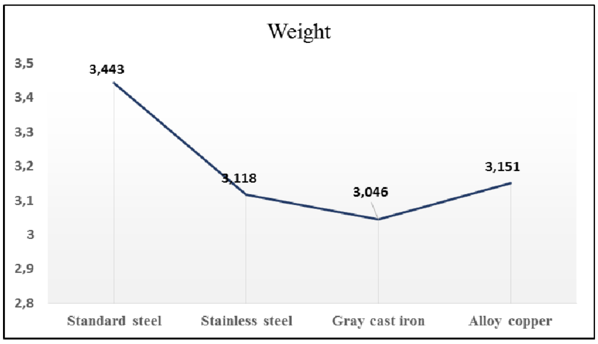

Figure 3. Values of weight in different materials.

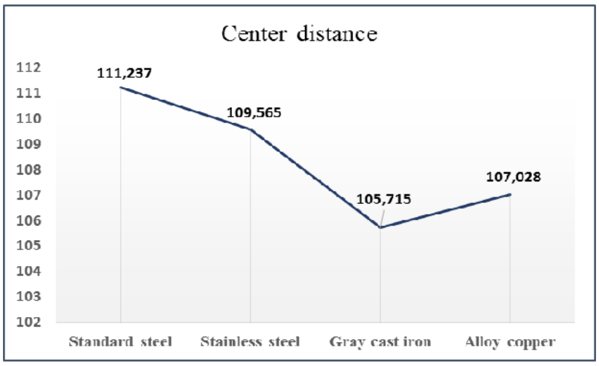

Figure 4. Values of center distance in different materials.

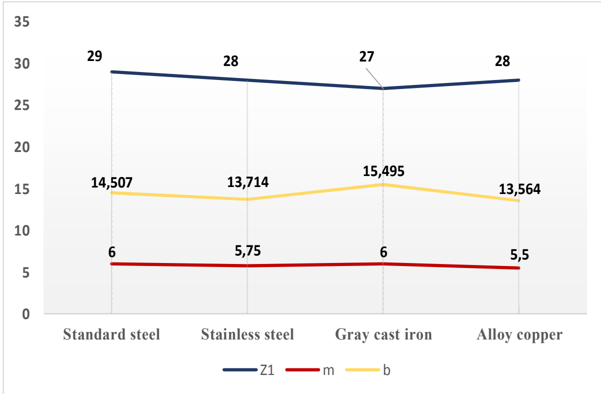

Figure 5. The optimal values of variables in different materials.

In the last graph, the optimal value of the number of teeth is presented in the case of gray cast iron. For the module and the tooth width, the alloy copper gives the best value. Therefore; the choice of materials is a task of the constructor and a structural analysis by the finite element method of the spur gears with different materials was necessary. It gives us a vision on the concentrations of stress, in order to minimize the propagation of cracks and to estimate the lifecycle of the system.

VI. CONCLUSION

Genetic algorithm approach is a relatively recent heuristic search method that is based on the idea of collaborative behavior and natural selection in biological populations. The objective of this paper was the optimization of the weight and the center distance of one pair of spur gear by the mean of the genetic algorithm technique.

The fitness function of gears weight was the best with a value of 3.046 kg for Gray cast iron. It was found to be equal to 3.443 for Standard steel, which present the poor result. The same diagnostic for the center distance it was maximal in the case of Gray cast iron and minimal in gears with Standard steel. The module present an average value equal to 5.75 mm through GA approach. The number of teeth was found to be equal to 28 in both cases stainless steel and alloy copper. It was maximal for gears with Standard style and minimal for the gray cast iron.

The results above determinate that a Genetic Algorithm is an important tool to optimize complex problems of mechanical system with a various parameters, objectives, and constraints.

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