Optimal Design of Compact One-Stage Spur Gear Reducer using Genetic Algorithm

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Abstract

The optimal design of external spur gear trains is a complex multi-objective problem that involves minimization of weight while maximizing fatigue strength and surface durability. This problem is further complicated by additional factors such as varied loading conditions, mounting conditions, variable speeds, thermal fatigue and so on. Designing a compact, lightweight spur gear train is advantageous since many power transmission applications require low weight [2], reduction of static loading on support members, and low manufacturing cost.

Typically, three geometric parameters are primarily considered as design variables for this problem. These are number of pinion teeth (N_p), diametral pitch (P), and tooth width (b) [3]. However, in most cases these values are chosen from past experiences which limit explorative studies. Furthermore, while hand chosen values might meet strength requirements, they do not guarantee an optimal reduction of weight or a compact design.

In this project, an optimization-based spur gear train design software was developed using a Genetic Algorithm (GA) that chooses values for (N_p, P, b) to minimize weight while ensuring maximum Gear-Tooth Bending Fatigue strength or Gear-Tooth Surface Durability strength is achieved for 10^6 cycles. For several steel alloys, it is demonstrated that the algorithm provides highly reliable configurations in both bending and surface durability analysis. This demonstrates the efficacy of this software for quickly analyzing effectiveness of different alloys in addressing a gear train design problem without requiring tedious manual hand calculations.

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Introduction

Gears are toothed members designed to transmit rotary motion from one shaft to another. When used in combinations, they may increase or decrease the torque or rotary speed of their connected shafts with power transmission efficiencies as high as 98% [5]. This, combined with their inherent durability, can make the design of gear combinations a desirable alternative to chains or belts. There are a wide variety of gears which may be chosen to achieve the required force transmission for a given application. Helical gears are typically used for high-speed designs due to better teeth meshing. Gear racks can convert rotational motion to linear motion. Worm gears can be used when quiet operation is desirable [5]. The spur gear, shown in Figure 1, is the most widely used, the simplest to use, and the simplest to analyze. Hence, design involving only these will be the focus of this project.

Figure 1: 1 Stage Spur Gear Example

Owing to the large number of parameters involved in gear train combinations, gearbox designs can become very complicated. If all parameters are considered, as many as 31 variables could be optimized for a given set of constraints [3]. Hence, it is important to only focus on optimizing the most important parameters while trying to determine the scope of a given gear design. This report demonstrates this concept through the optimization of a 1-Stage Spur Gear Reducer, in which the goal was to minimize the weight of the design by optimizing three design variables. The optimization algorithm designs under consideration the effects of bending and surface fatigue, which are common sources of failure for gear combinations. Various steel alloys were also considered for the pinion and gear design generations to demonstrate how much this material property may affect possible gear dimensions.

Significance

During the mechanical design process, it is vitally important to generate as many design alternatives as possible. To make these both a viable and competitive option, it is often desirable that the quality of information retrieved during investigations be as high as possible while simultaneously taking little time to get. This project demonstrates how a variety of possible alternatives based on basic material data may be generated for a gear reduction design while considering the effects of bending and surface fatigue, which are common sources of failure in these mechanisms. The code described in this report may be directly used during the design of any mechanism involving a simple 1-Stage Spur Gear reduction. In the case of LSU's capstone design, this can directly save a team time if they are trying to utilize a gear reduction within a gearbox or gear train. More importantly however, the concepts in the development of the algorithm this project uses may be applied to any engineering problem. This means that with the relevant engineering knowledge, this script may serve as a template to create specialized algorithms to optimize any mechanism for any application.

Background and Theory

Basic concepts and Nomenclature

Some of the basic concepts related to gears include the pitch circle, which is an imaginary circle that the gear teeth lie on, the pitch diameter, which is the diameter of the pitch circle, and the number of teeth, which determines velocity ratios.

Slight deviations in velocity ratios may occur due to manufacturing inaccuracies and tooth deflections, but acceptable tooth profiles are based on theoretical curves that meet the criterion of conjugate gear-tooth action. The pitch point, which is the point where the common normal to the surface at the point of contact intersects the line of centers, must remain constant as the gears rotate. A visualization of this is shown in Figure 2.

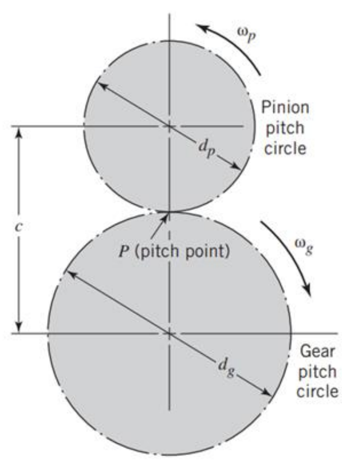

Figure 2: Friction Gear Pair Visualization

The gear ratio is the ratio of the number of teeth between two meshing gears, and it determines the speed and torque relationship between the input and output shafts. In general, gears with more teeth will provide a higher torque output but a lower rotational speed, while gears with fewer teeth will provide a higher rotational speed but a lower torque output.

The rotation of one pitch circle will cause rotation of the other at an angular-velocity ratio inversely proportional to their diameters, assuming no slippage occurs. In any pair of mating gears, the smaller one is called the pinion, and the larger one is called the gear. In equations, the subscript *p* is used to denote the pinion, and the subscript *g* is used to denote the gear.

$$\frac{\omega_p}{\omega_g} = -\frac{d_g}{d_p}$$

Equation 1: Velocity ratio between pinion and gear

Equation 1 uses the angular velocity, ω, and pitch diameter, d. The minus sign indicates that the two cylinders rotate in opposite directions.

When mounting gears to shafts, it is important to know the center distance of a gear. Error in this center distance can result in additional backlash of the gear mesh. If the center distance value is increased, the error would likewise increase. This results in the gear teeth not being able to mesh deeply enough, making the contact ratio decrease. The following equation is used to calculate the center distance. *d* refers to the pitch circle's diameter, and *r* is the pitch circle's radius.

$$c = \frac{d_p + d_g}{2} = r_p + r_g$$

Equation 2: "Center" distance between gears.

It is important to note that the "diameter" of a gear always refers to its pitch diameter. Mating gears of standard proportions generally have shorter teeth. The line of action is the defines the direction that force between mating teeth acts, neglecting friction. The path of contact is the locus of all points of tooth contact, and the face and flank portions of the tooth surface are divided by the pitch cylinder, which contains the pitch circle. Circular pitch is denoted as p.

$$p = \frac{\pi d}{N}$$

Equation 3: circular pitch (inches)

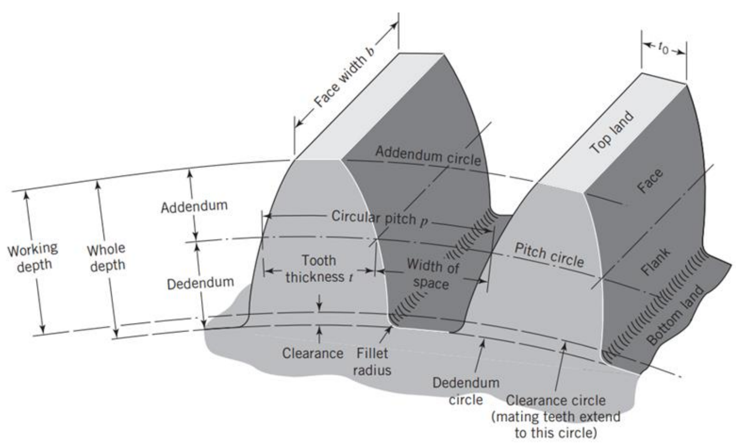

Figure 3: Common gear nomenclature

Gears are typically designated by a set of parameters, including the pitch diameter, number of teeth, pitch, and pressure angle. The pitch is the distance between corresponding points on adjacent teeth, and the pressure angle is the angle between the line of action and the tangent to the pitch circle at the point of contact. Diametral pitch, P, is defined as the number of teeth per inch of inch diameter. Gears are commonly made to an integral value of diametral pitch for Empirical units, which this paper uses.

$$P = \frac{N}{d}$$

Equation 4: Diametral Pitch (teeth per inch)

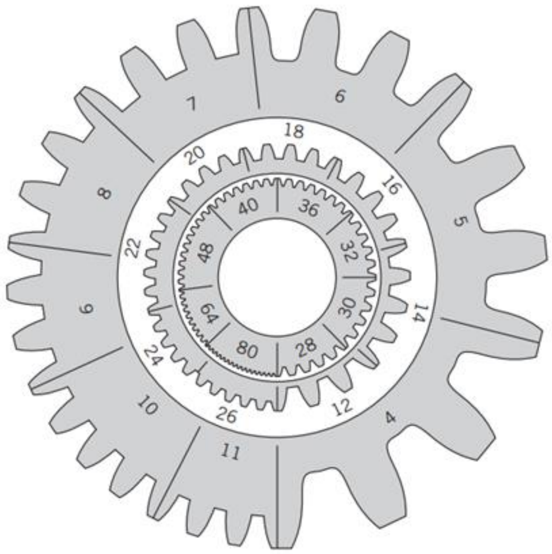

Figure 4: Actual size of gear teeth of various diametral pitches.

Figure 3 and Figure 4 both show the concept of gear tooth measurement units. Face width, *b*, is not standardized, but generally:

$$\frac{9}{P} < b < \frac{14}{p}$$

Equation 5: Face width

Wider face width makes it more challenging to manufacture and mount gears to ensure uniform contact. Gears made to standard systems are interchangeable and readily available in stock. However, mass-produced gears for specific applications commonly deviate from these standards to optimize performance.

Nowadays, the trend in gear production is more towards the use of specialized gears due to reduced cost with modern gear-cutting equipment and engineering design time with computer facilities. Standard nomenclature of gears is important to ensure that the correct gears are used in a particular application. Common gear nomenclature used in the United States is the diametral pitch system. Gears may also sometimes be classified based on their mounting position and direction of rotation.

Bending Fatigue

This type of failure is commonplace in gears. The reason for this is made evident by looking at the photo elastic pattern of two mating gears shown in Figure 5. Here, it is shown that the stress is always concentrated at the fillets at the base of the teeth in meshing gears, regardless of the direction that the force is applied from. Hence, the cracks which lead to bending fatigue failure almost always originate from these locations.

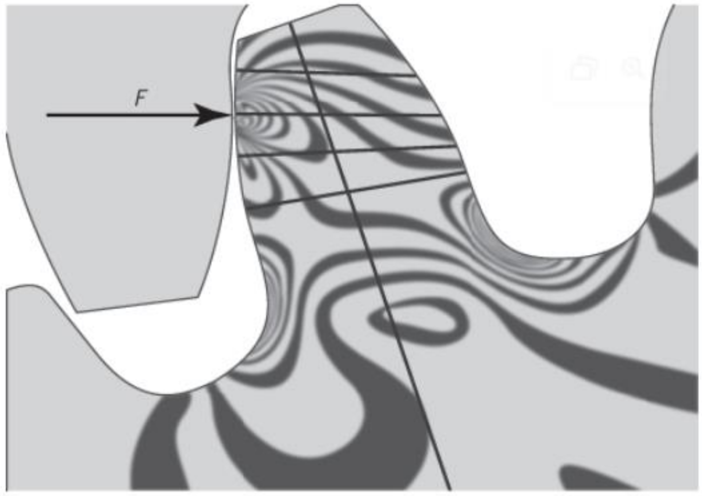

Figure 5: Photo elastic pattern of stress distribution in mating spur gears.

During bending analysis, it is safe to assume that the load being investigated is applied at the tip of a single gear tooth. This would happen if only one pair of teeth were in contact at a given time, which is closer to the case for cheaper, non-precision gears. If precision gears were used, then a phenomenon known as load sharing between gears would occur, lowering what the actual forces applied to each gear would be. However, the aforementioned single contact assumption could still be held for conservative estimates. This project assumes that no load sharing takes place between gears.

Whenever bending fatigue failure is being considered, the Lewis equation expressed in equation 6 is often used to determine the stress applied on each tooth. Here, F_t is the force tangent to the tip of the gear. P is the diametral pitch. b is the face width. J is the spur gear geometry factor. K_v is the velocity factor. K_0 is the overload factor. K_m is the mounting factor. This value, once determined, can be compared to the fatigue strength of the gear being analyzed to determine if the gear will fail.

$$\sigma = \frac{F_t P}{bJ}(K_v K_o K_m)$$

Equation 6: Lewis equation

The gear geometry factor considers stress concentration factors from the tooth fillets. It can change depending on how much load sharing happens between gears, the number of teeth, and the teeth profile. Figure 6 shows how this factor can change for 20-degree full depth gears, which is what this report uses.

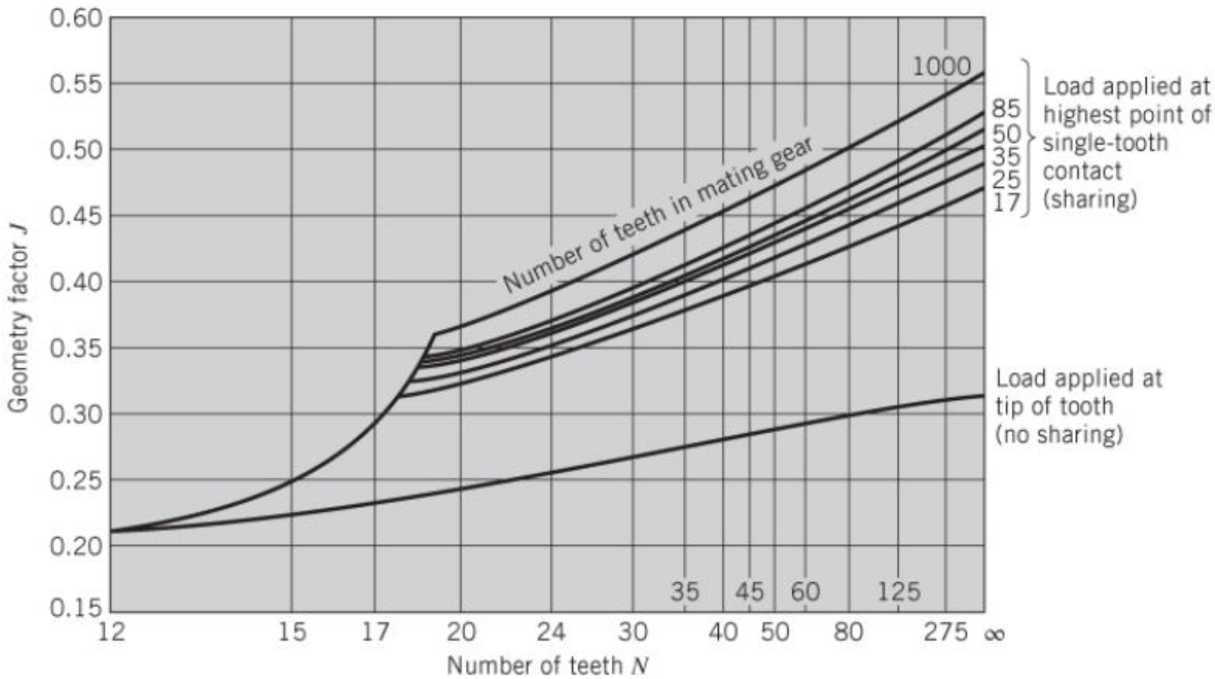

Figure 6 geometry factor J for standard spur gear for 20° full-depth teeth

The velocity factor indicates the severity of the impact between teeth as they engage together. This severity depends on the pitch line velocity and manufacturing accuracy of the gears. The faster the mating gears turn, the greater the impact on the colliding teeth. Likewise, the lower the manufacturing accuracy, the greater the backlash and subsequently the greater the impact on the meshed teeth. Figure 7 shows how different manufacturing operations and pitch line velocities can affect this factor. This project's analysis is based on gears which were formed by high precision cutting (type B).

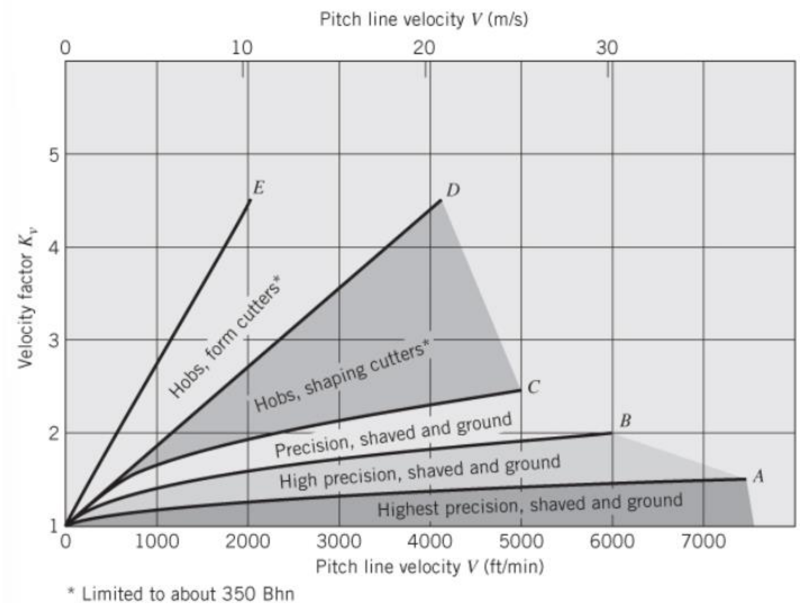

Figure 7 Velocity factor K_v ; figure accounts for the effects of tooth spacing and profile errors, tooth stiffness, and the velocity inertial, and stiffness of rotating parts.

The overload factor accounts for varying load conditions for the driving and load torques. Table 1 provides rough estimates for various loading conditions for the source of power and driven machinery. The source of power is the pinion gear, and the machinery is the gear for this paper.

Table 1 Overload Correction Factor K_0

Driven Machinery			
Source of Power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

The mounting factor reflects how accurately the meshed gears are aligned. Table 2 gives rough estimates for various face widths and presumed mounting conditions.

Table 2 Mounting Correction Factor K_m

	Face width (in)			
Support Characteristics	0 to 2	6	9	16+
Accurate mountings, precision gears, minimum deflection.	1.3	1.4	1.5	1.8
Not rigid mountings, less accurate gears	1.6	1.7	1.8	2.2
Accuracy and mounting that permit full face contact	>2.2			

Equation 7 is used to calculate the fatigue strength of a gear to compare with the stress value which could be found with Equation 6. S_n' is the Moore endurance limit. C_L is the load factor, 1, for bending loads. C_G is the gradient factor, 1, for a pitch greater than 5. It is 0.85 otherwise. C_S is the surface factor for the fillet of the gear being analyzed. It is assumed that this surface is machined during analysis unless otherwise specified. k_r is the reliability factor, which can be determined from Table 3. This project uses a reliability of 99.99% for bending fatigue analysis, and correspondingly, a factor of safety of 1.5. k_t is the temperature factor which is usually 1.0 below 160 $°F$. k_{ms} is the mean stress factor, which is 1.4 for input and output gears.

$$S_n = S_n' C_L C_G C_S k_r k_t k_{ms}$$

Equation 7: Endurance limit

Table 3 Reliability Correction Factor k_r, with Assumed Standard Deviation of 8%

Reliability (%)	50	90	99	99.9	99.99	99.9999
Reliability Factor	1.000	0.897	0.814	0.753	0.702	0.659

Surface Fatigue

Surface fatigue poses a greater risk to the cyclical longevity of mechanical components when compared to bending fatigue. Many materials, especially steels, tend to reach an endurance limit at 10^6 cycles at some stress X. If stress remains below that point X, the material should never fail. However, this is not the case for surface fatigue, as in this case there is no clear endurance limit. As the number of cycles goes to infinity, the amount of allowable surface fatigue continues to decrease without bound.

Surface fatigue becomes a balancing act when designing mechanical parts. When a part has a hard surface, it can resist wear more easily. But if the material is too hard, it cannot adjust to small bumps or scratches which can create more pressure in certain areas. This is why when a pair of gears are manufactured, one is made harder than the other so they can adjust to each other and run smoothly. The harder gear is usually the pinion, and the softer gear is usually the driven gear.

When parts are made to be very precise and smooth, it tends to result in a decrease of surface fatigue. However, manufacturing with this amount of precision and surface finish leads to substantial cost increases. If the designed parts create a lot of friction, then it can be beneficial to have tiny bumps or holes on one of the surfaces to hold lubricant and reduce friction. If there are strong forces between surfaces, it is better to design in ways that cause compressive stresses rather than tensile since this would limit crack propagation, ultimately leading to critical failure caused by fatigue and wear.

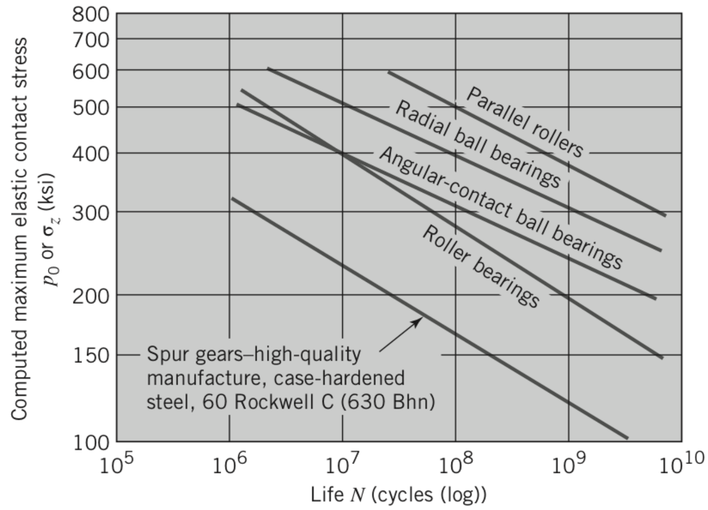

Figure 8 Average S-N curves for contact stresses – rollers, bearings, and spur gears, 10% failure probability

Project Parameters

Assumed Conditions

Prior to running the optimization, the user needs to choose the surface finish of the gears, analysis type (either bending or surface fatigue) and then set geometric parameters such as gear ratio, input speed of the pinon gear, amount of power to transmit, etc. as shown in the provided MATLAB live script (Appendix D). The user would then choose how many candidates (i.e., designs the genetic algorithm will be used in each 'generation'). This project assumes the following input, loading and geometric configurations are already defined. This analysis was inspired by Problem 15.5 from *The Fundamentals of Machine Components Design Textbook* [4], and the conditions were deemed acceptable by Professor Wahab.

1. **Gear combination type** = 1 stage gear reduction
2. **Gear Ratio** = 2
3. **Input (pinion) speed** = 1500 RPM
4. **Power transmitted** = 2.143 HP
5. **Gear type** = 20-degree full depth teeth (For both pinion and gear)
6. **Gear manufacturing method** = High Precision, Shaved, and Ground (Type B from Figure x).

   a. This is necessary because Brinell Hardness values greater than 350 were used in this project's analysis.
7. **Overload condition** = 1.5
   a. Light shock from the source of power and Moderate shock to driven machinery.
8. **Mounting condition** = assumed 1.8 (non-rigid mountings).
9. **Surface Finish** = Machined
10. F_0S = 1.5 at 99.99% reliability
11. k_r = 0.702 (99.99% for bending)
12. k_t = 1.0
13. k_{ms} = 1.4 (Input and driven gears)
14. C_L = 1.0
15. C_G = 1.0
16. Cl_i = 1.1
17. Cr_r = 1.0

Optimization Parameters

The system optimizes three design variables: number of teeth in pinion gear (N_P), diametral pitch (P) and gear tooth width (b). The following information is the upper and lower bounds for the design variables.

Maximum Allowed Diametral Pitch = 24 $\frac{teeth}{in}$
Minimum Allowed Diametral Pitch = 6 $\frac{teeth}{in}$
Maximum Allowed Pinion Teeth = 36 teeth
Minimum Allowed Pinion Teeth = 12 teeth
Maximum Allowed Face Width = 2.8 in.
Minimum Allowed Face Width = 0.45 in.
Maximum Weight Allowed = 210.77 lb.
Minimum Weight Allowed = 2.11 lb.

Materials Under Consideration

Table 4 Materials used under consideration; $\rho = 0.284\frac{lb}{in^3}$ *(Average of five steels, used for all cases)*

Material	Young's Modulus (psi)	Brinell Hardness (Bhn)
304 Stainless Steel	2.80e6	123
AISI 1045 Medium Carbon Steel	2.99e6	163
AISI 4340 Steel	2.78e6	217
AISI 4140 Steel	2.97e6	302
A2 Tool Steel	2.94e6	630

Methodology

The optimization method chosen for this project is called the Genetic Algorithm (GA). These are a class of search algorithms based on mechanics of natural selection and natural genetics. Genetic algorithms exploit the idea of the survival of the fittest and the interbreeding population to create a novel and innovative search strategy. A population of the strings representing solution to the specified problem is maintained by genetic algorithm, which then iteratively creates the new population from the old by ranking the strings and interbreeding the fittest to create the new strings, which are closer to the optimum solution to a specified problem. A flow chart showing a standard genetic algorithm is shown in Figure 9.

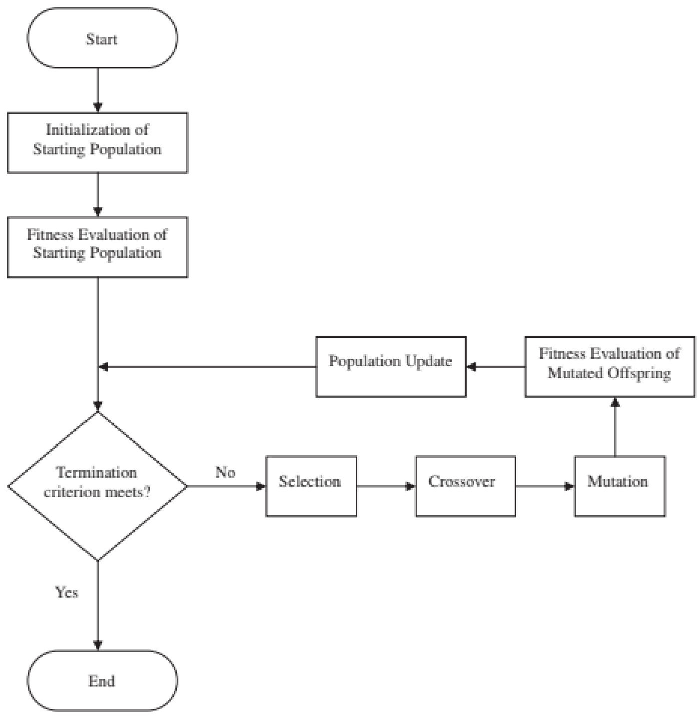

Figure 9 Flow Chart of Standard GA algorithm [1]

In the project's implementation, a candidate is chosen to be an optimal solution when it satisfies 10^6 life cycle analysis (either Bending or Surface Durability) and produces a system having minimum weight. This operation is performed by the function F_{obj}. If *W* is the weight of the entire gear reducer, then the optimization function is defined as

$$F_{obj} = F(Np, m, p) \ = minimize\ W \ = maximize\ (D * |W_{\max allow} - W_c|)$$

Equation 5 Optimization function

where W_c is the weight of the system for the current candidate and *D* is a scalar multiplier which is set to 1 when the current candidate passes fatigue analysis and 0.1 when either one or both pinion and driven gear fails the fatigue analysis. Weight of the system W_c is calculated as

$$W_c = \left(\frac{pi}{4}\right) * b * \rho * \left(\left(\frac{N_p}{P}\right)^2 + \left(\frac{N_g}{P}\right)^2\right)$$

Equation 6 Weight of the System

Where $N_g = gear\ ratio * N_p$. The procedure continues until the number of iterations i.e. generation is reached. Note that, after performing optimization step, the algorithm pulls the candidate having the minimum weight and then re-performs fatigue analysis for N_p, P and b values for that candidate and reports whether the pinion and driven gear is surviving 10^6 operations.

Results

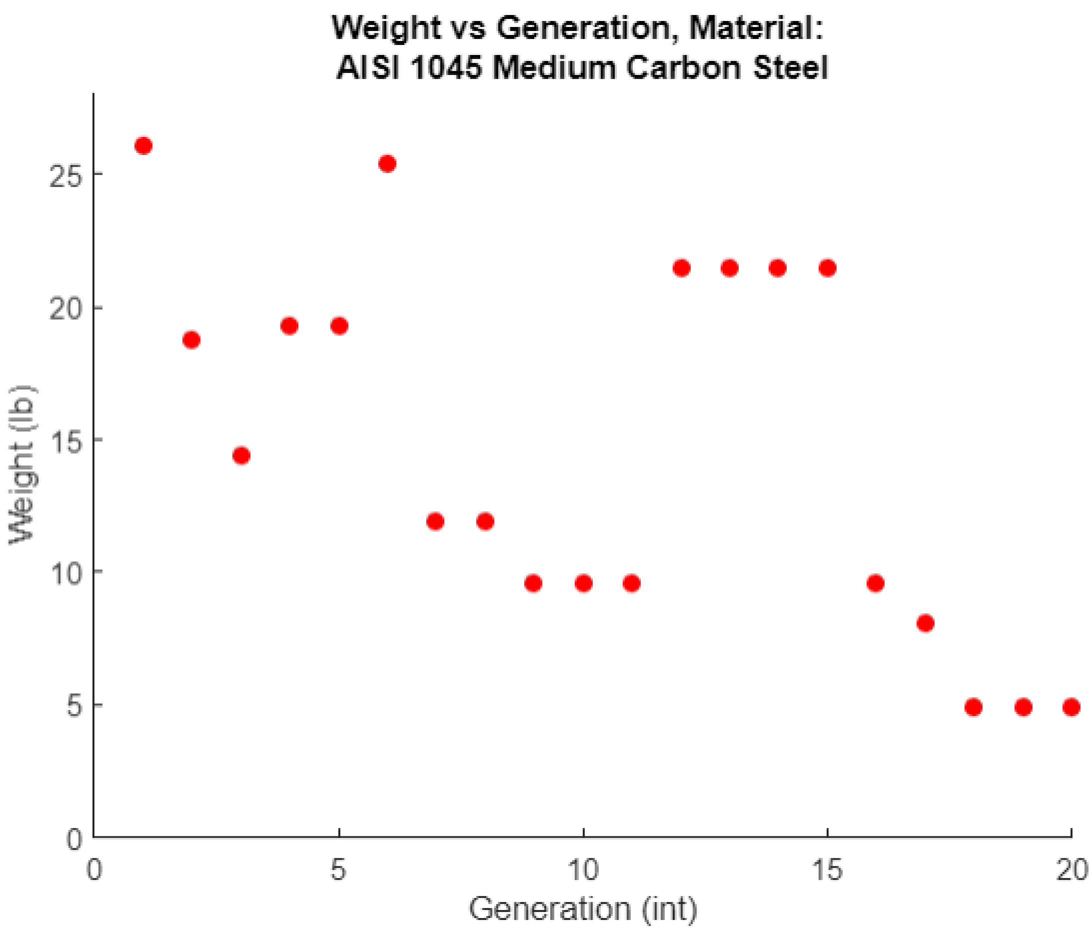

Figure 10: Weight Vs. Generation for AISI 1045 Medium Carbon steel

The results for 5 runs each having 20 iterations are tabulated below.

Results for Gear-Tooth Bending Fatigue Analysis

Table 5 Bending fatigue for 304 Stainless Steel

304 Stainless Steel

Trial	Np (teeth/in)	P (teeth)	b (in)	Weight (lb)	Fatigue Life (ksi)	Pinion Stress (ksi)	Gear Stress (ksi)
1	20	10	0.967	4.314	24.141	18.317	8.905
2	18	10	1.625	5.872	24.141	12.031	5.838
3	14	7	0.744	3.318	24.141	18.191	8.107
4	24	12	1.465	6.536	24.141	13.392	7.053
5	22	16	1.550	3.268	24.141	23.995	12.555
Average	20	11	1.270	4.662	24.141	17.185	8.482

Table 6 Bending fatigue for AISI 1045 Medium Carbon Steel

AISI 1045 Medium Carbon Steel							
Trial	Np (teeth/in)	P (teeth)	b (in)	Weight (lb)	Fatigue Life (ksi)	Pinion Stress (ksi)	Gear Stress (ksi)
1	24	14	1.116	3.659	31.621	23.689	12.442
2	24	14	1.389	4.553	31.621	19.036	9.998
3	24	14	1.350	4.424	31.621	19.593	10.291
4	22	12	0.912	3.417	31.621	23.351	12.279
5	22	13	0.807	2.577	31.621	30.807	16.177
Average	23	13	1.115	3.726	31.621	23.295	12.237

Table 7 Bending fatigue for AISI 4304 Steel

AISI 4340 Steel							
Trial	Np	P	b	Weight	Fatigue Life	Pinion Stress	Gear Stress
1	13	10	1.204	2.268	40.881	24.064	10.644
2	22	14	0.467	1.286	40.881	61.440	32.220

3	18	14	0.947	1.746	40.881	39.650	19.129
4	17	17	1.625	1.812	40.881	38.839	17.107
5	22	14	1.625	4.475	40.881	17.656	9.261
Average	18	14	1.174	2.317	40.881	36.330	17.672

Table 8 Bending fatigue for AISI 4140 Steel

AISI 4140 Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	15	8	1.339	5.246	52.808	12.274	5.464
2	21	12	1.425	4.867	52.808	16.902	8.197
3	22	16	1.568	3.305	52.808	23.723	12.412
4	24	24	0.995	1.11	52.808	75.750	39.433
5	15	12	0.842	1.467	52.808	46.048	20.341
Average	19	14	1.234	3.199	52.808	34.939	17.169

Table 9 Bending fatigue for A2 Tool Steel

A2 Tool Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	12	7	1.561	5.116	65.502	10.012	4.450
2	14	10	1.027	2.244	65.502	27.113	12.004
3	15	17	0.614	0.533	65.502	112.960	49.679
4	22	14	0.6596	1.817	65.502	43.504	22.814
5	15	13	0.826	1.226	65.502	48.450	21.398
Average	16	12	0.938	2.187	65.502	48.408	22.069

Averaged Best Bending Fatigue Data for Given Trial

Table 10 Average Bending Fatigue

Material	304 Stainless Steel	AISI 1045 Medium Carbon Steel	AISI 4340 Steel	AISI 4140 Steel	A2 Tool Steel
Np (teeth/in)	20	23	18	19	***16***
P (teeth)	11	13	14	14	***12***
b (in)	1.270	1.115	1.174	1.234	***0.938***
Weight (lb)	4.662	3.726	2.317	3.199	***2.187***
Fatigue life (ksi)	24.141	31.621	40.881	52.808	65.502
Pinion Stress (ksi)	17.185	23.295	36.330	34.939	48.408
Gear Stress (ksi)	8.482	12.237	17.672	17.169	22.069

Results for Gear-Tooth Surface Durability Strength Analysis

Table 11 Surface Fatigue for 304 Stainless Steel

304 Stainless Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	19	8	1.622	10.204	43.12	107.930	55.47
2	22	15	0.579	1.388	43.12	288.135	147.456
3	24	18	0.964	1.911	43.12	244.920	125.240
4	16	14	0.670	0.976	43.12	341.220	174.264
5	17	23	0.466	0.284	43.12	625.694	318.499
Average	20	16	0.860	2.953	43.12	321.580	164.186

Table 12 Surface Fatigue for AISI 1045 Medium Carbon Steel

AISI 1045 Medium Carbon Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	22	13	1.469	4.693	60.720	162.623	83.327
2	18	15	1.319	2.119	60.720	239.632	122.430
3	21	13	0.754	2.195	60.720	237.480	121.634
4	13	10	1.235	2.328	60.720	229.114	117.133
5	23	6	1.625	26.631	60.720	70.318	36.311
Average	19	11	1.280	7.593	60.720	187.833	96.167

Table 13 Surface Fatigue for AISI 4340 Steel

AISI 4340 Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	12	15	0.628	0.448	84.480	497.183	253.227
2	24	7	1.410	18.481	84.480	81.023	41.792
3	24	10	0.714	4.588	84.480	160.445	82.473
4	24	7	1.497	19.623	84.480	78.630	40.558
5	12	15	0.477	0.341	84.480	570.407	290.521
Average	19	11	0.945	8.696	84.480	277.538	141.714

Table 14 Surface Fatigue for AISI 4140 Steel

AISI 4140 Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	24	6	1.612	28.757	121.88	67.561	34.902
2	23	13	1.314	4.589	121.88	164.146	84.140
3	15	8	1.423	5.579	121.88	149.140	76.488
4	23	7	1.582	19.053	121.88	82.340	42.453
5	24	9	1.267	10.049	121.88	112.483	57.877
Average	22	9	1.440	13.605	121.88	115.134	59.172

Table 15 Surface Fatigue for A2 Tool Steel

A2 Tool Steel							
Trial	**Np (teeth/in)**	**P (teeth)**	**b (in)**	**Weight (lb)**	**Fatigue Life (ksi)**	**Pinion Stress (ksi)**	**Gear Stress (ksi)**
1	18	10	1.460	5.274	266.200	152.412	78.137
2	13	10	1.005	1.895	266.200	256.078	130.899
3	18	9	1.108	4.945	266.200	157.947	81.052
4	21	14	1.477	3.707	266.200	180.795	92.541
5	21	13	0.786	2.288	266.200	230.650	118.136
Average	18	11	1.167	3.622	266.200	195.576	100.153

Table 16 Averaged Best Surface Fatigue

Material	**304 Stainless Steel**	**AISI 1045 Medium Carbon Steel**	**AISI 4340 Steel**	**AISI 4140 Steel**	**A2 Tool Steel**
Np (teeth/in)	20	19	19	22	***18***

P (teeth)	16	11	11	9	***11***
b (in)	0.860	1.280	0.945	1.440	***1.167***
Weight (lb)	2.953	7.593	8.696	13.605	***3.622***
Fatigue life (ksi)	43.12	60.720	84.480	121.88	266.200
Pinion Stress (ksi)	321.580	187.833	277.538	115.134	195.576

Discussion

Bending Fatigue

As seen in the average bending fatigue data in Table 10, viable designs may be produced for all the given materials listed in this project. A2 Tool Steel had the lowest weight and correspondingly lowest face width and diametral pitch out of the materials considered. This can be attributed to the material's hardness, which was much greater than the second lightest design using AISI 4340 Steel. However, even though the difference in hardness was over 2.9 times, the difference in average weight between the designs was only 0.13 lbs. and the difference in average face width was 0.176 in. This shows that although a significantly harder material might produce a lighter, more compact design, these gains are marginal. The same can also be said regarding the affected fatigue life of these materials. Indeed, A2 Tool Steel, despite its higher hardness, only had a fatigue life 1.6 times greater than AISI 4340 Steel.

Looking at the raw data for each material, there are a wide range of possible solutions for each material. For this project, optimal bending fatigue designs were provided for 304 Stainless Steel, 1045 Medium Carbon Steel and 4340 Steel. All these materials' best designs were within 1% of their theoretical fatigue life for the respective material. Optimal designs were not generated for 4140 Steel and A2 Tool steel, however. The best working design for AISI 4140 steel was within 13% of its fatigue life. For A2 tool steel, the best design was within 26%. More trials would be needed to uncover the best designs for these materials. Some optimization design attempts failed for 4340 Steel, 4140 Steel, and A2 Tool Steel. In all cases presented, the pinion in the design failed before the gear. This kind of behavior is expected and explains why gearbox designers generally make the pinion out of a harder material than the gear. Looking at the weights of the failed and optimized designs for all materials, the best gear designs would be around 1.3 lbs. in weight.

Surface Fatigue

Surface fatigue is a common mode of failure in gears, resulting from the repeated application of stresses to the gear's surface. Although gears experience complex loading conditions, surface fatigue has been found to be the primary cause of failure, rather than bending fatigue. This is exemplified by the higher calculated success rate seen in the tables when designing with respect to bending rather than surface fatigue. This is due to the concentration of stresses at the surface's irregularities, where micro-cracks initiate and propagate until gear failure. This is why factors such as improving surface finish and increasing the hardness of the material are important when designing with respect to surface fatigue. Because all calculations were done utilizing the same parameters, for machined surface finish, the primary variable that will affect their fatigue life is their hardness value.

As seen in Table 16, the only two materials that were able to pass surface fatigue design criterion were AISI 4140 steel and A2 tool steel. This is directly attributed to the higher hardness values of A2 tool steel and AISI 4140 steel. A2 tool steel has over double the Bhn hardness value of AISI 4140 steel. This leads to the A2 tool steel having a significantly higher fatigue life of 266.200 ksi than the 121.88 ksi of AISI 4140 steel.

Weight

The results presented above show the minimization of weight problem formulated as maximization of a scalar value presented in Equation 9 is indeed correct. This is corroborated with observations that A2 Tool Steel, having the highest endurance strength, would yield the lightest gear train. This was also proven true for both bending and surface durability analysis. From Table 10, it was expected that as stronger alloys were used, more compact gears would be designed with lower weights due to higher strength of the materials. This hypothesis also turned out to be correct.

Validation

Validation was done by comparing the results from the optimization code to hand-written calculations (Appendix C).

Conclusion

From the results and discussion presented above, it can be concluded that the Genetic Algorithm based 1 stage external spur-gear design software formulated in this project can provide design teams with a range of viable alternatives for multiple materials given a specified loading problem. This can save time in early project development stages. There are multiple avenues for further improvement. Firstly, both the bending and surface durability analysis may be combined into another optimization function to improve reliability of the design further. Tooth deflection may also be added as a design constraint. Currently, the software only works for ferrous materials. It may also be an interesting exercise to expand its capabilities to non-ferrous materials.

Appendix

Appendix A: References

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3. Vanderplaats, Garret N., Xiang Chen, and Ning-tian Zhang. *Gear optimization*. No. NAS 1.26: 4201. NASA, 1988.
4. Juvinall, Robert C., and Kurt M. Marshek. *Fundamentals of machine component design*. John Wiley & Sons, 2020.
5. "Top." *KHK Gear Manufacturer*, https://khkgears.net/new/gear_knowledge/introduction_to_gears/types_of_gears.html.
6. *304 Stainless Steel*, https://www.matweb.com/search/datasheet.aspx?MatGUID=abc4415b0f8b490387e3c922237098da.
7. *AISI 1045 Medium Carbon Steel*, https://www.matweb.com/search/DataSheet.aspx?MatGUID=4b0553daf9c245e684f2199a48179d89
8. *AISI 4340 Steel,* https://www.matweb.com/search/DataSheet.aspx?MatGUID=fd1b43a97a8a44129b32b9de0d7d6c1a
9. *AISI 4140 Steel,* https://www.matweb.com/search/DataSheet.aspx?MatGUID=8b43d8b59e4140b88ef666336ba7371a&ckck=1
10. *A2 Tool steel,* https://www.matweb.com/search/DataSheet.aspx?MatGUID=5abd35ce6bd64254b6db980b83683f27

Appendix B: Gantt Chart Timeline

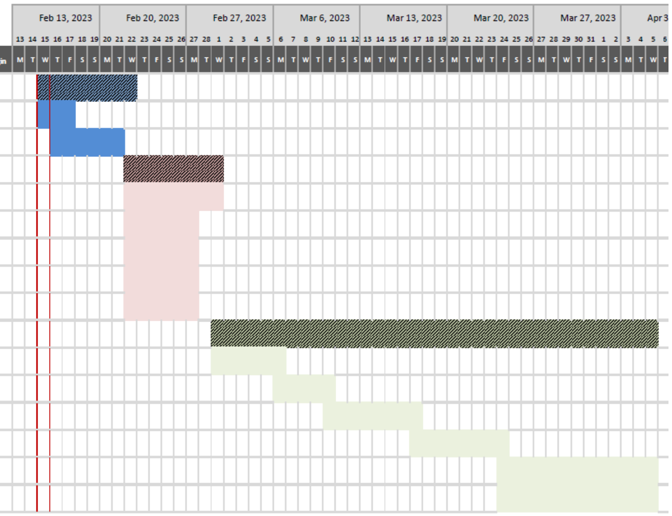
Gear Train Optimization Schedule

SIMPLE GANTT CHART by Vertex42.com
https://www.vertex42.com/ExcelTemplates/simple-gantt-chart.html

Group Number: #11
Project Team Members: Matthew Williams, Donovan Gegg, Seth Chiasson, Azmyn Kamal, Jennifer Nguyen

Project Start: Wed, 2/15/2023
Display Week: 1

TASK	ASSIGNED TO	PROGRESS	START	END	DAYS	Margin
GitHub			2/15/23	2/22/23	8	0
Code Uploaded to GitHub	Azmyn Kamal	0%	2/15/23	2/17/23	1	
Learn how to use GitHub	Matthew Williams, Seth Chiassion, Azmyn Kamal, Jennifer Jguyen	0%	2/16/23	2/21/23	6	
Perform Background Research/ Theoretical Analysis			2/22/23	3/1/23	8	0
Find and read relevant papers for reference	All	0%	2/22/23	3/1/23	8	
Determine Optimization Parameters	Donovan Gegg	0%	2/22/23	2/27/23	6	
Determine Typical Gear Stress Concentration Areas	Matthew Williams, Seth Chiassion	0%	2/22/23	2/27/23	6	
Find Example Gear Trains	Azmyn Kamal		2/22/23	2/27/23		
Select Materials for Gear Train Analysis	Jennifer Nguyen	0%	2/22/23	2/27/23	6	
Script Creation	All (THAT important. AND the process is linear)		3/1/23	4/5/23	36	0
Calculate theoretical optimization parameters by hand (if possible)	All	0%	3/1/23	3/6/23	6	
Code previous procedure in Matlab	All	0%	3/6/23	3/10/23	5	
Add Optimization functionality to code	All	0%	3/10/23	3/17/23	8	
Add New Material Options to Code	All	0%	3/17/23	3/24/23	8	
Time Permitting: Determine Critical elements in optimized gear train	All	0%	3/24/23	4/5/23	13	
Time Permitting: Determine Fatigue Life of optimized Gear Train	All	0%	3/24/23	4/5/23	13	



Appendix C: Hand-Written Calculations

304 S.S. Trial #1

$P = 20 \ \frac{teeth}{in}$
$N_p = 10$ teeth
$b = .967$ in
$W = 4.314$ lb
$n_p = 1500$ rpm

Assumptions / Given:
- Load sharing not expected: $J = .24$ • $K_m = 1.8$ • $K_t = 1$ • $K_{ms} = 1.4$ • $C_R = 1$ • $Bhn = 123$
- Fillet radius $= \frac{.35}{P} = 0.0175$ • $C_L = 1$ • $C_G = 1$ • $C_i = 1.1$ • C_s = machined
- Moderate shock to driven machinery, light shock w/ source of power • $F_t = 100$ lb
  $\hookrightarrow K_o = 1.50$

Velocity: $V = \frac{\pi N_p \cdot n_p}{12} = 392.69 \ ft/min$

$V = 392.69 \ ft/min$ yields $K_v \approx 1.7$

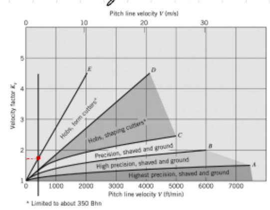


FIGURE 15.24 Velocity factor K_v. (Note: This figure, in a very rough way, is intended to account for the effects of tooth spacing and profile errors, tooth stiffness, and the velocity, inertia, and stiffness of the rotating parts.)

• Lewis Equation: $\sigma = \frac{F_t \cdot P}{b J} \cdot K_v \cdot K_o \cdot K_m$

$\sigma = \frac{100 \cdot 20}{.967 \cdot .24} \cdot 1.7 \cdot 1.5 \cdot 1.8 = 39.55 \ KSi = S_n$

• Infinite Life:

$S_n = S_n' C_L C_G C_s K_r K_t K_{ms}$
• $S_n' = 500 \cdot Bhn = 500 \cdot 123$
• $S_n' = 61{,}500 \ psi = 61.5 \ KSi$

$S_n = S_n' C_L C_G C_s K_r K_t \cdot K_{ms}$
$S_n = 61.5 \cdot 1 \cdot 1 \cdot .8 \cdot K_r \cdot 1 \cdot 1.4$
$S_n = 68.88 \ K_r$
$\frac{39.55}{68.88} = K_r = .574 \rightarrow \therefore$ Reliability % > 99.99 %

Table 15.3 **Reliability Correction Factor k_r, from Figure 6.19 with Assumed Standard Deviation of 8%**

Reliability (%)	50	90	99	99.9	99.99	99.999
Factor k_r	1.000	0.897	0.814	0.753	0.702	0.659

Appendix D: MATLAB Live Script Link to GitHub

https://github.com/HailtheWhale/Genetic_Algorithm_2D_Rosenbrock/tree/Mach_Des_II_Gear_Opt_Proj