Noise and errors are additive since the outputs are in series. Because the inputs are in parallel thermally, and because the thermocouples are essentially identical, the errors are also identical (or nearly so) and therefore the output error is n times the error (or noise) of a single thermocouple.

2.2.6 Hysteresis, Nonlinearity, and Saturation

Hysteresis (literally lag) is the deviation of the sensor's output at any given point when approached from two different directions (see **Figure 2.17**). Specifically, this means that the output at a given value of stimulus when it increases and when it decreases is different. For example, if temperature is measured, at a rated temperature of 50°C, the output might be 4.95 V when the temperature increases, but 5.05 V when the temperature decreases. This is an error of ±0.5% (for an OFS of 10 V in this idealized example). The sources of hysteresis are either mechanical (friction, slack in moving members), electrical (such as due to magnetic hysteresis in ferromagnetic materials), or due to circuit elements with inherent hysteresis. Hysteresis is also present in actuators and, in the case of motion, is more common than in sensors. There it may manifest itself as positioning errors. Also, hysteresis may be introduced artificially for specific purposes.

Nonlinearity may be either a property of a sensor (see, e.g., **Figure 2.1**) or an error due to deviation of a device's ideal, linear transfer function. A nonlinear transfer function is a property of the device and, as such, is neither good nor bad. One simply has to design with it or around it. However, a nonlinearity error is a quantity that influences the accuracy of the device. It must be known to the designer, must be taken into account, and possibly minimized. If the transfer function is nonlinear, the maximum deviation from linearity across the span is stated as the nonlinearity of the device. However, this measure of linearity is not always possible or desirable. Therefore there are various valid ways of defining the nonlinearity of a sensor or actuator. If the transfer function is close to linear, an approximate line may be drawn and used as the reference linear function. Sometimes this is done simply by connecting the end points (range values) of the transfer function (line 1 in **Figure 2.18**). Another method is to draw a least squares line through the actual curve (line 2 in **Figure 2.18**), usually by first selecting a reasonable number of points on the curve and then, given the selected (or measured) pairs of input and output values (x_i, y_i), draw the line $y = a + bx$ by calculating the slope a and the

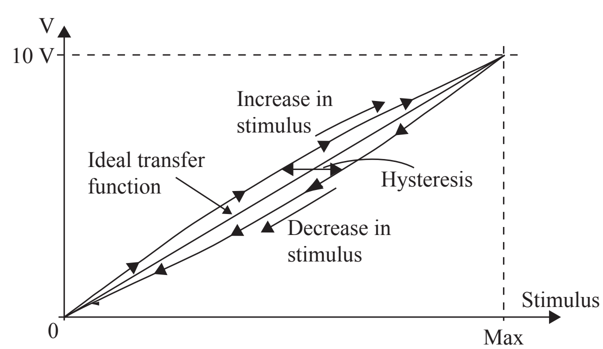

FIGURE 2.17 ■ Hysteresis in a sensor.

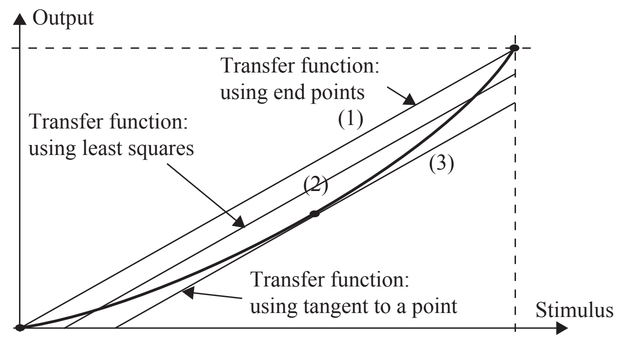

FIGURE 2.18 ■ Linear approximations of nonlinear transfer functions.

axis intercept b as shown in detail in **Appendix A**. The linear best fit to the data (see **Equation (A.15)**) is

$$a=\frac{n\sum_{i=1}^{n}x_iy_i-\left\{\sum_{i=1}^{n}x_i\right\}\left\{\sum_{i=1}^{n}y_i\right\}}{n\sum_{i=1}^{n}x_i^2-\left\{\sum_{i=1}^{n}x_i\right\}^2},\quad b=\frac{\left\{\sum_{i=1}^{n}y_i\right\}\left\{\sum_{i=1}^{n}x_i^2\right\}-\left\{\sum_{i=1}^{n}x_i\right\}\left\{\sum_{i=1}^{n}x_iy_i\right\}}{n\sum_{i=1}^{n}x_i^2-\left\{\sum_{i=1}^{n}x_i\right\}^2}. \quad (2.19)$$

This provides a "best" fit (in the least squares sense) through the points of the transfer function and can be used for purposes of measuring the nonlinearity of the actual sensor or actuator. If this method is used, nonlinearity is the maximum deviation from this line.

There are many variations on both of these methods. In some cases a sensor is only expected to operate in a small section of its span. In this case, either method may be applied for that portion of the span. Another method that is sometimes employed is to take a midpoint in this reduced range and draw a tangent to the transfer function through the selected point and use this tangent as the "linear" transfer function for purposes of defining nonlinearity (line 3 in **Figure 2.18**). Needless to say, each method results in different values for nonlinearity and, while these are valid, it is important for the user to know the exact method used.

It should also be noted that in spite of the foregoing, nonlinearity is not necessarily a "bad" thing or something that needs correction. In fact, there are instances in which a nonlinear response is superior to a linear response and there are sensors and actuators that are intentionally and carefully designed to be nonlinear. An example is the common potentiometer used as a volume control, especially in audio systems. Notwithstanding the fact that most current volume control systems tend to be digital and many are linear, our hearing is not linear—in fact, it is logarithmic. This allows the ear to respond to minute pressure changes (as low as 10^{-5} Pa) as well as high pressures (high power sound)—as high as 60 Pa. Normally the range is given between 0 and 130 dB. To accommodate this natural response, potentiometers for volume control were also designed as logarithmic to conform with our ears' response. Even some digital potentiometers are logarithmic. The following example discusses these issues a bit more, but the important point is that this nonlinear response has been designed on purpose to fit a particular need and in this case a nonlinear response is superior.