$\boxed{\dfrac{68}{75} = 90.7\%}$ Potential

$\dfrac{52}{75} \doteq \boxed{69.3\%}$ -23

$\boxed{16}$

$\boxed{\text{CALC ERRORS}}$

$\boxed{4}$??

1 Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. (−1) [4]

$$\left(2\cos\theta = 7 - \frac{3}{\cancel{\cos\theta}}\right) \times \cos\theta$$

$$2\cos\theta = 7\cos\theta - 3$$

$$\cancel{+3 = 7\cos\theta - 2\cos^2\theta}$$

$$\cancel{+3 = 5\cos\theta}$$

$$2\cos^2\theta - 7\cos\theta + 3 = 0$$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(+3)}}{4}$$

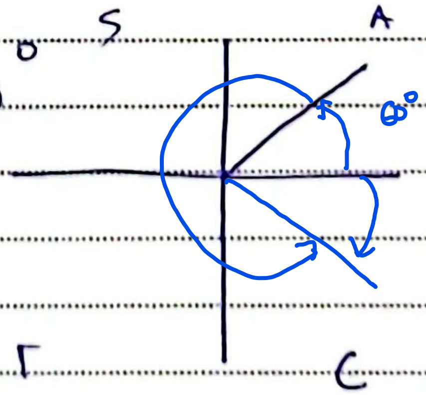
(3)

$$\frac{7+5}{4} \qquad \frac{7-5}{4}$$

$\cos\theta = 3$ or $\cos\theta = \dfrac{1}{2}$

$\cos^{-1}(3)$ not possible

$\cos^{-1}\left(\tfrac{1}{2}\right) = 60°$



$360 - 60 = 300°$

$-60°$

$\times \frac{1}{2}$

2 The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

The graph $y = f(x)$ has been shifted down 3 units in y direction

and $\boxed{\text{shrunk}}$ by the scale factor of 2 in x axis.

DON'T USE

Stretch in x-direction by $\left(\frac{1}{2}\right)$

The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

-2

$y = (2(5)) - 3$

$\cancel{y = 10 - 3}$

$y = \left(\frac{5}{2}\right) + 3$

$y = \frac{11}{2}$

$\frac{11}{2} = \left(\frac{x}{2}\right) + 3$

$\frac{5}{2} = \frac{x}{2}$

$x = 5$

$\left(5, \frac{11}{2}\right)$

$(5, 6)$ $+3$

$(5, 9)$ $\times 2$

$(10, 9)$

3 The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). **[2]**

~~$f(f(x))$~~

$f(5) = \frac{5+3}{5-1} = 2$

$ff(2) = \frac{2+3}{2-1} = 5$

(b) Find an expression for $f^{-1}(x)$. **[3]**

$f^{-1}(x) = y^{-1}$

$f(x) = y$

$y = \frac{x+3}{x-1}$

$yx - y = x + 3$

$yx - x = 3 + y$

$x(y-1) = 3 + y$

$x = \frac{3+y}{y-1}$

$f^{-1}(x) = \frac{3+x}{x-1}$

4 A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve. **[4]**

$\int 8(3x+2)^{-2}\,dx$

$\frac{1}{3} \times \frac{8(3x+2)^{-1}}{-1} + c = y$

$\frac{1}{3} \times \frac{8(3x+2)^{-1}}{-1} + c = y$

put value of x and y to find c

$-\frac{1}{3} + c = 5\frac{2}{3}$

$c = \frac{17}{3} + \frac{1}{3}$

$c = 6$

$\frac{8(3x+2)^{-1}}{-3} + 6 = y$

I CAN'T HELP?? → $\boxed{a, a+d, a+2d}$

5 The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. → $\frac{5\pi}{6}$ = $\boxed{\text{exact}}$ = multiples of π / $\sqrt{\ }$ **[3]** −3

$\frac{5\pi}{6}$

S A T $-\frac{\pi}{6}$

$a \approx 0.811$

$a + \cancel{d} = 2\cos x$

$a + \cancel{3}d = -6\sqrt{3}\sin x$

$a + d(5-1) = 10\cos x$

$a + 4d = 10\cos x$

① $a = 2\cos x$

② $a + 2d = -6\sqrt{3}\sin x$

③ $a + 4d = 10\cos x$

③ − ① $4d = 8\cos x$

$d = 2\cos x$

$2\cos x + d = -6\sqrt{3}\sin x$

$d = -6\sqrt{3}\sin x - 2\cos x$

$2\cos x + 4(-6\sqrt{3}\sin x - 2\cos x) = 10\cos x$

$2\cos x - 24\sqrt{3} - 8\cos x = 10\cos x$

$-24\sqrt{3}\sin x = 16\cos x$

$\cancel{-24.7}$

$\tan x = \frac{16}{-24\sqrt{3}}$

$\tan^{-1}\left(\frac{16}{-24\sqrt{3}}\right) = \cancel{-21.05}$ or $\cancel{-0.117}\pi^c$

-0.367π

$2\cos x + 4\cos x = -6\sqrt{3}\sin x$

$6\cos x = -6\sqrt{3}\sin x$

$\cos x = -\sqrt{3}\sin x$

$-\frac{1}{\sqrt{3}} = \tan x$

$x = \frac{5\pi}{6}$

(b) Hence find the exact sum of the first 25 terms of the progression. −3 **[~~3~~]**

$S_{25} = \frac{25}{2}\left(\cancel{2(\cos(-0.367\pi))} + \right.$

$a = 0.811$

$d = 8.6867$

$S_{25} = 12.5(2 \times 0.811 + 24(8.6867))$

$S_{25} = 2626.2$

$d = 2\cos x$ ← $\frac{5\pi}{6}$

$d = 2\cos\left(\frac{5\pi}{6}\right) = -\sqrt{3}$

$S_{25} = \frac{25}{2}\left[2(2\cos x) + (25-1)(2\cos x)\right]$

$= \frac{25}{2}\left[2(-\sqrt{3}) + 24(-\sqrt{3})\right]$

$= \frac{25}{2}\left[-2\sqrt{3} - 24\sqrt{3}\right]$

$= \frac{25}{2}\left[-26\sqrt{3}\right] = \boxed{-325\sqrt{3}}$

6 The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

$ar = 54$

$S_\infty = \frac{a}{1-r} = 243$

$r = \cdot > \frac{1}{2}$

$a = \frac{54}{r}$

$243 = \frac{\frac{54}{r}}{1-r}$

$243 - 243r = \frac{54}{r}$

~~243 - 243~~

$243r - 243r^2 = 54$

$0 = 243r^2 - 243r + 54 = 0$

$0 = 81r^2 - 81r + 18 = 0$

$0 = 9r^2 - 9r + 2 = 0$

$9r^2 - 6r - 3r + 2 = 0$

$3r(3r-2) - 1(3r-2) = 0$

$3r - 1 = 0$ or $3r - 2 = 0$

$r = \frac{1}{3}$ or $\boxed{r = \frac{2}{3}} > \frac{1}{2}$

	18	
6		3

$a \times \frac{2}{3} = 54$

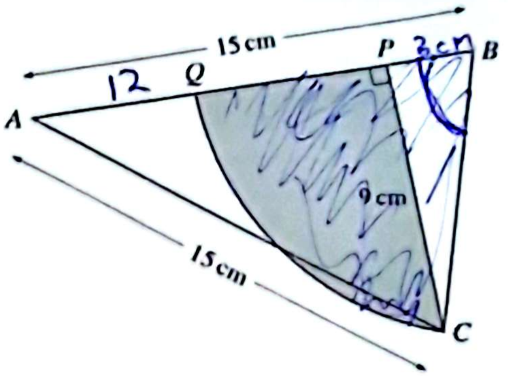
$a = 81$

$81r^9 = T_{10}$

$81\left(\frac{2}{3}\right)^9 = T_{10}$

$\frac{512}{243} = T_{10}$

7



In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

$15^2 - 9^2 = AP^2$

$18 \quad AP=$

$AP = 12 \quad PB = 3cm$

$\tan^{-1}\left(\frac{9}{3}\right) = \hat{B}$

$1.249 = \hat{B}$

1.25

(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

$\frac{\cancel{\pi}}{2}$

$BC^2 = 9^2 + 3^2$

$BC = 3\sqrt{10}$

$\frac{1}{2}(3\sqrt{10})^2 \times 1.25 = 56.25$

$56.25 - \text{Area of triangle } CPB$

$56.25 - \left(\frac{1}{2} \times 3 \times 9\right) = 42.75$

42.8

8 **(a)** It is given that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

$(1 \times 32) \quad - 80ax \quad + 80a^2x^2$

${}^5C_0(2)^5 + {}^5C_1(2)^4(-ax)^1 + {}^5C_2(2)^3(-ax)^2$

$32 - 80ax + 80a^2x^2$

$(4+2x)(32 - 80ax + 80a^2x^2)$

$-160ax^2 + 320a^2x^2 = -15x^2$

$-160a + 320a^2 = -15$

$320a^2 - 160a + 15 = 0$

$64a^2 - 32a + 3 = 0$

$a = \frac{3}{8}$ or $a = \frac{1}{8}$

$\frac{-(-32) \sqrt{(-32)^2 - 4(64)(3)}}{128}$

$\frac{32+16}{128}$ or $\frac{32-16}{128}$

$a = \frac{3}{8}$ or $\frac{1}{8}$

(b) It is given instead that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. ~~[4]~~ 2

$-160a + 320a^2 = -20$

$a = \frac{1}{4}$

$-160ax^2 + 320a^2x^2 = kx^2$

$-160a + 320a^2 - k = 0$

$320a^2 - 160a - k = 0$

$b^2 - 4aC = 0$

$(-160)^2 - 4(320)(-k) = 0$

$25600 - \cancel{1280} - \cancel{k} = 0$

$\cancel{k = 24320}$

$\cancel{+1280k}$

Complete it

$-4 \times 320 \times -k$

$\cancel{-1280}$ $-1280k$

$-\cancel{+}1280k = \cancel{2}5600$

$k = -20$

9 The volume $V\,\text{m}^3$ of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}(r - \frac{1}{2})^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\text{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

$r \geq 2$

$\frac{dv}{dt} = 1.5\,m^3$

$\frac{dr}{dt} = \frac{dv}{dt} \times \frac{dr}{dv}$

$\cancel{V = \frac{3}{2}(5.5 - \frac{1}{2})^3 - 1}$

$\frac{dv}{dr} = \frac{3}{2}(3)(r - \frac{1}{2})^2$

$\frac{9}{2}(r - \frac{1}{2})^2 = \frac{225}{2}$

$\frac{dr}{dv} = \frac{2}{225}$

$\frac{dr}{dt} = 1.5 \times \frac{2}{225}$

$\frac{dr}{dt} = \frac{1}{75}$ m per second

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

$\frac{dv}{dt} = \frac{dr}{dt} \times \frac{dv}{dr}$

$\frac{dv}{dr} = \frac{dr}{dt} \div \frac{dv}{dt}$

$\frac{dv}{dr} = 0.1 \div 1.5$

$\frac{dv}{dr} = \frac{1}{15}$

$\frac{9}{2}\left(r - \frac{1}{2}\right)^2 = \frac{1}{15}$

$\left(r - \frac{1}{2}\right)^2 = \frac{1}{15} \div \frac{9}{2}$

$\sqrt{\left(r-\frac{1}{2}\right)^2} = \sqrt{\frac{2}{135}}$

$r - \frac{1}{2} = \frac{\sqrt{30}}{45}$

$r = \frac{45 + 2\sqrt{30}}{90}$

$V = \frac{3}{2}\left(\left(\frac{45+2\sqrt{30}}{90} - \frac{1}{2}\right)^3 - 1\right)$

$\frac{dV}{dr} = \frac{dt}{dr} \times \frac{dv}{dt}$

$\frac{dV}{dr} = 10 \times 1.5$

$\frac{dv}{dr} = 15$

$\frac{9}{2}\left(r - \frac{1}{2}\right)^2 = 15$

$r - \frac{1}{2} = \frac{10}{3}$

$\left(r - \frac{1}{2}\right) = \sqrt{\frac{10}{3}}$

$r = \frac{3 + 2\sqrt{30}}{6}$

$V = \frac{3}{2}\left(\left(\frac{3+2\sqrt{30}}{6} - \frac{1}{2}\right)^3 - 1\right)$

$V = 8.13\,m^3$

10 The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

$y = x^2 + kx^{-1} + 2$

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. ~~[3]~~ -3

CANNOT HELP

$\frac{dy}{dx} = 2x^2 + kx^{-1} + 2$

$\frac{dy}{dx} = 2x - kx^{-2}$

$2x - 2kx^{-2}$

$2x - \frac{2k}{x^2} = 0$

$2(2) - \frac{2k}{2^2} = 0$

$-\frac{2k}{4} = -4$

$-2k = -16$

$k = 8 \quad 16$

CamScanner

APPROACH IS CORRECT.!!

$y = 2x - kx^{-2}$

(b) Determine the nature of the stationary point. ~~[2]~~ -2

$\frac{d^2y}{dx^2} = (2 + 2kx^{-3}) > 0$

$\frac{d^2y}{d^2x} = 2x - 2k\,x^{-2}$

$\cancel{x} +$

$2(1) - 2(-2)\,x^{-3}$

$2 + \frac{4}{x^3}$

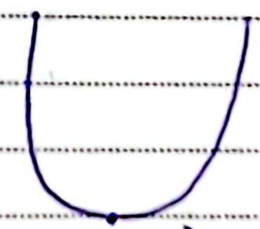
put $x = 2$ $\rightarrow 6$

$2 + \frac{4}{2^3} = \frac{5}{2}$

$\frac{5}{2} > 0$ so it is minimum

(c) Given that this is the only stationary point of the curve, find the range of f. ~~[2]~~ -1

~~f(x)~~

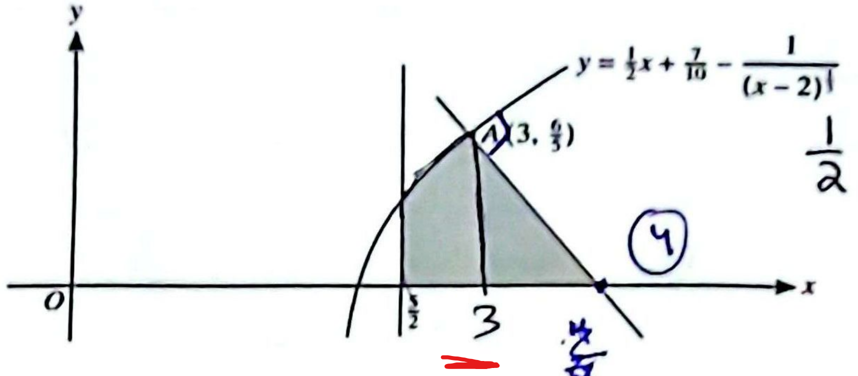


✓ (2, 10)

$x = 2,\ f(x) = 14$

$f(x) \geq 10$

11



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point A $(3, \frac{6}{5})$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

$\frac{dy}{dx} = \frac{1}{2}(x) + \frac{7}{10} - 1(x-2)^{-\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{2} - 1(-\frac{1}{3})(x-2)^{-\frac{1}{3}-1}$

$\frac{1}{2} + \frac{1}{3}(x-2)^{-\frac{4}{3}} = 0$

$\frac{1}{3}(x-2)^{-\frac{4}{3}} = -\frac{1}{2}$

$\frac{1}{(x-2)^{\frac{4}{3}}} = -\frac{3}{2}$

$\left((x-2)^{-\frac{4}{3}}\right)^3 = \left(-\frac{3}{2}\right)^3$

$\sqrt{(x-2)^{-4}} = \sqrt{-\frac{27}{8}}$

$\frac{1}{(x-2)^2} =$

$(x-2)^{-4} = -\frac{27}{8}$

$\frac{1}{(x-2)^4} = -\frac{27}{8}$

$\sqrt[3]{\frac{1}{-27}} = \sqrt[3]{\frac{(x-2)^4}{8}}$

$-\frac{1}{3} = \frac{x-2}{2}$

$-\frac{2}{3} = x - 2$

$x = \frac{4}{3}$ ~~gradient so~~

$$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{3}(x-2)^{-\frac{4}{3}}$$

put $x = 3$

gradient of curve at $x=3$ is $\frac{5}{6}$

gradient of normal $= -\frac{6}{5}$

$$y = -\frac{6}{5}x + c$$

$$\frac{6}{5} = -\frac{6}{5}(3) + c$$

$$\frac{6}{5} + \frac{18}{5} = c$$

$$\frac{24}{5} = c$$

$$y = \cancel{\frac{24}{5}} -\frac{6}{5}x + \frac{24}{5}$$

$$0 = -\frac{6}{5}(x) + \frac{24}{5}$$

$$-\frac{24}{5} = -\frac{6}{5}x$$

$$-\frac{24}{5} \div -\frac{6}{5} = x$$

$$4 = x$$

APPROACH ✓ ✓ -3 $3\frac{2}{6}$

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places.

upper graph - lower graph

$$\int \frac{x}{2} + 0.7 - \cancel{2}(x-2)^{-\frac{1}{3}}\,dx$$

$$\int_{\frac{5}{2}}^{4} \frac{x^2}{4} + 0.7x - \frac{(x-2)^{\frac{2}{3}}}{\frac{2}{3}}$$ ✓

$$\left|\frac{4^2}{4} + 0.7(4) - \frac{(4-2)^{\frac{2}{3}}}{\frac{2}{3}}\right| - \left|\frac{\left(\frac{5}{2}\right)^2}{4} + 0.7\left(\frac{5}{4}\right) - \frac{\left(\frac{5}{4}-2\right)^{\frac{2}{3}}}{\frac{2}{3}}\right|$$

$(4.41) - (2.36) = 2.042$ ✓

$2.042 + \boxed{\frac{1}{2} \times 1 \times \frac{6}{5}} = 2.642$

$\boxed{\text{Area } \Delta = 0.6}$

Write it separately.

2.642 unit square

$PR = 2\sqrt{30}$

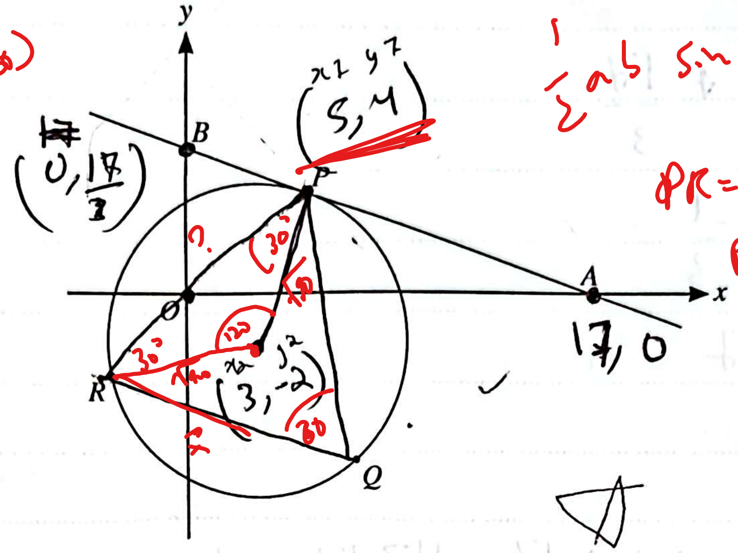
$\frac{1}{2}ab \times \sin(60)$

$\frac{1}{2} \times 2\sqrt{30} \times 2\sqrt{30}$

$\frac{1}{2}ab \sin C$

$PR = \sqrt{40^2 + 40^2 - 2(40)\cos(120)}$

$PR =$



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. **[5]**

$x^2 + y^2 - 6x + 4y = 27$

$(x-3)^2 + (y+2)^2 = 27 + 3^2 + 2^2$

$(x-3)^2 + (y+2)^2 = 40$

$\frac{-2-4}{3-5} = 3$

~~gradient of tangent~~ $-\frac{1}{3}$

~~$y = -\frac{1}{3}x + c$~~

$y = -\frac{1}{3}x + c$

$\frac{17}{3} = c$

~~$y = 3x + c$~~

~~$4 = 3(5) + c$~~

~~$4 - 15$~~ $= c$

~~$-9 = c$~~

~~$y = 3x - 9$~~

~~$0 = 3x - 9$~~

~~$9 = 3x$~~

~~$3 = x$~~

$$y = -\frac{1}{3}x + \frac{17}{3}$$

$$0 - \frac{17}{3} \div -\frac{1}{3} = x$$

$$17 = x$$

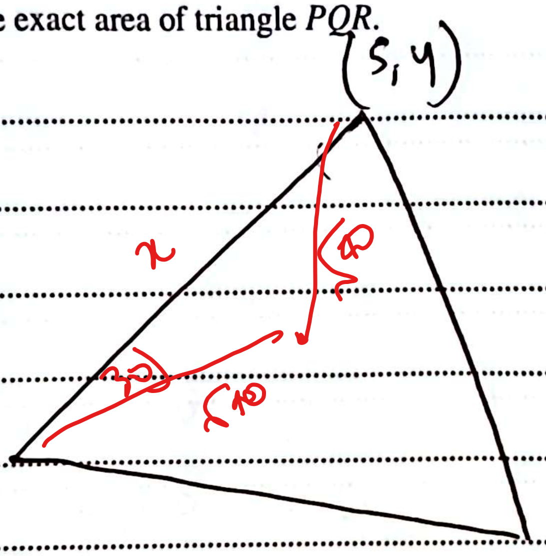
$$y = \frac{17}{3}$$

$$\frac{1}{2} \times 17 \times \frac{17}{3} = 48.17 \text{ unit square}$$

$$\left(\frac{289}{6}\right)$$

b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. **[3]**



−3

$30\sqrt{3}$

$\frac{x}{\sin 120} = \frac{\sqrt{40}}{\sin 30}$

x ?

$x = 2\sqrt{30}$

Area $= \frac{1}{2} \times 2\sqrt{30} \times 2\sqrt{30} \times \sin 60$

$= 30\sqrt{3}$