

**function** ALPHA-BETA-SEARCH( $state$ ) *returns an action*

**inputs:**  $state$ , current state in game

$v \leftarrow \text{MAX-VALUE}(state, -\infty, +\infty)$

**return** an *action* in  $\text{ACTIONS}(state)$  with value  $v$

---

**function** MAX-VALUE( $state, \alpha, \beta$ ) *returns a utility value*

**if**  $\text{TERMINAL-TEST}(state)$  **then return**  $\text{UTILITY}(state)$

$v \leftarrow -\infty$

**for**  $a$  in  $\text{ACTIONS}(state)$  **do**

$v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{Result}(s, a), \alpha, \beta))$

**if**  $v \geq \beta$  **then return**  $v$

$\alpha \leftarrow \text{MAX}(\alpha, v)$

**return**  $v$

---

**function** MIN-VALUE( $state, \alpha, \beta$ ) *returns a utility value*

**if**  $\text{TERMINAL-TEST}(state)$  **then return**  $\text{UTILITY}(state)$

$v \leftarrow +\infty$

**for**  $a, s$  in  $\text{SUCCESSORS}(state)$  **do**

$v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$

**if**  $v \leq \alpha$  **then return**  $v$

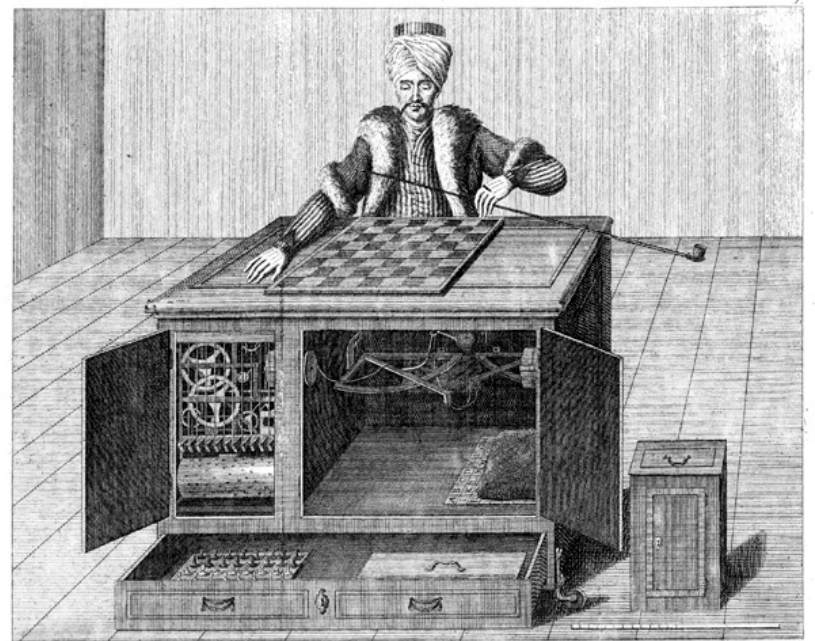
$\beta \leftarrow \text{MIN}(\beta, v)$

**return**  $v$

---

# Game-playing AIs: Games and Adversarial Search

**AIMA 5.1-5.5,  
AIMA 16.1-16.3**



*W. de Kempelen del. Che a Michel occidit. Basileae. P. G. Pary, fecit.  
Par Schach-Spieler, wie er vor dem Spiel zu sehen wird, so vor dem Spiel zu sehen wird. Le joueur d'Échecs, tel qu'on le montre avant le jeu, par devant.*

# Games: Outline of Unit

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## Part I: Games as Search

- Motivation
- Game-playing AI successes
- Game Trees
- Evaluation Functions

## Part II: Adversarial Search

- The Minimax Rule
- Alpha-Beta Pruning

# May 11, 1997

may 11th      game 6 : may 11 @ 3:00PM EDT | 19:00 GMT      kasparov 2.5 deep blue 2.5

Home   The match   The players   The technology   Community

## Deep Blue Wins 3.5 to 2.5

KASPAROV vs DEEP BLUE  
the rematch

With a dramatic victory in Game 6, Deep Blue won its six-game rematch with Champion Garry Kasparov

- OVERVIEW
- EVENT COVERAGE
- MATCH NEWS
- MAIN STORIES



**Commentary**  
**George Plimpton** on chess, Kasparov, and the limitations of computers  
[Read the article](#)



**Commentary**  
**Vishwanathan Anand** on the legacy of Kasparov vs. Deep Blue  
[Read the article](#)



**Club Kasparov**  
Visit the virtual home of the world's greatest chess player.



**Guest essays**  
Thoughts on chess, computers, and what it all means  
[Read the essays...](#)

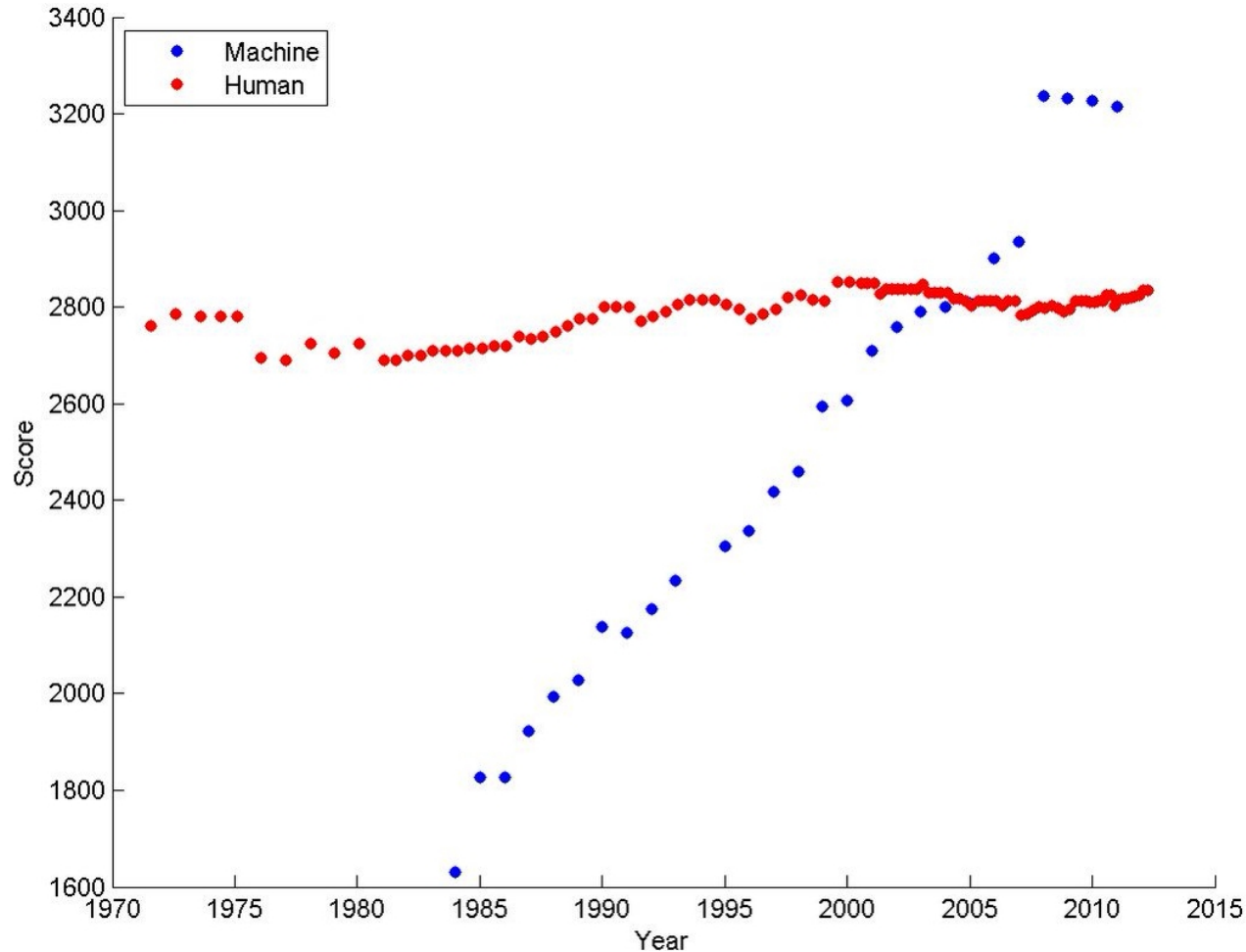


**Community**  
During the rematch, more than 20,000 people from 120 countries joined the community to talk about the match.



**Clips from the rematch**  
Video footage from the games  
[Highlights from the games](#)

# Ratings of human & computer chess champions



<https://srconstantin.wordpress.com/2017/01/28/performance-trends-in-ai/>

# AlphaGo seals 4-1 victory over Go grandmaster Lee Sedol

DeepMind's artificial intelligence astonishes fans to defeat human opponent and offers evidence computer software has mastered a major challenge

Steven Borowiec

Tuesday 15 March 2016 06.12 EDT



This article is 6 months old

Shares

613



Save for later



The world's top Go player, Lee Sedol, lost the final game of the Google DeepMind challenge match.  
Photograph: Yonhap/Reuters

Google DeepMind's AlphaGo program triumphed in its final game against South Korean Go grandmaster Lee Sedol to win the series 4-1, providing further evidence of the landmark achievement for an artificial intelligence program.

Lee started Tuesday's game strongly, taking advantage of an early mistake by AlphaGo. But in the end, Lee was unable to hold off a comeback by his opponent, which won a narrow victory.

(from The  
Guardian)

# The Simplest Game Environment

---

- ***Multiagent***
- ***Static:*** No change while an agent is deliberating
- ***Discrete:*** A finite set of percepts and actions
- ***Fully observable*** : An agent's sensors give it the complete state of the environment.
- ***Strategic:*** The next state is determined by the current state and the action executed by the agent and the actions of one other agent.



# Key properties of our sample games

---

- 1. Two players alternate moves**
  - 2. Zero-sum: one player's loss is another's gain**
  - 3. Clear set of legal moves**
  - 4. Well-defined outcomes (e.g. win, lose, draw)**
- **Examples:**
    - Chess, Checkers, Go,
    - Mancala, Tic-Tac-Toe, Othello ...



# More complicated games

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- **Most card games (e.g. Hearts, Bridge, etc.) and Scrabble**
  - Stochastic, not deterministic
  - Not fully observable: lacking in perfect information
- **Real-time strategy games, e.g. Warcraft**
  - Continuous rather than discrete
  - No pause between actions, don't take turns
- **Cooperative games**

# Pac-Man

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<https://youtu.be/-CbyAk3Sn9I>

# Formalizing the Game setup

---

1. Two players: **MAX** and **MIN**; **MAX** moves first.
  2. **MAX** and **MIN** take turns until the game is over.
  3. Winner gets award, loser gets penalty.
- **Games as search:**
    - *Initial state*: e.g. board configuration of chess
    - *Successor function*: list of (move,state) pairs specifying legal moves.
    - *Terminal test*: Is the game finished?
    - *Utility function*: Gives numerical value of terminal states.  
e.g. win ( $+\infty$ ), lose ( $-\infty$ ) and draw (0)
    - **MAX** uses search tree to determine next move.

# How to Play a Game by Searching

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- **General Scheme**

1. Consider all legal successors to the current state ('board position')
2. Evaluate each successor board position
3. Pick the move which leads to the best board position.
4. After **your opponent moves**, repeat.

- **Design issues**

1. Representing the 'board'
2. Representing legal next boards
3. Evaluating positions
4. Looking ahead

# Hexapawn: A very simple Game

---

- Hexapawn is played on a 3x3 chessboard



- **Only standard pawn moves:**
  1. A pawn moves forward one square onto an empty square
  2. A pawn “captures” an opponent pawn by moving diagonally forward one square, if that square contains an opposing pawn. The opposing pawn is removed from the board.

# Hexapawn: A very simple Game

---

- Hexapawn is played on a 3x3 chessboard

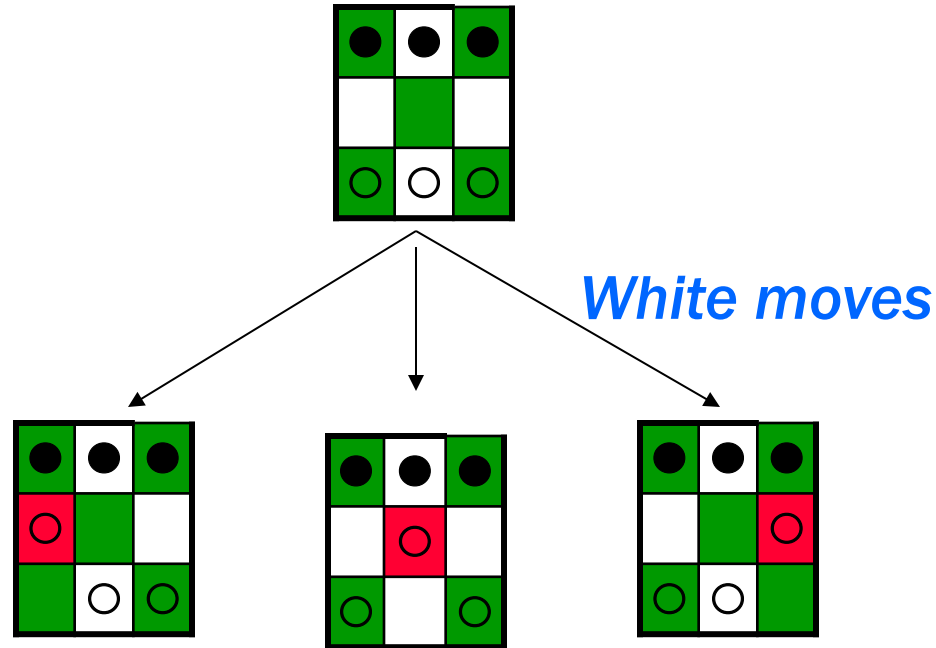


- **Player  $P_1$  wins the game against  $P_2$  when:**
  - One of  $P_1$ 's pawns reaches the far side of the board, or
  - $P_2$  cannot move because no legal move is possible.
  - $P_2$  has no pawns left.

*(Invented by Martin Gardner in 1962, with learning “program” using match boxes. Reprinted in “The Unexpected Hanging..”)*

# Hexapawn: Three Possible First Moves

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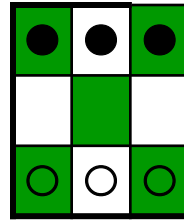
# Game Trees

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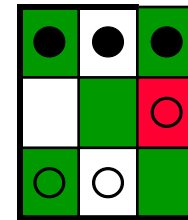
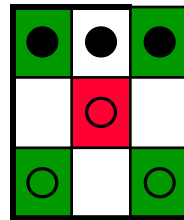
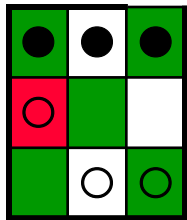
- **Represent the game problem space by a tree:**
  - Nodes represent 'board positions'; edges represent legal moves.
  - Root node is the first position in which a decision must be made.

# Hexapawn: Simplified Game Tree for 2 Moves

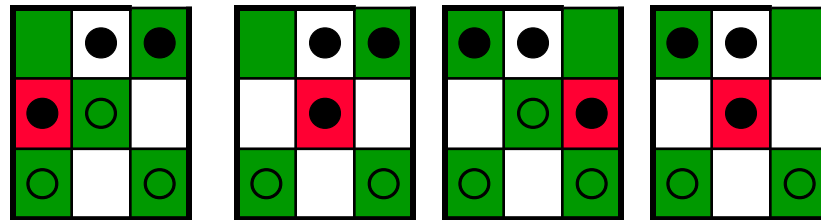
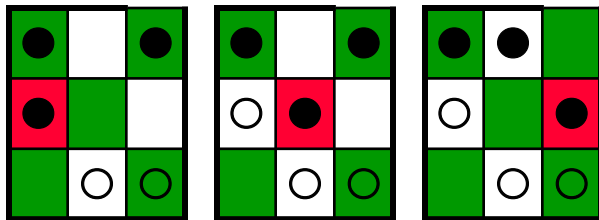
*White to move*



*Black to move*



...



*White to move*

# Adversarial Search

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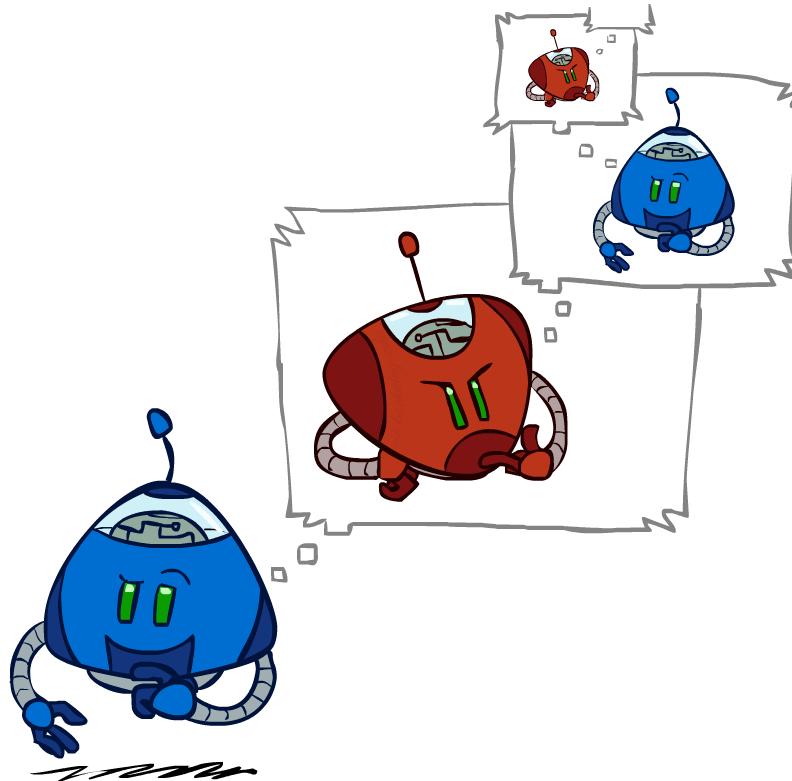


Image from Dan Klein

# Battle of Wits

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<https://www.youtube.com/watch?v=rMz7JBRbmNo>

# MAX & MIN Nodes : An egocentric view

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- Two players: MAX, MAX's opponent MIN
- *All play is computed from MAX's vantage point.*
- When MAX moves, MAX attempts to MAXimize MAX's outcome.
- When MAX's opponent moves, they attempt to MINimize MAX's outcome.

WE TYPICALLY ASSUME MAX MOVES FIRST:

- Label the root (level 0) MAX
- Alternate MAX/MIN labels at each successive tree level (*ply*).
- *Even levels* represent turns for MAX
- *Odd levels* represent turns for MIN

# Game Trees

---

- Represent the game problem space by a tree:
  - Nodes represent 'board positions'; edges represent legal moves.
  - Root node is the first position in which a decision must be made.
- **Evaluation function  $f$  assigns real-number scores to 'board positions' *without reference to path***
- **Terminal nodes represent ways the game could end, labeled with the desirability of that ending (e.g. win/lose/draw or a numerical score)**

# Evaluation functions: $f(n)$

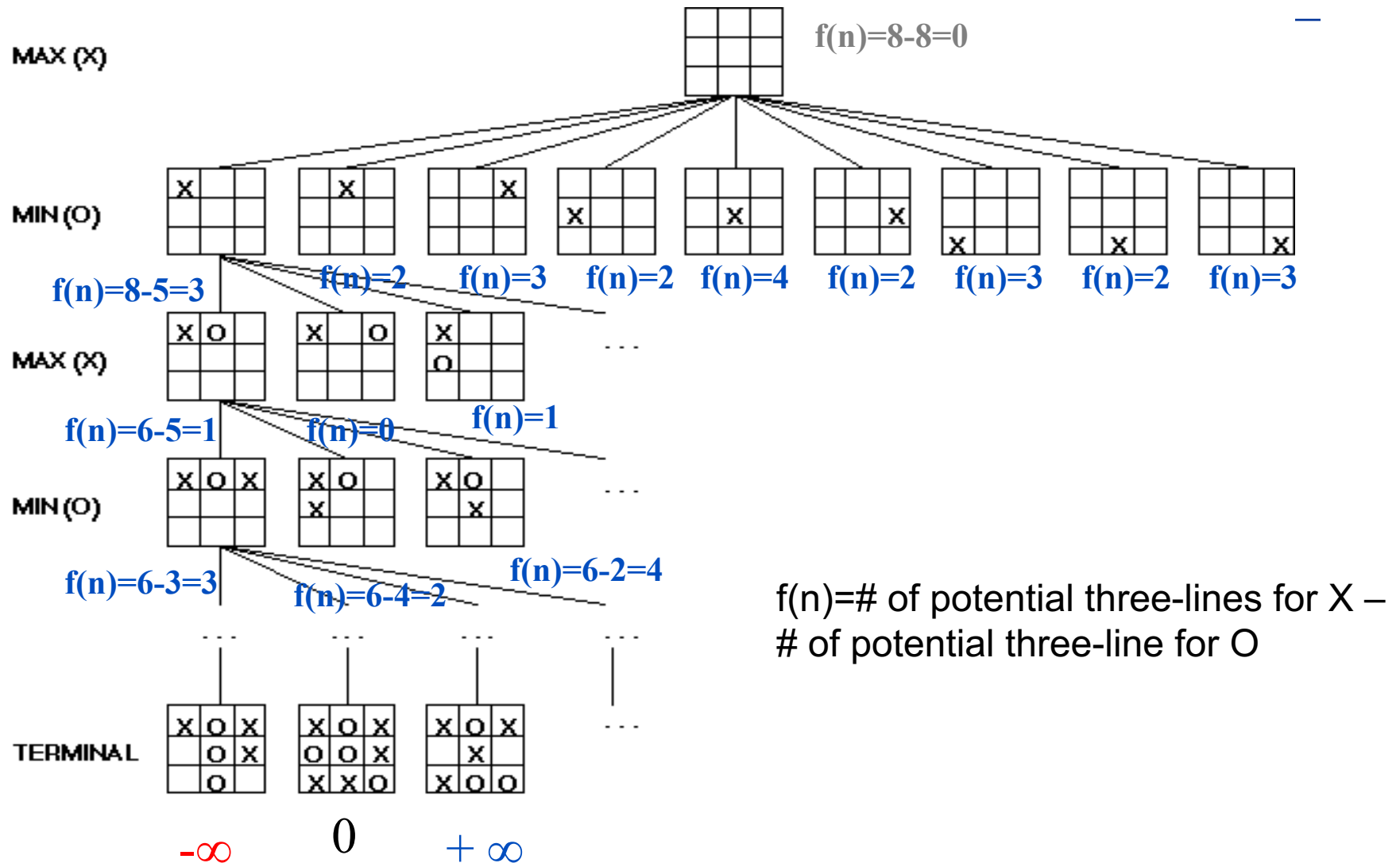
---

- Evaluates how good a ‘board position’ is
- Based on *static features* of that board alone
- Zero-sum assumption lets us use one function to describe goodness for both players.
  - $f(n) > 0$  if MAX is winning in position  $n$
  - $f(n) = 0$  if position  $n$  is tied
  - $f(n) < 0$  if MIN is winning in position  $n$
- Build using expert knowledge,
  - Tic-tac-toe:  $f(n) = (\text{\# of 3 lengths open for MAX}) - (\text{\# open for MIN})$

(AIMA 5.4.1)



# A Partial Game Tree for Tic-Tac-Toe



# Chess Evaluation Functions

- Claude Shannon argued for a chess evaluation function in a 1950 paper
- Alan Turing defined function in 1948:  
 *$f(n) = (\text{sum of } A\text{'s piece values}) - (\text{sum of } B\text{'s piece values})$*
- More complex: weighted sum of *positional* features:

$$\sum w_i \text{feature}_i(n)$$

- Deep Blue had >8000 features

Pawn	1.0
Knight	3.0
Bishop	3.25
Rook	5.0
Queen	9.0

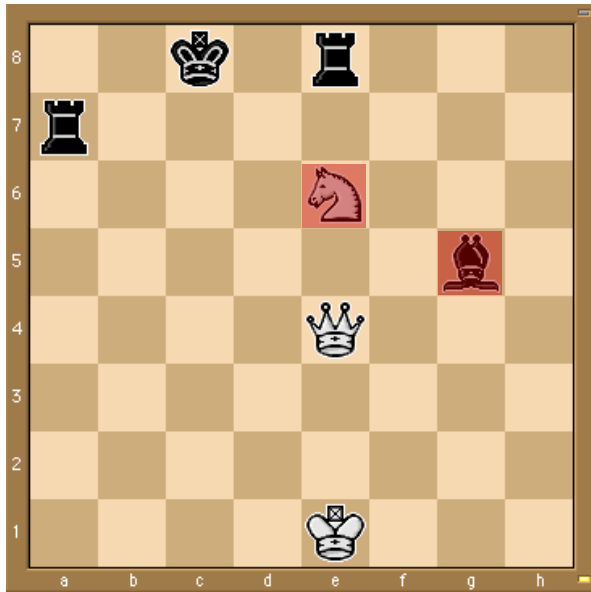
Pieces values for a simple Turing-style evaluation function often taught to novice chess players

**Positive:** rooks on open files, knights in closed positions, control of the center, developed pieces

**Negative:** doubled pawns, wrong-colored bishops in closed positions, isolated pawns, pinned pieces

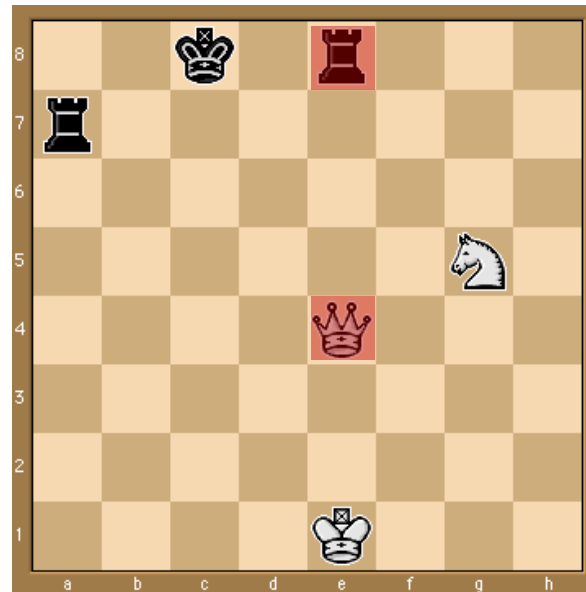
*Examples of more complex features*

# Some Chess Positions and their Evaluations



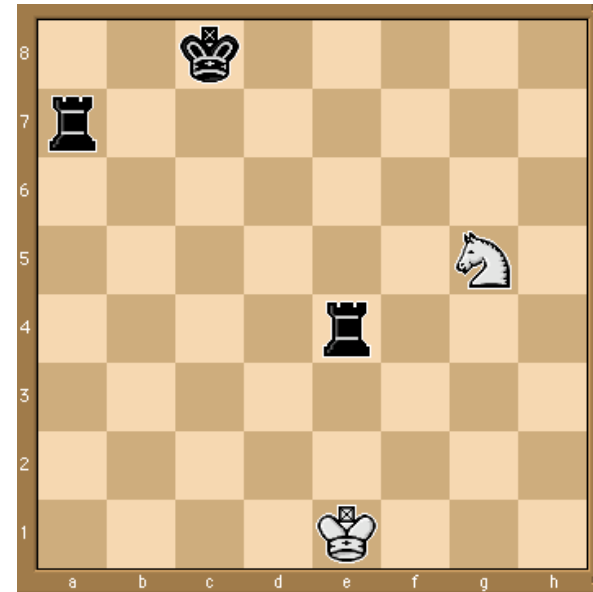
White to move

$$f(n) = (9+3) - (5+5+3.25) \\ = -1.25$$



... Nxg5??

$$f(n) = (9+3) - (5+5) \\ = 2$$



Uh-oh: Rxg4+

$$f(n) = (3) - (5+5) \\ = -7$$

*And black may force checkmate*

So, considering our opponent's possible responses would be wise.

---

# The Minimax Rule (AIMA 5.2)

# The Minimax Rule: “Don’t play hope chess”

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*Idea: Make the best move for MAX assuming that MIN always replies with the best move for MIN*

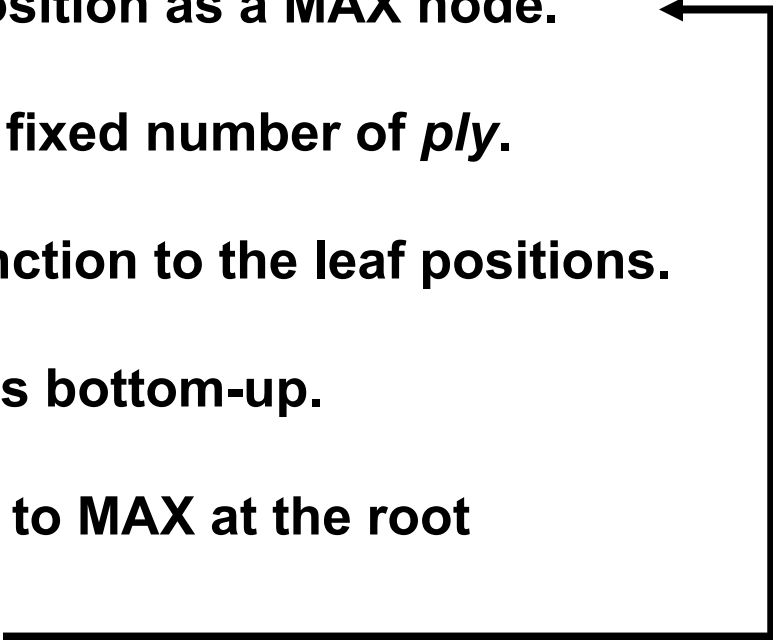
## Easily computed by a recursive process

- The ***backed-up value*** of each node in the tree is determined by the values of its children:
  - For a **MAX** node, the backed-up value is the ***maximum*** of the values of its children (*i.e. the best for MAX*)
  - For a **MIN** node, the backed-up value is the ***minimum*** of the values of its children (*i.e. the best for MIN*)

# The Minimax Procedure

---

**Until game is over:**

- 1. Start with the current position as a MAX node.**
  - 2. Expand the game tree a fixed number of *ply*.**
  - 3. Apply the evaluation function to the leaf positions.**
  - 4. Calculate back-up values bottom-up.**
  - 5. Pick the move assigned to MAX at the root**
  - 6. Wait for MIN to respond**
- 

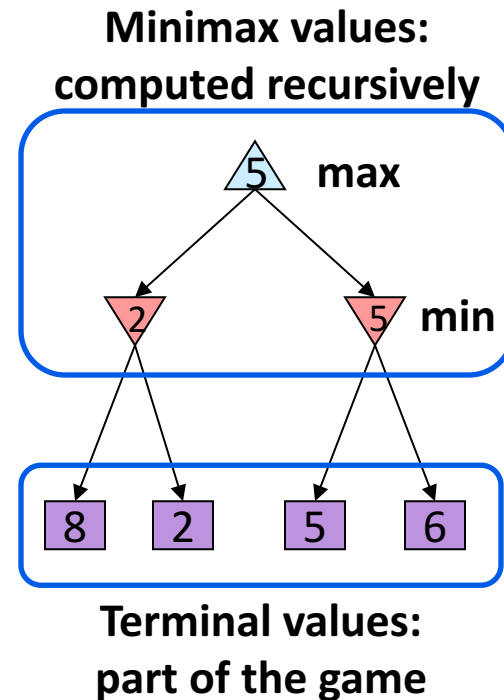
● MAX  
● MIN





# Adversarial Search (Minimax)

- **Minimax search:**
  - A state-space search tree
  - Players alternate turns
  - Compute each node's **minimax value**: the best achievable utility against a rational (optimal) adversary



# Minimax Implementation

---

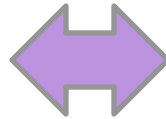
```
def max-value(state):
```

```
    initialize  $v = -\infty$ 
```

```
    for each successor of state:
```

```
         $v = \max(v, \text{min-value}(\text{successor}))$ 
```

```
    return  $v$ 
```



```
def min-value(state):
```

```
    initialize  $v = +\infty$ 
```

```
    for each successor of state:
```

```
         $v = \min(v, \text{max-value}(\text{successor}))$ 
```

```
    return  $v$ 
```

# Minimax Implementation

---

def max-value(state):

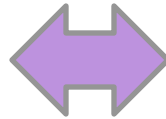
if the state is a terminal state:  
return the state's utility

initialize  $v = -\infty$

for each successor of state:

$v = \max(v, \text{min-value}(\text{successor}))$

return  $v$



def min-value(state):

if the state is a terminal state:  
return the state's utility

initialize  $v = +\infty$

for each successor of state:

$v = \min(v, \text{max-value}(\text{successor}))$

return  $v$

# Minimax Implementation (Dispatch)

**def value(state):**

if the state is a terminal state: return the state's utility  
if the next agent is **MAX**: return **max-value(state)**  
if the next agent is **MIN**: return **min-value(state)**

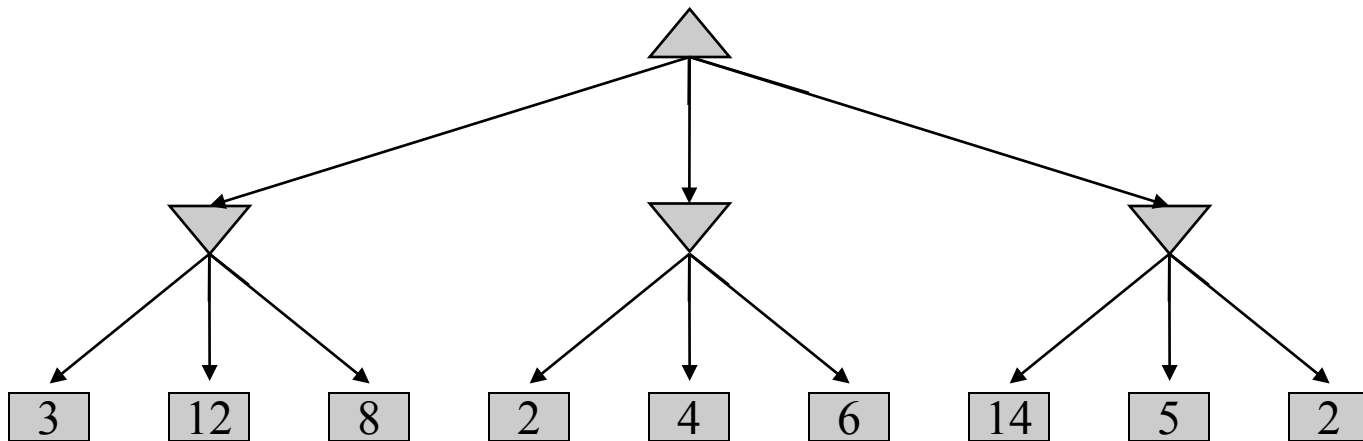
**def max-value(state):**

initialize  $v = -\infty$   
for each successor of state:  
     $v = \max(v, \text{value}(\text{successor}))$   
return  $v$

**def min-value(state):**

initialize  $v = +\infty$   
for each successor of state:  
     $v = \min(v, \text{value}(\text{successor}))$   
return  $v$

# Minimax Example



**def max-value(state):**

initialize  $v = -\infty$

for each successor of state:

$v = \max(v, \text{min-value}(\text{successor}))$

return  $v$



**def min-value(state):**

initialize  $v = +\infty$

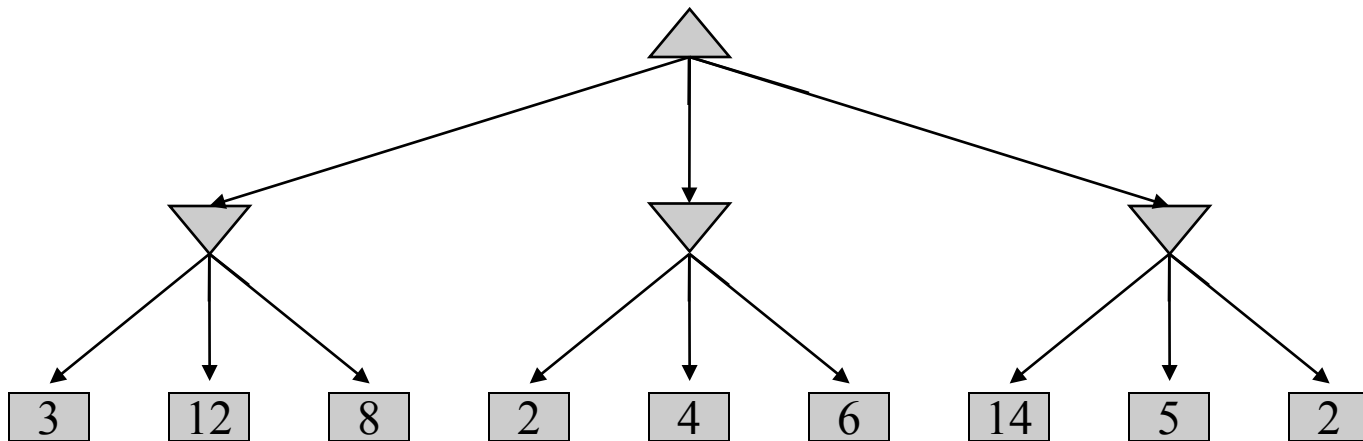
for each successor of state:

$v = \min(v, \text{max-value}(\text{successor}))$

return  $v$

# Minimax Example

---



**def max-value(state):**

if the state is a terminal state:  
return the state's utility

initialize  $v = -\infty$

for each successor of state:

$v = \max(v, \text{min-value}(\text{successor}))$

return  $v$

**def min-value(state):**

if the state is a terminal state:  
return the state's utility

initialize  $v = +\infty$

for each successor of state:

$v = \min(v, \text{max-value}(\text{successor}))$

return  $v$

## What if MIN does not play optimally?

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- **Definition of optimal play for MAX assumes MIN plays optimally:**
  - *Maximizes worst-case outcome* for MAX.
  - (Classic game theoretic strategy)
- **But if MIN does not play optimally, MAX will do even better.**
  - This theorem is not hard to prove



# Comments on Minimax Search

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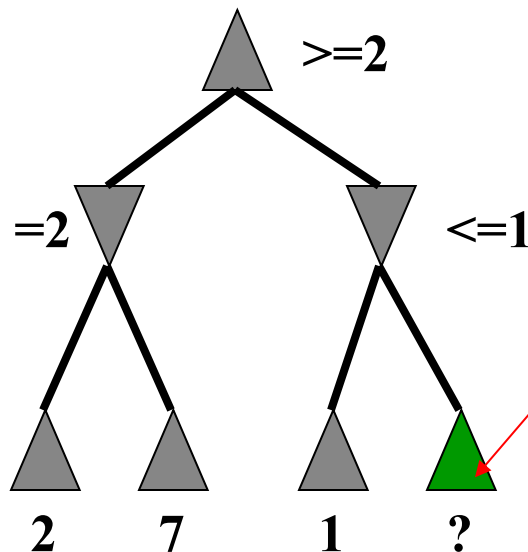
- **Depth-first search with fixed number of ply  $m$  as the limit.**
  - $O(b^m)$  time complexity – *As usual!*
  - $O(bm)$  space complexity
- **Performance will depend on**
  - the quality of the static evaluation function (expert knowledge)
  - depth of search (computing power and search algorithm)
- **Differences from normal state space search**
  - Looking to make *one* move only, despite deeper search
  - No cost on arcs – costs from backed-up static evaluation
  - MAX can't be sure how MIN will respond to his moves
- **Minimax forms the basis for other game tree search algorithms.**

---

# Alpha-Beta Pruning (AIMA 5.3)

# Alpha-Beta Pruning

- A way to improve the performance of the Minimax Procedure
- Basic idea: *“If you have an idea which is surely bad, don’t take the time to see how truly awful it is”* ~ Pat Winston



- We don't need to compute the value at this node.
- No matter what this is, it won't change the value of the root node.

# Alpha-Beta Pruning

---

- During Minimax, keep track of two additional values:
  - $\alpha$ : MAX's current *lower* bound on MAX's outcome
  - $\beta$ : MIN's current *upper* bound on MIN's outcome
- MAX will never allow a move that could lead to a worse score (for MAX) than  $\alpha$
- MIN will never allow a move that could lead to a better score (for MAX) than  $\beta$
- Therefore, stop evaluating a branch whenever:
  - When evaluating a MAX node: a value  $v \geq \beta$  is backed-up
    - MIN will never select that MAX node
  - When evaluating a MIN node: a value  $v \leq \alpha$  is found
    - MAX will never select that MIN node

# Alpha-Beta Implementation

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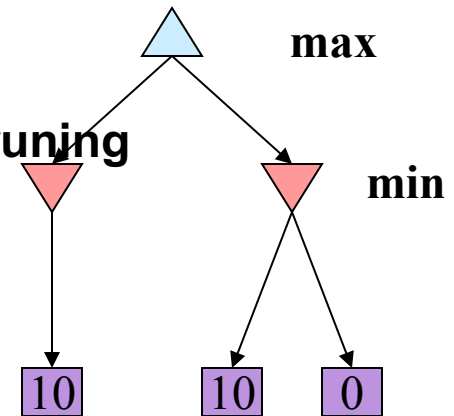
$\alpha$ : MAX's best option on path to root  
 $\beta$ : MIN's best option on path to root

```
def max-value(state,  $\alpha$ ,  $\beta$ ):  
    initialize  $v = -\infty$   
    for each successor of state:  
         $v = \max(v, \text{value}(\text{successor}, \alpha, \beta))$   
        if  $v \geq \beta$  return  $v$   
         $\alpha = \max(\alpha, v)$   
    return  $v$ 
```

```
def min-value(state,  $\alpha$ ,  $\beta$ ):  
    initialize  $v = +\infty$   
    for each successor of state:  
         $v = \min(v, \text{value}(\text{successor}, \alpha, \beta))$   
        if  $v \leq \alpha$  return  $v$   
         $\beta = \min(\beta, v)$   
    return  $v$ 
```

# Alpha-Beta Pruning Properties

- This pruning has **no effect** on minimax value computed for the root!
- **Values of intermediate nodes might be wrong**
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection
- **Good child ordering improves effectiveness of pruning**
- **With “perfect ordering”:**
  - Time complexity drops to  $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless...
- This is a simple example of **metareasoning** (computing about what to compute)



# Alpha-Beta Pruning

- Based on observation that for all viable paths utility value  $f(n)$  will be  $\alpha \leq f(n) \leq \beta$
- Initially,  $\alpha = -\text{infinity}$ ,  $\beta = \text{infinity}$



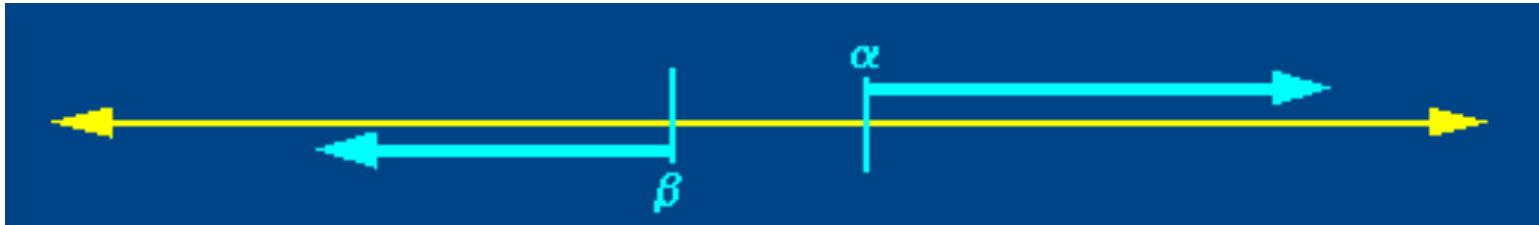
- As the search tree is traversed, the possible utility value window shrinks as  $\alpha$  increases,  $\beta$  decreases



# Alpha-Beta Pruning

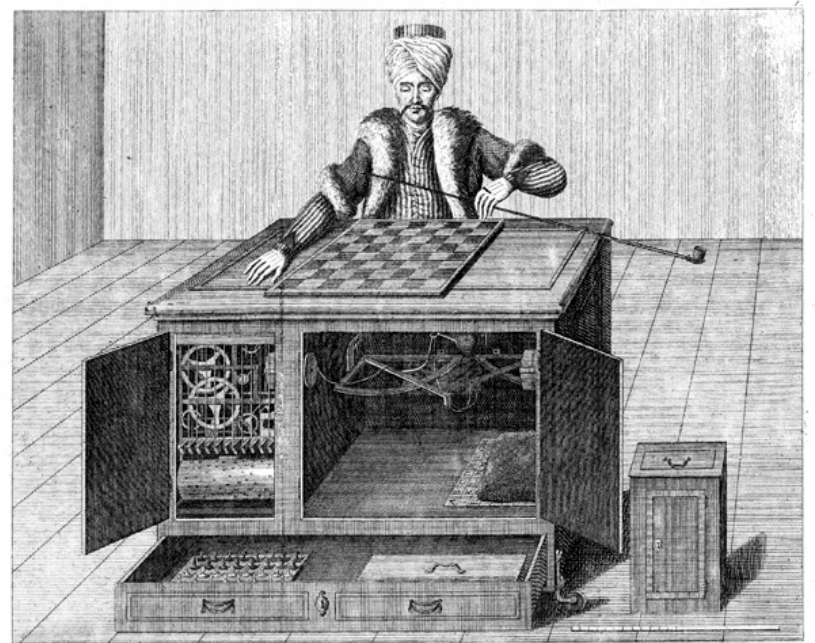
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- Whenever the current ranges of alpha and beta no longer overlap, it is clear that the current node is a dead end





# Games and Adversarial Search II



W. de Kempelen del. *Ch. a. Michel sculp. Basilea.* P. G. Pary, fec.  
*Par Schach-Spieler, wie er vor dem Spiel gezeigt wird, so vor dem Spiel. Le joueur d'Échecs, tel qu'on le montre avant le jeu, par devant.*

---

# Alpha-Beta Pruning (AIMA 5.3)

Some slides adapted from Richard Lathrop, USC/ISI, CS 271

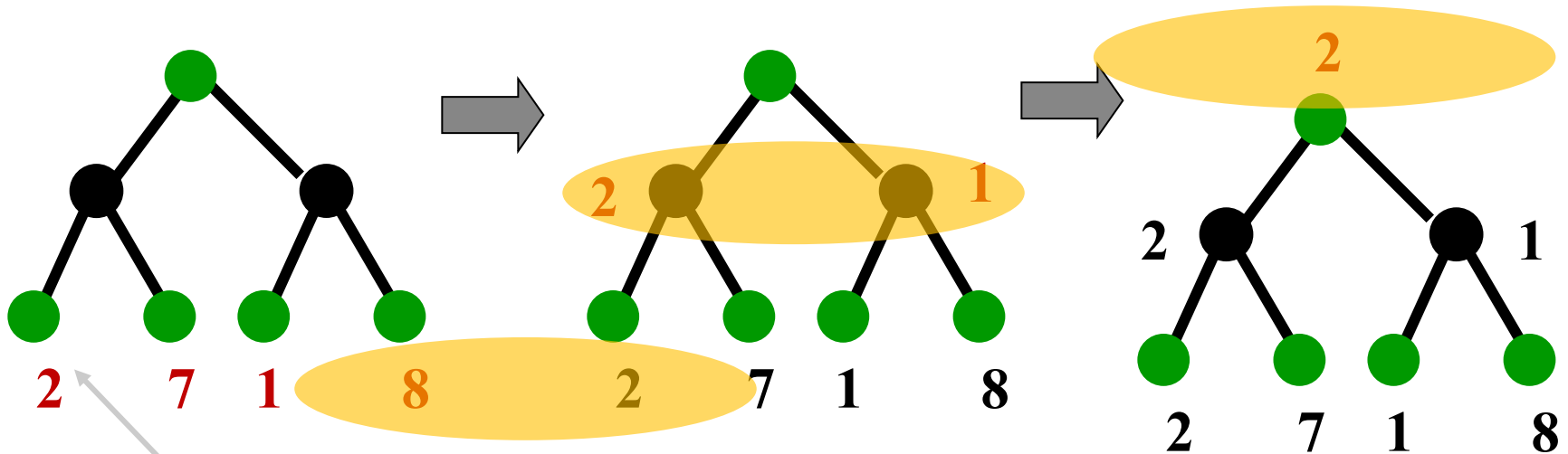
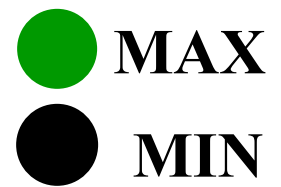
# Review: The Minimax Rule

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*Idea: Make the best move for MAX **assuming that MIN always replies with the best move for MIN***

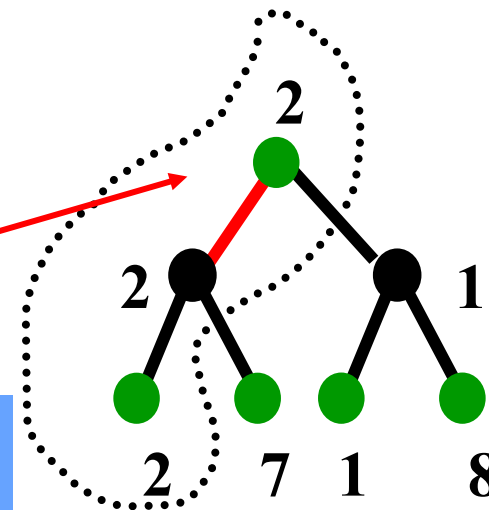
1. Start with the current position as a MAX node.
2. Expand the game tree a fixed number of *ply*.
3. Apply the evaluation function to all leaf positions.
4. Calculate back-up values bottom-up:
  - For a **MAX** node, return the *maximum* of the values of its children (*i.e. the best for MAX*)
  - For a **MIN** node, return the *minimum* of the values of its children (*i.e. the best for MIN*)
5. Pick the move assigned to MAX at the root
6. Wait for MIN to respond and REPEAT FROM 1

# 2-ply Example: Backing up values



Evaluation function value

This is the move  
selected by minimax



*New point:  
Actually calculated by DFS!*

# Minimax Algorithm

---

**function** MINIMAX-DECISION(*state*) *returns an action*

**inputs:** *state*, current state in game

$v \leftarrow \text{MAX-VALUE}(\text{state})$

**return** an *action* in SUCCESSORS(*state*) with value  $v$

---

**function** MAX-VALUE(*state*) *returns a utility value*

**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow -\infty$

**for**  $a, s$  in SUCCESSORS(*state*) **do**

$v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$

**return**  $v$

---

**function** MIN-VALUE(*state*) *returns a utility value*

**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow \infty$

**for**  $a, s$  in SUCCESSORS(*state*) **do**

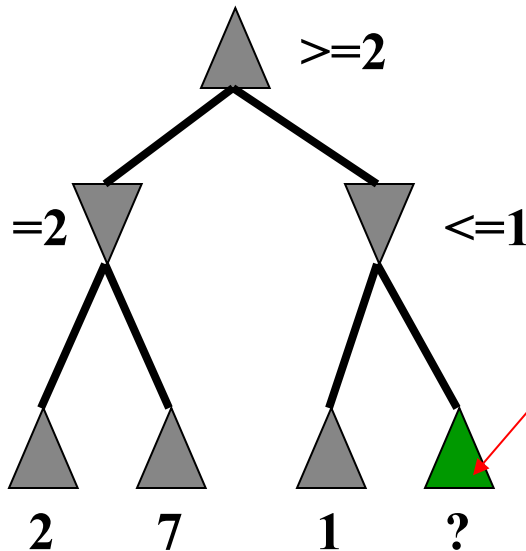
$v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s))$

**return**  $v$

---

# Alpha-Beta Pruning

- A way to improve the performance of the Minimax Procedure
- Basic idea: *“If you have an idea which is surely bad, don’t take the time to see how truly awful it is”* ~ Pat Winston
  - Assuming left-to-right tree traversal:
  - We don’t need to compute the value at this node!
  - No matter what it is it can’t effect the value of the root node.



# Alpha-Beta Pruning II

---

- During Minimax, keep track of two additional values:
  - $\alpha$ : current *lower* bound on MAX's outcome
  - $\beta$ : current *upper* bound on MIN's outcome
- MAX will never choose a move that could lead to a worse score (for MAX) than  $\alpha$
- MIN will never choose a move that could lead to a better score (for MAX) than  $\beta$
- Therefore, stop evaluating a branch whenever:
  - When evaluating a MAX node: a value  $v \geq \beta$  is backed-up
    - MIN will never select that MAX node
  - When evaluating a MIN node: a value  $v \leq \alpha$  is found
    - MAX will never select that MIN node

# Alpha-Beta Pruning IIIa

- Based on observation that for all viable paths utility value  $f(n)$  will be  $\alpha \leq f(n) \leq \beta$
- Initially,  $\alpha = -\text{infinity}$ ,  $\beta = \text{infinity}$



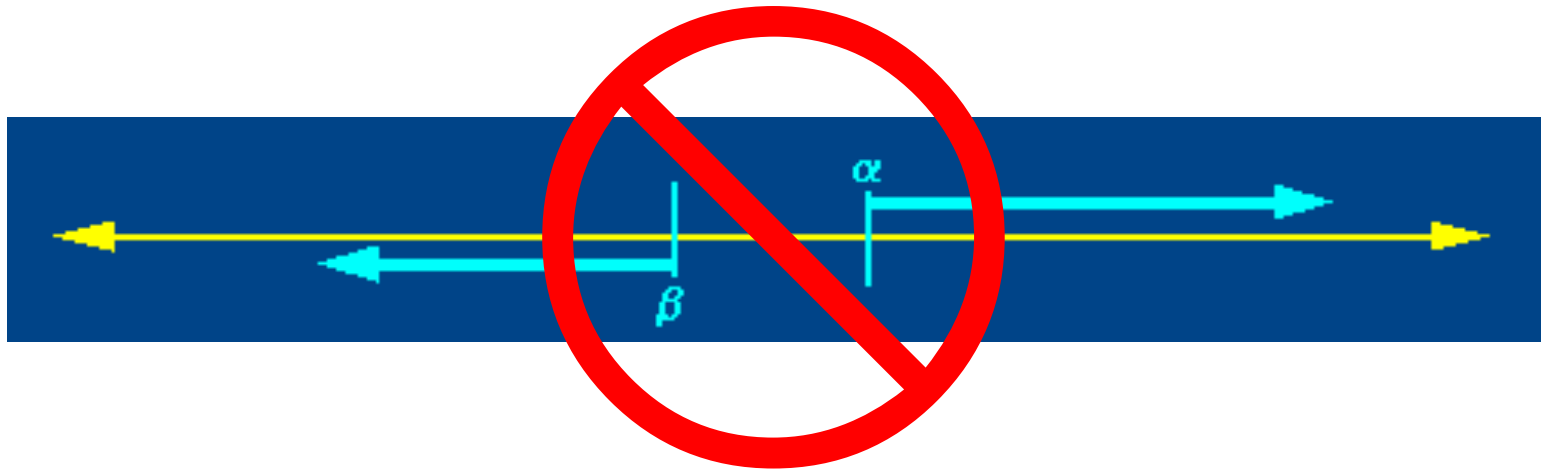
- As the search tree is traversed, the possible utility value window shrinks as  $\alpha$  increases,  $\beta$  decreases





# Alpha-Beta Pruning IIb

- Whenever the current ranges of alpha and beta no longer overlap ( $\alpha \geq \beta$ ), it is clear that the current node is a dead end, so it can be pruned



# Alpha-beta Algorithm: In detail

---

- **Depth first search (usually bounded, with static evaluation)**
  - only considers nodes along a single path from root at any time

$\beta$   
↓

$\alpha$  = current **lower** bound on MAX's outcome  
(initially,  $\alpha$  =  $-\infty$ )

$\beta$  = current **upper** bound on MIN's outcome  
(initially,  $\beta$  =  $+\infty$ )

↑  
 $\alpha$

- **Pass current values of  $\alpha$  and  $\beta$  **down** to child nodes during search.**
- **Update values of  $\alpha$  and  $\beta$  during search:**
  - MAX updates  $\alpha$  at MAX nodes
  - MIN updates  $\beta$  at MIN nodes
- **Prune remaining branches at a node whenever  $\alpha \geq \beta$**

# When to Prune

---

Prune whenever  $\alpha \geq \beta$ .

$\beta$   
↓

- Prune below a Max node when its  $\alpha$  value becomes  $\geq$  the  $\beta$  value of its ancestors.
  - **Max nodes update**  $\alpha$  based on children's returned values.
  - Idea: Player MIN at node above won't pick that value anyway, since MIN can force a worse value.

↑  
 $\alpha$

- Prune below a Min node when its  $\beta$  value becomes  $\leq$  the  $\alpha$  value of its ancestors.
  - **Min nodes update**  $\beta$  based on children's returned values.
  - Idea: Player MAX at node above won't pick that value anyway; she can do better.

# Pseudocode for Alpha-Beta Algorithm

---

**function** ALPHA-BETA-SEARCH(*state*) **returns** *an action*

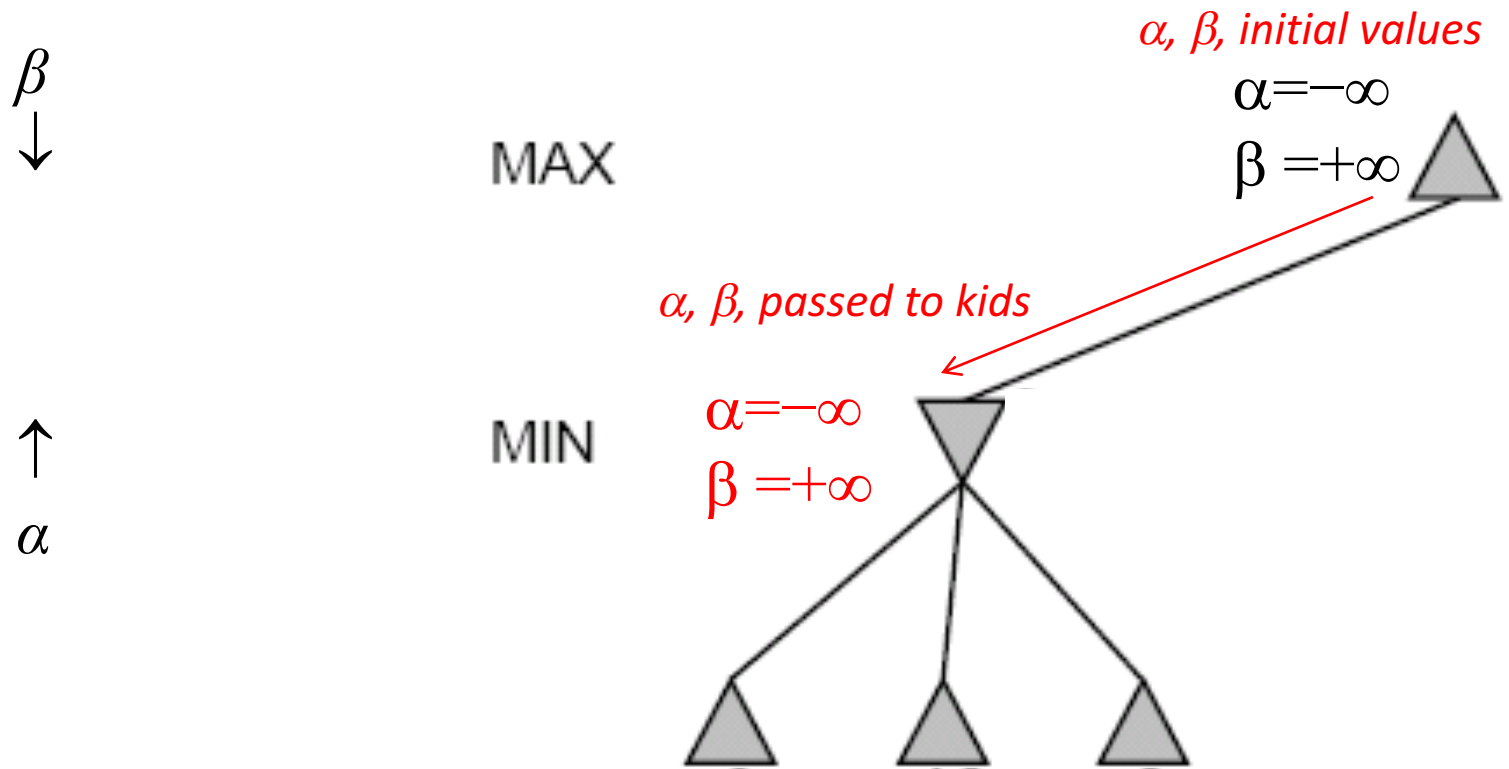
**inputs:** *state*, current state in game

$v \leftarrow \text{MAX-VALUE}(\textit{state}, -\infty, +\infty)$

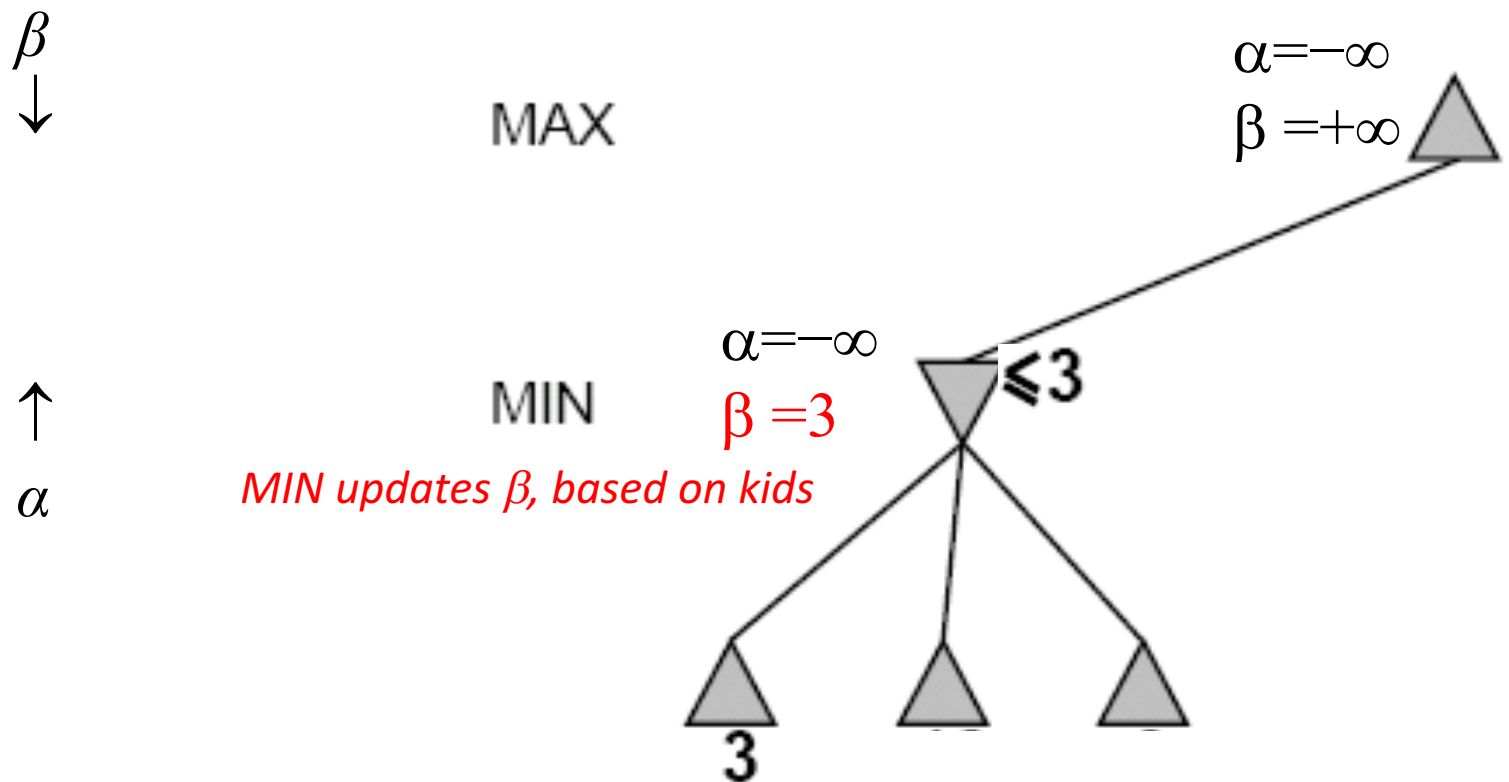
**return** *an action* in ACTIONS(*state*) with value  $v$

# An Alpha-Beta Example

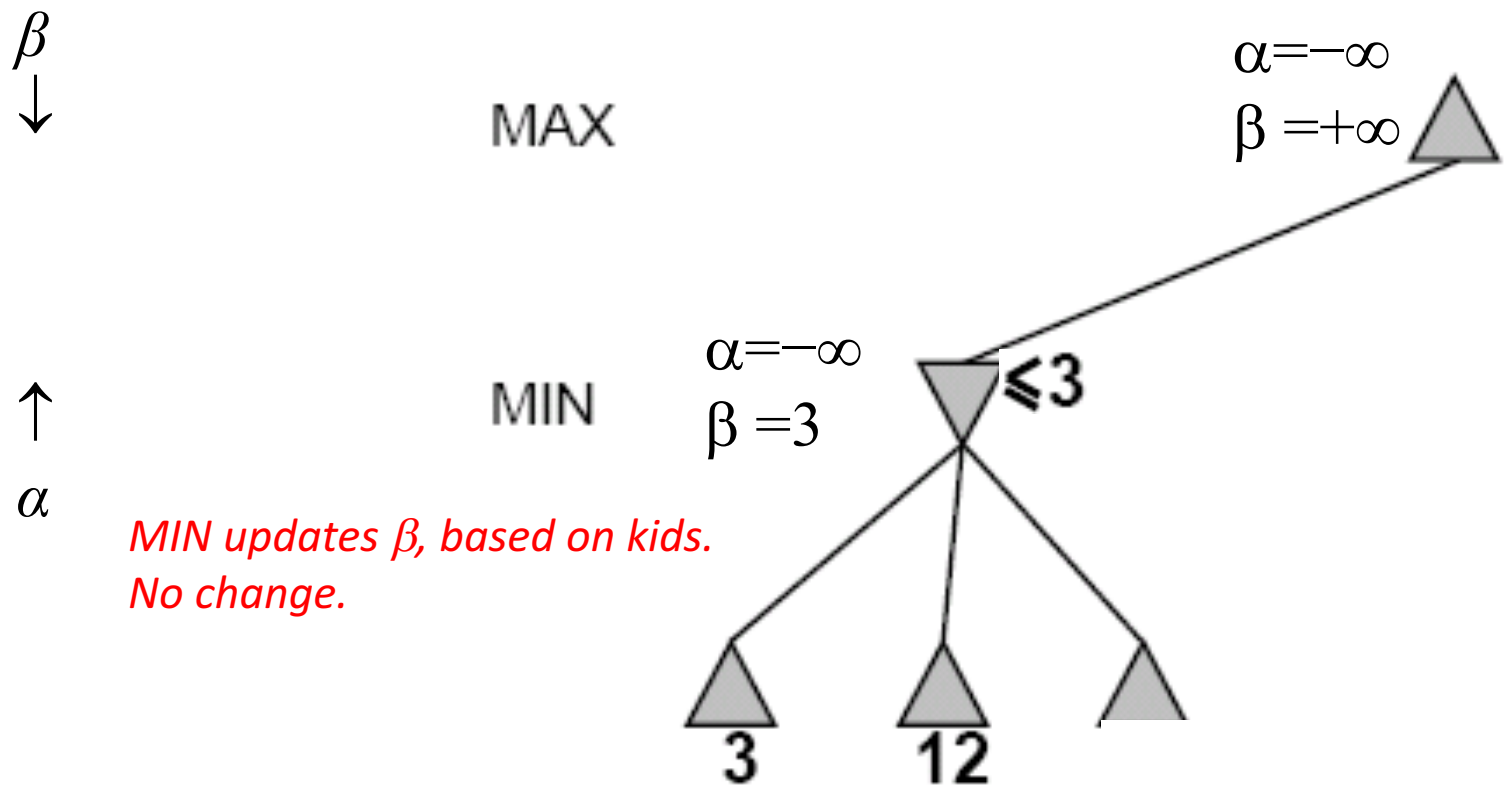
Do DF-search until first leaf



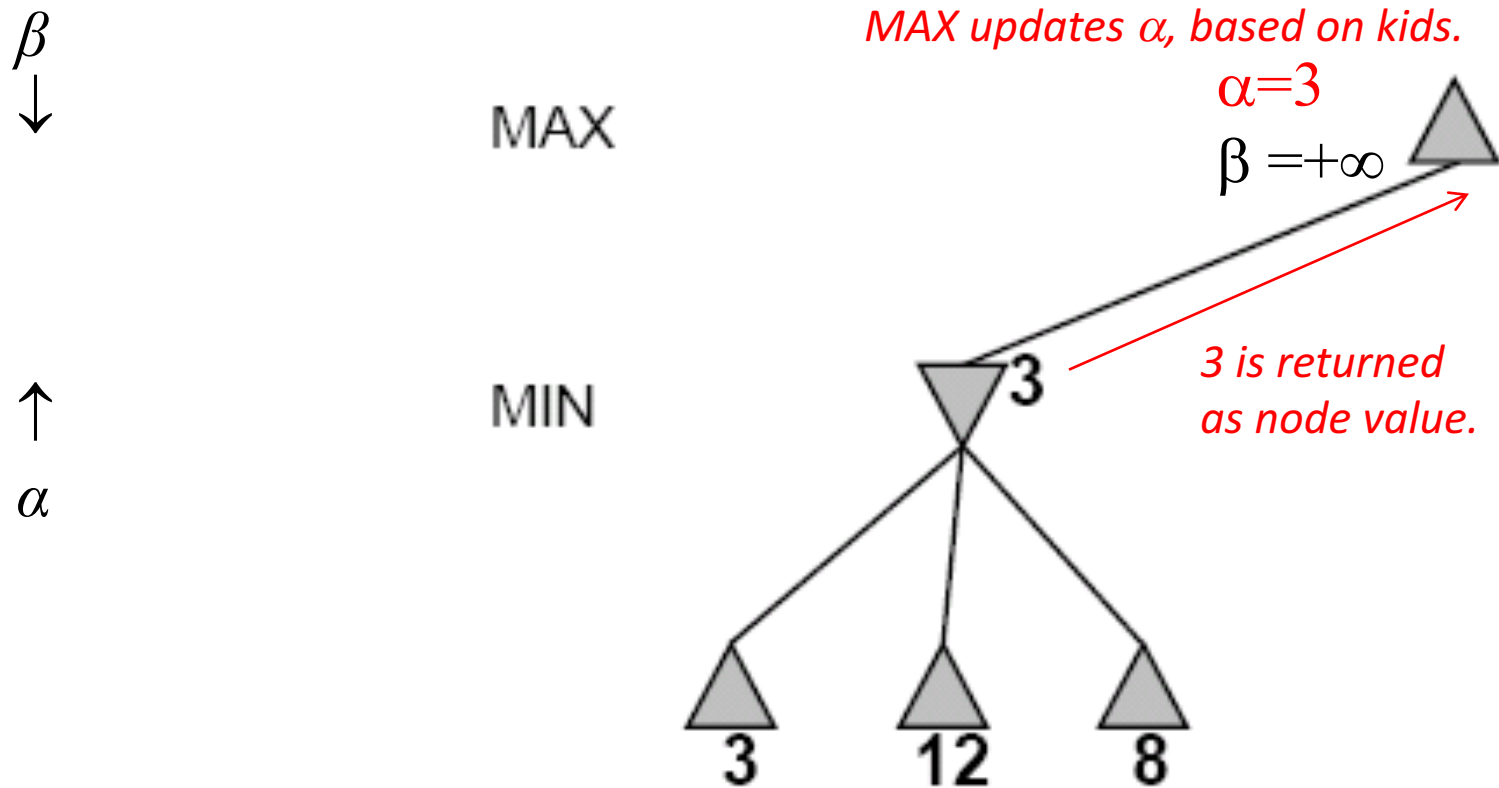
# Alpha-Beta Example (continued)



# Alpha-Beta Example (continued)

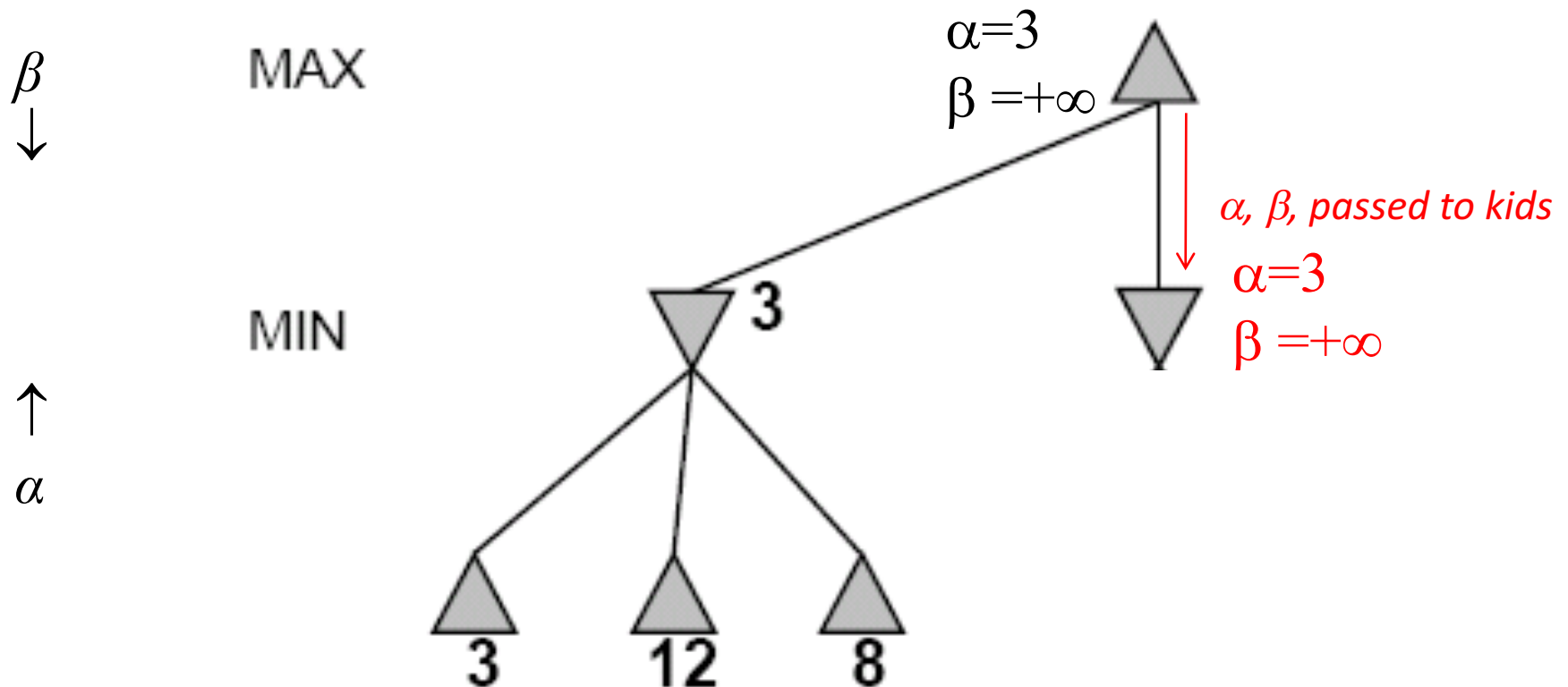


# Alpha-Beta Example (continued)

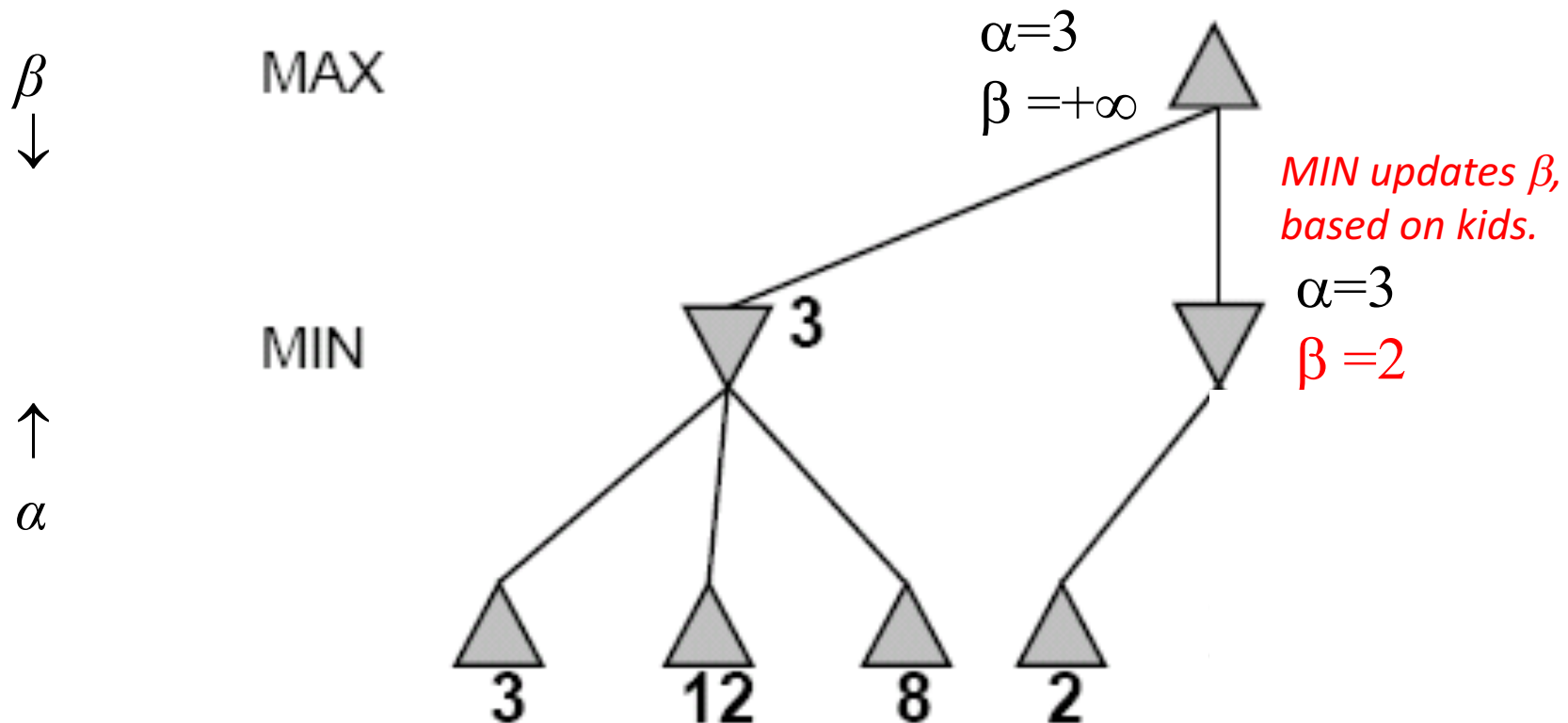




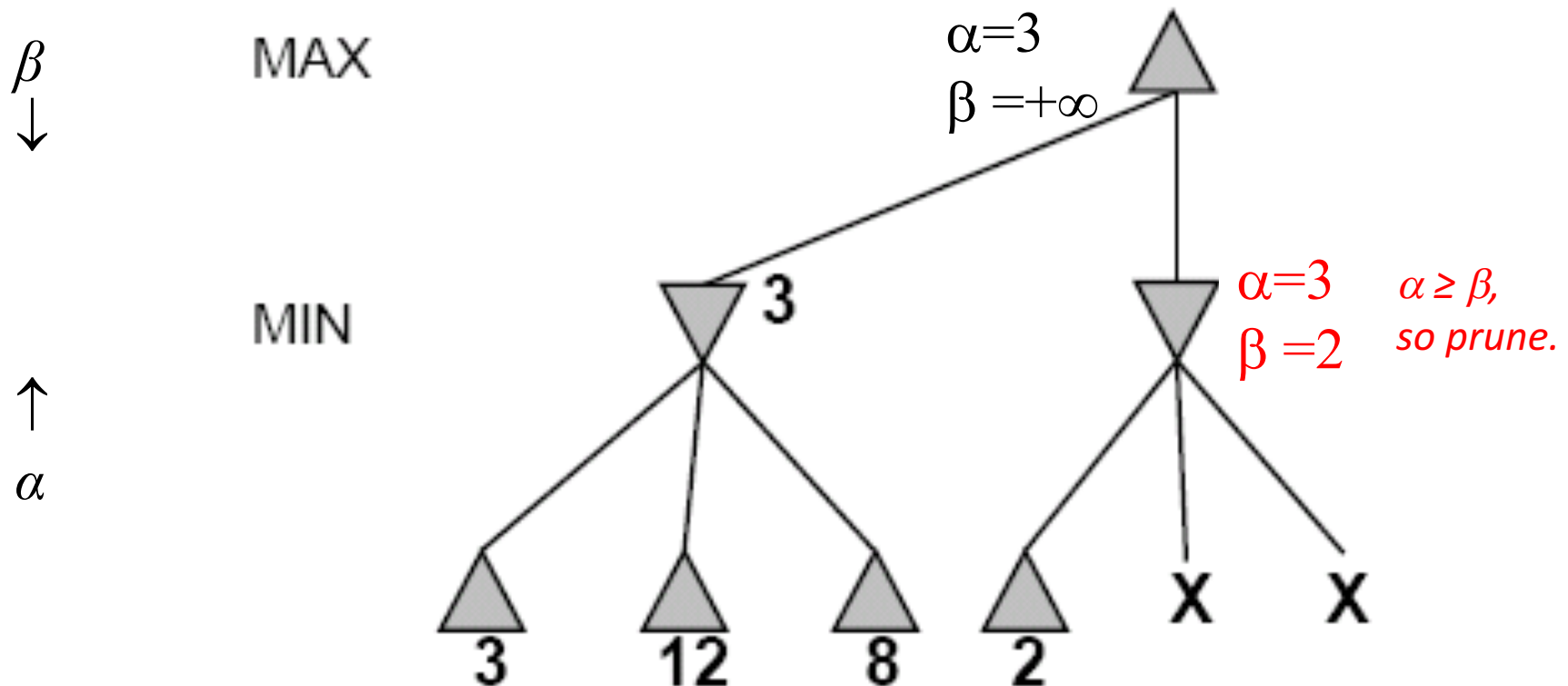
# Alpha-Beta Example (continued)



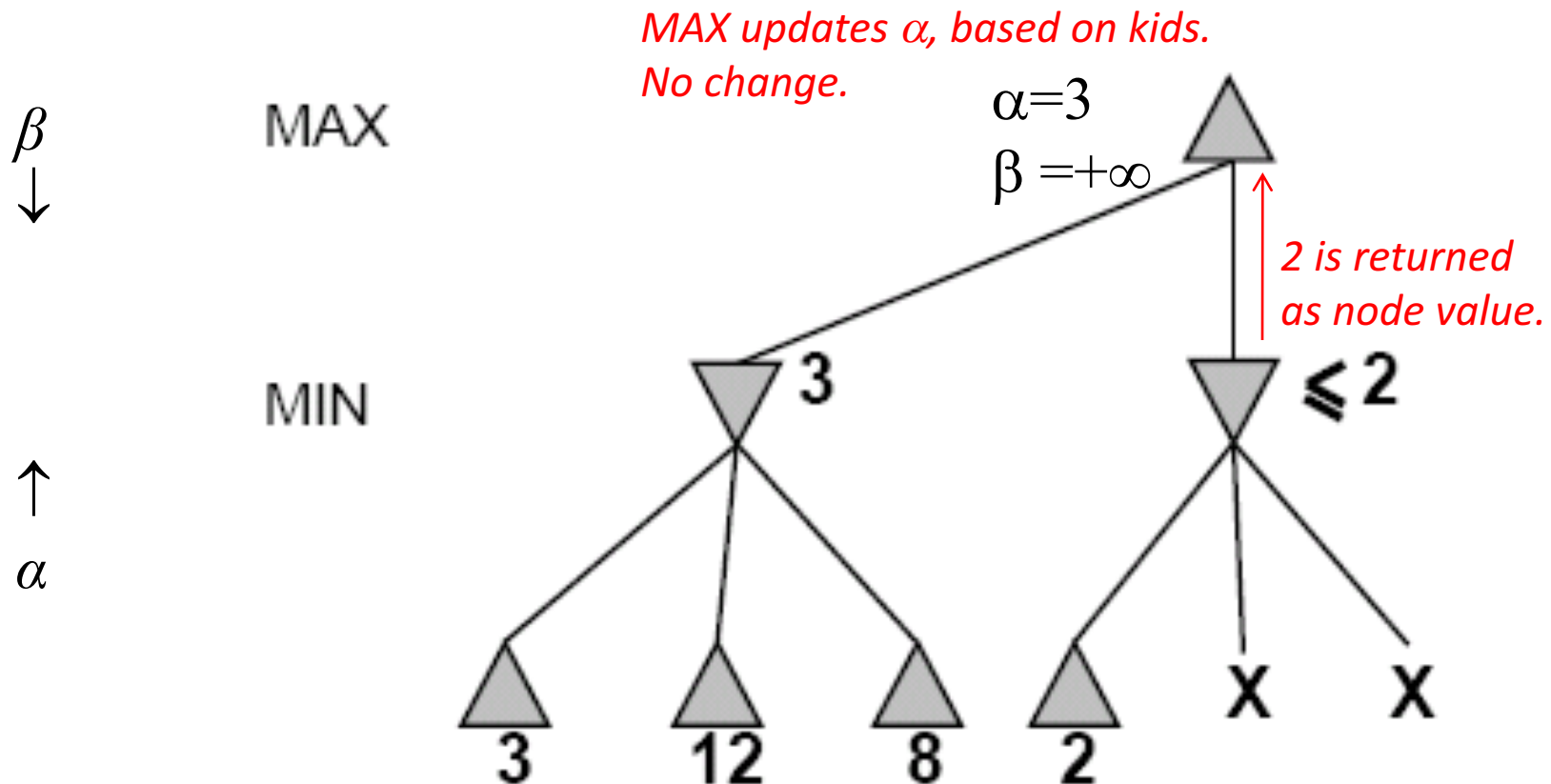
# Alpha-Beta Example (continued)



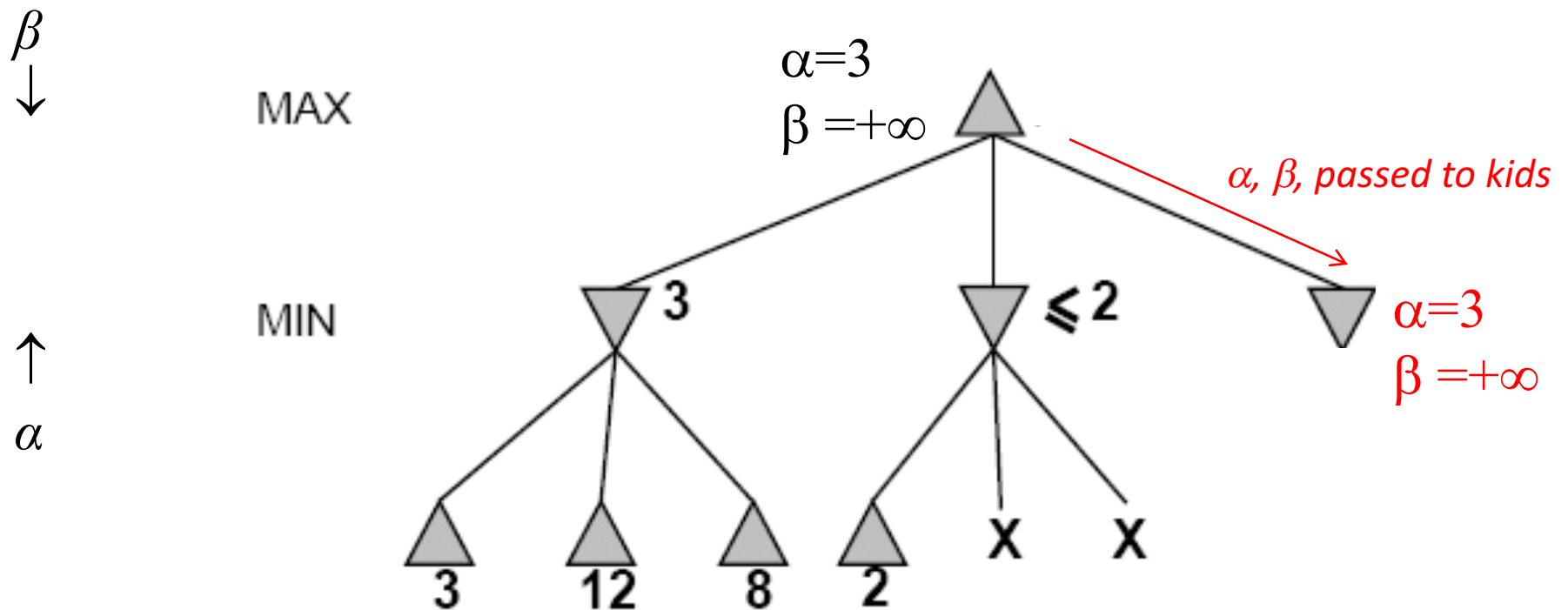
# Alpha-Beta Example (continued)



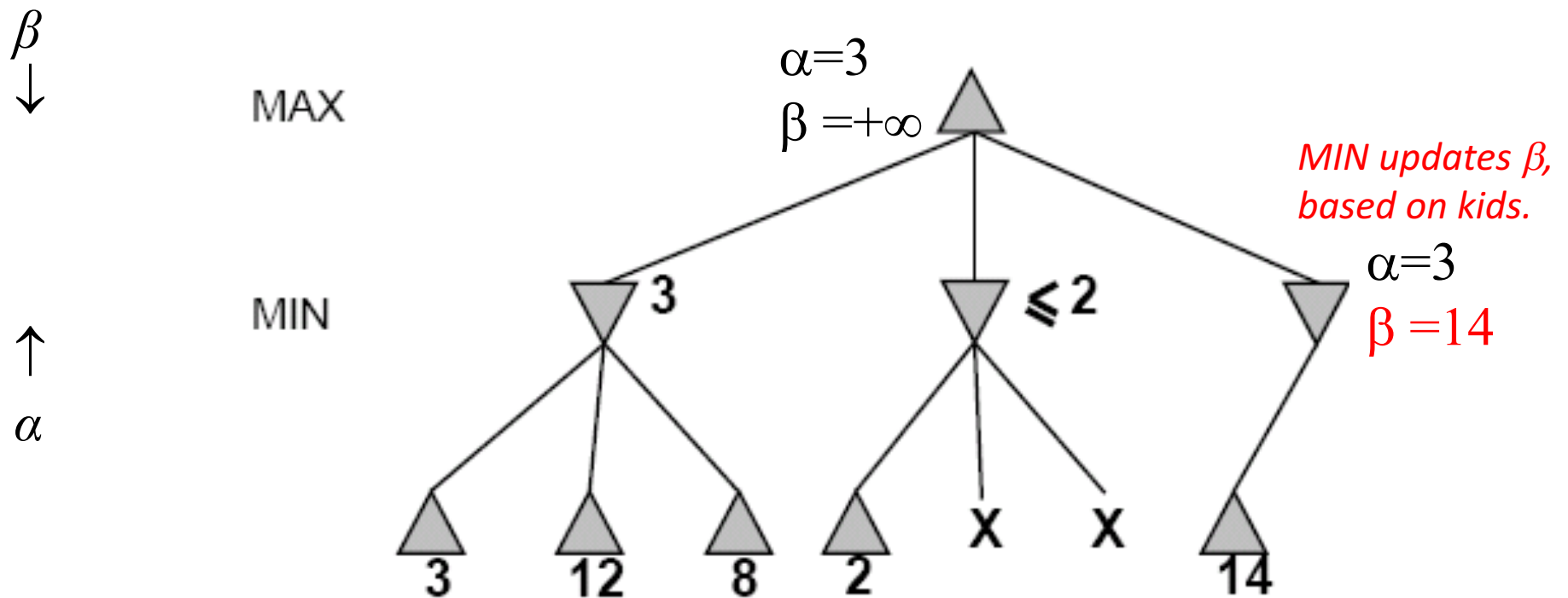
# Alpha-Beta Example (continued)



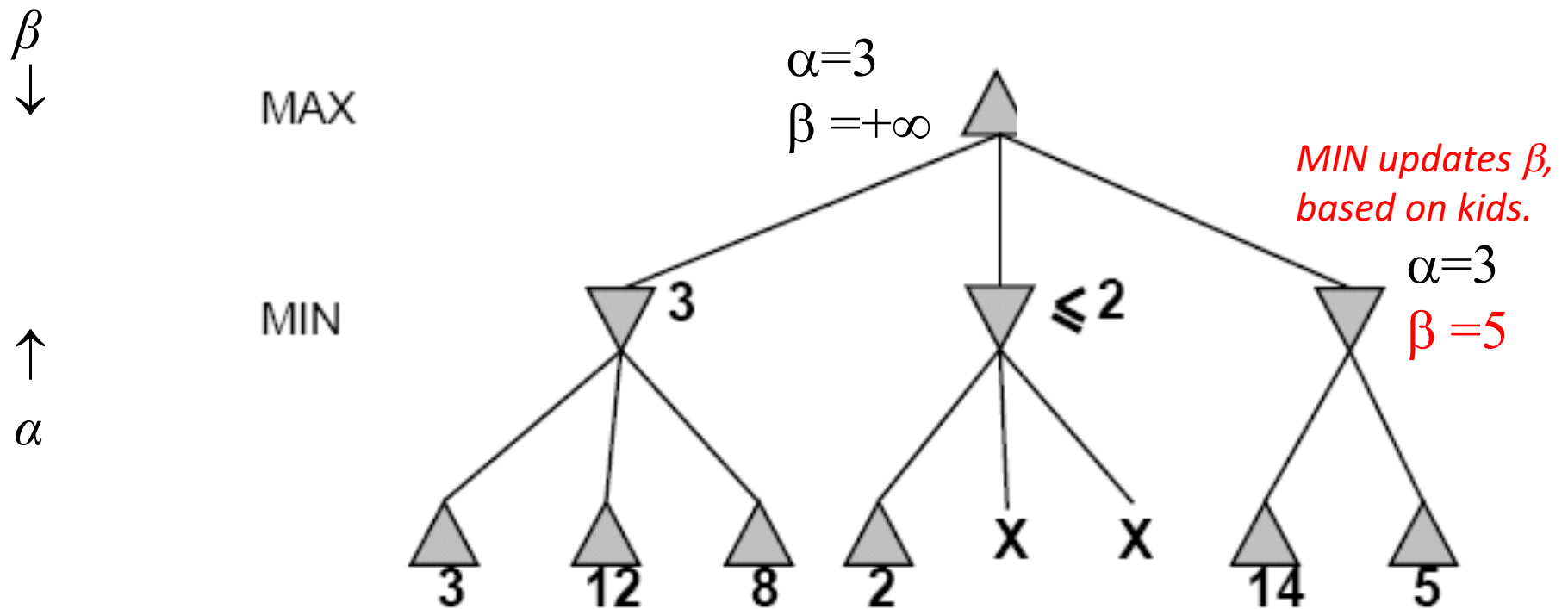
# Alpha-Beta Example (continued)



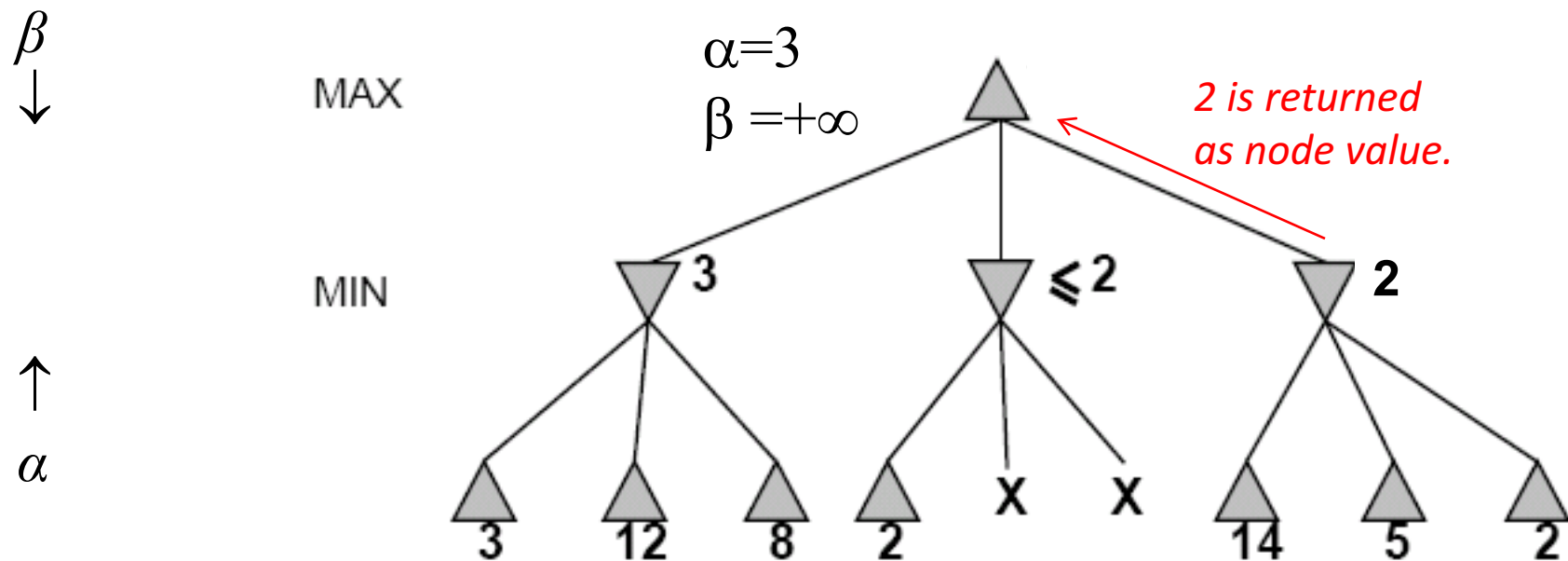
# Alpha-Beta Example (continued)



# Alpha-Beta Example (continued)



# Alpha-Beta Example (continued)





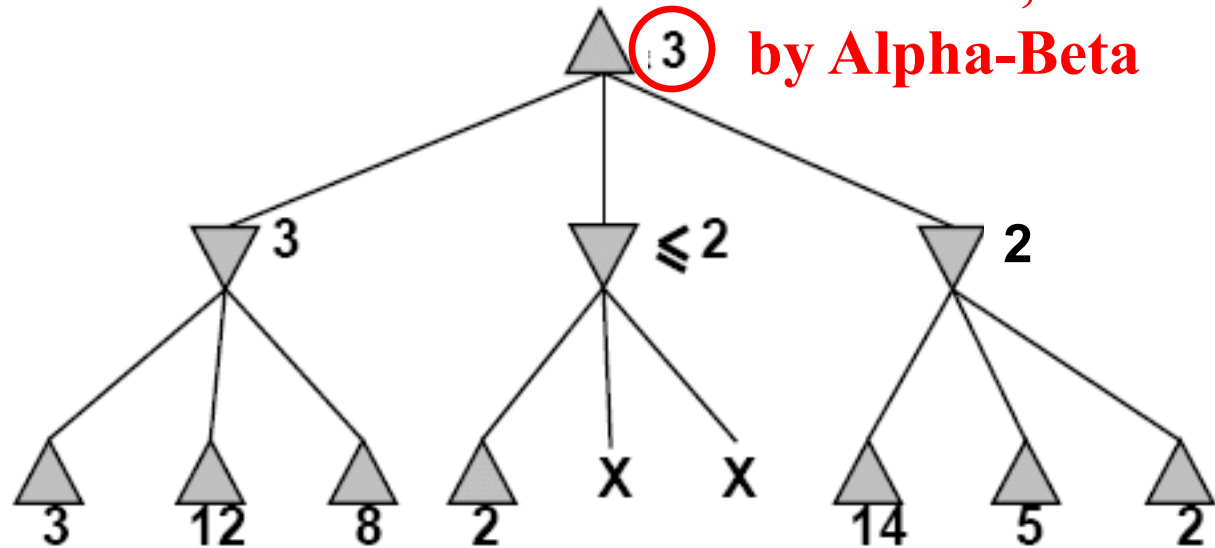
# Alpha-Beta Example (continued)

$\beta$   
↓

MAX

MIN

↑  
 $\alpha$



**Max now makes it's  
best move, as computed  
by Alpha-Beta**

# Alpha-Beta Algorithm Pseudocode

---

**function** ALPHA-BETA-SEARCH(*state*) **returns** *an action*

**inputs:** *state*, current state in game

$v \leftarrow \text{MAX-VALUE}(\text{state}, -\infty, +\infty)$

**return** an *action* in  $\text{ACTIONS}(\text{state})$  with value  $v$

---

**function** MAX-VALUE(*state*,  $\alpha$ ,  $\beta$ ) **returns** *a utility value*

**if**  $\text{TERMINAL-TEST}(\text{state})$  **then return**  $\text{UTILITY}(\text{state})$

$v \leftarrow -\infty$

**for**  $a$  in  $\text{ACTIONS}(\text{state})$  **do**

$v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{Result}(s, a), \alpha, \beta))$

**if**  $v \geq \beta$  **then return**  $v$

$\alpha \leftarrow \text{MAX}(\alpha, v)$

**return**  $v$

# Alpha-Beta Algorithm II

---

---

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 
```

# Effectiveness of Alpha-Beta Pruning

---

- **Guaranteed to compute same root value as Minimax**
- **Worst case:** no pruning, same as Minimax ( $O(b^d)$ )
- **Best case:** when each player's best move is the first option examined, examines only  $O(b^{d/2})$  nodes, allowing to search twice as deep!

# When best move is the first examined, examines only $O(b^{d/2})$ nodes....

---

- So: run Iterative Deepening search, sort by value returned on last iteration.
- So: expand captures first, then threats, then forward moves, etc.
- **$O(b^{d/2})$**  is the same as having a branching factor of  $\sqrt{b}$ ,
  - Since  $(\sqrt{b})^d = b^{d/2}$
  - e.g., in chess go from  $b \sim 35$  to  $b \sim 6$
- **For Deep Blue, alpha-beta pruning reduced the average branching factor from 35-40 to 6, as expected, doubling search depth**

# Real systems use a few more tricks

---

- **Expand the proposed solution a little farther**
  - Just to make sure there are no surprises
- **Learn better board evaluation functions**
  - E.g., for backgammon
- **Learn model of your opponent**
  - E.g., for poker
- **Do stochastic search**
  - E.g., for go

## Read AIMA Chapter 16.1-16.3

