# Bayesian Data Analysis

Passenger satisfaction with Logistic Regression

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Our dataset contains information about the passengers satisfaction during a trip of an airlines Company. This target variable is accompanied by other variables that describe the status of the trip. Our goal in this case study is to predict the satisfaction of a customer depending on predictor variables by means of a classical and Bayesian logistic regression.

**SATISFACTION**: It has 1 if satissfied. Otherwise, it is assigned to 0.

GOAL: study how trip status variables affect the probability of being satissfied, which is the dependent variable.



## DATA EXPLORATION & PREPROCESSING

Let us first comprehend our data through a brief exploratory data analysis.

```
# Read data
data = read.csv('passengers.csv', header = TRUE)
head(data)
```

##		satisfaction	Gender	Age	Class	Flight.Distance	Seat.comfort
##	1	satisfied	${\tt Female}$	53	Eco	519	4
##	2	${\tt dissatisfied}$	${\tt Female}$	25	Eco	1145	3
##	3	satisfied	${\tt Female}$	47	Eco	421	0
##	4	satisfied	Male	46	Business	3305	5

```
satisfied Female 53 Business
                                                    2097
## 6
        satisfied Male 59
                                    Eco
                                                    1826
     Departure.Arrival.time.convenient Food.and.drink Gate.location
## 1
## 2
                                       2
                                                       3
## 3
                                       1
                                                       1
                                                                       3
## 4
                                       5
                                                                       5
                                       3
## 5
                                                       3
                                                                       3
## 6
                                       4
     Inflight.wifi.service Inflight.entertainment Online.support
                           1
## 2
                           4
                                                   3
## 3
                          1
                                                   1
                                                                   1
## 4
                                                   4
## 5
                          2
## 6
                          3
                                                   3
     Ease.of.Online.booking On.board.service Leg.room.service Baggage.handling
                            3
                                              3
                                                                4
## 2
                            4
                                              1
                                                                                  4
                                                                4
## 3
                                              3
                            1
                                                                5
                                                                                  3
## 4
                           2
                                              2
                                                                2
                                                                                  2
## 5
                           5
                                              5
                           3
                                              1
## 6
                                                                5
     Checkin.service Cleanliness Online.boarding Departure.Delay.in.Minutes
## 1
                    4
                                 3
## 2
                    1
                                 3
                                                  4
                                                                              24
## 3
                    1
                                 5
                                                  1
                                                                               0
## 4
                    4
                                 2
                                                  3
                                                                               0
## 5
                    3
                                 5
                                                  4
                                                                               0
                    5
                                                  3
                                                                               0
##
     Arrival.Delay.in.Minutes
## 1
                             53
## 2
                              9
                              0
## 3
## 4
                              0
## 5
                              7
## 6
```

## summary(data)

```
satisfaction
                         Gender
                                             Age
                                                           Class
## Length:2000
                      Length: 2000
                                         Min. : 7.00
                                                        Length: 2000
                                                        Class : character
   Class :character
                      Class : character
                                         1st Qu.:28.00
   Mode :character
                      Mode :character
                                         Median :40.00
                                                        Mode :character
##
                                         Mean
                                               :39.52
##
                                         3rd Qu.:51.00
##
                                         Max.
                                               :80.00
## Flight.Distance Seat.comfort
                                   Departure.Arrival.time.convenient
## Min. : 51
                   Min.
                          :0.000
                                   Min.
                                         :0.000
## 1st Qu.:1293
                   1st Qu.:2.000
                                   1st Qu.:2.000
## Median :1895
                   Median :3.000
                                   Median :3.000
                   Mean :2.837
## Mean :1937
                                   Mean
                                        :2.994
   3rd Qu.:2537
                   3rd Qu.:4.000
                                   3rd Qu.:4.000
## Max. :6099
                   Max.
                          :5.000
                                   Max. :5.000
```

```
Food.and.drink
                    Gate.location
                                     Inflight.wifi.service Inflight.entertainment
                                             :0.000
##
           :0.000
    Min.
                     Min.
                            :1.000
                                     Min.
                                                             Min.
                                                                    :0.000
    1st Qu.:2.000
                     1st Qu.:2.000
                                      1st Qu.:2.000
                                                             1st Qu.:2.000
   Median :3.000
                     Median :3.000
                                     Median :3.000
                                                             Median :4.000
##
##
    Mean
           :2.837
                     Mean
                            :3.033
                                     Mean
                                             :3.212
                                                             Mean
                                                                    :3.373
##
    3rd Qu.:4.000
                     3rd Qu.:4.000
                                      3rd Qu.:4.000
                                                             3rd Qu.:4.000
##
    Max.
           :5.000
                     Max.
                            :5.000
                                     Max.
                                             :5.000
                                                             Max.
                                                                    :5.000
##
    Online.support
                     Ease.of.Online.booking On.board.service Leg.room.service
##
    Min.
           :1.000
                     Min.
                            :1.000
                                             Min.
                                                    :1.00
                                                               Min.
                                                                      :0.000
##
    1st Qu.:3.000
                     1st Qu.:2.000
                                             1st Qu.:3.00
                                                               1st Qu.:2.000
    Median :4.000
                     Median :4.000
                                             Median:4.00
                                                               Median :4.000
##
    Mean
           :3.513
                     Mean
                            :3.494
                                             Mean
                                                    :3.49
                                                               Mean
                                                                      :3.496
##
    3rd Qu.:5.000
                     3rd Qu.:5.000
                                             3rd Qu.:5.00
                                                               3rd Qu.:5.000
                                                    :5.00
##
   Max.
           :5.000
                     Max.
                            :5.000
                                             Max.
                                                               Max.
                                                                      :5.000
    Baggage.handling Checkin.service
                                       Cleanliness
                                                       Online.boarding
##
    Min.
           :1.000
                      Min.
                             :1.000
                                       Min.
                                              :1.000
                                                       Min.
                                                               :1.000
##
    1st Qu.:3.000
                      1st Qu.:3.000
                                       1st Qu.:3.000
                                                       1st Qu.:2.000
    Median :4.000
                      Median :3.000
                                       Median :4.000
                                                       Median :4.000
##
   Mean
           :3.717
                      Mean
                             :3.341
                                       Mean
                                              :3.721
                                                       Mean
                                                               :3.361
##
    3rd Qu.:5.000
                      3rd Qu.:4.000
                                       3rd Qu.:5.000
                                                       3rd Qu.:4.000
##
    Max.
           :5.000
                      Max.
                             :5.000
                                       Max.
                                              :5.000
                                                       Max.
                                                               :5.000
    Departure.Delay.in.Minutes Arrival.Delay.in.Minutes
##
    Min.
           : 0.00
                                Min.
                                        : 0.00
    1st Qu.:
##
             0.00
                                1st Qu.:
                                          0.00
##
   Median: 0.00
                                Median: 0.00
   Mean
           : 15.83
                                Mean
                                       : 16.05
##
    3rd Qu.: 12.25
                                3rd Qu.: 13.00
                                        :470.00
    Max.
           :435.00
                                Max.
all_columns <- names(data);all_columns</pre>
##
    [1] "satisfaction"
                                              "Gender"
##
    [3] "Age"
                                              "Class"
    [5] "Flight.Distance"
                                              "Seat.comfort"
    [7] "Departure.Arrival.time.convenient"
                                              "Food.and.drink"
##
   [9] "Gate.location"
                                              "Inflight.wifi.service"
## [11] "Inflight.entertainment"
                                              "Online.support"
## [13] "Ease.of.Online.booking"
                                              "On.board.service"
## [15] "Leg.room.service"
                                              "Baggage.handling"
## [17] "Checkin.service"
                                              "Cleanliness"
                                              "Departure.Delay.in.Minutes"
## [19] "Online.boarding"
## [21] "Arrival.Delay.in.Minutes"
```

# ## [1] 2000 21

dim(data)

It seems that our data has initially 2000 observations and 21 variables. Let us explain our variables:

- satisfaction: Factor variable indicating whether the customer was satisfied or unsatisfied.
- **Gender**: Gender of the customer.
- **Age**: Age of the customer.

- Class: Factor variable indicating the type of class the customer was traveling in. (Eco,Eco Plus,Business)
- Flight.distance: Distance of the flight (in kms)
- Seat.comfort: Satisfaction level of seat comfort
- Departure. Arrival.time.convenient: Satisfaction level of departure/arrival time convenience.
- Food.and.drink: Satisfaction level of food and drink.
- Gate.location: Satisfaction level of gate location.
- Inflight.wifi.service: Satisfaction level of the inflight WiFi service.
- Inflight.entertainment: Satisfaction level of inflight entertainment.
- Online.support: Satisfaction level of online support.
- Ease of Online booking: Satisfaction level of online booking.
- On.board.service: Satisfaction level of on-board service.
- Leg.room.service: Satisfaction level of leg room service.
- Baggage.handling: Satisfaction level of baggage handling.
- Checkin.service: Satisfaction level of the checkin service.
- Cleanliness: Satisfaction level of cleanliness.
- Online.boarding: Satisfaction level of online boarding.
- Arrival.Delay.in.Minutes: Arrival delay in minutes.

Features corresponding to satisfaction level are in range (0-5).

```
# Check for NA's in the entire dataset
null= any(is.na(data)); null
```

```
## [1] FALSE
```

There are no missing values so we can proceed with our analysis.

## 'Class' are not numeric and were ignored

```
#Let us plot the correlation
# Load necessary libraries
library(GGally)

## Warning: package 'GGally' was built under R version 4.0.5

## Loading required package: ggplot2

## Warning: package 'ggplot2' was built under R version 4.0.5

## Registered S3 method overwritten by 'GGally':
## method from
## +.gg ggplot2

library(ggplot2)
ggcorr(data, label = T)
## Warning in ggcorr(data, label = T): data in column(s) 'satisfaction', 'Gender',
```

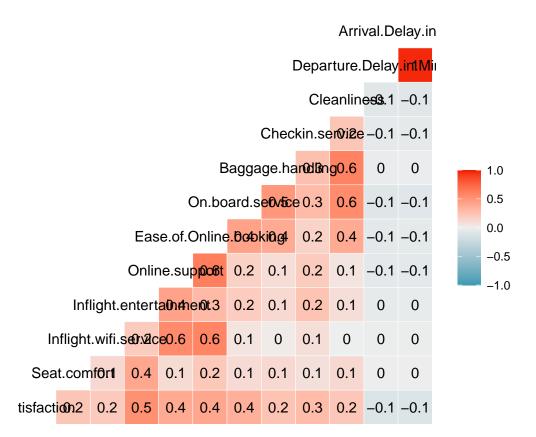


As we can see there are several variables that may not add value when predicting the binary variable satisfaction, so we will remove them.

```
# Remove variables that do not add much value
data <- subset(data, select = -c(Age, Gender, Food.and.drink, Leg.room.service, Gate.location,Class, Fl
# Encode 'satisfaction' column as 0 and 1
data$satisfaction <- ifelse(data$satisfaction == "satisfied", 1, 0)</pre>
```

This is the new correlation with the remaining variables we will use in our task.

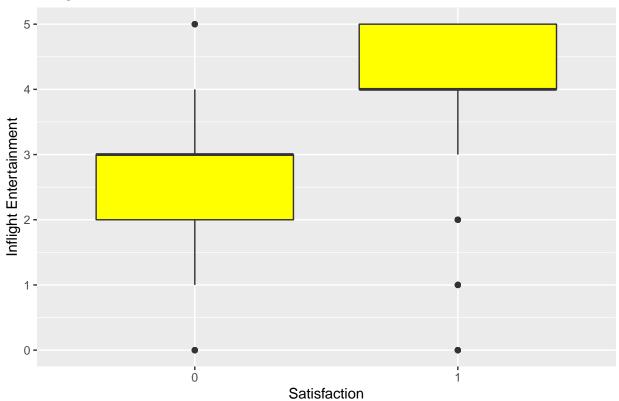
```
# Load necessary libraries
library(GGally)
library(ggplot2)
ggcorr(data, label = T)
```



Now, let us visualize some boxplots to check how the different predictors behave with the satisfaction class. For instance, satisfied customers tend to give a higher score for the entertainment provided during the flight.

```
# Boxplot for Inflight.entertainment vs Satisfaction
ggplot(data, aes(x = factor(satisfaction), y = Inflight.entertainment)) +
   geom_boxplot(fill="yellow") +
   labs(title = "Inflight Entertainment vs Satisfaction", x = "Satisfaction", y = "Inflight Entertainment")
```

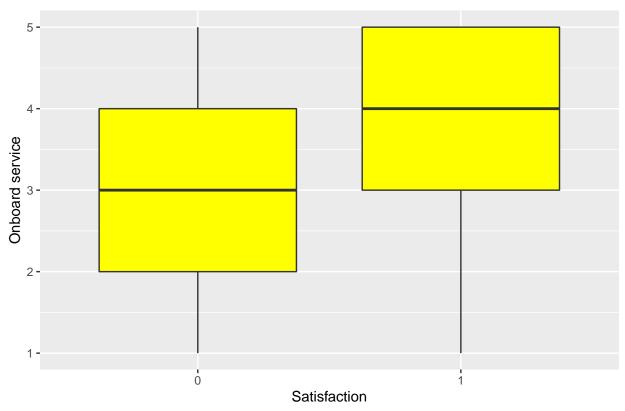




Regarding the onboard service, the median for satisfied customers is 4 while for unsatisfied ones, is 3.

```
# Boxplot for On.board.service vs Satisfaction
ggplot(data, aes(x = factor(satisfaction), y = On.board.service)) +
  geom_boxplot(fill="yellow") +
  labs(title = "Onboard service vs Satisfaction", x = "Satisfaction", y = "Onboard service")
```

## Onboard service vs Satisfaction



## CLASSICAL LOGISTIC REGRESSOR

First of all, we will try a classical logistic regression which shows the relationship between some trip status variables and the dependent one, satisfaction.

For that, we begin firstly with the complete model to see which are the variables that are significant when predicting the satisfaction.

```
classic <- glm (satisfaction ~ .,data = data, family = binomial)
summary(classic)</pre>
```

```
##
## Call:
## glm(formula = satisfaction ~ ., family = binomial, data = data)
##
## Deviance Residuals:
##
       Min
                 1Q
                      Median
                                    3Q
                                            Max
  -2.7281
           -0.7295
                      0.3173
                               0.6271
                                         3.2558
##
##
## Coefficients:
##
                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                              -6.402865
                                           0.354879 -18.042 < 2e-16 ***
## Seat.comfort
                                           0.049908 0.194
                               0.009668
                                                              0.8464
```

```
## Inflight.wifi.service
                             -0.073970
                                         0.061749 -1.198
                                                            0.2309
## Inflight.entertainment
                              0.844564
                                         0.057717 14.633 < 2e-16 ***
## Online.support
                              0.165342
                                         0.064553
                                                   2.561
                                                            0.0104 *
## Ease.of.Online.booking
                                         0.075245
                                                    5.413 6.19e-08 ***
                              0.407317
## On.board.service
                              0.367125
                                         0.058751
                                                    6.249 4.14e-10 ***
## Baggage.handling
                              0.056835
                                         0.065530
                                                   0.867
                                                            0.3858
## Checkin.service
                              0.247932
                                         0.049501
                                                    5.009 5.48e-07 ***
## Cleanliness
                             -0.044159
                                         0.069477
                                                   -0.636
                                                            0.5250
## Departure.Delay.in.Minutes -0.004323
                                         0.005881
                                                   -0.735
                                                            0.4623
## Arrival.Delay.in.Minutes
                             -0.002035
                                         0.005747 - 0.354
                                                            0.7232
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 2747.4 on 1999
                                      degrees of freedom
## Residual deviance: 1807.5 on 1988 degrees of freedom
## AIC: 1831.5
## Number of Fisher Scoring iterations: 5
```

As we can see from the summary, the variables that have lowest p-values are those that add info to the model:

- Inflight.entertainment
- · Ease.of.Online.booking
- On.board.service
- Checkin.service

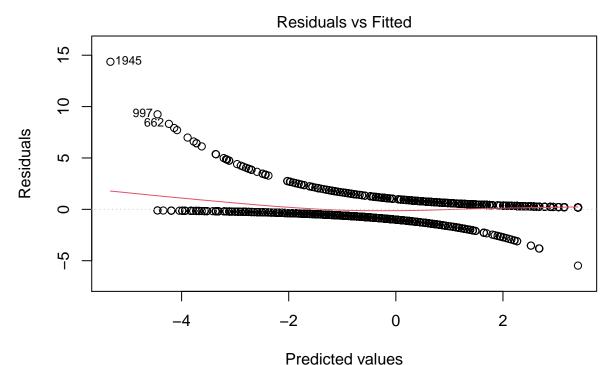
Therefore, these are the variables that we will use as predictors in our logistic regression case study.

```
# Fit logistic regression model
classic <- glm(satisfaction ~ Inflight.entertainment + On.board.service + Ease.of.Online.booking</pre>
               + Checkin.service, data = data, family = binomial)
# Summary of the model
summary(classic)
##
## Call:
## glm(formula = satisfaction ~ Inflight.entertainment + On.board.service +
       Ease.of.Online.booking + Checkin.service, family = binomial,
##
##
       data = data)
##
## Deviance Residuals:
                      Median
                                   ЗQ
##
                 10
                                           Max
                                        3.2662
## -2.6205 -0.7457
                      0.3297
                               0.6528
##
## Coefficients:
                          Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                          -6.41345
                                      0.30214 -21.226 < 2e-16 ***
## Inflight.entertainment 0.87843
                                      0.05085 17.276 < 2e-16 ***
## On.board.service
                                      0.05025
                                               7.245 4.33e-13 ***
                           0.36405
```

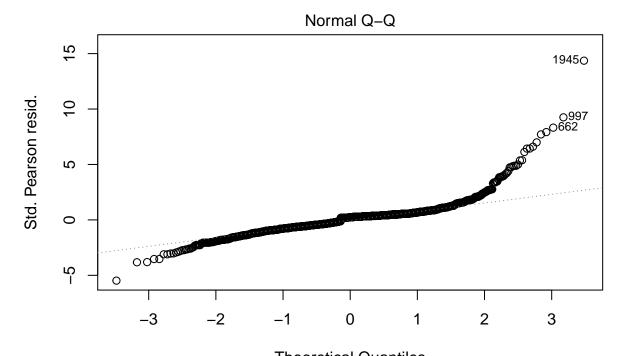
```
## Ease.of.Online.booking 0.46237
                                      0.04977
                                                9.290 < 2e-16 ***
## Checkin.service
                           0.25797
                                      0.04800
                                                5.374 7.69e-08 ***
##
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
##
   (Dispersion parameter for binomial family taken to be 1)
##
##
##
      Null deviance: 2747.4 on 1999
                                       degrees of freedom
## Residual deviance: 1832.9
                             on 1995
                                       degrees of freedom
  AIC: 1842.9
##
## Number of Fisher Scoring iterations: 5
```

From the summary, Beta1 is 0.87 which menas that for every unit the Inflight.entertainment increases, the estimated probability of being satisfied increases 0.87. Also, Beta3 is 0.46237 and that means that when the Ease of online booking increases 1 unit, it makes probability of being satisfied increase by 0.46237.

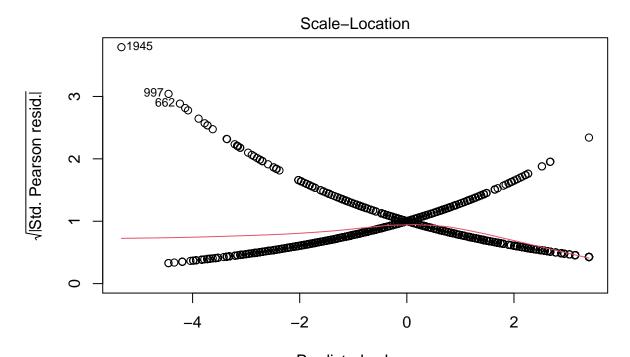
## plot(classic)



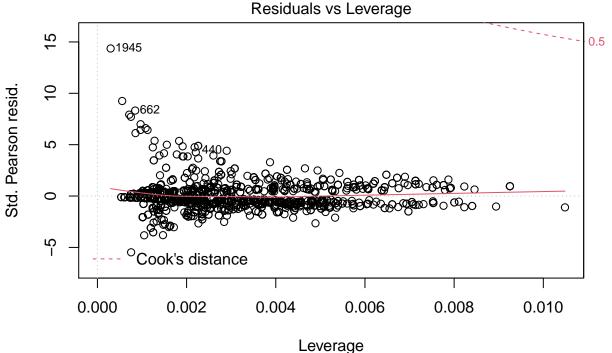
glm(satisfaction ~ Inflight.entertainment + On.board.service + Ease.of.Onli ...



Theoretical Quantiles glm(satisfaction ~ Inflight.entertainment + On.board.service + Ease.of.Onli ...



Predicted values glm(satisfaction ~ Inflight.entertainment + On.board.service + Ease.of.Onli ...



glm(satisfaction ~ Inflight.entertainment + On.board.service + Ease.of.Onli ...

#### Some remarks:

- In the Residuals vs Fitted plot, as the residuals are centered around the horizontal dashed line at 0, it indicates that the model does not systematically overestimate or underestimate the response.
- The normal Q-Q plot helps to assess the normality of residuals, however as we are in a logistic regression problem we should focus more on the overall fit and other diagnostics like residuals vs fitted plots.

Let us now introduce some new data to the model and predict the probability of being satisfied given the following scores:

```
# New data for prediction
new_data <- data.frame(
    Inflight.entertainment = 5,
    On.board.service = 4,
    Ease.of.Online.booking = 5,
    Checkin.service = 3
)

# Predict using the model
predicted_values <- predict(classic, newdata = new_data, type = "response")

# Output the predicted values
print(predicted_values)</pre>
```

## 0.9255775

A probability of 0.9255774 of being satisfied has been obtained. Below, we will test also this new data with bayesian approaches such as MCMC and R2OPENBUGS.

## BAYESIAN APPROACH

## MONTE CARLO MARKOV CHAIN

Now, let us proceed but instead using a bayesian approach and the MCMC (Monte Carlo Markov Chain) to sample from the posteriors as we do not have a way to compute directly the posterior.

First we begin also with the full model to search for the important predictors.

```
library(MCMCpack)
## Warning: package 'MCMCpack' was built under R version 4.0.5
## Loading required package: coda
## Loading required package: MASS
## ##
## ## Markov Chain Monte Carlo Package (MCMCpack)
## ## Copyright (C) 2003-2024 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
## ##
## ## Support provided by the U.S. National Science Foundation
## ## (Grants SES-0350646 and SES-0350613)
## ##
logit = MCMClogit(satisfaction ~ ., data = data, thin=10, burnin=1000, mcmc=21000)
summary(logit)
##
## Iterations = 1001:21991
## Thinning interval = 10
## Number of chains = 1
```

```
## Sample size per chain = 2100
##
## 1. Empirical mean and standard deviation for each variable,
##
      plus standard error of the mean:
##
                                             SD Naive SE Time-series SE
                                  Mean
                             -6.455596 0.376268 0.0082108
## (Intercept)
                                                               0.0199088
## Seat.comfort
                              0.009002 0.050864 0.0011099
                                                               0.0024956
## Inflight.wifi.service
                             -0.075140 0.059555 0.0012996
                                                               0.0029505
## Inflight.entertainment
                             0.856372 0.057966 0.0012649
                                                               0.0028328
## Online.support
                              0.162570 0.064967 0.0014177
                                                               0.0034620
```

```
## Ease.of.Online.booking
                               0.411779 0.077069 0.0016818
                                                                 0.0040361
                               0.370942 0.059816 0.0013053
## On.board.service
                                                                 0.0031389
## Baggage.handling
                               0.055443 0.069938 0.0015262
                                                                 0.0037839
## Checkin.service
                               0.249091 0.048673 0.0010621
                                                                 0.0025293
## Cleanliness
                              -0.043639 0.071394 0.0015579
                                                                 0.0037847
## Departure.Delay.in.Minutes -0.004090 0.005853 0.0001277
                                                                 0.0002876
## Arrival.Delay.in.Minutes
                              -0.002281 0.005701 0.0001244
                                                                 0.0002808
## 2. Quantiles for each variable:
##
##
                                  2.5%
                                             25%
                                                        50%
                                                                   75%
                                                                           97.5%
                              -7.15864 -6.709179 -6.460007 -6.1972916 -5.686710
## (Intercept)
## Seat.comfort
                              -0.08635 -0.029819 0.008797
                                                             0.0453673
                                                                        0.109170
## Inflight.wifi.service
                                                                        0.037455
                              -0.20854 -0.114324 -0.074672 -0.0339491
## Inflight.entertainment
                               0.74372 0.815504
                                                  0.856495
                                                             0.8952359
                                                                        0.965617
## Online.support
                               0.03481
                                        0.118892
                                                  0.163165
                                                             0.2043972
                                                                        0.292724
## Ease.of.Online.booking
                               0.25895
                                        0.356606
                                                  0.410609
                                                             0.4656417
                                                                        0.560862
## On.board.service
                               0.25384
                                        0.329749
                                                  0.373114
                                                             0.4146641
                                                                        0.481782
## Baggage.handling
                              -0.07650 0.005668
                                                  0.057177
                                                             0.1007723
                                                                        0.195555
## Checkin.service
                               0.15822 0.215198 0.250204
                                                             0.2824293
                                                                        0.342922
## Cleanliness
                              -0.17305 -0.091713 -0.046598
                                                             0.0046239
                                                                        0.101704
## Departure.Delay.in.Minutes -0.01571 -0.008015 -0.004107 -0.0005644
                                                                        0.008248
## Arrival.Delay.in.Minutes
                              -0.01390 -0.005856 -0.002098 0.0014895
                                                                        0.008441
```

From this sumamry we can verify that the variables used before in the classical model are the significant ones as they do not contain 0 in their quantiles. The rest of variables do contain 0 in ther ci's and therefore they are not statistically significant when predicting the satisfaction variable.

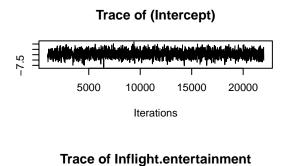
Let us implement now the model for the chosen predictors and see its convergence:

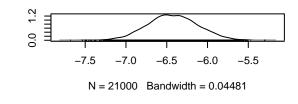
```
## Iterations = 1001:22000
## Thinning interval = 1
## Number of chains = 1
## Sample size per chain = 21000
##
## 1. Empirical mean and standard deviation for each variable,
##
      plus standard error of the mean:
##
##
                                        SD Naive SE Time-series SE
                          -6.4277 0.30939 0.0021350
## (Intercept)
                                                           0.008961
## Inflight.entertainment 0.8814 0.05121 0.0003534
                                                           0.001455
## On.board.service
                           0.3657 0.05132 0.0003542
                                                           0.001438
## Ease.of.Online.booking
                           0.4627 0.04861 0.0003354
                                                           0.001337
## Checkin.service
                           0.2569 0.04846 0.0003344
                                                           0.001344
##
## 2. Quantiles for each variable:
##
                              2.5%
##
                                       25%
                                               50%
                                                       75%
                                                             97.5%
```

```
## (Intercept)
                           -7.0315 -6.6357 -6.4310 -6.2198 -5.8136
## Inflight.entertainment 0.7824
                                    0.8458
                                            0.8814
                                                    0.9150
                                                            0.9842
## On.board.service
                           0.2664
                                            0.3654
                                                    0.3997
                                                             0.4693
                                    0.3315
## Ease.of.Online.booking
                           0.3669
                                            0.4619
                                                    0.4952
                                                             0.5599
                                    0.4305
                           0.1603
## Checkin.service
                                    0.2245
                                            0.2569
                                                    0.2886
                                                             0.3524
```

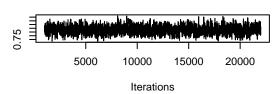
Let us check model convergence and see the trace plots and posterior densities for the intercept and betas:

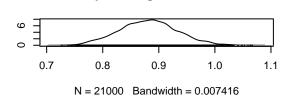
## plot(logit)



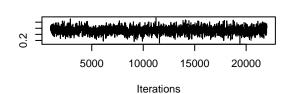


**Density of (Intercept)** 

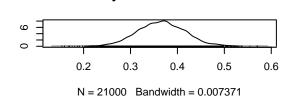




**Density of Inflight.entertainment** 

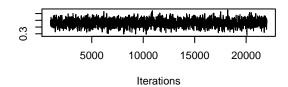


Trace of On.board.service

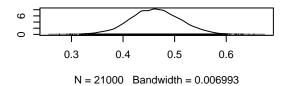


**Density of On.board.service** 

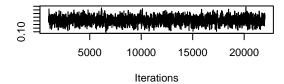
## Trace of Ease.of.Online.booking



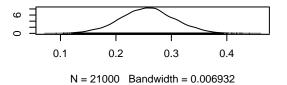
## Density of Ease.of.Online.booking



## **Trace of Checkin.service**



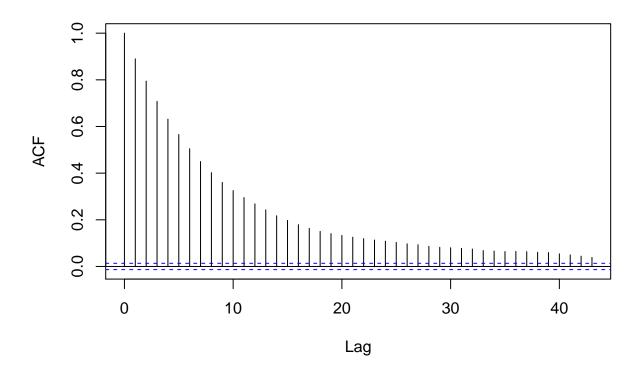
## **Density of Checkin.service**



Let us also look for the autocorrelation:

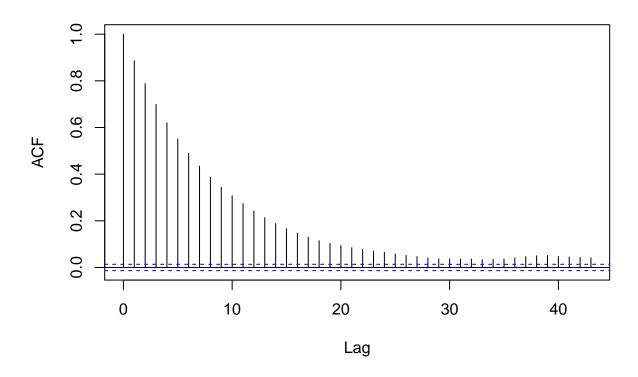
acf(logit[,1])

# Series logit[, 1]



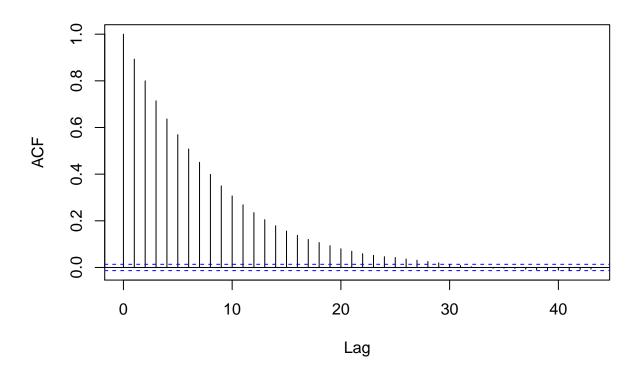
acf(logit[,2])

Series logit[, 2]



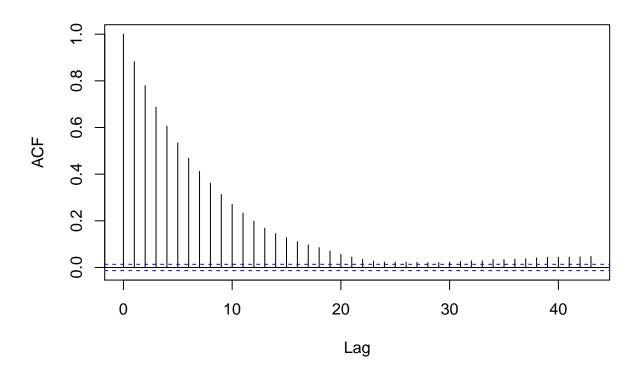
acf(logit[,3])

Series logit[, 3]



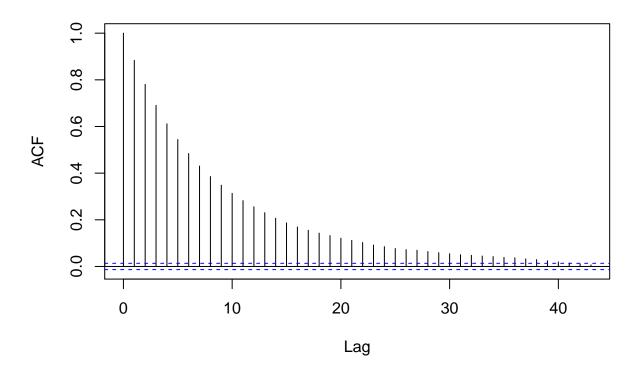
acf(logit[,4])

Series logit[, 4]



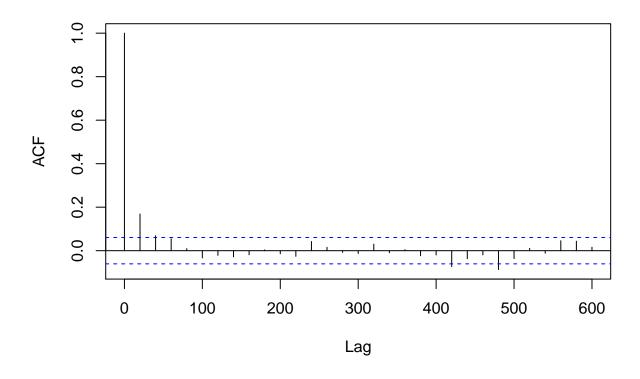
acf(logit[,5])

# Series logit[, 5]



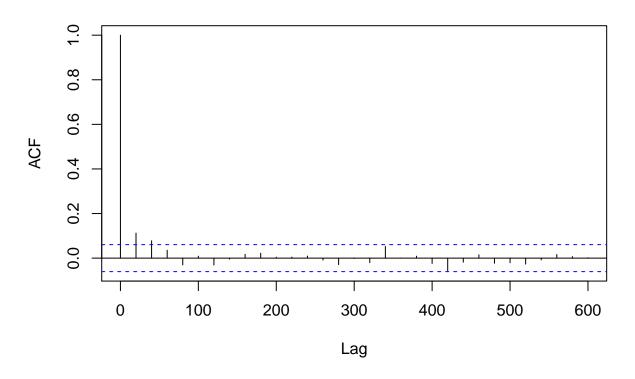
As it does not converge perfectly, we use thining of 20 iterations to achieve a better convergence and remove the autocorrelation

Series logit[, 1]



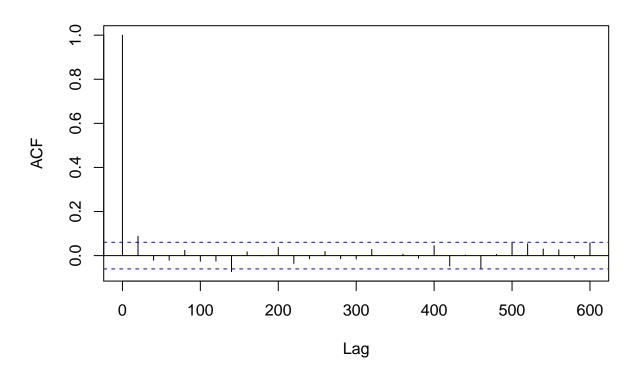
acf(logit[,2])

Series logit[, 2]



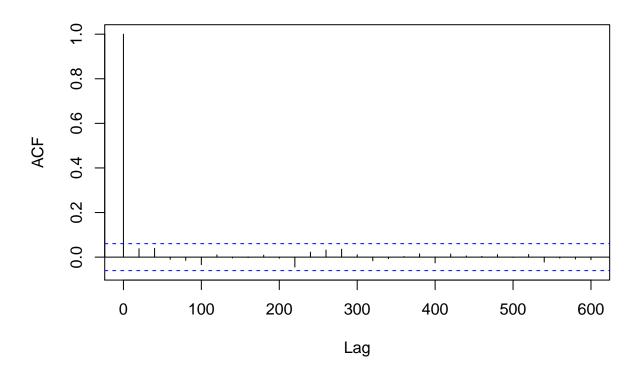
acf(logit[,3])

Series logit[, 3]



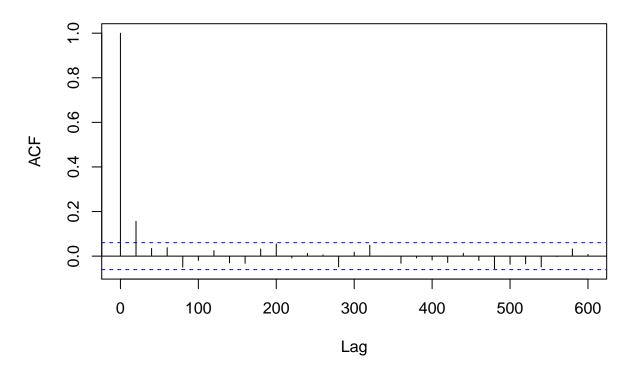
acf(logit[,4])

Series logit[, 4]



acf(logit[,5])

# Series logit[, 5]

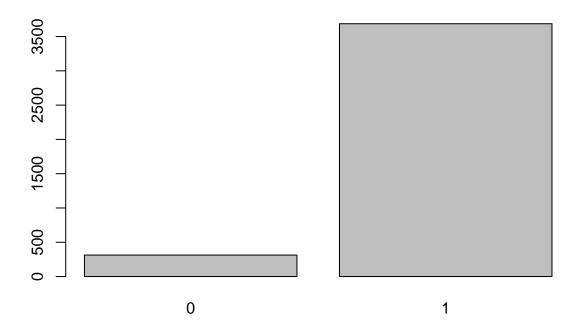


Now, we can play with the posterior distribution for some unseen data and check how the predictive distribution would be:

```
library(boot)
set.seed(1)

Inflight.entertainment = 5
On.board.service=4
Ease.of.Online.booking=5
Checkin.service=3

satisfaction.prob=inv.logit(logit[,1]+Inflight.entertainment*logit[,2] + On.board.service*logit[,3] + E
pred=rbinom(4000,size=1,prob=satisfaction.prob)
barplot(table(pred))
```



## table(pred)

```
## pred
## 0 1
## 313 3687
```

We can see that for the new unseen data we gave to the model, the predictive sample composed by 4000 passengers states that 3687 passengers will be satisfied and 313 ones, disatissfied.

Trace plots and posterior density can be obtained with:

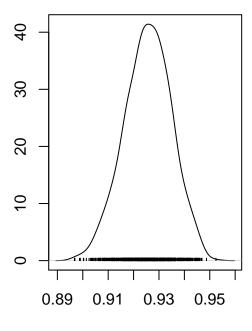
## plot(satisfaction.prob)

## Trace of var1

# 0.90 0.94

5000

# Density of var1



N = 1050 Bandwidth = 0.002433

From this plotted posterior we can see that the probability of being satisfied is high, as it is centered around 0.93, which makes sense when cheking the data sequence we inputed to the model (5,4,5,3) which has generally high rates for these trip status variables.

## **R2OPENBUGS**

In this part of the case study, we will implement the bayesian approach but with the OPENBUGS package:

## library(R2OpenBUGS)

## Warning: package 'R2OpenBUGS' was built under R version 4.0.5

15000

**Iterations** 

```
# Define function
logit.bayes <- function() {
    for (i in 1:N) {
        y[i] ~ dbern(p[i])
        logit(p[i]) <- beta0 + beta1 * Inflight.entertainment[i] + beta2 * On.board.service[i] + beta3 * Ea
    }
    beta0 ~ dnorm(0.0, 1.0E-6)
    beta1 ~ dnorm(0.0, 1.0E-6)
    beta2 ~ dnorm(0.0, 1.0E-6)
    beta3 ~ dnorm(0.0, 1.0E-6)
    beta4 ~ dnorm(0.0, 1.0E-6)
}</pre>
```

```
# Prepare data for observed data and prior parameters
data_list <- list(</pre>
   N = nrow(data),
    y = data$satisfaction, #variable to predict
    Inflight.entertainment = data$Inflight.entertainment,
    On.board.service = data$On.board.service,
    Ease.of.Online.booking = data$Ease.of.Online.booking,
    Checkin.service = data$Checkin.service
)
# Specify initial values for parameters
inits <- function() {</pre>
    list(beta0 = 1, beta1 = 0, beta2 = 0, beta3 = 0, beta4 = 0)
# Run OpenBUGS
output <- bugs(data = data_list,</pre>
                   inits = inits,
                   parameters.to.save = c("beta0", "beta1", "beta2", "beta3", "beta4"),
                   model.file = logit.bayes,
                   n.chains = 1,
                   n.iter = 10000,
                   debug = F)
```

Let us inspect a summary of the output

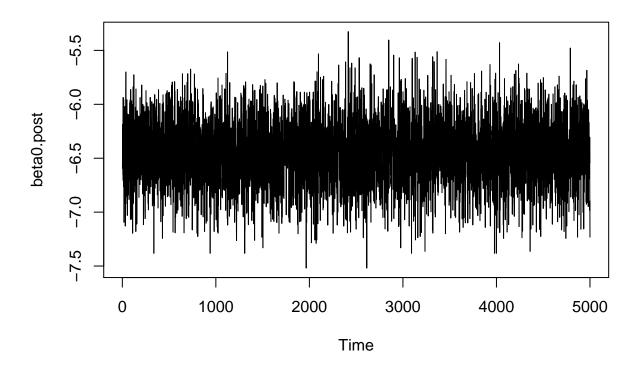
## summary(output)

```
##
                Length Class Mode
## n.chains
                 1 -none- numeric
## n.iter
                   1 -none- numeric
## n.burnin
                    1 -none- numeric
## n.thin
                   1 -none- numeric
## n.keep
                   1 -none- numeric
                1 -none- numeric
## n.sims
## sims.array 30000 -none- numeric
## sims.list
                6 -none- list
## sims.matrix 30000 -none- numeric
                   42 -none- numeric
## summary
                    6 -none- list
## mean
## sd
                    6 -none- list
## median
                   6 -none- list
                   6 -none- character
## root.short
## long.short
                   6 -none- list
## dimension.short 6 -none- numeric
## indexes.short
                   6 -none- list
## last.values
                   1 -none- list
## isDIC
                   1 -none- logical
## DICbyR
                  1 -none- logical
## pD
                   1 -none- numeric
## DIC
                   1 -none- numeric
                 1 -none- character
## model.file
```

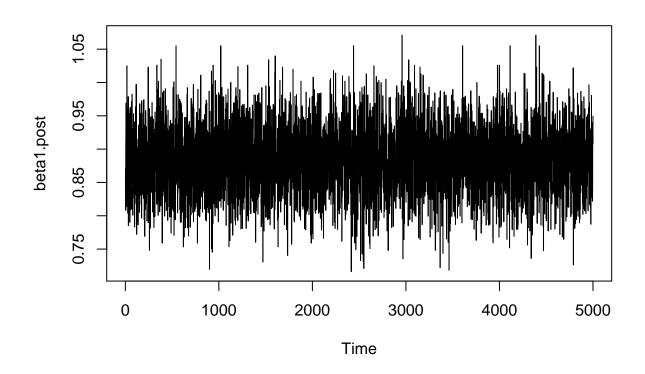
Let us now check the traces of the posterior betas:

```
beta0.post <-output$sims.list$beta0
beta1.post <-output$sims.list$beta1
beta2.post <-output$sims.list$beta2
beta3.post <-output$sims.list$beta3
beta4.post <-output$sims.list$beta4

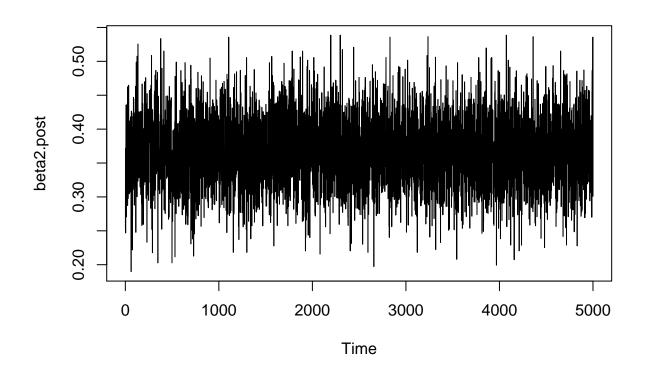
ts.plot(beta0.post)</pre>
```



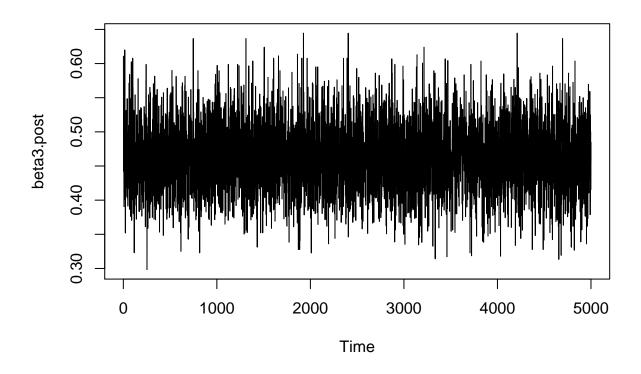
```
ts.plot(beta1.post)
```



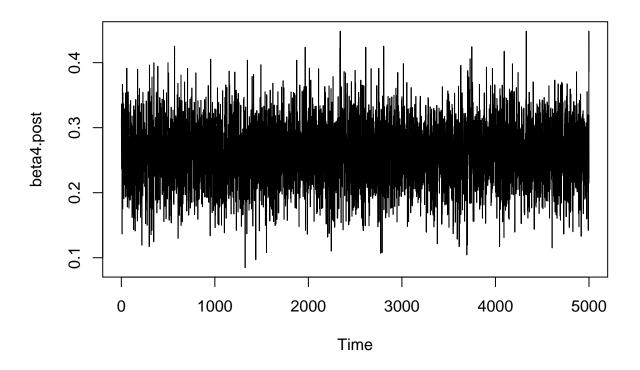
ts.plot(beta2.post)



ts.plot(beta3.post)



ts.plot(beta4.post)



We can see that the traces usually do converge

Finally, we can obtain the means and the 95% CI's of the betas by:

```
mean(beta0.post); quantile(beta0.post,c(0.025,0.975))
## [1] -6.449846
##
        2.5%
                 97.5%
## -7.072075 -5.864000
mean(beta1.post); quantile(beta1.post,c(0.025,0.975))
## [1] 0.8837521
                 97.5%
##
        2.5%
## 0.7864975 0.9824000
mean(beta2.post); quantile(beta2.post,c(0.025,0.975))
## [1] 0.3654114
##
       2.5%
               97.5%
## 0.270295 0.470300
```

```
mean(beta3.post); quantile(beta3.post,c(0.025,0.975))

## [1] 0.4643171

## 2.5% 97.5%

## 0.3704 0.5656

mean(beta4.post); quantile(beta4.post,c(0.025,0.975))

## [1] 0.2599757

## 2.5% 97.5%

## 0.1643975 0.3546075
```

Now, we look for the predictive distribution given the provided data using OpenBugs:

0.8871 0.9201 0.9262 0.9259 0.9320 0.9529

And, this is the plot of this probability density:

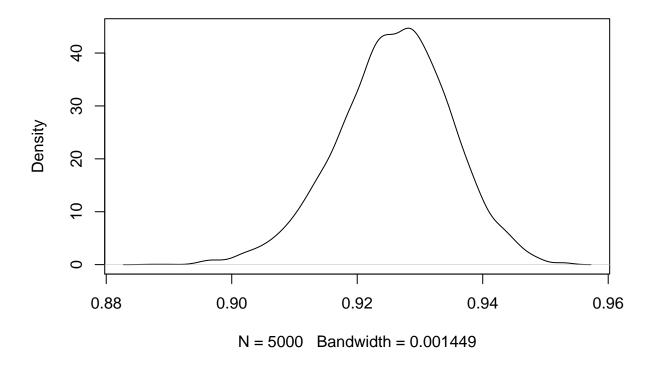
##

```
#New set of data
Inflight.entertainment = 5
On.board.service=4
Ease.of.Online.booking=5
Checkin.service=3
satisfaction.prob=inv.logit(beta0.post+Inflight.entertainment*beta1.post + On.board.service*beta2.post
summary(satisfaction.prob)
## Min. 1st Qu. Median Mean 3rd Qu. Max.
```

These values represent the posterior mean, standard deviation and 95% CI for the satisfaction probability.

```
plot(density(satisfaction.prob))
```

# density.default(x = satisfaction.prob)



Again, we can see that our posterior probability of being satisfied is centered around 0.93 similarly with the MCMC seen previously. It is remarkably high, as it is correctly predicting the probability of being satisfied minding that the values inputted to the model are high ratings (5,4,5,3)

## CONCLUSIONS

In this case study, we analyzed the factors influencing customer satisfaction using logistic regression through three distinct **approaches**: - the classical frequentist approach, - the Bayesian approach using Markov Chain Monte Carlo (MCMC), - and the Bayesian approach using R2OpenBUGS.

Our aim was to predict customer satisfaction based on the predictors: Inflight entertainment, On-board service, Ease of online booking, and Check-in service.

## Classical Frequentist Logistic Regression

Using the classical frequentist approach, we fitted a logistic regression model and obtained the maximum likelihood estimates of the regression coefficients. The main findings are:

- Model Fit: The model provided a good fit to the data, with the coefficients indicating the direction and magnitude of the influence of each predictor on customer satisfaction.
- Significance of Predictors: Inflight entertainment, On-board service, Ease of online booking, and Checkin service were all significant predictors of customer satisfaction.

## Bayesian Logistic Regression using MCMC

In the Bayesian approach using MCMC, we incorporated prior beliefs about the parameters and used MCMC to estimate the posterior distributions of the regression coefficients. The main findings are:

- Posterior Distributions: This approach provided a full posterior distribution for each parameter, giving a richer understanding of the uncertainty around the estimates.
- Priors and Posteriors: The use of non-informative priors ensured that the results were driven mainly by the data. The posterior distributions of the parameters were consistent with the frequentist estimates but provided additional information about the uncertainty and potential variability of the estimates.
- Credible Intervals: The 95% credible intervals for the regression coefficients indicated that all predictors
  had a positive and significant impact on customer satisfaction, aligning with the frequentist approach
  results.

## Bayesian Logistic Regression using R2OpenBUGS

Using R2OpenBUGS, we implemented the Bayesian logistic regression model, combining the flexibility of BUGS language with the MCMC approach. The main findings are:

- Posterior Distributions: Similar to the MCMC approach, R2OpenBUGS provided posterior distributions for each parameter, reinforcing the robustness of our findings.
- Results Consistency: The results from R2OpenBUGS were consistent with both the frequentist approach and the Bayesian MCMC approach, validating the reliability of the insights.

## Comparative Insights

- Parameter Estimates: All three methods yielded similar point estimates for the regression coefficients, indicating consistent results across different approaches.
- Uncertainty: The Bayesian approaches (both MCMC and R2OpenBUGS) offered a more comprehensive view of uncertainty through posterior distributions and credible intervals, whereas the frequentist approach provided standard errors and confidence intervals.

#### **Final Conclusion**

Both the classical frequentist and Bayesian approaches (MCMC and R2OpenBUGS) confirmed that *Inflight* entertainment, On-board service, Ease of online booking, and Check-in service are significant predictors of customer satisfaction. The Bayesian approaches, while computationally more demanding, provided deeper insights into the parameter uncertainties and were consistent with the results obtained from the frequentist approach.