Problem Description: Workforce Scheduling

A service company operates in three shifts (Morning, Afternoon, Night) and needs to determine the number of employees assigned to each shift. The objective is to minimize labor costs while ensuring sufficient staffing levels for service demand.

Decision Variables:

1. x_1: Number of employees assigned to the Morning shift.
2. x_2: Number of employees assigned to the Afternoon shift.
3. x_3: Number of employees assigned to the Night shift.
4. x_4: Overtime hours for the Morning shift.
5. x_5: Overtime hours for the Afternoon shift.
6. x_6: Overtime hours for the Night shift.
7. x_7: Number of part-time employees assigned to the Morning shift.
8. x_8: Number of part-time employees assigned to the Afternoon shift.
9. x_9: Number of part-time employees assigned to the Night shift.
10. x_{10}: Number of temporary employees assigned to the Morning shift.
11. x_{11}: Number of temporary employees assigned to the Afternoon shift.
12. x_{12}: Number of temporary employees assigned to the Night shift.
13. x_{13}: Number of on-call employees for the Morning shift.
14. x_{14}: Number of on-call employees for the Afternoon shift.
15. x_{15}: Number of on-call employees for the Night shift.
16. x_{16}: Number of employees on vacation during the Morning shift.
17. x_{17}: Number of employees on vacation during the Afternoon shift.
18. x_{18}: Number of employees on vacation during the Night shift.
19. x_{19}: Number of employees on training during the Morning shift.
20. x_{20}: Number of employees on training during the Afternoon shift.

Regenerate

Model Sets:

- S: Set of shifts (Morning, Afternoon, Night).
- E: Set of employee types (Regular, Part-time, Temporary, On-call, Vacation, Training).

Parameters:

- C_e: Cost per hour for each employee type $e \in E$.
- D_s: Service demand for each shift $s \in S$.
- M_s: Maximum overtime hours allowed for each shift $s \in S$.
- U_e: Maximum allowed number for each employee type $e \in E$ (Part-time, Temporary, On-call, Vacation, Training).

Variables:

- X_{es}: Number of employees of type e assigned to shift s.

Objective Function:

$$\text{Minimize } Z = \sum_{e \in E} \sum_{s \in S} C_e \cdot X_{es}$$

Constraints:

1. **Shift Coverage Constraints:**

$$\sum_{e \in E} X_{es} = D_s \quad \text{for all } s \in S$$

2. **Overtime Constraints:**

$$X_{4s} \leq M_s \quad \text{for all } s \in S$$

3. **Employee Type Constraints:**

$$X_{7s}, X_{8s}, X_{9s} \leq U_{\text{Part-time}} \quad \text{for all } s \in S$$

$$X_{10s}, X_{11s}, X_{12s} \leq U_{\text{Temporary}} \quad \text{for all } s \in S$$

$$X_{13s}, X_{14s}, X_{15s} \leq U_{\text{On-call}} \quad \text{for all } s \in S$$

$$X_{16s}, X_{17s}, X_{18s} \leq U_{\text{Vacation}} \quad \text{for all } s \in S$$

$$X_{19s}, X_{20s} \leq U_{\text{Training}} \quad \text{for all } s \in S$$

4. **Non-negativity Constraints:**

$$X_{es} \geq 0 \quad \text{for all } e \in E, s \in S$$

Objective Function:

Minimize labor costs:

$$\text{Minimize } Z = 8 \cdot (x_1 + x_2 + x_3) + 12 \cdot (x_4 + x_5 + x_6) + 6 \cdot (x_7 + x_8 + x_9) + 10$$

Constraints:

1. **Shift Coverage Constraints:**
   - Morning: $x_1 + x_7 + x_{10} + x_{13} - x_{16} - x_{19} = \text{Morning Demand}$
   - Afternoon: $x_2 + x_8 + x_{11} + x_{14} - x_{17} - x_{20} = \text{Afternoon Demand}$
   - Night: $x_3 + x_9 + x_{12} + x_{15} - x_{18} = \text{Night Demand}$
2. **Overtime Constraints:**
   - Morning: $x_4 \leq 5$
   - Afternoon: $x_5 \leq 5$
   - Night: $x_6 \leq 5$
3. **Part-time Constraints:**
   - $x_7, x_8, x_9 \leq 3$
4. **Temporary Employee Constraints:**
   - $x_{10}, x_{11}, x_{12} \leq 2$
5. **On-call Employee Constraints:**
   - $x_{13}, x_{14}, x_{15} \leq 4$
6. **Vacation Constraints:**
   - $x_{16}, x_{17}, x_{18} \leq 1$
7. **Training Constraints:**
   - $x_{19}, x_{20} \leq 1$
8. **Non-negativity Constraints:**
   - $x_i \geq 0$ for all i

Let's modify the workforce scheduling problem to include binary variables to represent whether an employee is assigned to a particular shift. This modification will introduce logical and conditional constraints that require the use of binary variables.

Decision Variables (Modified):

1. X_{es}: Continuous variable representing the number of employees of type e assigned to shift s.
2. Y_{es}: Binary variable indicating whether an employee of type e is assigned to shift s (1 if assigned, 0 otherwise).

Objective Function (Modified):

$$\text{Minimize } Z = \sum_{e \in E} \sum_{s \in S} C_e \cdot X_{es}$$

Constraints (Modified):

1. **Shift Coverage Constraints:**

$$\sum_{e \in E} X_{es} = D_s \quad \text{for all } s \in S$$

$$Y_{es} \leq X_{es} \quad \text{for all } e \in E, s \in S$$

2. **Overtime Constraints:**

$$X_{4s} \leq M_s \quad \text{for all } s \in S$$

3. **Employee Type Constraints:**

$$X_{7s}, X_{8s}, X_{9s} \leq U_{\text{Part-time}} \quad \text{for all } s \in S$$

$$X_{10s}, X_{11s}, X_{12s} \leq U_{\text{Temporary}} \quad \text{for all } s \in S$$

$$X_{13s}, X_{14s}, X_{15s} \leq U_{\text{On-call}} \quad \text{for all } s \in S$$

Regenerate

$$X_{13s}, X_{14s}, X_{15s} \leq U_{\text{On-call}} \quad \text{for all } s \in S$$

$$X_{16s}, X_{17s}, X_{18s} \leq U_{\text{Vacation}} \quad \text{for all } s \in S$$

$$X_{19s}, X_{20s} \leq U_{\text{Training}} \quad \text{for all } s \in S$$

Binary Variable Constraints:

$$Y_{es} \in \{0, 1\} \quad \text{for all } e \in E, s \in S$$

Some useful links:

- Modeling and optimization of a weekly workforce with Python and Pyomo | by Christian Carballo Lozano | Towards Data Science
- Applied Optimization in Python Using the Pyomo Library | by Emre Uzel | Python in Plain English