Mini project - A communication problem

Handed out: Wednesday September 10
Deadline: Friday October 3

In 1996, the French telecommunications giant TranzPhone was granted the rights to build Poland's national mobile network. After extensive evaluations, TranzPhone and the Polish government concluded that Poland could be divided into four areas to be developed separately. TranzPhone then established four companies responsible for development in each area.

The company responsible for development in the northeastern part of Poland is called MobiPol. Since its establishment in 1998, the work has progressed steadily and is expected to be completed by September 2003.

The northwestern part of Poland consists mostly of rural areas, with only a few larger cities. P.Com, which is developing this area, experienced some problems in the first two years. The problems were mainly due to difficulties in finding interpreters who understood the specific dialect spoken in northwestern Poland. The setbacks turned around when P.Com hired an interpreter, and the work is expected to be completed by May 2003.

The third area is located in the southeastern part of the country, bordered by the Bug and Vistula rivers. A well-functioning mobile network in both Ukraine and Slovakia made the development in this part of Poland completely smooth, and the development was completed as early as August 2001.

In the fourth area, problems began even before the work had started. The mayor of Kraków realized early on that the city would not be included in the southeastern region, which would have led to a rapid network expansion in the city. In protest, he boycotted several important planning meetings. As a result, the plans for a fully developed network by 2003 had to be shelved. The situation further escalated when the mayors of Warsaw and Wrocław threatened to exclude Kraków from the mobile network. The situation took a new turn when the mayor of Kraków died, officially due to heart failure. No evidence exists, but strong indications suggest that the death was not entirely natural. The new mayor was a reform politician who immediately took the initiative to resume the planning process.

The company responsible for this fourth area, MobiPhlex, thus has busy days ahead. MobiPhlex has divided the work of developing southwestern Poland into three phases: First, a main line is laid from Warsaw to Jelenia Góra, then all major cities are connected to the main line, and finally, the base stations are connected to cells in the cities. Your task is to assist MobiPhlex with the first two phases.

A few years ago, the EU's Achilles support program provided capital to upgrade the telecommunications network in Poland. In southwestern Poland, most work was done on the Radom - Kielce and Gliwice - Opole routes. The Polish government therefore wants these two routes to be used as part of the main line.

MobiPhlex has divided southwestern Poland into three areas. Area 1 is the region around Warsaw, area 2 includes the area around Kraków, while area 3 consists of Poznań and its surroundings. MobiPhlex has planned for the main line to start in Warsaw. The line then stretches through

area 1 to Radom, where it connects to the existing network. In area 2, the line starts in Kielce and continues to Gliwice, while Opole and Jelenia Góra are the start and end points in area 3, respectively. The existing telecommunications network is used between Gliwice and Opole.

MobiPhlex and the Polish state have jointly developed some conditions that must be met for the total coverage of southwestern Poland to be considered satisfactory. These are:

- Besides Warsaw and Radom, the main line must pass through two more cities in area 1.
- Besides Kielce and Gliwice, the main line must pass through three more cities in area 2.
- In addition to Opole and Jelenia Góra, the main line must pass through three more cities in area 3.
- Since information can be sent both ways in the main line, two cities should not be connected in more than one way: If the line runs from A to B, it cannot simultaneously run from B to A.
- To stimulate local growth in the area around Konin, the Polish government has decided that the line must pass through at least one of the cities Konin and Kalisz.
- To decentralize the region west of Kraków, the Polish Ministry of Industry wants the main line to pass through at most one of the cities Bytom, Sosnowiec, and Katowice.
- For the same reason, the line cannot pass through both Wodzisław Śląski and Jastrzębie-Zdrój.
- To connect Poland closer to Germany, the main line must pass through at least one of the cities Poznań, Zielona Góra, and Leszno.

The map (Appendix 1) shows the division of southwestern Poland into the three areas. The table in Appendix 2 shows the coordinates of each city.

Tasks

a) **Introduction** Write a concise problem description where all your assumptions are clearly stated. The problem description should be enough for someone else to formulated a mathematical model of the problem.

b) **Mathematical formulation** Present a mathematical formulation of the problem. All notation should be clearly defined. All constraints and the objective function should be properly explained. The formulation should be an integer linear program.

c) **Implementation** Implement your mathematical formulation in Python + Gurobi. All data should be read from one or more data files. The implementation should output the following:

- The length of the main cable.
- The cities connected to the main cable in order. Starting from Warsaw.

After the main line is installed, the cities that the line does not pass through must be connected. This is done by laying a Russian copper cable between the respective cities and at least one of the cities Warsaw, Kielce, and Opole. In these three cities, corresponding switching stations are

installed. The switching stations are developed by the Lithuanian conglomerate GPZ and are designed so that several smaller standard units can be connected in parallel to form complete stations. Each standard unit has a capacity of 400 TB/s. In Opole, one such unit are installed, while the number of units is two and three for Warsaw and Kielce, respectively. The estimated capacity requirement for each city is presented in Appendix 2.

TranzPhone has provided the following additional information about the problem:

- The Russian copper cable is of rather poor quality, with a maximum capacity of 200 TB/s. Thus, no city can send more than 200 TB/s to a switching station.
- Due to the copper cable's construction with many sub-threads, it is possible to purchase cables for exactly the capacity needed. The Polish state buys the cable from the Belgian Jean-Pierre Rijn, who specializes in reselling Russian copper. After tough negotiations in Brussels, the price ended up at 1000 EUR per coordinate unit and TB/s. Thus, for example, a cable that is 3.4 coordinate units long with a capacity of 80 TB/s would cost 272,000 EUR.
- Due to the risk of interference, it is not possible to lay parallel cables. For cities where 200 TB/s is not sufficient, the cable must therefore be laid to more than one switching station.
- A UFO association with strong ties to Łódź's political elite warns about the effects an invasion from Mars would have on the city's infrastructure. To minimize these effects, politicians in Łódź have decided that a maximum of 40% of the total information flow from the city can be sent in a single cable.
- Due to the quality of the copper cable, the amount of information that can be sent decreases with distance. The American researcher Pete Shannon has found that the relationship between the amount of information that can be sent (x) (measured in TB/s) and the length of the copper cable (d) (expressed in coordinate units) can be written as:

$$x \leq 2\pi \min(35, 70 - d) \tag{1}$$

Tasks

d) Mathematical formulation Present a mathematical formulation of the problem. All notation should be clearly defined. All constraints and the objective function should be properly explained. The formulation should be a continuous linear program.

e) Implementation Implement your mathematical formulation in Python + Gurobi. All data should be read from one or more data files. The implementation should output enough information so that the questions following below can be answered:

- What are the costs to connect the remaining cities to the switching stations?
- What capacity should be installed between Poznań and Wałbrzych and the various switching stations?
- How much of the total capacity at the switching station in Kielce is utilized?

f) Sensitivity analysis Output sensitivity information from the model so that the following questions can be answered

- The demand forecasts for Konin are unfortunately very uncertain. Konin's local bureaucrats suspect that the demand has been miscalculated by up to 10%. However, an upset opposition politician claims that the miscalculation could be as much as 20%. How is the total cost affected by a miscalculation of 10% and 20%, respectively?
- The solution depends on the level of precision used for the constant π. Within what interval can π lie for the current basis to remain optimal?

Output

The delivery of the project is a written report, Python files with the implemented models and data files. The report should answer each task according to the following format:

- Introduction with problem description
- Mathematical formulation - main line
- Output from the implementation and results
- Mathematical formulation - connecting cities
- Sensitivity analysis
- A map showing the main line, and the connections with capacities

In addition, the Python code should be delivered.

Appendix

Figure 1: The division of Poland

Table 1: The names of the cities in the different areas together with their expected capacity requirements

	x	y	TB/s
Area 1			
Warszawa	100	65	-
Skierniewice	92	62	197
Lódz	85	58	317
Konin	68	64	236
Sieradz	72	55	178
Piotrków	89	51	139
Radom	102	50	288
Area 2			
Kielce	98	42	-
Czestochowa	79	40	278
Bytom	76	31	163
Gliwice	73	30	105
Sosnowiec	80	29	143
Kraków	89	25	331
Bielsko	79	20	276
Katowice	78	28	263
Jastrzębie-Zdrój	74	23	137
Wodzisław Śląski	71	23	119
Area 3			
Poznań	54	68	251
Zielona Góra	39	60	121
Leszno	51	58	84
Kalisz	65	58	207
Legnica	45	47	176
Wrocław	54	45	133
Jelenia Góra	40	42	167
Wałbrzych	46	39	129
Opole	67	37	-