Mini project - A communication problem

Part 1

Task A – Problem description

MobiPhlex needs to design a main telecommunication line connecting Warsaw to Jelenia Góra through southwestern Poland as the first phase of mobile network development. The line must utilize existing EU-funded infrastructure while satisfying government requirements for regional coverage and economic development.

Geographic Context

- **Area 1**: Region around Warsaw
- **Area 2**: Region around Kraków
- **Area 3**: Region around Poznań

The main line follows a prescribed route: Warsaw → Area 1 → Radom → [existing network] → Kielce → Area 2 → Gliwice → [existing network] → Opole → Area 3 → Jelenia Góra.

Fixed Infrastructure Requirements

1. **Mandatory endpoints**: Warsaw (start) and Jelenia Góra (end)
2. **Required anchor cities**: Radom, Kielce, Gliwice, and Opole must be included
3. **Existing connections**: The line must use the EU-funded Radom-Kielce and Gliwice-Opole routes

Coverage Requirements

- **Area 1**: At least 2 additional cities beyond Warsaw and Radom
- **Area 2**: At least 3 additional cities beyond Kielce and Gliwice
- **Area 3**: At least 3 additional cities beyond Opole and Jelenia Góra

Policy Constraints

1. **Economic development**: At least one of Konin or Kalisz must be included
2. **Decentralization (Kraków region)**: At most one of Bytom, Sosnowiec, or Katowice
3. **Decentralization (industrial region)**: At most one of Wodzisław Śląski or Jastrzębie-Zdrój
4. **German connectivity**: At least one of Poznań, Zielona Góra, or Leszno

Technical Constraints

- **Bidirectional communication**: No two cities can be connected in both directions simultaneously
- **Connectivity**: Cities can only be connected if they are in the same area, except for the mandatory inter-area connections (Radom-Kielce and Gliwice-Opole)

Assumptions

1. **Distance metric**: Construction cost is proportional to the Euclidean distance of the line.
2. **Network topology**: The main line forms a simple path (no branches or cycles)
3. **Area restrictions**: Direct connections between areas are only allowed via the designated anchor points
4. **Feasibility**: A solution satisfying all constraints is guaranteed to exist

Summary: Design a main telecommunication line connecting Warszawa to Jelenia Góra, passing through specific cities while minimizing total distance and satisfying various constraints.

Task B - Mathematical formulation

Sets

- *V*: Set of all cities (vertices), indexed by $i, j \in \{0, 1, \dots, n-1\}$
- *E*: Set of allowed arcs (within-area connections + inter-area bridges)
- V_1, V_2, V_3: Cities in areas 1, 2, and 3 respectively
- $E_{in}(j)$: Set of cities *i* such that $(i, j) \in E$ (incoming arcs to city *j*)
- $E_{out}(i)$: Set of cities *j* such that $(i, j) \in E$ (outgoing arcs from city *i*)

Parameters

- d_{ij}: Euclidean distance between cities *i* and *j*
- *s*: Source city (Warszawa)
- *t*: Destination city (Jelenia Góra)

Key city indices

- Anchors: RAD (Radom), KIE (Kielce), GLI (Gliwice), OPO (Opole)
- Policy cities: KON (Konin), KAL (Kalisz), BYT (Bytom), SOS (Sosnowiec), KAT (Katowice), WOD (Wodzisław Śl.), JAS (Jastrzębie-Zdrój), POZ (Poznań), ZIE (Zielona Góra), LES (Leszno)

Decision variables

- $x_{ij} \in \{0,1\}$: 1 if arc *(i, j)* is used in the main line, 0 otherwise, for all $(i, j) \in E$
- $y_i \in \{0,1\}$: 1 if city *i* is included in the main line, 0 otherwise, for all $i \in V$

Objective function

Minimize the total distance of the main line:

$$\min \sum_{(i,j)\in E} d_{ij} \cdot x_{ij}$$

Constraints

Flow and path structure

- Source constraint (only outflow): $\sum_{i\in E_{in}(s)} x_{is} = 0,\ \sum_{j\in E_{out}(s)} x_{sj} = 1,\ \ y_s = 1$
- Destination constraint (only inflow): $\sum_{i\in E_{in}(t)} x_{it} = 1,\ \sum_{j\in E_{out}(t)} x_{tj} = 0,\ \ y_t = 1$
- Other cities (inflow=outflow):

  For all $j \in V \setminus \{s,t\}$: $\sum_{i\in E_{in}(j)} x_{ij} = y_j,\ \sum_{k\in E_{out}(j)} x_{jk} = y_j$

Arc direction constraint - Prevent using both directions of the same connection:

$$x_{ij} + x_{ji} \le 1 \quad \forall (i,j),(j,i) \in E$$

Area inclusion requirements – Minimum number of cities (excluding anchors) from each area:

- Area 1: $\sum_{i\in V_1\setminus\{s,\mathrm{RAD}\}} y_i \ge 2$
- Area 2: $\sum_{i\in V_2\setminus\{\mathrm{KIE,GLI}\}} y_i \ge 3$
- Area 3: $\sum_{i\in V_3\setminus\{\mathrm{OPO},t\}} y_i \ge 3$

Anchor City Requirements

Force inclusion of specific cities: $y_{\mathrm{RAD}} = 1, \quad y_{\mathrm{KIE}} = 1, \quad y_{\mathrm{GLI}} = 1, \quad y_{\mathrm{OPO}} = 1$

Policy constraints

- At least one of Konin or Kalisz: $y_{\mathrm{KON}} + y_{\mathrm{KAL}} \ge 1$
- At most one of Bytom, Sosnowiec, or Katowice: $y_{\mathrm{BYT}} + y_{\mathrm{SOS}} + y_{\mathrm{KAT}} \le 1$
- At most one of Wodzisław Śl. or Jastrzębie-Zdrój: $y_{\mathrm{WOD}} + y_{\mathrm{JAS}} \le 1$
- At least one of Poznań, Zielona Góra, or Leszno: $y_{\mathrm{POZ}} + y_{\mathrm{ZIE}} + y_{\mathrm{LES}} \ge 1$

Inter-Area Bridge Requirements - Force exactly one direction of each required inter-area connection:

- Radom-Kielce bridge: $x_{\mathrm{RAD,KIE}} + x_{\mathrm{KIE,RAD}} = 1$
- Gliwice-Opole bridge: $x_{\mathrm{GLI,OPO}} + x_{\mathrm{OPO,GLI}} = 1$

Summary

The allowed arc set *E* includes all arcs *(i, j)* where cities *i* and j are in the same area, in addition to the given inter-area arcs (RAD,KIE), (KIE,RAD), (GLI,OPO), (OPO,GLI).

The solution forms a simple path from Warszawa to Jelenia Góra.

Task C - Implementation

The code and data used is attached in the hand-in. The output is given below.

```
Optimal main line length: 149.16 coordinate units

Main line path (in order): Warszawa → Skierniewice → Piotrków → Radom →
Kielce → Kraków → Bielsko → Jastrzębie-Zdrój → Gliwice → Opole → Kalisz →
Leszno → Legnica → Jelenia Góra
```

It can be verified that this order of cities satisfies all the constraints. Since the model converged to a feasible solution, this must therefore be the optimal solution, i.e. the shortest feasible path.

Part 2

Task D – Mathematical formulation

We define a linear program to connect all cities not on the main line to one or more of the switching stations (Warszawa, Kielce, Opole).

Sets

- R: set of cities not on the main line (from part 1)
- S = Warszawa, Kielce, Opole: set of switching stations

Parameters

- D_i: demand of city *i* (int TB/s)
- d_{is}: distance between city *i* and station *s*
- Station capacities
  - $Cap_{Warszawa}$ = 800 TB/s
  - Cap_{Kielce} = 1200 TB/s
  - Cap_{Opole} = 400 TB/s
- Cable cost per TB/s: $c_{i,s} = 1000 \cdot d_{i,s}$

Decision variables

$$f_{i,s} \geq 0\text{: flow (in TB/s) sent from city } i \in \text{T to station } s \in S$$

Objective function

Minimize total cable cost: $\min \sum_{i \in R} \sum_{s \in S} c_{i,s} \ f_{i,s}$

Constraints

- Demand satisfaction

$$\sum_{s \in S} f_{i,s} = D_i \quad \forall i \in R$$

- Cable upper bound

$$0 \leq f_{i,s} \leq 200 \quad \forall i \in R, \forall s \in S$$

- Cable attenuation limit

$$f_{i,s} \leq 2\pi min(35, 70 \ - \ d_{i,s}) \quad \forall i \in R, \forall s \in S$$

- Station capacity

$$\sum_{i \in R} f_{i,s} \leq \text{Cap}_s \quad \forall s \in S$$

- Łódź special rule (max 40% of demand per cable)

$$f_{\text{Łódź},s} \leq 0.4 \cdot D_{\text{Łódź}} \quad \forall s \in S$$

Task E – Implementation

Output:

```
1. TOTAL CONNECTION COSTS
   Total cost to connect all remaining cities: 66,696,434.43 EUR

   Breakdown by city:
   - Łódź: 6,466,519.79 EUR
   - Konin: 7,742,401.65 EUR
   - Sieradz: 5,253,098.46 EUR
   - Czestochowa: 6,367,667.76 EUR
   - Bytom: 4,009,269.88 EUR
   - Sosnowiec: 3,175,115.27 EUR
   - Katowice: 5,832,517.30 EUR
   - Wodzisław Śl.: 3,928,802.62 EUR
   - Poznań: 12,075,624.98 EUR
   - Zielona Góra: 7,094,001.84 EUR
   - Wrocław: 2,030,156.89 EUR
   - Wałbrzych: 2,721,257.98 EUR

2. CAPACITY INSTALLATIONS
   Required capacities between cities and switching stations:

   Poznań connections:
   - Poznań -> Warszawa: 150.18 TB/s
   - Poznań -> Kielce: 100.82 TB/s
   - Poznań -> Opole: 0.00 TB/s (no connection)

   Wałbrzych connections:
   - Wałbrzych -> Warszawa: 0.00 TB/s (no connection)
   - Wałbrzych -> Kielce: 0.00 TB/s (no connection)
   - Wałbrzych -> Opole: 129.00 TB/s

3. SWITCHING STATION UTILIZATION
   Kielce switching station:
   - Utilized capacity: 1200.00 TB/s
   - Total capacity: 1200 TB/s
   - Utilization rate: 100.0%
   Warszawa switching station:
   - Utilized capacity: 731.00 TB/s
   - Total capacity: 800 TB/s
   - Utilization rate: 91.4%
   Opole switching station:
   - Utilized capacity: 400.00 TB/s
   - Total capacity: 400 TB/s
   - Utilization rate: 100.0%
```

Task F – Sensitivity analysis

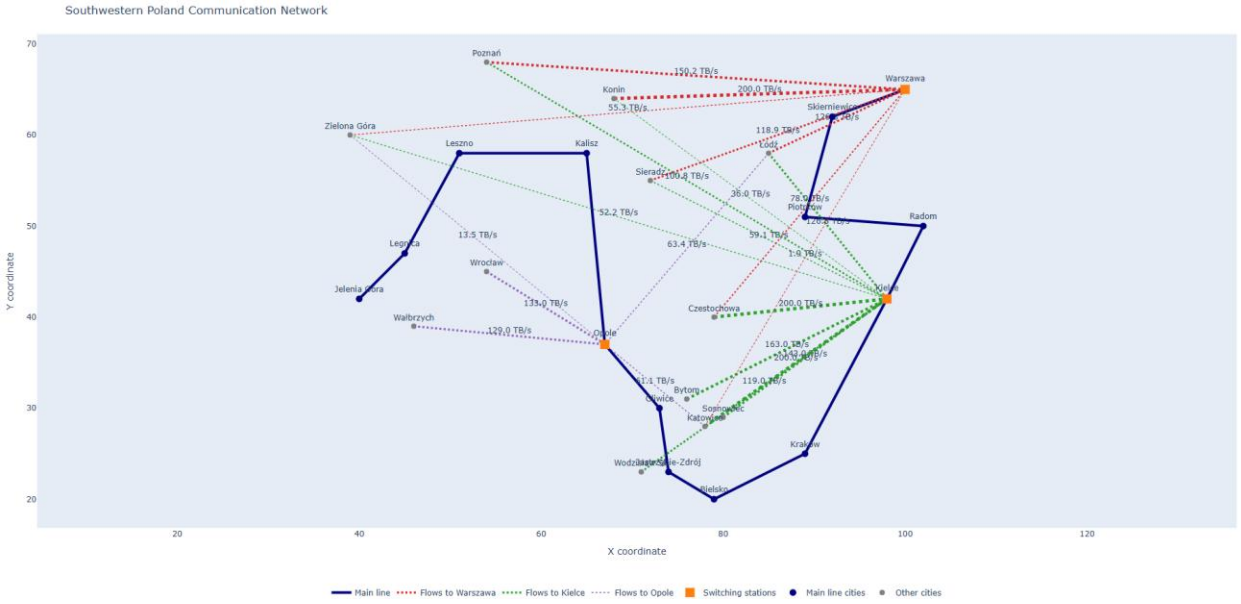
```
4. SENSITIVITY ANALYSIS

   Konin Demand Sensitivity:
   Konin current demand: 236.0 TB/s
   Konin shadow price: 37865.4043 EUR per TB/s
   +/-10% demand change (+/-23.6 TB/s): Total cost shifts by +/-893,623.54
EUR
   +/-20% demand change (+/-47.2 TB/s): Total cost shifts by +/-1,787,247.08
EUR (outside valid sensitivity range)

   pi Parameter Sensitivity:
   Current pi: 3.141593
   pi can vary within [2.767443, 3.249037] without changing the basis.
```

Map of solution

The code will generate an interactive map of the solution. A screenshot is provided below.



AI disclaimer

Large Language Models have been used to help write code and verify the correctness of the mathematical formulations.