UNIT - IV

22/02/18

Normal Forms

Minimization of Given Grammar:

A Grammar is minimized by implementing following:

1. Removing epsilon productions
2. Removing Unit productions
3. Removing useless symbols/productions

1. Removing epsilon productions:

A production in the form of $\alpha \longrightarrow \beta$ is said to be epsilon production if β is epsilon. Inorder to minimize it first of all it have to remove the epsilon production, then we have to substitute epsilon in place of that corresponding Non-Terminal and rewrite the productions.

Eg: * Remove epsilon productions from the given grammar.

$S \rightarrow ASB \mid AB \mid aAb$

$A \rightarrow Aa \mid \epsilon$

$B \longrightarrow Cb \mid b$

$C \longrightarrow a \mid \epsilon$

Sol: Step-1: In the given grammar we have 2 Epsilon productions i.e., A, C

Step-2: So we have to remove the ϵ productions by rewriting the ϵ in place of A, C

$S \rightarrow ASB / AB / aAb$

$A \rightarrow Aa / \epsilon$

$B \rightarrow Cb / b$

$C \rightarrow a$

$S \rightarrow SB / B / ab$

2. Removing Unit productions:

A production in the form of $\alpha \rightarrow \beta$ is said to be Unit production if β is only one Non-Terminal

* Once we identified the unit production then we have to remove the Unit production, in place of that we are writing corresponding Non-Terminal productions.

Eg: ① $A \rightarrow Aa$

② $B \rightarrow C$

③ $A \rightarrow a$

In the above example, production ② is said to be Unit production

Eg: * $S \rightarrow AB / aS / BS / C$ ~~ad~~

$A \rightarrow AB / D$

$B \rightarrow b / bc$

$C \rightarrow D / a$

$D \rightarrow b$

In the above example, there are 3 unit productions in S, A, C. So in place of that we are writing corresponding non-terminal productions.

$S \rightarrow AB \mid aS \mid BS \mid \phi \mid a \mid b$.

$A \rightarrow AB \mid b$.

3. Removing Useless symbols/productions:

A production in the form of $\alpha \rightarrow \beta$ is become Useless if any one of the following conditions are satisfied:

1. A Non-Terminal (or) Production is not reachable from starting symbol of the grammar.
2. A Non-Terminal (or) production is not deriving any string (combination of terminals).

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Eg: $S \rightarrow AB \mid aA \mid BC \mid a$

$A \rightarrow Aa \mid AC$

$B \rightarrow Bc \mid b$

$C \rightarrow D \mid Aa$

$D \rightarrow DE$

Sol: $S \rightarrow AB \mid aA \mid BC \mid a$

$A \rightarrow Aa \mid Ac$

$B \rightarrow Bc \mid b$

Reduce (or) Minimize the given grammar

S ⟶ AB/BC/Aa/b

A ⟶ Aa/AC/ε

B ⟶ CD/b

C ⟶ aC/ε

D ⟶ DE/a

E ⟶ ε

Sol: Step: 1 - Eliminating ε productions in the given grammar

S ⟶ AB/BC/Aa/b/B/a

A ⟶ Aa/AC/a/c

B ⟶ CD/b

C ⟶ aC/a

D ⟶ DE/a/D

In the given grammar we are having 3 ε productions A ⟶ ε, C ⟶ ε, E ⟶ ε

Step: 2 - Eliminating Unit productions In this there are 2 unit productions but one is directly cancelled

S ⟶ B

D ⟶ ~~D~~

Step: 3 - Eliminating of useless productions

$S \rightarrow AB/BC/Aa/b/CD/a$

$A \rightarrow Aa/AC/a/c$

$B \rightarrow CD/b$

$C \rightarrow aC/a$

$D \rightarrow DE/a$

In this there is one useless production ~~is~~ ~~there~~ i.e., $D \rightarrow DE$ that is removed.

$S \rightarrow AB/BC/Aa/b/CD/a$

$A \rightarrow Aa/AC/a/c$

$B \rightarrow CD/b$

$C \rightarrow ac/a$

$D \rightarrow a$

Finally the reduced grammar is

Normal Forms: 27/02/18

Normal Forms are of 2 types:

1. Chomsky Normal Form (CNF)
2. Greibach Normal Form (GNF)

1) Chomsky Normal Form (CNF): A grammar is said to be in CNF if all productions in the given form

Non-terminal $\rightarrow$ Non-terminal. Non-terminal

Non-terminal $\rightarrow$ terminal

Eg: $S \rightarrow AA$

$A \rightarrow a/b$

2) Greibach Normal Form: A grammar is said to be in greibach normal form if all the production are in the given form.

Non Terminal $\rightarrow$ Terminal any number of Non-Terminal.

Eg: $S \rightarrow aB/a$

$B \rightarrow b$.

* $S \rightarrow ABC$ (Convert to CNF)

$S \rightarrow XC$

$X \rightarrow AB$.

* $S \rightarrow ABCD$.

$S \rightarrow XY$

$X \rightarrow AB$

$Y \rightarrow CD$.

Convert to GNF

* $S \rightarrow aAB$.

$C_a \rightarrow a$.

$S \rightarrow C_aAB$

$S \rightarrow XB$

$X \rightarrow C_aA$

* Convert to CNF

* $S \rightarrow aabc$

$C_a \rightarrow a$

$C_b \rightarrow b$

$C_c \rightarrow c$

$S \rightarrow C_a C_a C_b C_c$

$S \rightarrow XY$

$X \rightarrow C_aC_a$

$Y \rightarrow C_bC_c$

* ~~$S \rightarrow AaBbCc$~~

* $S \rightarrow AaBCb$

$S \rightarrow AaBCb$

consider $C_a \rightarrow a$

$C_b \rightarrow b$

$S \rightarrow AC_aBCC_b$

$S \rightarrow XY$

$X \rightarrow AC_a$

$Y \rightarrow ZC_b$

$Z \rightarrow BC$

* Convert the given grammar into chomskey Normal form

$S \rightarrow aBS \mid aA \mid a$

$A \rightarrow BCa \mid CD$

$C \rightarrow aab \mid CE$

$D \rightarrow b \mid aa$

$E \rightarrow c$

Sol:- In the given grammar the CNF productions are $S \rightarrow a$, $A \rightarrow CD$, $C \rightarrow CE$, $D \rightarrow b$, $E \rightarrow c$

* $S \rightarrow aBS$

$C_a \rightarrow a$

$S \rightarrow C_aBS$

$S \rightarrow XS$

$X \rightarrow C_aB$

[∵ $S \rightarrow aA$
$S \rightarrow C_aA$].

* $A \rightarrow BC_a$

$A \rightarrow BCC_a$

$A \rightarrow YC_a$

$Y \rightarrow BC$

* $C \rightarrow aab$

$C_b \rightarrow b$.

$C \rightarrow C_aC_aC_b$.

$C \rightarrow ZC_b$

$Z \rightarrow C_aC_a$

* $D \rightarrow aa$

$D \rightarrow C_aC_a$

The C.N.F

$S \rightarrow XS \mid C_aA \mid a$

$X \rightarrow C_aB$

$C_a \rightarrow a$

$A \rightarrow YC_a \mid CD$

$Y \rightarrow BC$

$C \rightarrow ZC_b \mid CE$

$Z \rightarrow C_aC_a$

$C_b \rightarrow b$

$D \rightarrow C_aC_a \mid b$

$E \rightarrow c$

* Convert the given grammar into CNF.

$S \rightarrow aaabA \mid aBb \mid a$

$A \rightarrow CDEaB \mid b$

$B \rightarrow BaAb$

$C \rightarrow cA$

$D \rightarrow Ea$

$E \rightarrow b$

Sol: In the given grammar the CNF productions are $S \rightarrow a$, $A \rightarrow b$, $E \rightarrow b$.

* $S \rightarrow aaabA$

Consider $C_a \rightarrow a$

$C_b \rightarrow b$

$S \rightarrow C_aC_aC_aC_bA$

$S \rightarrow XY$

$X \rightarrow C_aC_a$

$Y \rightarrow ZA$

$Z \rightarrow C_aC_b$

$S \rightarrow aBb$

$S \rightarrow C_aBC_b$

$S \rightarrow PC_b$

$P \rightarrow C_aB$

In $A \to CDEaB$

$A \to CDEC_aB$

$A \to QR$

$Q \to CD$

$R \to TB$

$T \to EC_a$

In $B \to BaAb$

$B \to BC_aAC_b$

$B \to UV$

$U \to BC_a$

$V \to AC_b$

In $C \to cA$

consider $C_c \to c$

$C \to C_cA$

In $D \to Ea$

$D \to EC_a$

GNF

$\therefore$ The CNF productions for given grammar follows:

$S \to XY \mid S \to PC_b \mid a$

$X \to C_aC_a \mid P \to C_aB$

$Y \to ZA$

$Z \to C_aC_b$

$A \to QR \mid b$

$Q \to CD$

$R \to TB$

$T \to EC_a$

$B \to UV$

$U \to BC_a$

$V \to AC_b$

$C \to C_cA$

$D \to EC_a$

$E \to b$

GNF:

Inorder to convert given grammar into GNF we have to apply following 2 rules.

1. lemma 1 - Substitution rule

Eg: $S \rightarrow AB$

$A \rightarrow a$

$S \rightarrow aB$

$A \rightarrow a$

2. lemma 2 - Elimination of left recursion

Eg: ① $A \rightarrow A\alpha / \beta$

(or)

$A \rightarrow A\alpha (\beta_1, \beta_2, \ldots \beta_n)$

$A \rightarrow \beta A'$ $\quad$ $A \rightarrow (\beta_1 A', \beta_2 A', \ldots \beta_n A')$

$A' \rightarrow \alpha A' / \epsilon$ $\quad$ $A' \rightarrow \alpha A' / \epsilon$

② $\underset{A}{A} \rightarrow \underset{A\ \alpha}{Aab} / \underset{\beta_1}{a} / \underset{\beta_2}{b}$

$A \rightarrow aA' / bA'$

$A' \rightarrow abA' / \epsilon$

* Convert the given grammar into GNF

$S \rightarrow ABA$

$A \rightarrow aA / \epsilon$

$B \rightarrow bB / \epsilon$

Sol: $S \rightarrow ABA$

$A \rightarrow aA / \epsilon$

$B \rightarrow bB / \epsilon$

not minimized

Eliminate ϵ productions

$S \rightarrow ABA \mid BA \mid AB \mid B \mid AA \mid A$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

Eliminate unit productions

$S \rightarrow ABA \mid BA \mid AB \mid bB \mid b \mid AA \mid aA \mid a$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$.

no useless productions

grammar is minimized.

In the above grammar the GNF productions are $S \rightarrow bB$, $S \rightarrow b$, $S \rightarrow aA$, $S \rightarrow a$, $A \rightarrow aA$, $A \rightarrow a$

$B \rightarrow bB$, $B \rightarrow b$.

In $S \rightarrow ABA$

Sub A in S

$S \rightarrow aABA \mid aBA$

$S \rightarrow BA$

sub B in S

$S \rightarrow bBA \mid bA$

$S \rightarrow AB$

Sub A is S

$S \rightarrow aAB \mid aB$.

$S \rightarrow AA$

sub A in S

$S \rightarrow aAA \mid aA$

finally the GNF productions of given grammar is

* Convert the given grammar into GNF

$A_1 \rightarrow A_2 A_1 / a$

$A_2 \rightarrow A_1 A_2$

sol: $A_1 \rightarrow A_2 A_1 / a$

$A_2 \rightarrow A_1 A_2$

sub A_1 in A_2

$\underset{A}{\underline{A_2}} \rightarrow \underset{A}{\underline{A_2}} \underset{\alpha}{\underline{A_1 A_2}} / \underset{\beta}{\underline{aA_2}}$

$A \rightarrow \beta A'$

$A' \rightarrow \alpha A' / e$

$A_2 \rightarrow aA_2 Z$

$Z \rightarrow A_1 A_2 Z / \epsilon$

Eliminating ϵ productions

$A_2 \rightarrow aA_2 Z / aA_2$

$Z \rightarrow A_1 A_2 Z / A_1 A_2$

Sub A_2 in A_1

$A_2 \rightarrow A_2 A_1 / a$

$A_1 \rightarrow aA_2 ZA_1 / aA_2 A_1 / a$

sub A_1 in Z

$Z \rightarrow aA_2 ZA_1 A_2 Z / aA_2 A_1 A_2 Z / aA_2 ZA_1 A_2 / aA_2 A_1 A_2 / aA_2$

Finally the GNF productions are

$A_1 \rightarrow aA_2 ZA_1 / aA_2 A_1 / a$

$A_2 \rightarrow aA_2 Z / aA_2$

$Z \rightarrow aA_2 ZA_1 A_2 Z / aA_2 A_1 A_2 Z / aA_2 ZA_1 A_2 / aA_2 A_1 A_2 / aA_2$

(or)

$A_1 \rightarrow A_2 A_1 / a$

$A_2 \rightarrow A_1 A_2$

sub A_2 in A_1

$\underset{A}{\underline{A_1}} \rightarrow \underset{A}{\underline{A_1}}\, \underset{\alpha}{\underline{A_2 A_1}} \,/\, \underset{\beta}{\underline{a}}$

$A \rightarrow \beta A'$ $\qquad$ $A_1 \rightarrow aZ$

$A' \rightarrow \alpha A' / \epsilon$ $\qquad$ $Z \rightarrow A_2 A_1 Z / \epsilon$

eliminating ϵ productions.

$A_1 \rightarrow aZ / a$

$Z \rightarrow A_2 A_1 Z / A_2 A_1$

sub A_1 in A_2

$A_2 \rightarrow aZ A_2 / a A_2$

sub A_2 in Z

$Z \rightarrow aZA_2 A_1 Z / aA_2 A_1 Z / aZA_2 A_1 / aA_2 A_1$

Finally the GNF productions are

$A_1 \rightarrow aZ$

$A_2 \rightarrow aZA_2 / aA_2$

$Z \rightarrow aZA_2 A_1 Z / aA_2 A_1 Z / aZA_2 A_1 / aA_2 A_1$

* $A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 / b$

$A_3 \rightarrow A_1 A_2 / a$

i)

Scanned by CamScanner

02/03/18

* Convert the given grammar into CNF

$S \rightarrow AACD$

$A \rightarrow aAb / \epsilon$

$C \rightarrow aC / a$

$D \rightarrow aDa / bab / \epsilon$

Sol :- Not minimized

1. eliminate ϵ productions, two ϵ productions

$A \rightarrow \epsilon$, $D \rightarrow \epsilon$

$S \rightarrow AACD / ACD / CD / AAC / AC / C$

$A \rightarrow aAb / ab$

$C \rightarrow aC / a$

$D \rightarrow aDa / bDb / aa / bb$

2. eliminate unit productions.

$S \rightarrow C$

$S \rightarrow AACD / ACD / CD / AC / AAC / aC / a$

$A \rightarrow aAb / ab$

$C \rightarrow aC / a$

$D \rightarrow aDa / bDb / aa / bb$

3. useless productions

No useless productions

* $S \rightarrow AACD$

$S \rightarrow XY$

$X \rightarrow AA$

* $Y \rightarrow CD$

* $S \rightarrow ACD$

$S \rightarrow AY$

* $S \rightarrow AAC$

$S \to XC$

$S \to aC$

$S \to C_aC$

$C_a \to a$

$A \to aAb$

$A \to C_aAC_b$

$C_b \to b$

$A \to ZC_b$

$Z \to C_aA$

$A \to ab$

$A \to C_aC_b$

$C \to aC$

$C \to C_aC$

$D \to aDa$

$D \to C_aDC_a$

$D \to PC_a$

$P \to C_aD$

$D \to bDb$

$D \to C_bDC_b$

$D \to QC_b$

$Q \to C_bD$

$D \to aa$

$D \to C_aC_a$

$D \to bb$

$D \to C_bC_b$

Finally CNF of given grammar is

$S \to XY / AY / XC / CD / AC / C_aC / a.$

$X \to AA$

$Y \to CD$

$A \to$

* Convert the given grammar into greibach normal form

$S \to AB$

$A \to BS / b$

$B \to SA / a$

Sol:-

&. Give grammar is minimized

Sub A in S.

$S \to BSB / bB$.

Sub S in B.

$\underset{A}{B} \to \underset{A}{B}\underset{\alpha}{SBA} / \underset{\beta_1}{bBA} / \underset{\beta_2}{a}$

$A \to A\alpha / \beta_1 / \beta_2$

$A \to \beta_1 A' / \beta_2 A'$

$A' \to \alpha A' / \epsilon$

$B \to bBAZ / aZ / bBA / a$ sub A in S

sub S in Z

$Z \to SBAZ / SBA$

Sub B in A

$A \to bBAZS / aZS / bBAS / aS / b$

sub A in S

$S \to bBAZSB / aZSB / bBASB / aSB / bB$.

sub S in Z

$Z \to bBAZSBBAZ / aZ\ bBAZSBBA / aZSBBAZ /$

$aZSBBA / bBASBBAZ / bBASBBA / aSBBAZ /$

$aSBBA / bBBAZ / bBBA$

The GNF of given grammar is

$S \rightarrow bBAZSB|$

Pumping Lemma principle for Context free Languages: 06/03/18

Pumping lemma principle is based on pigeon hole principle & it is used to show give language is not a context free language.

In pumping lemma principle we are considering a string $z = pqrst$

where $|prs| \leq n$

$|qs| \geq 1$

and we are considering $z = pq^i r s^i t \quad \forall i \geq 0$

for any i value if $z \notin L$ (L is a given language) then the language is not a context free language by using pumping lemma principle

* Check whether given language is context free language (or) not by using pumping lemma principle

$L = \{a^n b^n c^n / n \geq 0\}$

Sol: Given $L = \{a^n b^n c^n / n \geq 0\}$

$L = \{\epsilon, abc, aabbcc, aaabbbccc, \ldots\}$

$z = \underline{a}_p\underline{a}_q\underline{bb}_r\underline{c}_s\underline{c}_t = pqrst$

$z = pq^i r s^i t$

$i = 1 \Rightarrow z = pq^1 r s^1 t = aabbcc \in L$

$i = 2 \Rightarrow z = pq^2 r s^2 t = aaabbccc \notin L$

$\therefore z \notin L$

$\therefore$ Given language is not a context free language by using pumping lemma principle

* Show that given language is not a context free language by using pumping lemma principle

$L = \{a^i b^j c^k / i < j \ \& \ j < k\}$
$i, j, k \geq 1$

Sol: Given $L = \{a^i b^j c^k / i < j \ \& \ j < k\}$
$i, j, k \geq 1$

$L = \{abbccc, aabbbcccc, \ldots\}$

$Z =$

$i=1 \Rightarrow Z = pq^i r s^i t = aabbbcccc \in L$

$i=2 \Rightarrow Z = pq^i r s^i t = aaabbbcccccc \notin L$

Properties of context Free Languages:

1. Context Free languages are closed under Union, if L_1 is CFL & L_2 is CFL then $L_1 \cup L_2$ is also a context Free language.
2. Context Free languages are closed under concatenation operation if L_1 is CFL & L_2 is CFL then $L_1 \cdot L_2$ is also a context Free language.
3. Context Free languages are closed under closure operation, if L is a CFL the L^* is also a context Free language
4. Context Free languages are not closed under intersection operation if L_1 is a CFL & L_2 is a CFL then $L_1 \cap L_2$ is always not a context Free language.
5. Content Free languages are not closed under complementation operation, if L is CFL then its complement $\bar{L}$ is not a Context Free language.