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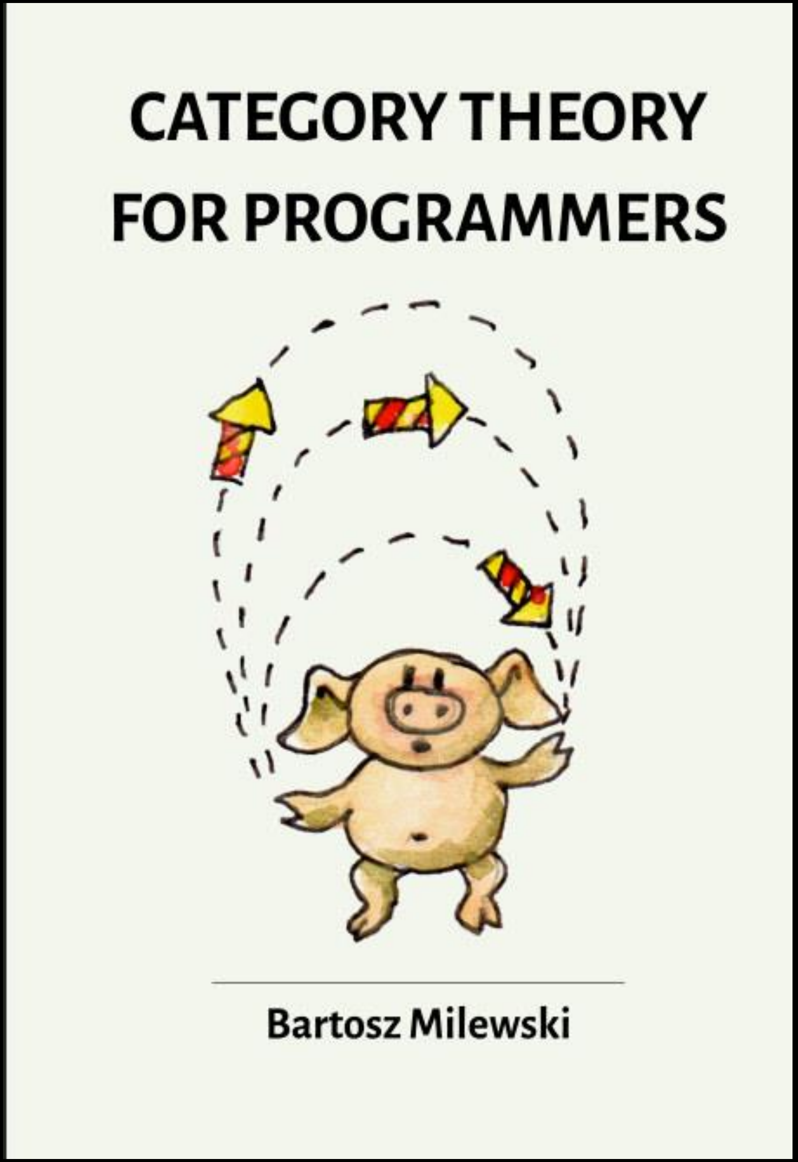
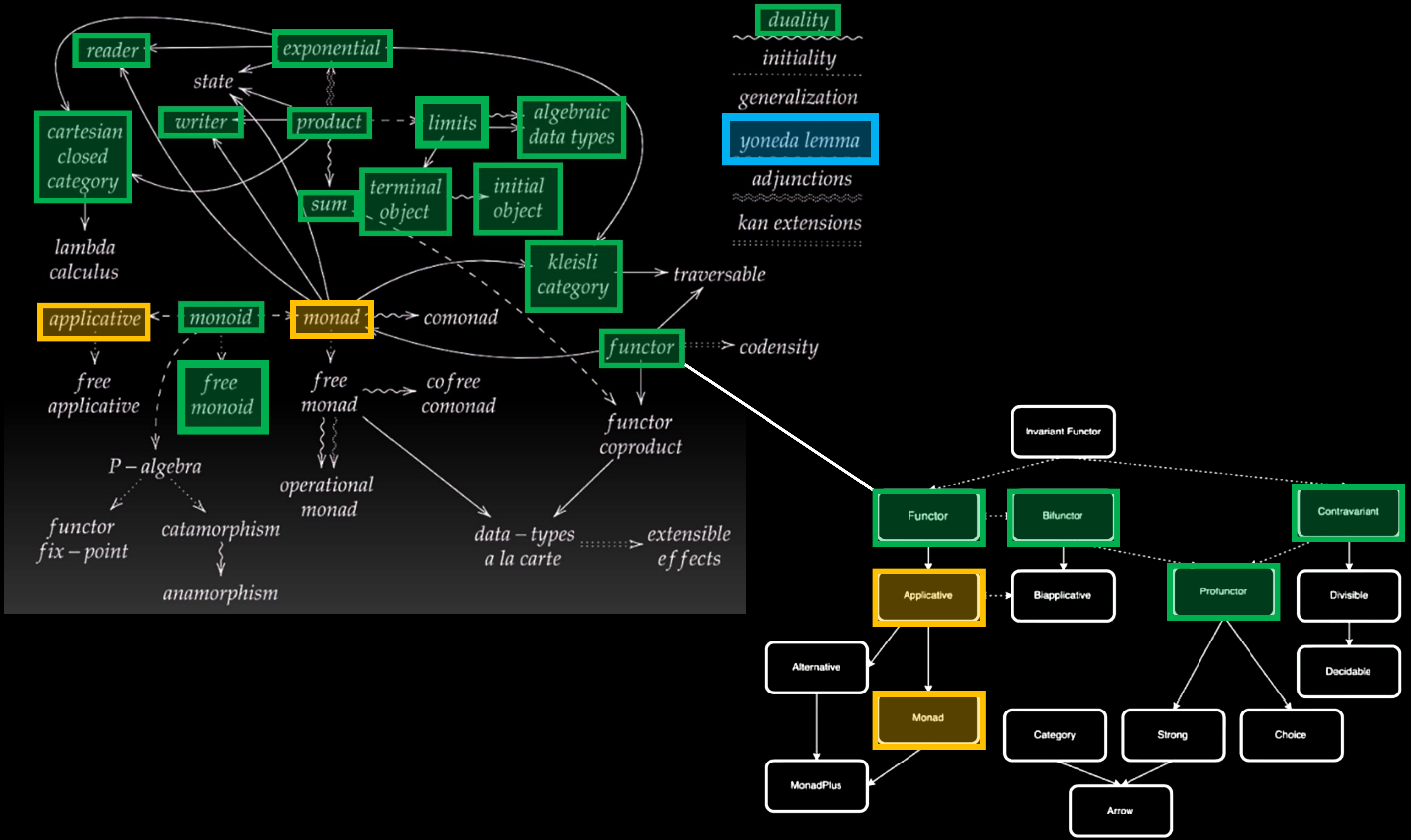


Category Theory for Programmers

Chapter 15: The Yoneda Lemma

The Tools for Thought



Most constructions in category theory are generalizations of results from other more specific areas of mathematics. Things like products, coproducts, monoids, exponentials, etc., have been known long before category theory. They might have been known under different names in different branches of mathematics. A Cartesian product in set theory, a meet in order theory, a conjunction in logic — they are all specific examples of the abstract idea of a categorical product.

The setting for the Yoneda lemma is an arbitrary category **C** together with a functor F from **C** to **Set**. We've seen in the previous section that some **Set**-valued functors are representable, that is isomorphic to a hom-functor. The Yoneda lemma tells us that all **Set**-valued functors can be obtained from hom-functors through natural transformations, and it explicitly enumerates all such transformations.

The Yoneda lemma tells us that a natural transformation between a hom-functor and any other functor F is completely determined by specifying the value of its single component at just one point! The rest of the natural transformation just follows from naturality conditions.

$$F :: C \to Set$$

$$\alpha :: [C, Set](C(a,-), F) \simeq Fa$$

$$\begin{tikzcd}
a \arrow[d, "f"'] \arrow[r] \arrow[dr] & C(a,a) \arrow[r, "C(a,f)"] \arrow[d, "\alpha_x"] & C(a,y) \arrow[d, "\alpha_y"] \\
y & Fa \arrow[r, "Ff"] & Fy
\end{tikzcd}$$

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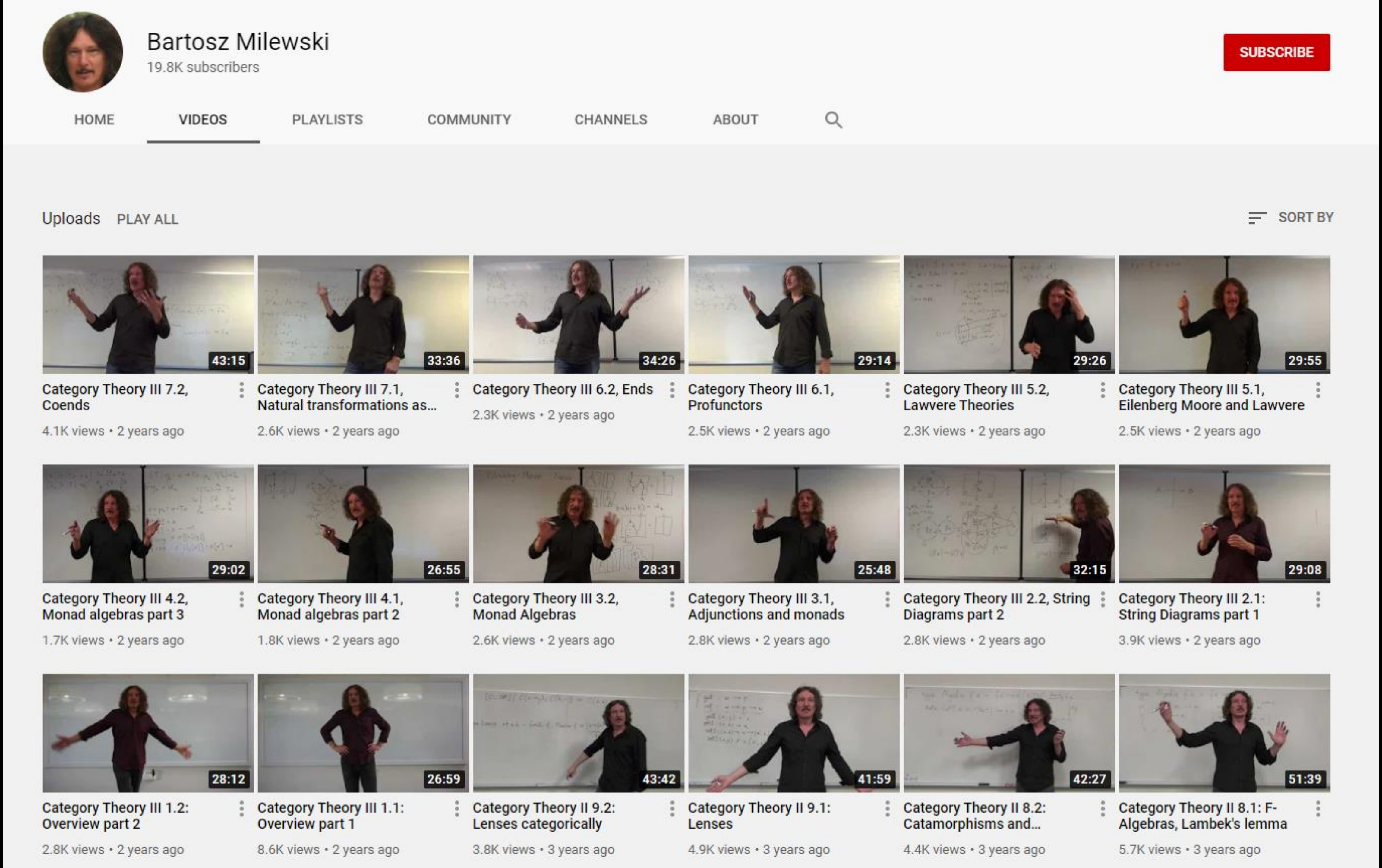
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