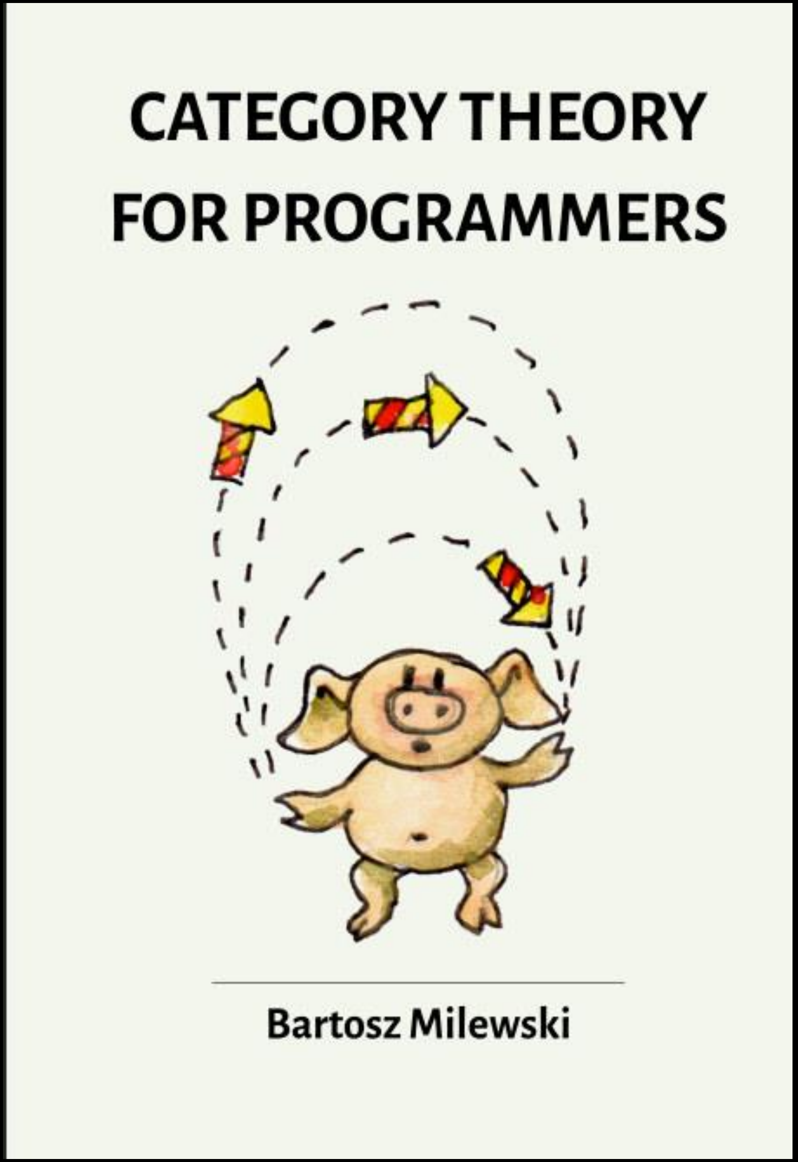
Java

Meetup

Friendly Environment Policy

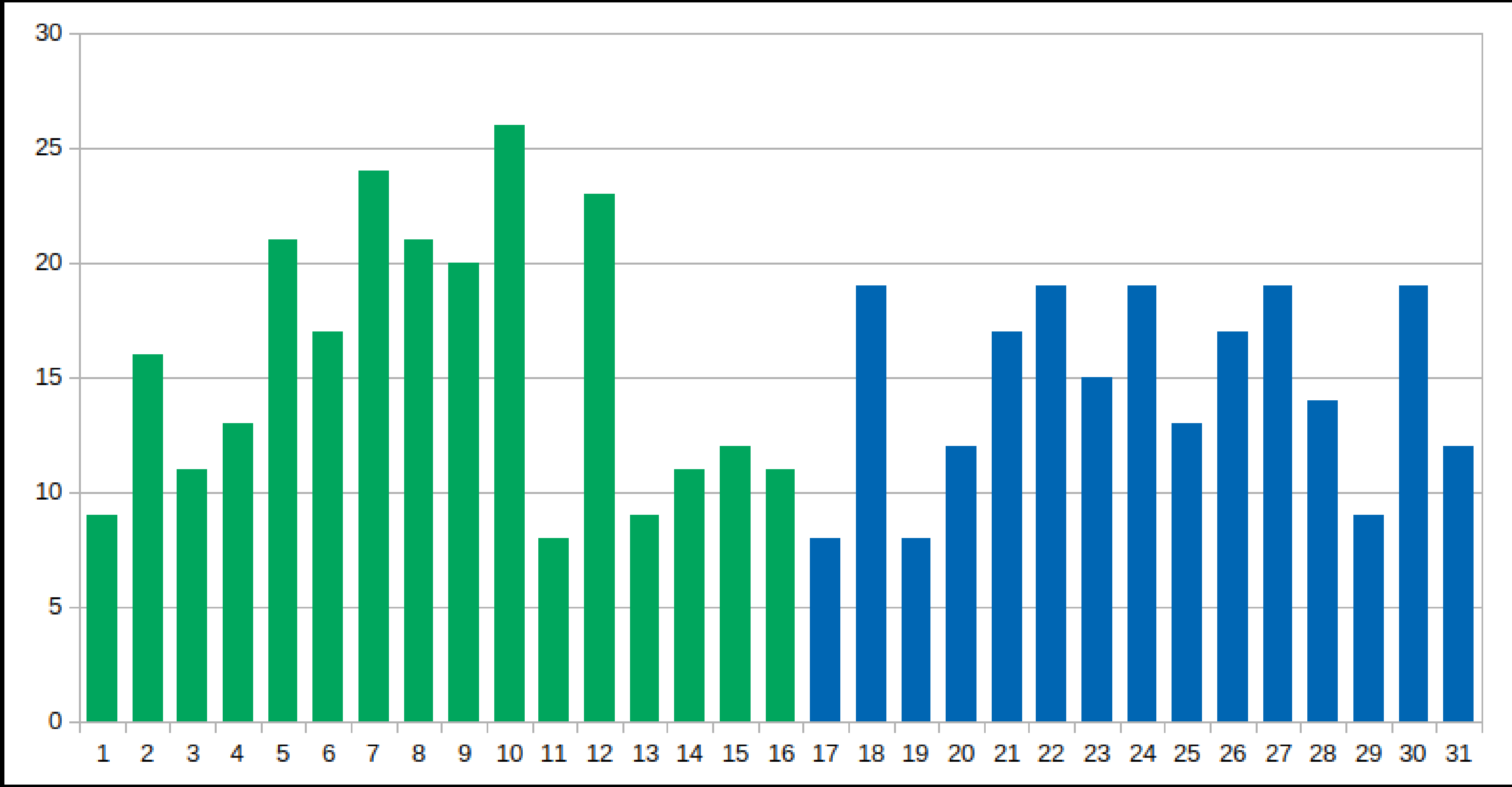
Berlin Code of Conduct

DISCORD



Category Theory for Programmers

Chapter 16: The Yoneda Embedding


30
25
20
15
10
5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

16 Yoneda Embedding **243**
16.1 The Embedding 246
16.2 Application to Haskell 247
16.3 Preorder Example 248
16.4 Naturality 250
16.5 Challenges 251

$$[C, Set](C(a,-), F) \simeq \underline{F}\,\underline{a}$$

$$\text{forall } x \,.\, (a \to x) \to F\,x \simeq F\,a$$

$$\alpha :: (a \to x) \to [x] \simeq [a]$$

$$\alpha\ f = \text{fmap}\ f\ [a_0 \,..\, a_n]$$

WE'VE SEEN PREVIOUSLY that, when we fix an object a in the category $\mathbf{C}$, the mapping $\mathbf{C}(a, -)$ is a (covariant) functor from $\mathbf{C}$ to $\mathbf{Set}$.

$$x \to \mathbf{C}(a, x)$$

(The codomain is $\mathbf{Set}$ because the hom-set $\mathbf{C}(a, x)$ is a *set*.) We call this mapping a hom-functor — we have previously defined its action on morphisms as well.

Now let's vary a in this mapping. We get a new mapping that assigns the hom-*functor* $\mathbf{C}(a, -)$ to any a.

$$a \to \mathbf{C}(a, -)$$

16.1 The Embedding

The (contravariant) functor we have just described, the functor that maps objects in $\mathbf{C}$ to functors in $[\mathbf{C}, \mathbf{Set}]$:

$$a \to \mathbf{C}(a, -)$$

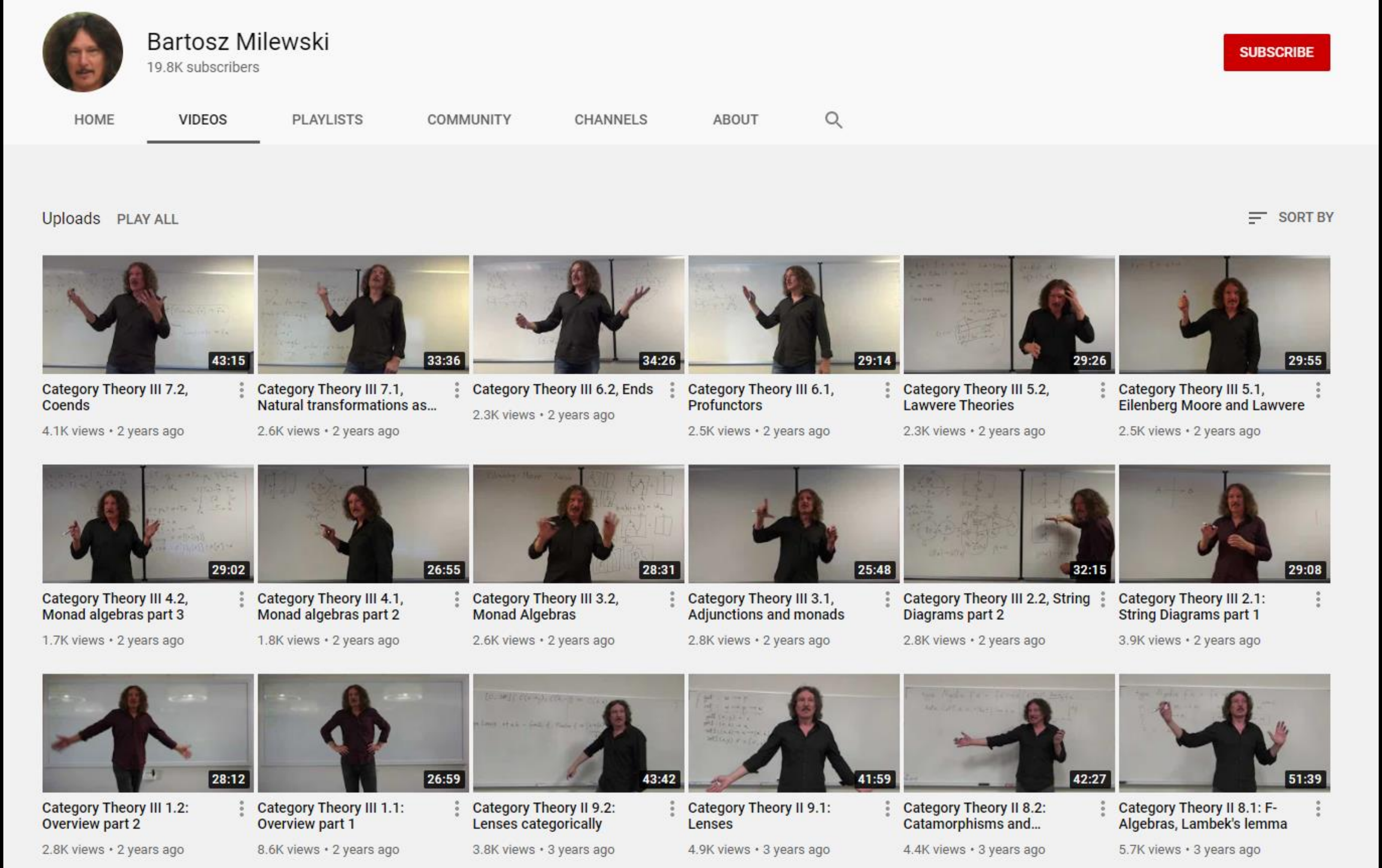
defines the *Yoneda embedding*. It *embeds* a category $\mathbf{C}$ (strictly speaking, the category $\mathbf{C}^{op}$, because of contravariance) inside the functor category $[\mathbf{C}, \mathbf{Set}]$. It not only maps objects in $\mathbf{C}$ to functors, but also faithfully preserves all connections between them.

16.2 Application to Haskell

In Haskell, the Yoneda embedding can be represented as the isomorphism between natural transformations amongst reader functors on the one hand, and functions (going in the opposite direction) on the other hand:

```
forall x. (a -> x) -> (b -> x) ≅ b -> a
```

(Remember, the reader functor is equivalent to `((->) a)`.)


Bartosz Milewski
19.8K subscribers
SUBSCRIBE
HOME
VIDEOS
PLAYLISTS
COMMUNITY
CHANNELS
ABOUT
Uploads
PLAY ALL
SORT BY
43:15
Category Theory III 7.2, Coends
4.1K views • 2 years ago
33:36
Category Theory III 7.1, Natural transformations as...
2.6K views • 2 years ago
34:26
Category Theory III 6.2, Ends
2.3K views • 2 years ago
29:14
Category Theory III 6.1, Profunctors
2.5K views • 2 years ago
29:26
Category Theory III 5.2, Lawvere Theories
2.3K views • 2 years ago
29:55
Category Theory III 5.1, Eilenberg Moore and Lawvere
2.5K views • 2 years ago
29:02
Category Theory III 4.2, Monad algebras part 3
1.7K views • 2 years ago
26:55
Category Theory III 4.1, Monad algebras part 2
1.8K views • 2 years ago
28:31
Category Theory III 3.2, Monad Algebras
2.6K views • 2 years ago
25:48
Category Theory III 3.1, Adjunctions and monads
2.8K views • 2 years ago
32:15
Category Theory III 2.2, String Diagrams part 2
2.8K views • 2 years ago
29:08
Category Theory III 2.1: String Diagrams part 1
3.9K views • 2 years ago
28:12
Category Theory III 1.2: Overview part 2
2.8K views • 2 years ago
26:59
Category Theory III 1.1: Overview part 1
8.6K views • 2 years ago
43:42
Category Theory II 9.2: Lenses categorically
3.8K views • 3 years ago
41:59
Category Theory II 9.1: Lenses
4.9K views • 3 years ago
42:27
Category Theory II 8.2: Catamorphisms and...
4.4K views • 3 years ago
51:39
Category Theory II 8.1: F-Algebras, Lambek's lemma
5.7K views • 3 years ago

Java

Meetup