



COC Berlin Code of Conduct





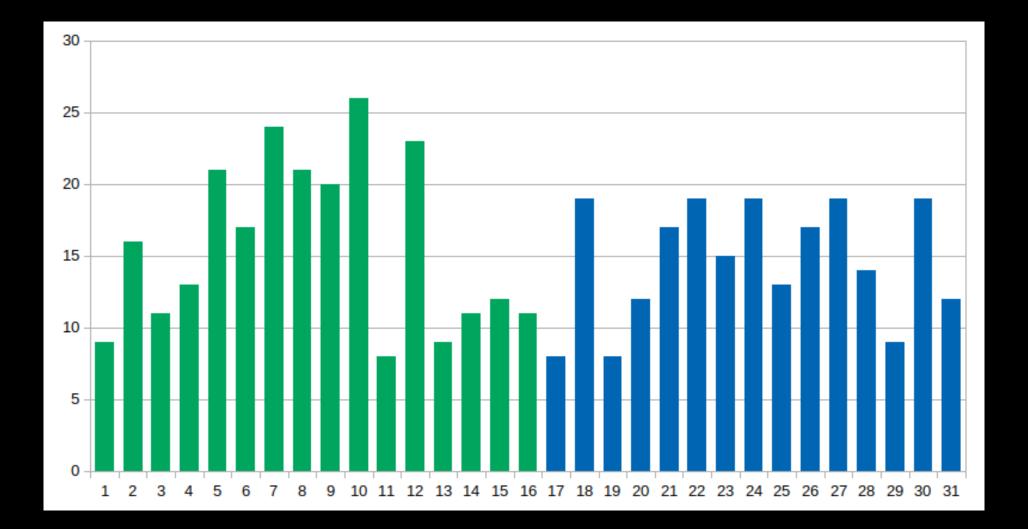
## CATEGORY THEORY FOR PROGRAMMERS



Bartosz Milewski

## Category Theory for

## Programmers



Main Concepts	Supporting Concepts	
Universal Construction (technique)		
A) Objects	Naturality Condition	
B) Morphisms Also known as "arrows"	Isomorphism	
C) <b>Category</b> (A + B)	Hom-Set	
D) <b>Functor</b> (C + B) Bifunctor, Profunctor, Contravariant Functor Product, Coproduct    Maybe, List, Reader	Hom-Functor	
E) Natural Transformation (D + B)	Yoneda Lemma	
	Yoneda Embedding	

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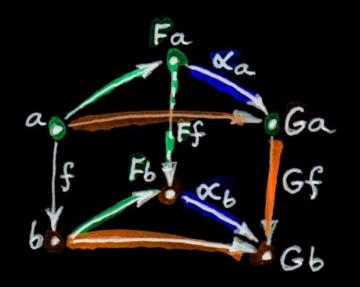
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O FAR I've been glossing over the meaning of function types. A function type is different from other types.

Take Integer, for instance: It's just a set of integers. Bool is a two element set. But a function type  $a \to b$  is more than that: it's a set of morphisms between objects a and b. A set of morphisms between two objects in any category is called a hom-set. It just so happens that in the category **Set** every hom-set is itself an object in the same category —because it is, after all, a *set*.

the category. We don't want to make artificial connections between objects. So it's *natural* to use existing connections, namely morphisms. A natural transformation is a selection of morphisms: for every object a, it picks one morphism from Fa to Ga. If we call the natural transformation  $\alpha$ , this morphism is called the *component* of  $\alpha$  at a, or  $\alpha_a$ .

$$\alpha_a :: Fa \to Ga$$



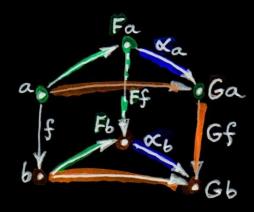
Now we have two ways of getting from Fa to Gb. To make sure that they are equal, we must impose the *naturality condition* that holds for any f:

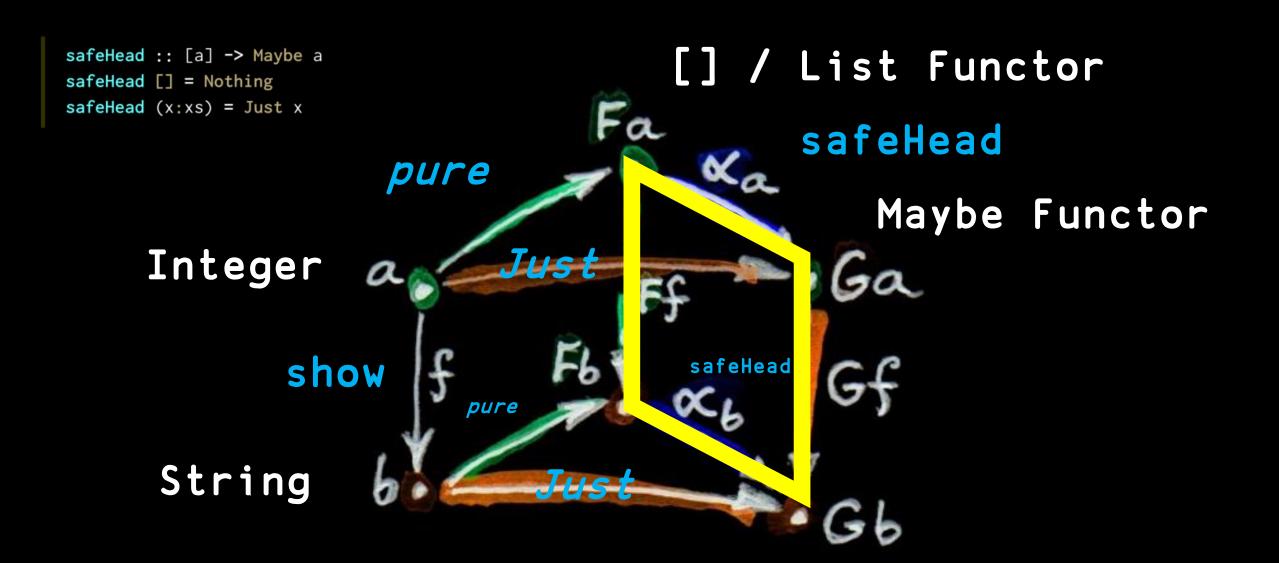
$$Gf \circ \alpha_a = \alpha_b \circ Ff$$

Chapter 10: Natural Transformations, Page 157

Let's see a few examples of natural transformations in Haskell. The first is between the list functor, and the Maybe functor. It returns the head of the list, but only if the list is non-empty:

```
safeHead :: [a] -> Maybe a
safeHead [] = Nothing
safeHead (x:xs) = Just x
```





Chapter 10: Natural Transformations, Page 162

Chapter 11-16 // TODO

