CIV499 - Thesis Presentation

Application of service level requirement and mixed integer programming for predicting optimal inventory rebalancing route for bike sharing systems

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Outline

- Introduction & Background
- Literature Reviews
- Research Question
- Model Design and Methods
- Case Study
- Findings
- Model Limitation & Potential Errors
- Future Works & Research
- Key Takeaways & Learning

Introduction & Background

Bike sharing around the world

- First introduced in Netherlands in summer 1965
- Largest bike sharing system - Hangzhou
- Until April 2013, 500+ cities
- 535+ bike-sharing programs (375 programs, 2011)
- Volume of 517,000 bikes (236,000 bikes, 2011)

Generation of Bike sharing systems

- First generation – White Bikes systems
- Second generation – Coin deposit systems
- Third generation – Information technology based systems
- Forth generation – Demand responsive, multimodal systems

Bike sharing systems in Canada

- Montreal, Toronto, Ottawa, Vancouver
- January 2014, Bixi filed bankruptcy protection
- Bike Share Toronto is launched in April 2014

Literature Review

Past studies on Bike Sharing Systems in different cities:

Hangzhou

– Early Adoption and Behavioral Responses

Vancouver

–Seasonal Autoregression Model Using Weather Variables

Montreal

– Impact Evaluation of a Public Bicycle Share Program

Tel Aviv, Israel

– Optimal Inventory Management

Boston & Washington DC

– Inventory Rebalancing and Vehicle Routing

London

– Health Impact Modelling Study

Research Question

How to reduce operation cost by following the minimum travelling route at the same time maintain a high service quality for users?

Bike station inventory rebalancing model

Part I: Service Level Requirement Bounds (application)

Find bike return and pickup rate

Generate service level requirement bounds

- M/M/1/K queuing system, Morse (1958, p. 64)

Calculate non-self-sufficient probability matrix P(i, t)

Part II: Minimum Travelling Routing Problem

Case 1) With real time bike station status information

- Decide time to initiate rebalancing
- Set of stations to visit
- Use TSP algorithm to find the optimal route

Case 2) Without real time bike station status information

- Predict optimal set of stations to visit at certain time
- Use TSP algorithm to find the optimal route

Model Variables and Parameters

4.1 Variables:

$x(i,t) - a\ binary\ variable\ represents\ whether\ a\ station\ i\ will\ be\ rebalance\ at\ time\ t$

$y(t) - a\ binary\ varianel\ represents\ whether\ there\ will\ be\ a\ rebalace\ at\ time\ t$

4.2 Parameters:

$d(i,t)$ – distance between station i and j; 0 is the starting location, not a bike station

$p(i,t) - probability\ that\ station\ i\ at\ time\ t\ is\ out\ of\ service\ bounds$

n – total number of stations in the bike sharing system

lb – minimum number of stations to visit by the maintenance truck

ub – maximum number of station to visit by the maintenance truck

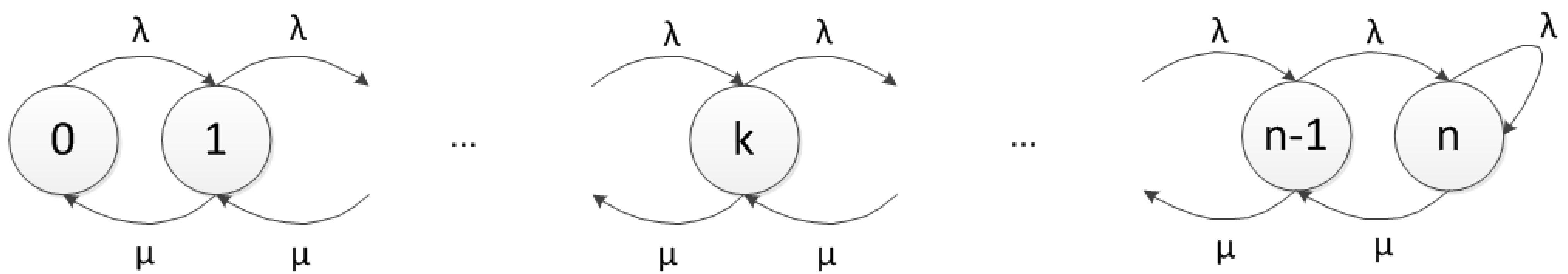
t – time in hours

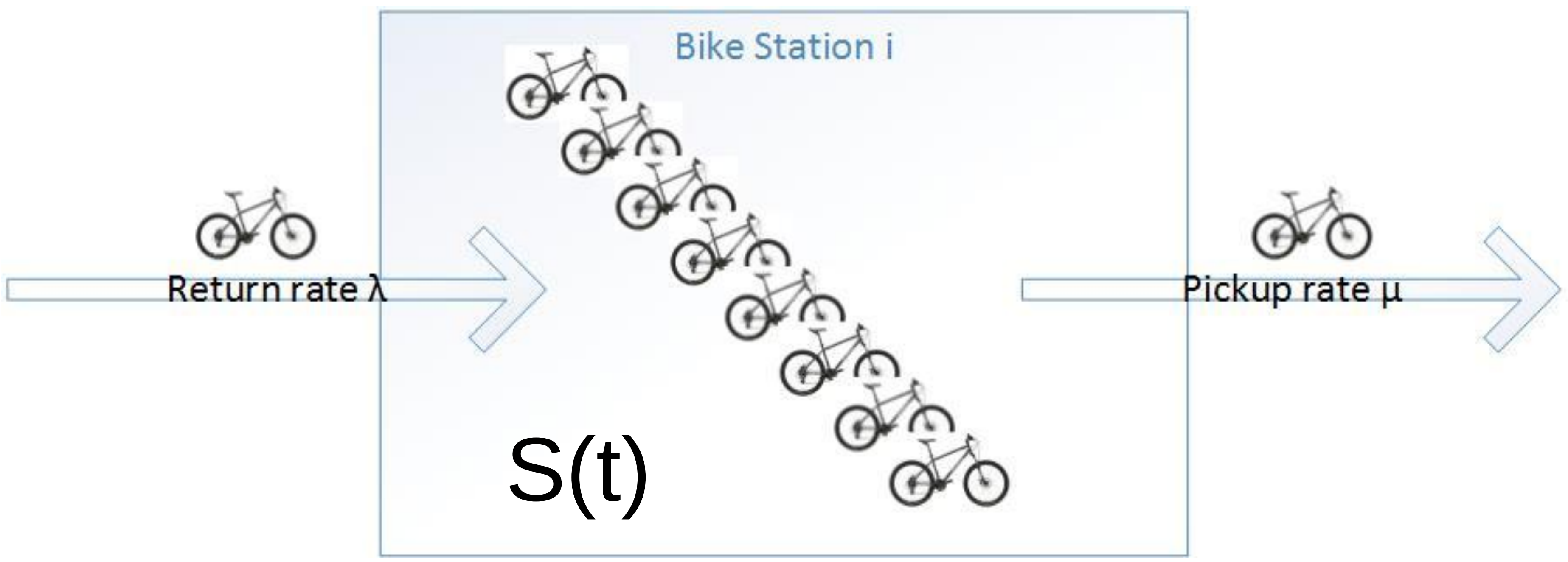
$\beta_{sla} - service\ level\ requirement\ to\ initiate\ rebalancing$

$M_{min} - minimum\ number\ of\ rebalancing\ allowed\ in\ a\ day$

$M_{max} - mimum\ numer\ of\ rebalancing\ allowed\ in\ a\ day$

M/M/1/K Queuing System





Part I: Service Level Requirement Bounds

There are two criteria introduced for customer's satisfaction of a specific bike station i :

$$\frac{E[Satisfied\ bike\ pickup\ requests]}{E[Total\ bike\ pickup\ requests]} \geq \beta_i^{pickup}$$

$$\frac{E[Satisfied\ bike\ return\ requests]}{E[Total\ bike\ return\ requests]} \geq \beta_i^{return}$$

$\beta_i^{pickup}, \beta_i^{return} \in [0,1]$

$$p_i(s,\sigma,t) = \Pr\left(S_i(t) = \sigma \mid S_i(0) = s\right)$$

$$E[Satisfied\ bike\ pickup\ requests] = \int_0^T \mu_i.(1 - p_i(s,0,t))dt$$

$$E[Total\ bike\ pickup\ requests] = \mu_i T$$

Part I: Service Level Requirement Bounds (cont.)

$$\frac{E[Satisfied\ bike\ pickup\ requests]}{E[Total\ bike\ pickup\ requests]} = \frac{\int_0^T \mu_i \cdot (1 - p_i(s,0,t))dt}{\mu_i T}$$

$$= 1 - \frac{1}{T}\int_0^T p_i(s,0,t)dt$$

$$= 1 - g_i(s,0) \geq \beta_i^{pickup}$$

And similarly we could get

$$\frac{E[Satisfied\ bike\ return\ requests]}{E[Total\ bike\ return\ requests]} = 1 - g_i(s,K_i) \geq \beta_i^{return}$$

$$s_i^{min} = \min\{ s \in \{0, \dots, K_i\}: 1 - g_i(s,0) \geq \beta_i^{pickup}\}$$

$$s_i^{max} = \max\{ s \in \{0, \dots, K_i\}: 1 - g_i(s,K_i) \geq \beta_i^{return}\}$$

Non-self-sufficient Probability Matrix

$$s_i^{min} = \min\{\, s \in \{0, \dots, K_i\}: 1 - g_i(s,0) \geq \beta_i^{pickup}\}$$

$$s_i^{max} = \max\{\, s \in \{0, \dots, K_i\}: 1 - g_i(s,K_i) \geq \beta_i^{return}\}$$

$$P(i,t) = \begin{bmatrix} p(0,1) & p(0,2) & \cdots & p(0,22) & p(0,23) \\ \vdots & & \ddots & & \vdots \\ p(n,1) & p(n,2) & \cdots & p(n,22) & p(n,23) \end{bmatrix}$$

$$i \in [1, \dots, n] \qquad t \in [0, \dots, 23]$$

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Minimum Travelling Route Problem

Case 1) With real time bike station status information

- Decide time to initiate rebalancing
- Set of stations to visit
- Use TSP algorithm to find the optimal route

Case 2) Without real time bike station status information

- Predict optimal set of stations to visit at certain time
- Use TSP algorithm to find the optimal route

Operator with real time bike station status information

Decide time to initiate rebalancing & Set of stations to visit

- Management decision
- Company policy
- Availability of maintenance trucks and operator
- Metrics or measures using probability matrix

Use TSP algorithm to find the optimal route

Simplified model to find minimum travelling route

Travelling Salesman Problem

Objective: Minimizing total routing distance of maintenance truck

$$\sum_{i \in V(t)} \sum_{i \neq j, j \in V(t)} d(i,j) * k(i,j)$$

Subject to:

$$\sum_{i \neq j, i \in V(t)} k(i,j) = 1$$

$$\sum_{i \neq j, j \in V(t)} k(i,j) = 1$$

$$u_i - u_j + card[V(t)] * x(i,j) \leq card[V(t)] - 1 \quad i \neq j; i,j \in V(t)$$

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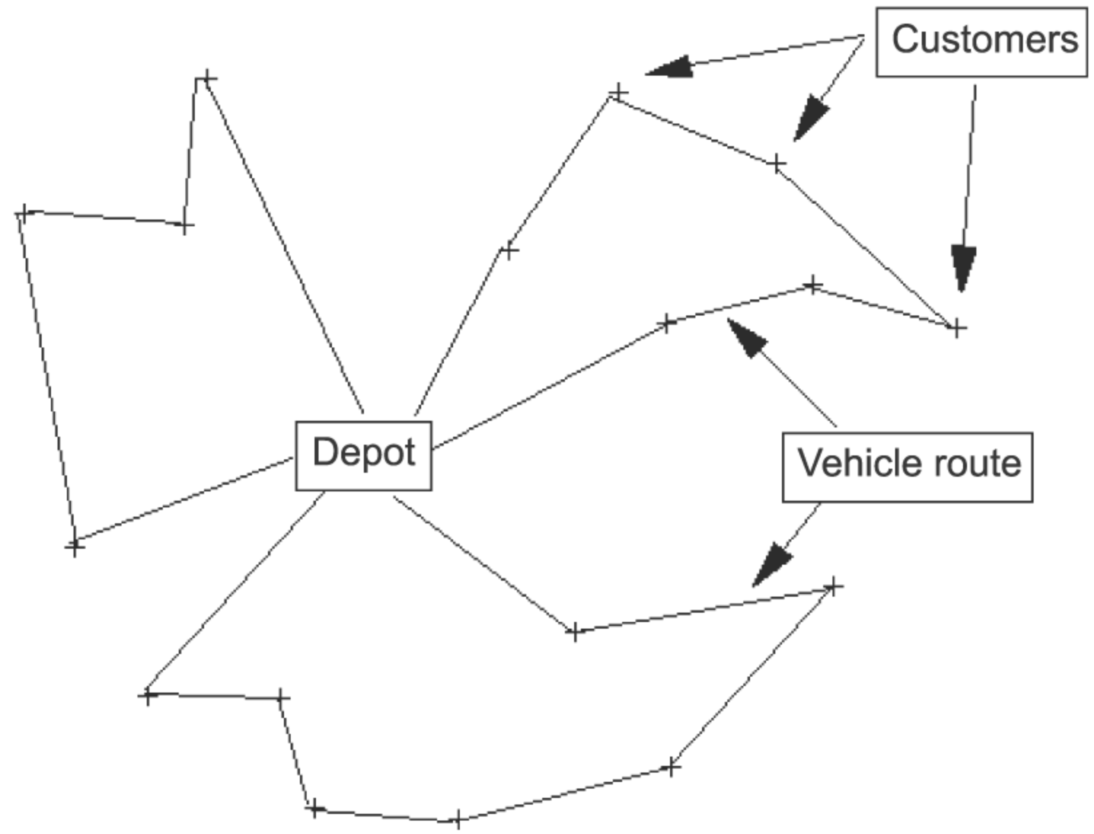
Operator without real time bike station status information

Predict optimal set of stations to visit at certain time

1. *Based on given parameters (beta, lb, Mmin, etc.)*
2. *Non-self-sufficient probability matrix*
3. *Vehicle routing with stochastic demand*
4. *Finding expected minimum travelling route*
5. *Decide the set of stations to visit at time t*

Use TSP algorithm to find the optimal route

Vehicle routing with stochastic demand



$$E[L_\tau^h] = \sum_{i=1}^{n} d(0,i)p_i \prod_{r=1}^{i-1}(1-p_r)$$

$$+ \sum_{i=1}^{n} d(i,0)p_i \prod_{r=i+1}^{n}(1-p_r)$$

$$+ \sum_{i=1}^{n} \sum_{j=i+1}^{n} d(i,j)p_i p_j \prod_{r=i+1}^{j-1}(1-p_r)$$

MIP model for optimal set of stations to visit at time t

Objective: minimize expected routing length

$$L = \sum_{t=0}^{23}\sum_{j=1}^{n} x(j,t)d(0,i)p(j,t)\prod_{r=1}^{i-1}(1-p(r,t)) + \sum_{t=0}^{23}\sum_{i=1}^{n} x(i,t)d(i,0)\,p(i,t)\prod_{r=i+1}^{n}(1-p(r,t))$$

$$+\sum_{t=0}^{23}\sum_{i=1}^{n}\sum_{j=i+1}^{n} x(i,t)x(j,t)d(i,j)\,p(i,t)p(j,t)\prod_{r=i+1}^{j-1}(1-p(r,t))$$

$$y_t = \{0,1\} \quad \forall\, t = 0,\dots,23$$

Subject to:

$$x(i,t) = \{0,1\} \quad \forall\, i = 1,\dots,n;\; t = 0,\dots,23$$

$$x(0,t) = 1 * y(t)$$

$$\forall\, t = 0,\dots,23 \quad (maintenance\ truck\ leaves\ the\ base)$$

$$y(t) * lb \le \sum_{i=1}^{n} x(i,t) \le y(t) * ub$$

$$\forall\, t = 0,\dots,23 \quad (limit\ on\ number\ of\ stations\ to\ visit\ by\ maintence\ truck)$$

$$M_{min} \le \sum_{t=0}^{23} y(t) \le M_{max}$$

$$(number\ of\ maintenance\ per\ day)$$

$$x(i,t) * p(i,t) \ge x(i,t) * \beta_{sla}$$

$$\forall\, i = 1,\dots,n\; t = 0,\dots,23 \quad (rebalancing\ condition)$$

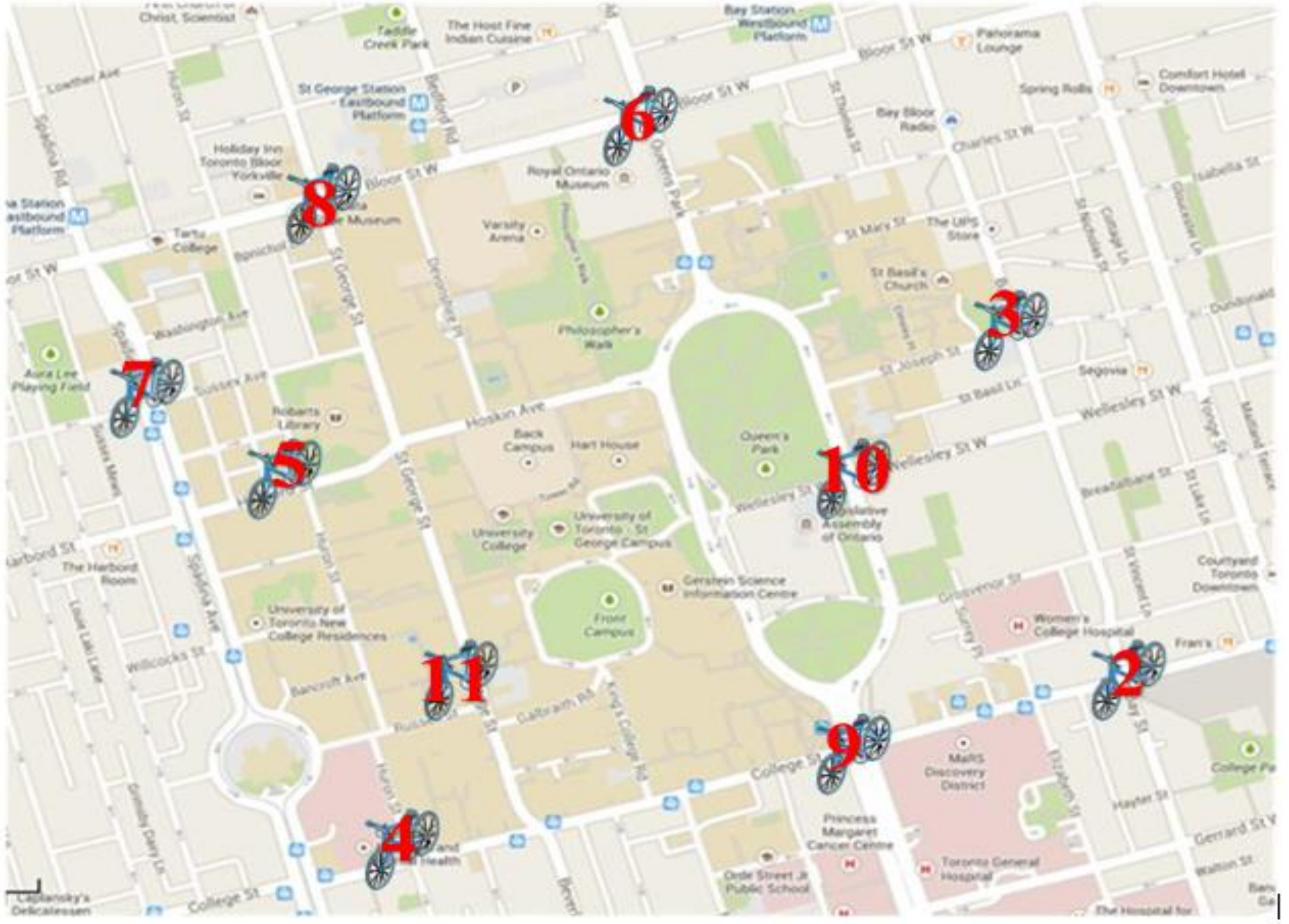
Case Study on 10 bike stations


```
<stations lastUpdate="1398050281329" version="2.0">
  <station>
    <id>1</id>
    <name>Jarvis St/ Carleton St</name>
    <terminalName>7055</terminalName>
    <lastCommWithServer>1398049564282</lastCommWithServer>
    <lat>43.66207</lat>
    <long>-79.37617</long>
    <installed>true</installed>
    <locked>false</locked>
    <installDate/>
    <removalDate/>
    <temporary>false</temporary>
    <public>true</public>
    <nbBikes>3</nbBikes>
    <nbEmptyDocks>12</nbEmptyDocks>
    <latestUpdateTime>1398047154967</latestUpdateTime>
  </station>
```

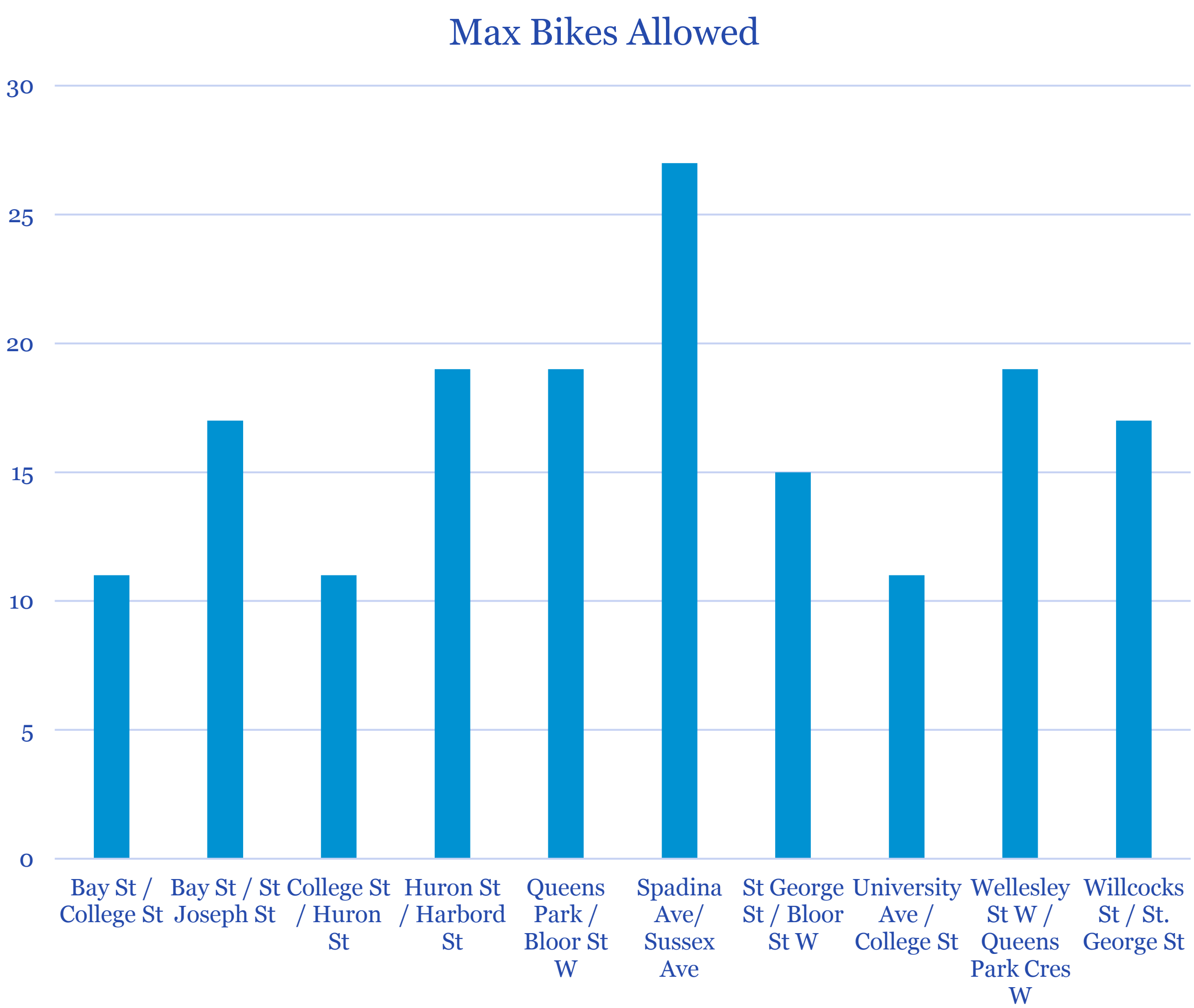

Data source

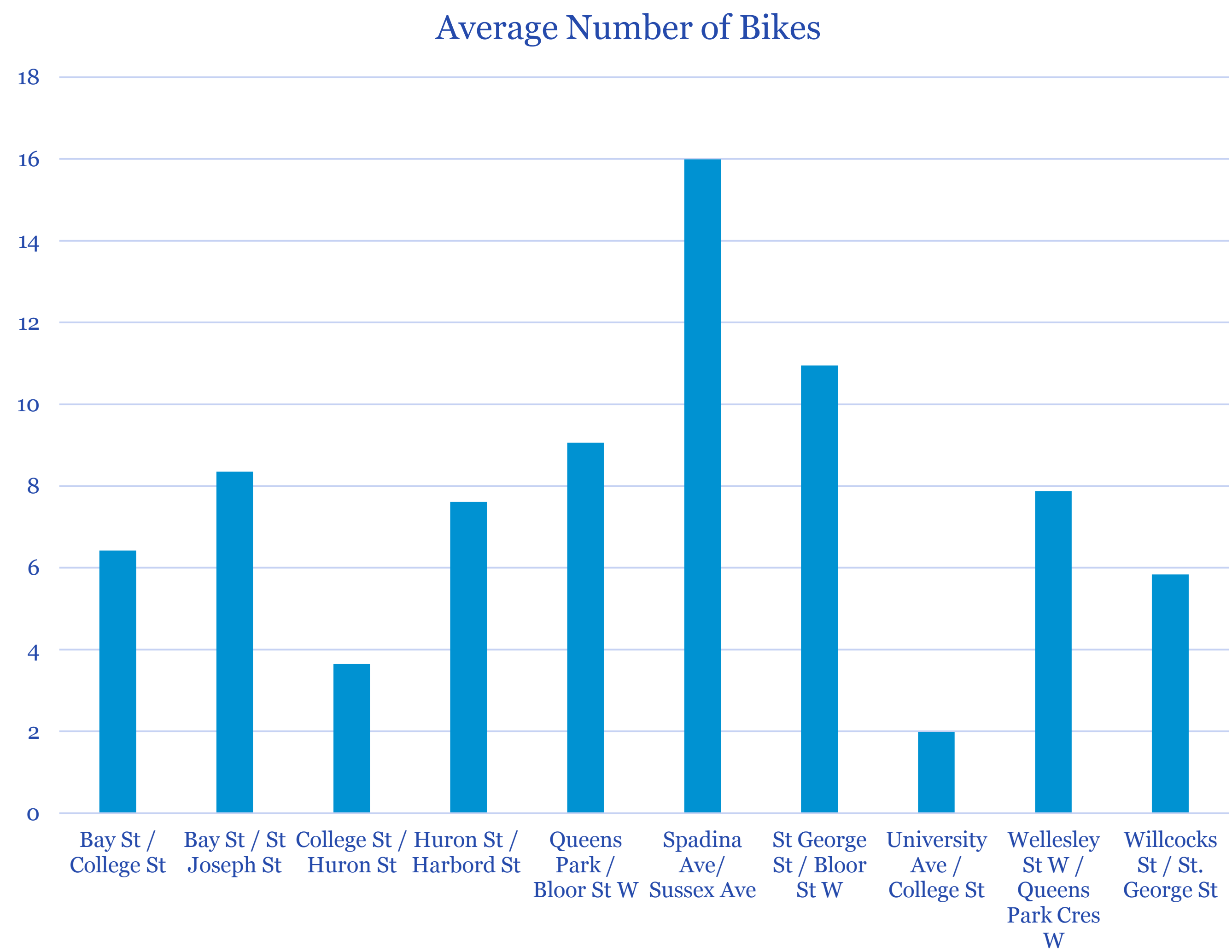
- Open Data Toronto
- Montreal's Bike sharing system data
- Simulation

Bike Stations	Number Assigned
Base	1
Bay St / College St	2
Bay St / St Joseph St	3
College St / Huron St	4
Huron St / Harbord St	5
Queens Park / Bloor St W	6
Spadina Ave/ Sussex Ave	7
St George St / Bloor St W	8
University Ave / College St	9
Wellesley St W / Queens Park Cres W	10
Willcocks St / St. George St	11



Bike Station Status Information





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Return and Pickup rate simulation

Return and Pickup rate

Hourly	Hourly Return	Hourly Pickup
mean	2.044172932	2.025898079
Variance	1.464381685	1.292726396

$$\lambda_{i,Toronto} = \lambda_{i,Montreal} + stdev(\lambda_{i,Montreal}) * \varepsilon_i$$

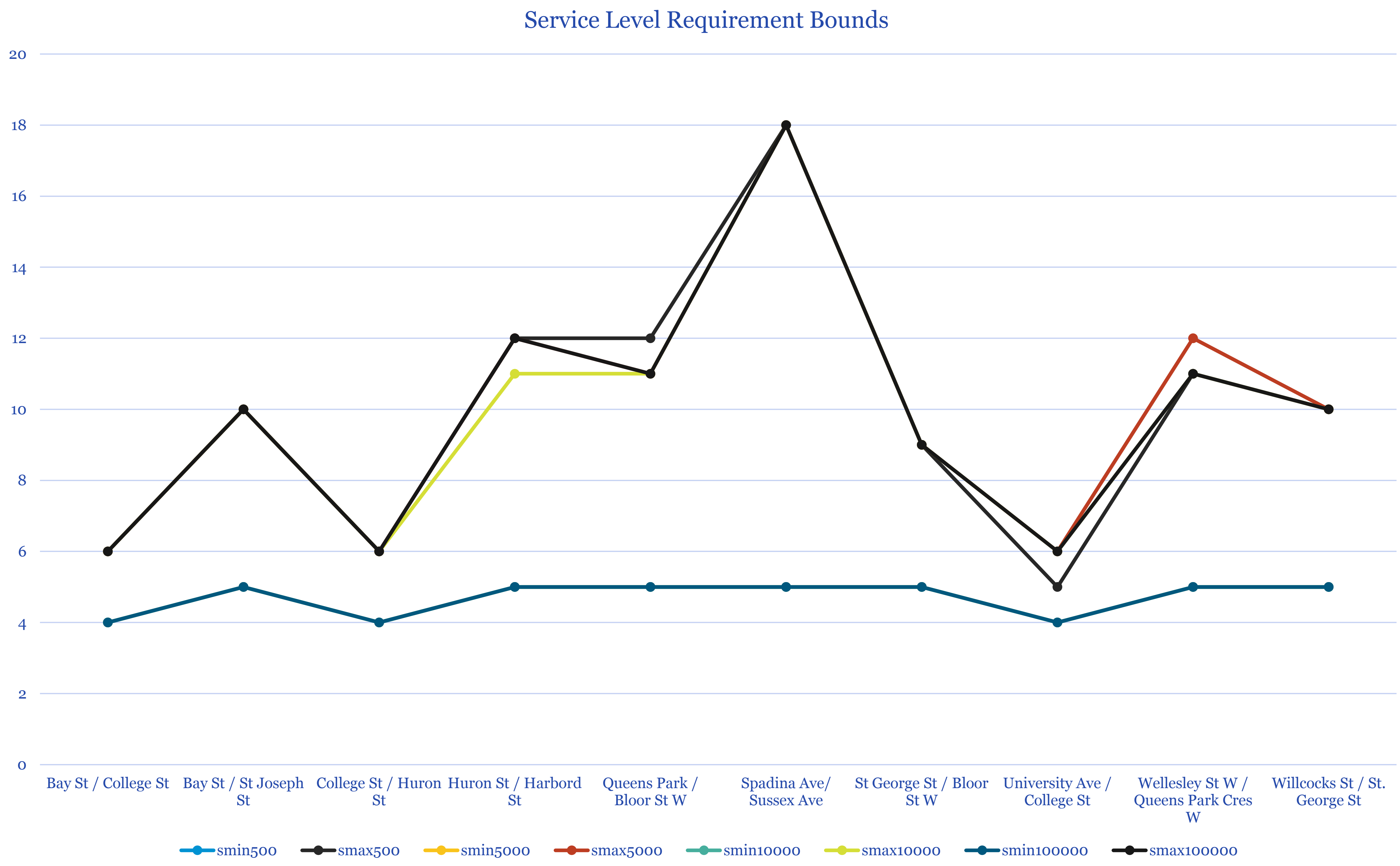
$$\mu_{i,Toronto} = \mu_{i,Montreal} + stdev(\mu_{i,Montreal}) * \varepsilon_i$$

$$\varepsilon_i \sim N(0,1)$$

Simulated Service Level Requirement Bounds

Number	Station	Max bike	smin500	smax500	smin5000	smax5000	smin10000	smax10000
2	Bay St / College St	11	4	6	4	6	4	6
3	Bay St / St Joseph St	17	5	10	5	10	5	10
4	College St / Huron St	11	4	6	4	6	4	6
5	Huron St / Harbord St	19	5	12	5	12	5	**11**
6	Queens Park / Bloor St W	19	5	**12**	5	11	5	11
7	Spadina Ave/ Sussex Ave	27	5	18	5	18	5	18
8	St George St / Bloor St W	15	5	9	5	9	5	9
9	University Ave / College St	11	4	**5**	4	6	4	6
10	Wellesley St W / Queens Park Cres W	19	5	11	5	12	5	11
11	Willcocks St / St. George St	17	5	10	5	10	5	10

Simulated Service Level Requirement Bounds (cont.)



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Probability & Distance Matrix

Non-self-sufficient probability matrix

- Bike station status information is pulled online hourly
- Considering self-sufficient when bike inventory is within bounds
- A week's data is used to calculate probability

Distance matrix

- Starting base - Learning Enrichment Foundation (LEF) training centre
  - *Lawrence & Jane*
- Google map shortest distance between bike stations are used

Model Case 2 Study Result

$$M_{min} = 1, M_{max} = 24, lb = 3, ub = 10, \beta_{sla} = 0.9$$

x [*,*] (tr)

:	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	1	0	1	1	0	0
14	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0

```
y [*] :=
 1 0    4 0    7 0   10 0   13 1   16 0   19 0   22 0
 2 0    5 0    8 0   11 0   14 0   17 0   20 0   23 0
 3 0    6 0    9 0   12 0   15 0   18 0   21 0   24 0
```

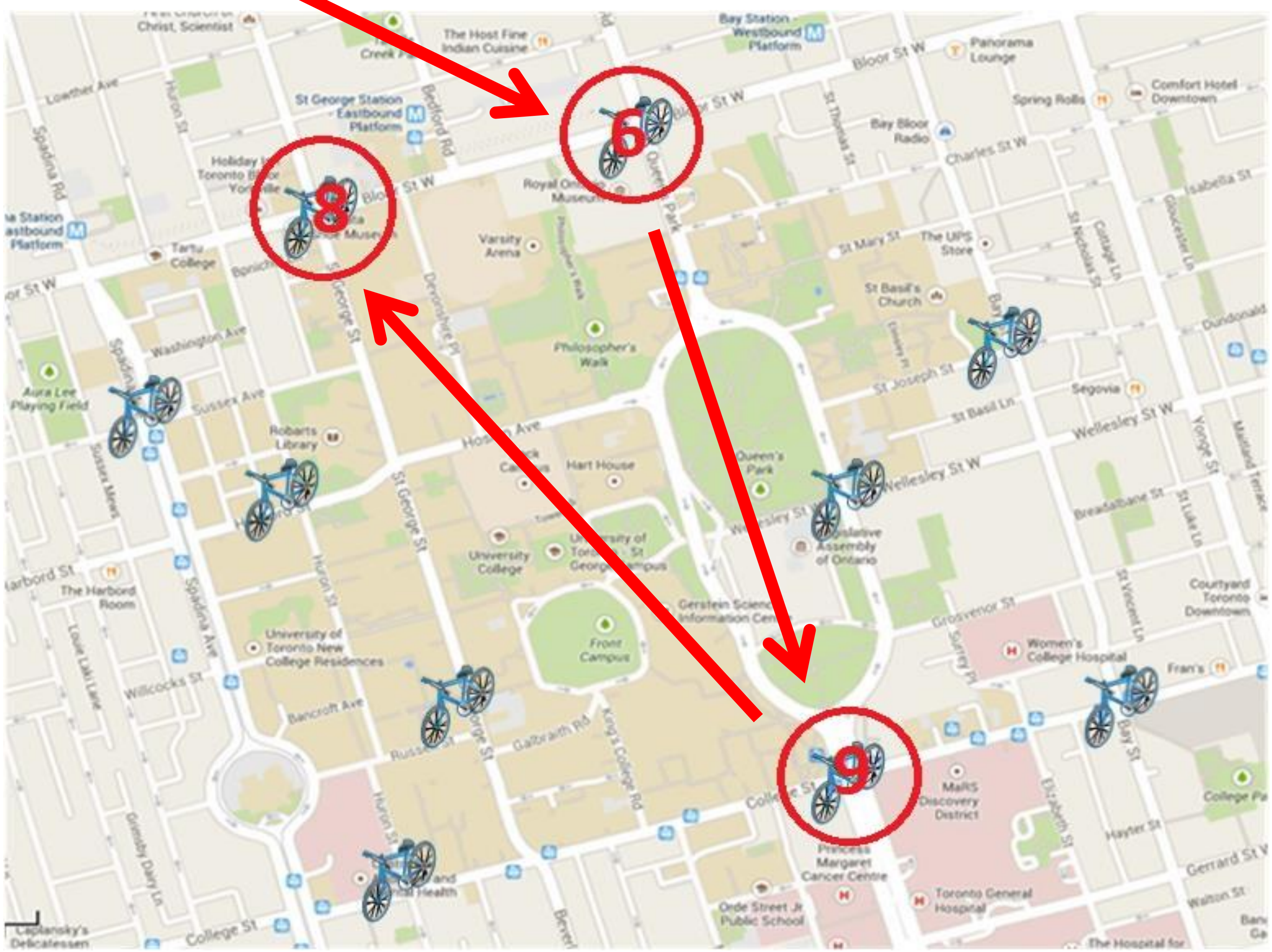
```
ampl: include bixi.run;
Solution determined by presolve;
objective total_cost = 16400.11992.
ampl:
```

```
ampl: include tsp.run;
Gurobi 5.6.0: optimal solution; objective 22719
X :=
1 6   1
1 8   1
1 9   0
6 8   0
6 9   1
8 9   1
;
```

Minimum Rebalancing Travelling Route

1

1-6-9-8-1 Total Distance: 22,719m



Hourly Non-Self-Sufficient Probaility

Simulated Out-SLA Probability	Station #	Hourly Average
Base	1	1
Bay St / College St	2	0.275901
Bay St / St Joseph St	3	0.202855
College St / Huron St	4	0.391187
Huron St / Harbord St	5	0.020769
Queens Park / Bloor St W	**6**	**0.934965**
Spadina Ave/ Sussex Ave	7	0
St George St / Bloor St W	**8**	**0.939356**
University Ave / College St	**9**	**1**
Wellesley St W / Queens Park Cres W	10	0
Willcocks St / St. George St	11	0

Sensitivity Analysis on Service Level Requirement

$$M_{min} = 1, M_{max} = 24, lb = 3, ub = 10, \beta_{sla} = (*)$$

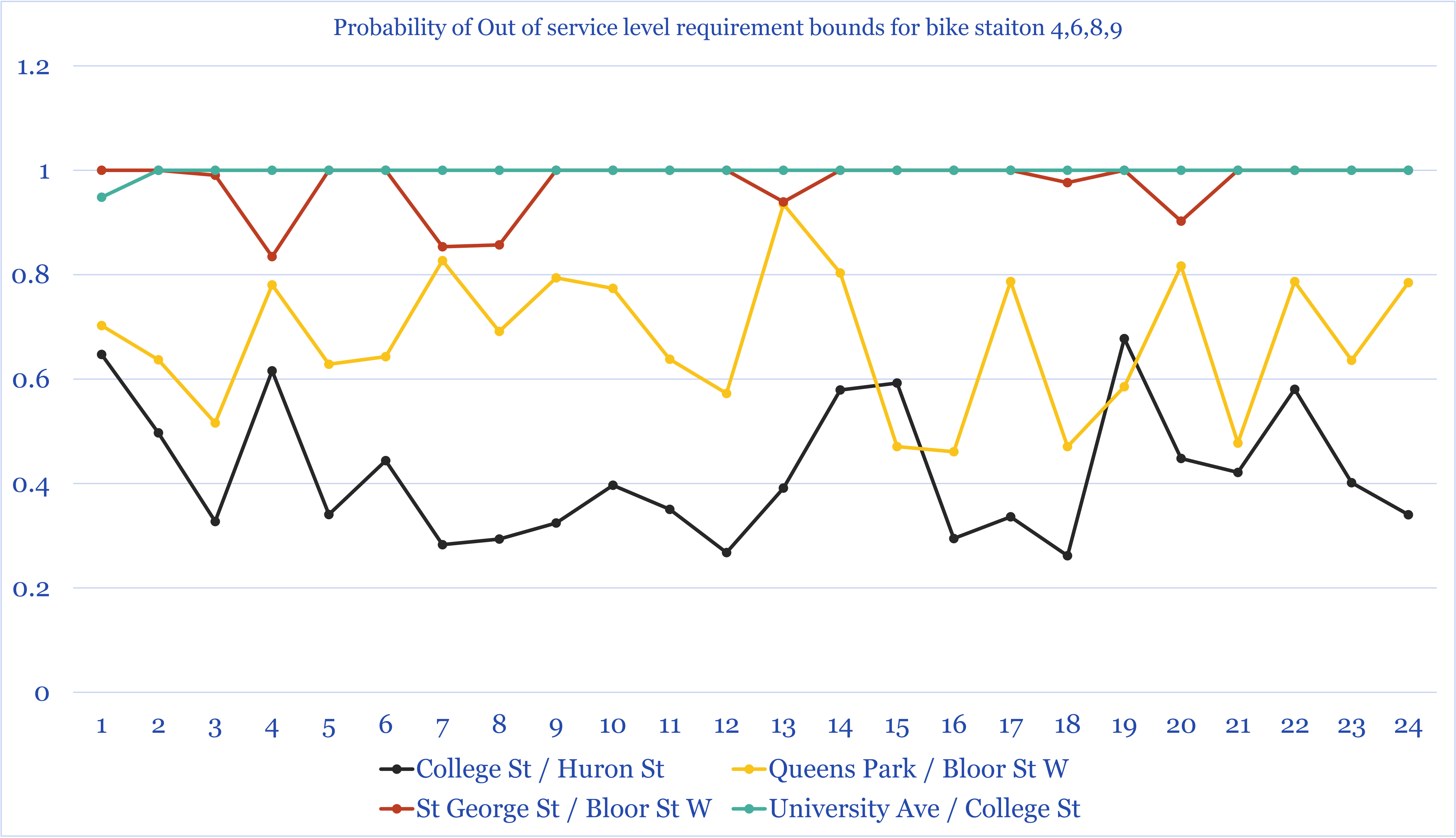
Beta SLA	Min = 1, lb = 3	t	Optimal route	Minimum Distance
0.5	1,4,6,8	14	1-4,4-6,6-8,8-1	22574
0.55	1,4,6,8	14	1-4,4-6,6-8,8-1	22574
0.6	1,4,6,8	4	1-4,4-6,6-8,8-1	22574
0.65	1,6,8,9	14	1-6,6-9,9-8,8-1	22719
0.7	1,6,8,9	14	1-6,6-9,9-8,8-1	22719
0.8	1,6,8,9	14	1-6,6-9,9-8,8-1	22719
0.9	1,6,8,9	13	1-6,6-9,9-8,8-1	22719
0.95	N/A	N/A	N/A	N/A

Sensitivity Analysis on Minimum Stations to Visit

$$M_{min} = 1, M_{max} = 24, lb = (*), ub = 10, \beta_{sla} = 0.6$$

lb	Beta SLA = 0.6, Min = 1,	t	Minimum route	Minimum Distance	% change in Distance
1	8	13	1-8,8-1	20600	N/A
2	1,6,8	14	1-6,6-8,8-1	21650	5.10%
3	1,4,6,8	4	1-4,4-6,6-8,8-1	22574	4.27%
4	1,4,6,8,9	4	1-4,4-9,9-6,6-8,8-1	22650	0.34%
5	N/A	N/A	N/A	N/A	N/A

Bike Station Inventory Rebalancing Timing



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Model Limitation & Potential Errors

Assumptions

1. Rebalancing in one hour
2. Always having enough capacity to satisfy rebalance
3. Montreal's data is similar to Toronto
4. Distribution of bike return and pickup
5. Service level requirement bounds can always be found
6. Bike stations are independent of each other

Data

1. City Data difference
2. Simulation should not be used when real data is around
3. Seasonal and stochastic bike return and pickup
4. Probability matrix shifts
5. Model
6. Optimal route can not always be used
7. Computation limit

Future Work & Research

1. Optimal rebalancing route with truck capacity limit
2. Consider seasonal and stochastic return and pickup rate
3. Linkage between bike stations
4. Continuous data gathering
5. Predicting patterns in usage and proactively rebalance bike station inventory

Key Takeaway & Learning

1. It is possible to reduce operation cost by following the minimum travelling route at the same time maintain a high service quality for users

2. Optimization models would only give accurate result if you feed them with high quality data

3. More openness of Toronto Bike Sharing System's data would help to improve the bike sharing system of Toronto

Thanks!

Questions?

Email: yuhong.liu@mail.utoronto.ca

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Backup

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Solving M/M/1/K queuing system, Morse (1958, p. 64)

LEMMA 1. *A closed-form expression for $g_i(s,\sigma)$ exists.*

Proof of Lemma 1. Morse (1958, p. 64) presents a transient solution for the $M/M/1/N$ queue (note that $N = C_i$):

$$p_i(s,\sigma,t) = \pi_i(\sigma) + \frac{2\rho_i^{\frac{1}{2}(\sigma-s)}}{C_i+1}\sum_{m=1}^{C_i} K_{i,m}e^{-k_{i,m}t}$$

with

$$\rho_i = \frac{\lambda_i}{\mu_i}$$

$$\pi_i(\sigma) = \begin{cases} \frac{1}{C_i+1} & \text{if } \rho_i = 1 \\ \frac{1-\rho_i}{1-\rho_i^{C_i+1}}\rho_i^\sigma & \text{otherwise} \end{cases}$$

$$K_{i,m} = \left(\frac{\mu_i}{k_{i,m}}\right)\left(\sin\frac{ms\pi}{C_i+1} - \sqrt{\rho_i}\sin\frac{m(s+1)\pi}{C_i+1}\right)\left(\sin\frac{m\sigma\pi}{C_i+1} - \sqrt{\rho_i}\sin\frac{m(\sigma+1)\pi}{C_i+1}\right)$$

$$k_{i,m} = \lambda_i + \mu_i - 2\sqrt{\lambda_i\mu_i}\cos\left(\frac{m\pi}{C_i+1}\right)$$

The antiderivative $P_{s,\sigma}(t)$ follows naturally:

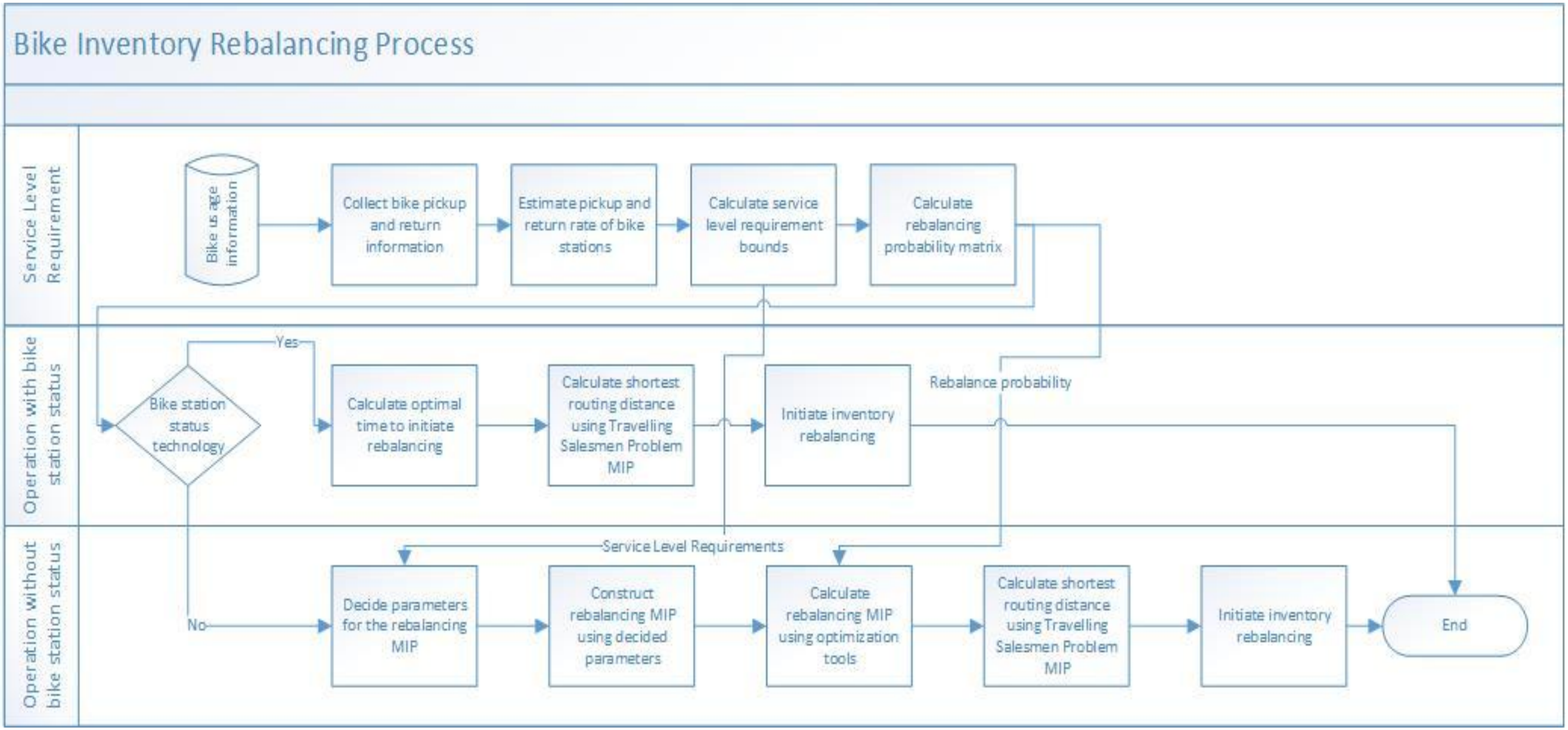
$$\begin{aligned} P_i(s,\sigma,t) &= \int p_i(s,\sigma,t)\,\mathrm{d}t \\ &= \pi_i(\sigma)t - \frac{2\rho_i^{\frac{1}{2}(\sigma-s)}}{C_i+1}\sum_{m=1}^{C_i} K_{i,m}\frac{e^{-k_{i,m}t}}{k_{i,m}}. \end{aligned}$$

Thus, $g_i(s,\sigma) = \frac{1}{T}\int_0^T p_i(s,\sigma,t)\,\mathrm{d}t = \frac{1}{T}\left(P_i(s,\sigma,T) - P_i(s,\sigma,0)\right)$. $\square$

Model Assumptions

- The simulated bike return and pickup rate are accurate measures
- Distance data between stations could be calculated using existing map tools
- The shortest distance path between bike stations could always be used by maintenance truck
- Bike pickup and return follows standard Poisson arrival process
- Bike station serves like a M/M/1/K queuing system, customers arrive independently and one at a time
- Service level requirement is given to the maintenance operator
- Probability of bike station out of service bounds are calculated using historical data gathered over time
- Depending on type of policy, there are maximum and minimum rebalancing allowed in a day
- Rebalancing are assume to be finished within an hour
- Assume maintenance truck will have enough capacity to always satisfy the rebalancing needs.
- Maintenance truck will not have access to real-time bike station information (model 2 only)

Bike Inventory Rebalancing Process



UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING