

Report of Assignment 5

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Haodong Liao & Fan Gao

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1 Preface

This is a \LaTeX report written by Haodong Liao and Fan Gao, students of UESTC who participated in the summer school of UCPH. More codes of this summer school are at [Repository](#).

2 Introduction

Lambda calculus (also written as λ -calculus) is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution¹, it is one of the three fundamental computational models and is the mother of all functional programming languages.

In this assignment, we needed to implement an interpreter for terms of untyped λ calculus as presented in the given slides. We used methods like pattern-matching, tuple and so on to meet the requirements and we had to say that figured out the logic of recursion was very challenging for us, thankfully, things went well.

3 Analysis and Design

3.1 Term

According to [1], the objects of study in λ -calculus are λ - *terms*, and the set of *pre-terms* has three kinds of elements: *variable*, *abstraction* and *application*, so it was not hard for us to implement the embedded domain specific language *Term*, for the grammar *pre-term* with the given slide *Lambda calculus AST*.

3.2 Substitute

In this sub-assignment, we needed to implement the function *substitute* which performed the substitution.

Since we could ignore the free variable side condition, there were three branches needed to be handled.

- (i) Var x: if x was equaled to the given substitute argument, then we substituted it with the target argument.
- (ii) Abs (x,y): according to the structure of abstraction, x was a variable, so we needed to compare y with the given substitute argument and called this function recursively.
- (iii) App (x,y): same as abstraction, but it required to substitute both x and y recursively with the given argument(if they were equivalent).

List.fold function *updates an accumulator iteratively by applying f to each element in lst*.

So there were two branches needed to be handled, one is an empty list, the other is a list with element(s). For an empty list, the return value couldn't be constrained to list and should be as same as the accumulator. For a list with element(s), fold function should be called recursively, the accumulator should always be the result of argument function which had parameters of the accumulator and the current element of the list. The key part of the pseudocode is shown as following:

```
1 Fold function accumulator list =
2   match list with
3   | emptylist -> return accumulator
4   | list with element(s) ->
5       Fold function (result of function with
6       current accumulator and element) restList
```

```
1 List.filter: f:( 'T -> bool) -> lst: 'T list -> 'T list
```

¹https://en.wikipedia.org/wiki/Lambda_calculus

According to [2], List.filter function *returns a new list with all the elements of lst for which f evaluates to true.*

Same as fold function, there were two branches needed to be handled. For an empty list, the return value should be the empty list itself. For a list with element(s), filter function should be called recursively according to the bool expression, if the value was true, not only should we call the filter function recursively, the current element should be a part of the result list. If the value of bool expression is false, there was no need to care about the current element. The key part of the pseudocode is shown as following:

```
1  Filter boolexpression list =
2    match list with
3    | emptylist -> return emptylist
4    | list with element(s) ->
5      if boolexpression is true
6      then
7        concat current element
8        call Filter function with boolexpression and rest list
9      else
10     call Filter function with boolexpression and rest list
```

4 Program description

My implementation of fold function was as follows:

```
1  let rec myFold (f: 'b -> 'a -> 'b) (acc: 'b) (lst: 'a list) : 'b =
2    match lst with
3    | [] -> acc
4    // [] -> [] // second time wrong
5    | elm::rest ->
6      // let result = myFold f acc rest @ (f acc elm) // first time wrong
7      let result = myFold f (f acc elm) rest
8      result
```

As it shows, to my point of view, the place I made mistake was the key of this function, for an empty list or a list with element(s), a problem I met was that I constrained the type of return value. It was a subtle but vital problem.

My implementation of filter function was as follows:

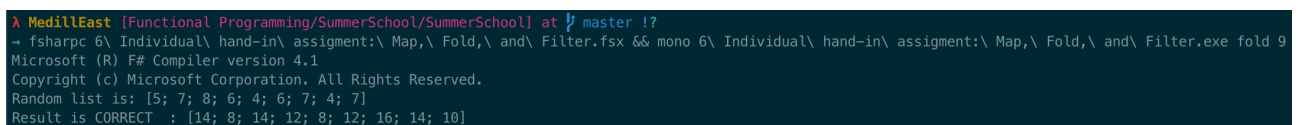
```
1  let rec myFilter (p: 'a -> bool) (lst: 'a list) : 'a list =
2    // List.filter p lst
3    match lst with
4    | [] -> []
5    | elm::rest ->
6      if (p elm) then [elm] @ (myFilter p rest)
7      else myFilter p rest
```

I struggled in the code of a list with element(s) when I started to solve this problem, and the point confused me was that I got bogged down with the detail of recursion and ignored the branches of boolean expressions. Things went smoothly when I rearranged my thoughts.

5 Evaluation

The testing environment was macOS Mojave 10.14.5 system with iTerm and Microsoft (R) F# Compiler version 4.1.

I compiled and tested the fold and filter function with parameter *fold 9* and *filter 9*, and the result were correct and showed in Figure 1 and Figure 2 separately:



```
λ MedillEast [Functional Programming/SummerSchool/SummerSchool] at master !?
→ fsharpc 6\ Individual\ hand-in\ assignment:\ Map,\ Fold,\ and\ Filter.fsx && mono 6\ Individual\ hand-in\ assignment:\ Map,\ Fold,\ and\ Filter.exe fold 9
Microsoft (R) F# Compiler version 4.1
Copyright (c) Microsoft Corporation. All Rights Reserved.
Random list is: [5; 7; 8; 6; 4; 6; 7; 4; 7]
Result is CORRECT : [14; 8; 14; 12; 8; 12; 16; 14; 10]
```

Figure 1: Testing of fold function

```
λ MedillEast [Functional Programming/SummerSchool/SummerSchool] at master !?  
→ fsharpc 6\ Individual\ hand-in\ assignment:\ Map,\ Fold,\ and\ Filter.fsx && mono 6\ Individual\ hand-in\ assignment:\ Map,\ Fold,\ and\ Filter.exe filter 9  
Microsoft (R) F# Compiler version 4.1  
Copyright (c) Microsoft Corporation. All Rights Reserved.  
Random list is: [9; 0; 8; 8; 4; 0; 1; 5; 8]  
Result is CORRECT : [9; 8; 8; 5; 8]
```

Figure 2: Testing of filter function

6 Conclusion

In this assignment, I implemented the *myFold* and *myFilter* functions with my own recursive method. I stuck at the beginning of my writing, but things went smoothly when I rearranged my thoughts and wrote down the pseudocode. It's easy to get bogged down in the details of a program, but we should take a top-down functional programming approach and thinking more about what to do than how to do it.

References

- [1] Morten Heine Sørensen and Pawel Urzyczyn. *Lectures on the Curry-Howard isomorphism*, volume 149. Elsevier, 2006.
- [2] Jon Spurring. *Learning to Program with F#*. 2019.