Mathematical Depiction on Optical Characteristic of WaterWave

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***Abstract*—Omnipresent water has been well studied for its physical, chemistry and biological property. Besides, its electric polarity also equipped water with unique optical characteristics. I believe anyone here has ever saw the glittering drops on petals, colored rainbow after the watery sky, and using tranquil lake us mirror. However, the water is not always flat, many times, mostly powered by wind, and something like the shake of the earth, we "see" water wave. Well, the weird thing comes here, we did not actuary "see" the water, but we see the optical effect of water, like distortion, reflection, or refraction. Mind that we didn't mention the diffusion here, as any object that not emitting visible light themselves are visible most because of diffusion. In this paper, we focused on the spacial and time-domain periodicity of water wave, and study it's optical characteristics from a reflection event happened at a place for a single light to the water wave as a whole.**

I. Introduction

Without much study of the shape of water wave, we first intuitively assume water wave is a one-dimension cosine wave, and we study the light path where the light emitted from the source reflect only once at the interface of water and air, by study this naive model. Over the study of this naive model, we gain a basic understanding of the how water wave became visible in our eyes, and also's it spacial and time-domain periodicity.

After study the single light, pace, we consider apply the object-image model to water wave scenario. At any given point, we can calculate the normal vector and curvature, and model the small area of water-air interface as a spherical mirror, together, we can get the full image of the object over water. By this way, we can get the image at any give time, as water wave is periodic, we need only to calculate the image over one time period and we can get the full information of water wave geometrically.

Over two models, we can re-visualize the effect of water wave provide the light source and the shape of water. However, problems remain unsolved:

- Times of Reflection: Each time a light reflect at the point, it generate refraction beside the reflection, and where the reflection light goes is hard to say. Besides, whether the light will reflect once or times not upper bounded. Light from a singular source is easier to trace, but when it comes to the whole subject, the computation goes to infinity.
- Polarity and Intensity: Over the analysis before, we calculate the range of the image to be see geometrically regardless of the intensity, polarity.
- Dispersion: the dielectric coefficient of water may vary over wavelength of light, this may cause trouble when calculating light path over compound light containing wide range of wavelength. Besides, the light intensity distribution over polarization direction varies. Hopefully, considering only the degree of reflection light, it is equal over wavelength. So they could be considered geometrically equivalent under reflection regardless of polarity and intensity.
- Paralleled Light: we have being consider the paralleled light source over the model, which models well for Celestial objects such as sun and moon. However, it's too special, and we need to apply it to more generalized scenerios.

II. Naive Model

A. Considering a Light Path From Light Source to Human Eye

For simplicity, we naively assume there exist a one dimensional cosine water wave (1):

$$h = A\cos(\frac{2\pi}{\lambda}x - \omega t) \tag{1}$$

in which h is the height of the wave, x is a measure of distance, t measures time, A present the Amplitude of the wave, λ is the wavelength, and ω is the angular frequency.

Consider the light path from the light source to Human Eye as shown in "Fig. 1", denote the degree from eye to to the ground as α, the degree from light source to the ground as β, then the tangent line has a degree to the ground γ which satisfies(2):

$$\gamma = \frac{1}{2}(\alpha - \beta) \tag{2}$$

Remember that β is x dependent, and α can be a constant if the light source is paralleled. As the water way is time-domain periodic, over one period $T = \frac{2\pi}{\omega}$, the light can enter you eye again from the same place, resulting a periodic light pulse in your eye at degree β, thus the image you see over the water has a same angular frequency ω_i equal to ω the angular frequency of water wave.

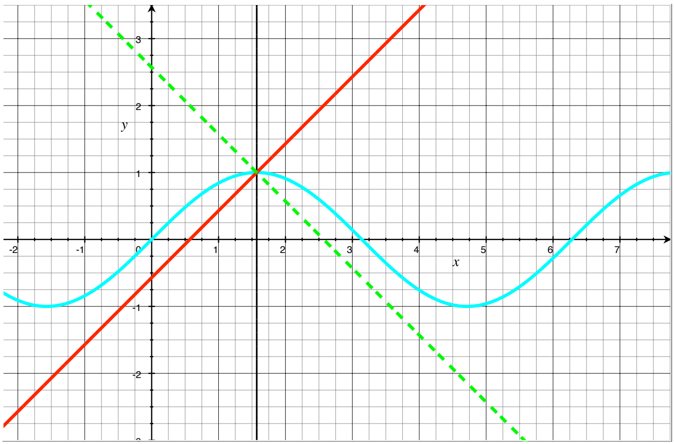

Fig. 1. Example of a figure caption.

Lets now consider the boundary of degree you can see the image of light source which reflect on the interface of water-air only once. The limit of the gradient of a water wave follow the fluctuation equation (1) is:

$$\delta = \arctan\left(-\frac{2\pi}{\lambda}A\right) \tag{3}$$

which must satisify (4):

$$\|\delta\| \geq \|\gamma\| \tag{4}$$

remember that β is x dependent, so does γ, therefore, we can calculate the range of degree you can see the one-time reflection image.

However the model doesn't suit that well

III. Ease of Use

A. Maintaining the Integrity of the Specifications

The IEEEtran class file is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts are prescribed; please do not alter them. You may note peculiarities. For example, the head margin measures proportionately more than is customary. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. Please do not revise any of the current designations.

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A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

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- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
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- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{5}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(5)", not "Eq. (5)" or "equation (5)", except at the beginning of a sentence: "Equation (5) is . . ."

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

BibTeX does not work by magic. It doesn't get the bibliographic data from thin air but from .bib files. If you use BibTeX to produce a bibliography you must send the .bib files.

LaTeX can't read your mind. If you assign the same label to a subsubsection and a table, you might find that Table I has been cross referenced as Table IV-B3.

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Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won't be any anyway) and it might stop a wanted equation number in the surrounding equation.

E. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).
- Do not use the word "essentially" to mean "approximately" or "effectively".
- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [7].

F. Authors and Affiliations

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

G. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

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H. Figures and Tables

a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. 2", even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of

Fig. 2. Example of a figure caption.

quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

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