ECE 227 *Project*: Timik Multi-layer Network Analysis

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Abstract—**This project investigates node centrality and community structure in a multilayer social network constructed from the Timik.pl dataset, which captures user interactions across five distinct modalities: friendships, messages, campaigns, transactions, and visits. We analyze three classical centrality measures—degree, betweenness, and eigenvector—both within individual layers and in aggregated multilayer representations. The results reveal consistent core-periphery structures within layers, but considerable divergence across them, indicating that each interaction type highlights different aspects of user influence.**

For community detection, we adopt a spectral clustering method based on the averaged normalized Laplacian across layers. This approach produces low-dimensional embeddings that capture both intralayer and interlayer structural patterns. Visualizations of the embedding reveal clearly separated clusters in both 2D and 3D, confirming the value of integrating multiple interaction types.

Overall, the findings highlight the structural complexity of real-world social systems and demonstrate how multilayer analysis can uncover patterns that single-layer methods would miss. We also provide the source code of this project on GitHub: https://github.com/MeditatorE/Timik-Multi-layer-Network-Analysis

I. Introduction

Many real-world systems exhibit complex relational structures that cannot be adequately captured by a single-layer network. In such systems, the same entities often interact through multiple distinct types of connections. A classic example is an online platform where users can maintain social friendships, send messages, and conduct transactions—each of these forming a different type of interaction layer. Multilayer or multiplex network models provide a unified framework to represent such systems by considering each relation type as a separate layer, while maintaining consistent node identities across layers.

Identifying influential nodes and uncovering community structures are central tasks in network science. However, traditional centrality and community detection methods typically assume a single-layer structure and may fail to capture key cross-layer dynamics. In a multilayer setting, a node's influence can stem from cumulative or synergistic effects across layers. Likewise, community structures may emerge not solely from tight connections within a single layer, but from composite connectivity across multiple layers.

This project aims to explore node centrality and community structure within a multiplex network constructed from the publicly available dataset hosted by Harvard Dataverse Sapiezynski et al. (2019), which includes user-level interactions over five modalities. By applying both layer-wise and integrated multilayer analysis methods, we examine how structural roles differ between single-layer and composite representations. Our analysis employs weighted and supra-graph-based centrality measures. For community detection, we adopt a multilayer spectral clustering approach that computes layer-wise normalized Laplacians, aggregates them, and applies KMeans clustering to the resulting spectral embedding.

II. Related Work

The study of node importance in multilayer networks has received increasing attention in recent years. Traditional centrality metrics—degree, betweenness, and eigenvector centrality—have been generalized to capture the added complexity of multilayer systems. De Domenico et al. De Domenico et al. (2015) proposed a supra-graph framework to compute shortest paths and eigenvector scores across layers, allowing one to quantify influence that spans different modalities of interaction. Similarly, Brodka et al. Brodka et al. (2011) and Solá et al. Solá et al. (2013) introduced layer-wise weighted aggregations to maintain interpretability while integrating inter-layer effects. Eigenvector centrality has also been extended through supra-adjacency matrices and multilayer decomposition Tudisco et al. (2018).

In our work, we adopt a multilayer spectral clustering framework for community detection, rather than modularity-based methods such as Louvain. This approach leverages normalized graph Laplacians computed per layer. The Laplacians are then averaged across layers to produce a unified representation that integrates structural signals from all views Dickison et al. (2016). Spectral decomposition of the averaged Laplacian yields a low-dimensional embedding, to which KMeans clustering is applied to infer community labels.

Compared to single-layer clustering, this method allows us to capture latent patterns that are only apparent when multiple interaction types are considered jointly. It also avoids the combinatorial complexity of optimizing modularity across layers, and is particularly suitable for symmetric, binary multiplex networks like ours.

III. Models and Preliminaries

A. Degree Distribution

The distribution of degree is an important property of network. Our approach is to compute the node degree of each layer respectively, and then fit the resulting distributions with two natural distribution, power law distribution and exponential distribution. Among them, Power law distribution is expected to excellently fit the trend, as it is well-suited for capturing the heavy-tailed nature often observed in real-world networks.

To examine the consistency of node importance across layers, we computed the overlap of the top 10% high-degree nodes in each network layer. This overlap reflects the extent to which influential nodes are shared across different types of interactions or relationships. A high overlap suggests that certain nodes consistently play a central role across multiple layers, while a low overlap indicates that different layers highlight different influential nodes. This has important implications for designing intervention strategies and understanding multiplex network structure.

B. Single and Multilayer Centrality

Centrality measures are fundamental tools in network analysis for quantifying the structural importance of individual nodes. De Domenico et al. (2015) In this study, we focus on three of the most widely used centrality metrics: degree centrality, betweenness centrality, and eigenvector centrality. For each metric, we provide definitions in both single-layer and multilayer network settings, and describe how these measures are computed and interpreted across layers. This allows us to analyze the role and influence of nodes within and across different network layers.

Degree Centrality: Degree centrality is a fundamental measure that quantifies the local influence of a node by counting the number of direct connections it has. In multilayer networks, Degree Centrality reflects how well-connected a node is across multiple types of relationships or interaction layers. Brodka et al. (2011) We describe its formulation in both single-layer and multilayer networks.

Degree Centrality in a Single-Layer Network. Let $G = (V, E)$ be an unweighted, undirected network with $n = |V|$ nodes. The normalized degree centrality of a node $v \in V$ is defined as:

$$C_D^{(G)}(v) = \frac{\deg(v)}{n-1}$$

where $\deg(v)$ denotes the degree of node v, i.e., the number of edges incident to v. The normalization ensures that $C_D^{(G)}(v) \in [0,1]$.

Degree Centrality in a Multilayer Network. In a multilayer network $\mathcal{M}$ composed of L layers $\{G^{[1]}, G^{[2]}, \ldots, G^{[L]}\}$, each layer $G^{[\ell]} = (V^{[\ell]}, E^{[\ell]})$ is a single-layer network defined over the same set of nodes V (i.e., $V^{[\ell]} = V$ for all ℓ).

The degree centrality of node $v \in V$ in layer ℓ is given by:

$$C_D^{(G^{[\ell]})}(v) = \frac{\deg^{[\ell]}(v)}{n-1}$$

where $\deg^{[\ell]}(v)$ denotes the degree of v in layer ℓ.

The *aggregated multilayer degree centrality* is computed by summing the weighted per-layer centralities:

$$C_D^{(\mathcal{M})}(v) = \sum_{\ell=1}^{L} w^{[\ell]} \cdot C_D^{(G^{[\ell]})}(v)$$

where $w^{[\ell]} \in [0,1]$ is a weight factor reflecting the relative importance of layer ℓ, and $\sum_{\ell=1}^{L} w^{[\ell]} = 1$ if normalized.

Betweenness Centrality: Betweenness centrality is a global measure that quantifies the extent to which a node acts as a bridge along the shortest paths between other nodes. Freeman (1977) It reflects a node's potential control over information flow through the network.

Betweenness Centrality in a Single-Layer Network. Let $G = (V,E)$ be a connected, unweighted graph. The betweenness centrality of a node $v \in V$ is defined as:

$$C_B^{(G)}(v) = \sum_{\substack{s \neq v \neq t \\ s,t \in V}} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is the total number of shortest paths from node s to node t, and $\sigma_{st}(v)$ is the number of those paths that pass through v.

Betweenness Centrality in a Multilayer Network. In a multilayer network $\mathcal{M}$ composed of L layers $\{G^{[1]}, G^{[2]}, \ldots, G^{[L]}\}$, shortest paths may traverse intra-layer and inter-layer edges. Therefore, computing betweenness centrality requires constructing a *supra-graph* that integrates all layers. De Domenico et al. (2013)

Let $\mathcal{G} = (V_{\mathcal{M}}, E_{\mathcal{M}})$ be the supra-graph of the multilayer network, where $V_{\mathcal{M}} = V \times \{1,2,\ldots,L\}$ denotes layer-dependent node copies, and $E_{\mathcal{M}}$ contains both intra-layer and inter-layer edges.

Then, the multilayer betweenness centrality of node $v \in V$ is computed as:

$$C_B^{(\mathcal{M})}(v) = \sum_{\substack{s \neq t \\ s,t \in V_{\mathcal{M}}}} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is the number of shortest paths from s to t in the supra-graph, and $\sigma_{st}(v)$ is the number of such paths that include *any replica* of node v (i.e., (v,ℓ) for some layer ℓ).

Unlike degree centrality, betweenness centrality in multilayer networks cannot be computed as a weighted sum over individual layers, since shortest paths may traverse interlayer edges, and path dependencies are inherently nonlinear.

Eigenvector Centrality: Eigenvector centrality is a fundamental spectral centrality measure that evaluates a node's importance based not only on how many connections it has, but also on the centrality of the nodes it is connected to. This recursive idea captures influence through a network more effectively than degree-based metrics, and is widely used in applications ranging from social networks to biological systems. Newman (2010)

Eigenvector Centrality in a Single-Layer Network. Given a graph $G = (V,E)$ with $n = |V|$ nodes and adjacency matrix $A \in \mathbb{R}^{n \times n}$, the eigenvector centrality of node v_i is defined as the i-th entry of the leading eigenvector $\mathbf{x} \in \mathbb{R}^n$ of A, corresponding to the largest eigenvalue λ:

$$C_E^{(G)}(v_i) = x_i, \quad \text{where } A\mathbf{x} = \lambda \mathbf{x}$$

Equivalently, this can be written in scalar form as:

$$C_E^{(G)}(v_i) = \frac{1}{\lambda} \sum_{j \in V} A_{ij} \cdot C_E^{(G)}(v_j)$$

Eigenvector Centrality in Multilayer Networks. There are two common approaches to generalizing eigenvector centrality in multilayer settings.

(a) Layer-wise Weighted Aggregation. The simple extension computes eigenvector centrality in each layer $G^{[\ell]} = (V^{[\ell]}, E^{[\ell]})$ and combines the results via a weighted sum Solá et al. (2013):

$$C_E^{(\mathcal{M})}(v_i) = \sum_{\ell=1}^{L} w^{[\ell]} \cdot C_E^{(G^{[\ell]})}(v_i)$$

(b) Supra-adjacency Matrix. Another approach constructs a supra-adjacency matrix $\mathcal{A} \in \mathbb{R}^{nL \times nL}$ that connects node-layer pairs (i,ℓ) via intra-layer and inter-layer edges. Let $\mathbf{x} \in \mathbb{R}^{nL}$ be the principal eigenvector of $\mathcal{A}$:

$$\mathcal{A}\mathbf{x} = \lambda \mathbf{x}$$

The multilayer eigenvector centrality of node v_i is then obtained by aggregating its scores across all layers Tudisco et al. (2018):

$$C_E^{(\mathcal{M})}(v_i) = \sum_{\ell=1}^{L} x_{(i,\ell)}$$

This method involves high computational cost due to the size of the supra-adjacency matrix, especially in large-scale multilayer networks.

C. Community Detection

We adopt a spectral clustering method tailored for multilayer networks. The core idea is to aggregate structural information across layers using the graph Laplacian, then identify communities based on the geometric structure of the resulting embedding.

The process consists of the following steps. For each interaction layer v, we construct a binary, symmetric adjacency matrix $A^{(v)} \in \mathbb{R}^{n \times n}$, where n is the number of nodes. From this, we compute the symmetric normalized Laplacian:

$$L^{(v)} = I - D^{(v)^{-\frac{1}{2}}} A^{(v)} D^{(v)^{-\frac{1}{2}}}$$

where $D^{(v)}$ is the degree matrix of layer v, and I is the identity matrix.

To integrate multiple views, we compute the uniform average of all layer-wise Laplacians:

$$\bar{L} = \frac{1}{V} \sum_{v=1}^{V} L^{(v)}$$

This averaged Laplacian preserves the shared node structure while incorporating connectivity from all layers.

Next, we perform eigendecomposition of $\bar{L}$ and select the k eigenvectors corresponding to the smallest non-zero eigenvalues. These form a matrix $U \in \mathbb{R}^{n \times k}$, where each row u_i serves as a low-dimensional representation of node i.

Finally, we apply KMeans clustering to the rows of U, assigning each node a community label by minimizing intra-cluster variance:

$$\min_{C} \sum_{i=1}^{n} \left\| u_i - \mu_{c(i)} \right\|^2$$

where $\mu_{c(i)}$ denotes the centroid of cluster $c(i)$. The result is a partition of the node set into k disjoint communities, informed by aggregated multilayer structure.

IV. Dataset Description

We used the publicly available **Timik.pl Multilayer Interaction Dataset** Sapiezynski et al. (2019), hosted on the Harvard Dataverse platform [1]. This dataset captures detailed user-level interactions on a Polish social network platform between 2007 and 2012. It offers a rich multilayer structure, with different types of social, behavioral, and transactional relations among users.

File	Lines	Size	Short description
campaigns	4,981	189 KB	Semicolon delimited file containing five spreading processes in Timik.pl
friends	12,416,809	474 MB	Semicolon delimited file containing the friendship relation between Timik.pl users
logins	14,688,527	455 MB	Semicolon delimited file containing date and time of each user login to the service
messages	26,134,695	999 MB	Semicolon delimited file containing the messages (without text) between Timik.pl users
transactions	538,597	21.7 MB	Semicolon delimited file containing the virtual currency transactions between Timik.pl users
visits	12,660,735	479 MB	Semicolon delimited file containing the visits of Timik.pl in private rooms of other users

Figure 1: Overview of Timik.pl dataset layers, sizes, and descriptions

[1] https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/V6AJRV

We constructed a multiplex network using five of these layers: `friends`, `messages`, `campaigns`, `transactions`, and `visits`. Each layer was preprocessed into a binary, symmetric adjacency matrix, with users as nodes and interactions as edges.

V. Results

A. Degree Distribution Analysis

The Figure 2 shows the node degree distribution of a multi-layers network, with each subplot corresponding to a distinct layer: **friends**, **visits** and **transactions**. The distribution of all layers closely match power law distribution, with a heavy tail in the Linear-Linear plot in the left column. In addition, the Log-Log plots in the right column exhibit a near-linear trend, both plot indicating potential power-law behavior. The slope of three layers are **-1.77, -1.42, -1.94** respectively. These negative exponents suggest most of the nodes have small degree and high-degree only take up an extremely small part. This trend aligns with our real-life observations.

Notably, the different slopes suggest variation in the dynamics across layers: the **visits** layer, with its flatter slope, indicates a more uniform participation across nodes, while the steeper slope in the **transactions** layer implies stronger centralization, where a few nodes dominate interactions.

Finally, minor deviations from linearity at the tail end of the log-log plots could be attributed to statistical noise due to the sparsity of high-degree nodes. Overall, the observed trends align well with expectations for scale-free networks and highlight structural distinctions among the different interaction layers.

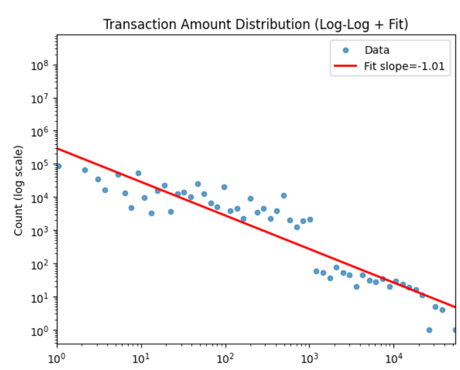


Figure 3: Transaction Amount Distribution

In contrast to the degree distribution analysis, Figure 3 focuses on the distribution of transaction amounts across the network. The log-log plot reveals

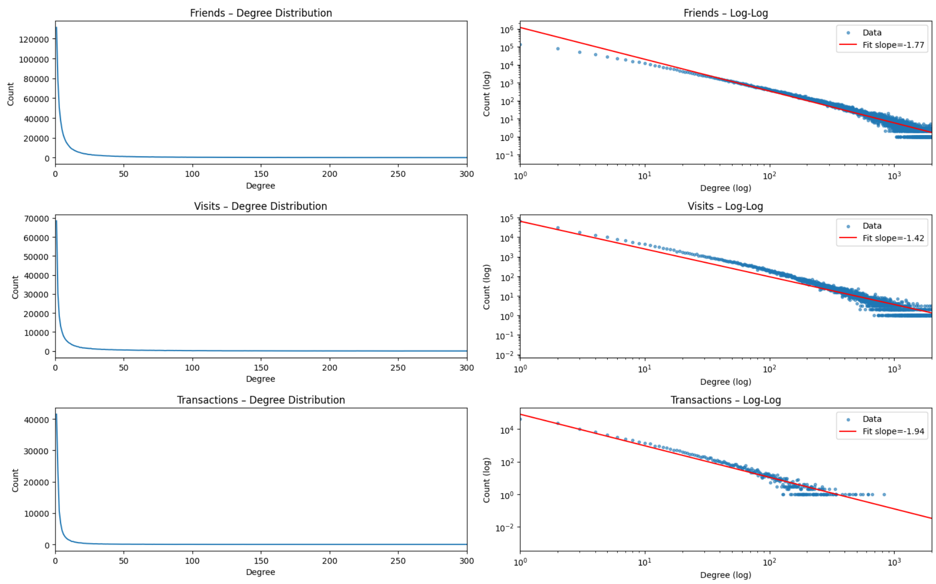


Figure 2: Distribution Analysis

a strong linear trend, with a fitted slope of approximately **-1.01**, indicating a heavy-tailed, power-law-like behavior in transaction magnitudes. This implies that while the vast majority of transactions involve relatively small amounts, a few exceptionally large transactions dominate the overall volume.

This observation complements the degree distribution analysis by highlighting not only who connects to whom, but also the intensity or weight of those connections. Together, they provide a comprehensive view of both structure and function of the network.

B. High-Degree Node Overlap Across Layers

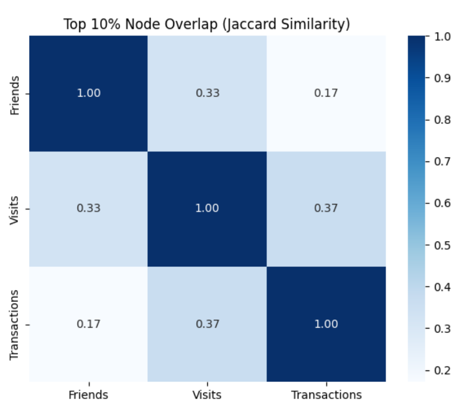


Figure 4: High-Degree Node Overlap Heatmap

To assess the consistency of node importance across different interaction layers, we compute the Jaccard similarity of the top 10% high-degree nodes for each pair of network layers, as shown in Figure 4. The heatmap reveals moderate overlap between layers, with the highest similarity observed between *Visits* and *Transactions* (0.37), followed by *Friends–Visits* (0.33), and the lowest between *Friends–Transactions* (0.17).

These values indicate that while some high-importance nodes are shared across layers, each layer also has a distinct set of influential nodes. The overlap of high degree node is not as high as prediction. The relatively low overlap between *Friends* and *Transactions* suggests that social ties do not strongly predict economic interactions.

C. Single Layer Centrality

Before investigating multilayer centrality measures, we first examine the centrality of nodes within each individual layer of the network. We compute the three centrality measures introduced in Section III - degree, betweenness, and eigenvector centrality - for each of the five network layers, respectively. For each metric, we extract and visualize the top 100 most central nodes in each layer.

The resulting visualizations are shown in Figure 5 and 6. In each subfigure, node size reflects centrality magnitude. To reduce visual clutter, we apply the sparse layout for clearer visualization. Furthermore, the top 10 central nodes in each network are annotated with their node IDs to facilitate cross-layer and cross-metric comparisons. The three columns visualize degree, betweenness, and eigenvector centrality, respectively, using the plasma, coolwarm, and winter colormaps to indicate relative centrality values. Each row corresponds to one of the five network layers.

The visualizations reveal clear structural differences across network layers. Each layer exhibits a distinct graph topology, and the sets of most central nodes differ significantly between layers, indicating that each interaction type captures a unique facet of the overall system. For example, the top-ranked nodes in the *campaigns* layer differ from those in the *friends* or *transactions* layers, suggesting that influence, social ties, and transactional roles are distributed differently across modalities.

Within each layer, however, different centrality measures tend to highlight overlapping sets of highly central nodes, albeit with differences in ranking. For instance, in the *campaigns* layer, node 17884 has the highest degree centrality, ranks among the top 10 in betweenness, and is third in eigenvector centrality. Meanwhile, node 18056 is ranked highest in both betweenness and eigenvector centrality within the same layer. Similarly, in the *friends* layer, node 193674 holds the top position in both betweenness and eigenvector centrality, and is second in degree. The *transactions* layer presents an even more concentrated structure: all three centrality measures identify node 16453 as the dominant hub, with no other nodes approaching its centrality, suggesting that this node functions as a transactional hub.

Overall, these patterns underscore two key characteristics of the multilayer network: (1) **intralayer centrality coherence**, where different metrics within a layer often point to a consistent core of important nodes, and (2) **interlayer centrality divergence**, where the top central nodes differ markedly across layers, reflecting functional heterogeneity. This supports the view that each layer captures distinct dynamics or roles within the broader system, and reinforces the value of a multilayer framework for uncovering these complementary structures.

D. Multilayer Centrality

To gain a holistic understanding of node influence in the network, we computed three classical centrality measures—degree, betweenness, and eigenvector—in a multilayer context De Domenico et al. (2015); Tudisco et al. (2018). Our approach follows two general principles: (1) aggregating per-layer measures using normalized weights and (2) incorporating inter-layer connectivity through a supra-graph construction.

We focused on the top 100 nodes with the highest centrality values under each measure, using the aggregated results across all five layers: campaigns, friends, messages, transactions, and visits. For visualization, we embedded each node in a three-dimensional space using (degree, betweenness, eigenvector) as coordinate axes. This enabled us to jointly examine different notions of centrality for the same node set.

Figure 7, 8, and 9 respectively display the top 100 nodes under degree, betweenness, and eigenvector centrality, plotted in 3D space. Each point represents a user, and its position indicates its relative importance under the three centrality measures.

These visualizations reveal several structural patterns:

- A small cluster of nodes dominates all three measures, suggesting multi-role hub users.
- Degree and eigenvector centralities are more positively correlated, while betweenness often diverges—highlighting nodes that act as structural bridges without necessarily having many neighbors Solá et al. (2013).
- Some users with mid-level degree nevertheless achieve high eigenvector centrality, indicating indirect influence via well-connected neighbors.

Overall, the results show that considering all three centrality types jointly in a multilayer framework enables a more nuanced view of influence that is otherwise missed in single-layer or single-metric analyses.

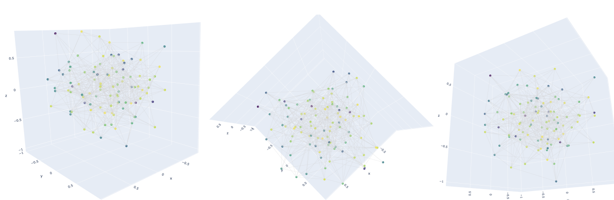


Figure 7: Top 100 users in multilayer degree centrality (3D projection)

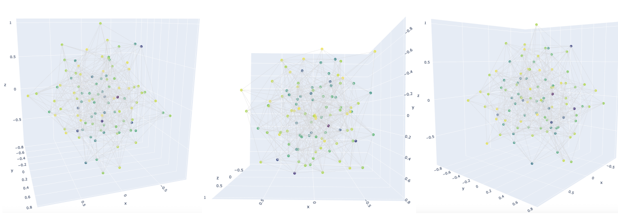
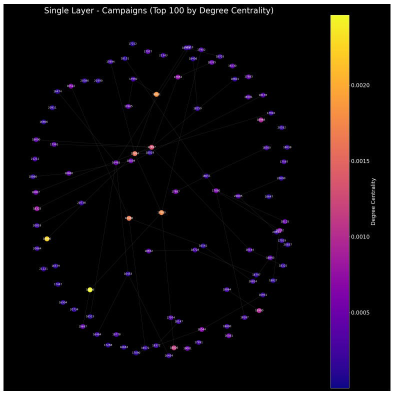
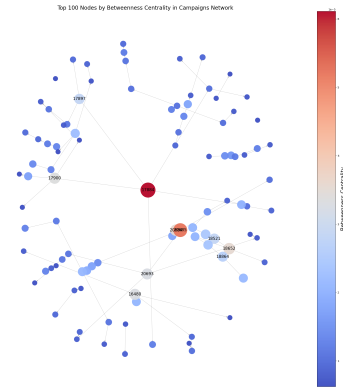


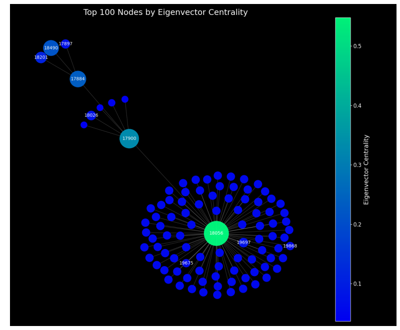
Figure 8: Top 100 users in multilayer betweenness centrality (3D projection)



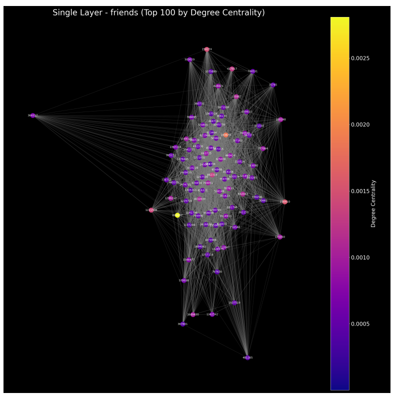
(a) Degree - Layer 1 - Campaigns



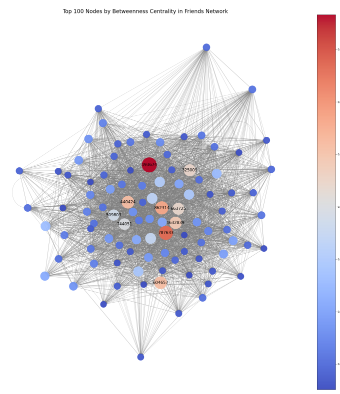
(b) Betweenness - Layer 1 - Campaigns



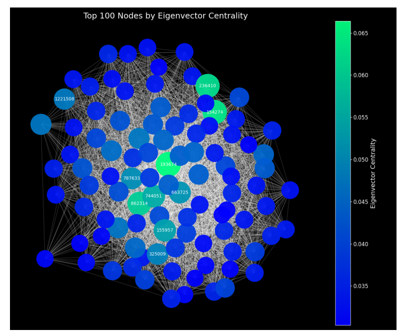
(c) Eigenvector - Layer 1 - Campaigns



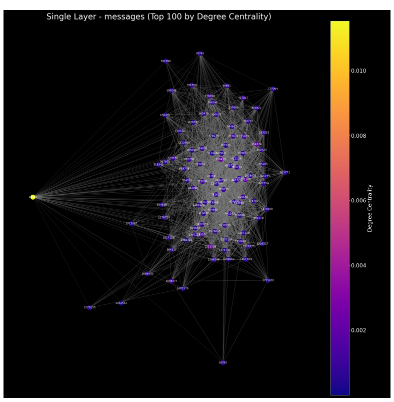
(d) Degree - Layer 2 - Friends



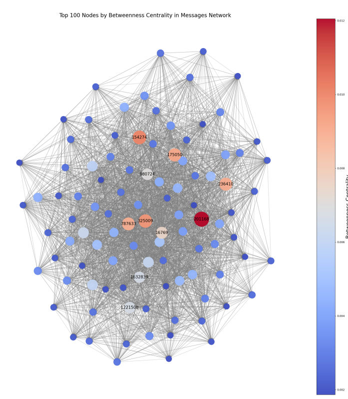
(e) Betweenness - Layer 2 - Friends



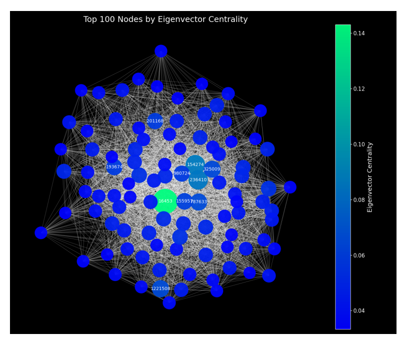
(f) Eigenvector - Layer 2 - Friends



(g) Degree - Layer 3 - Messages



(h) Betweenness - Layer 3 - Messages



(i) Eigenvector - Layer 3 - Messages

Figure 5: Top 100 central nodes under degree, betweenness, and eigenvector centrality in Layers 1–3.

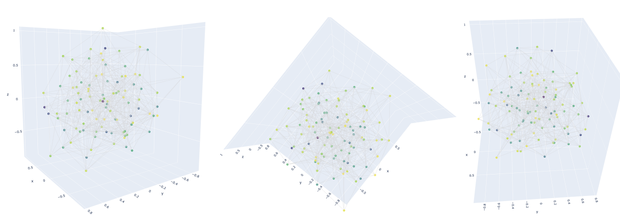
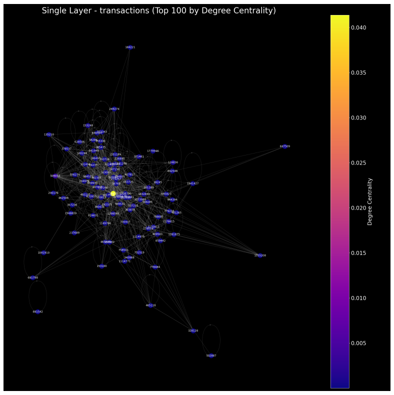


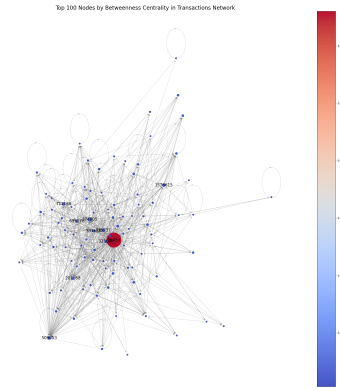
Figure 9: Top 100 users in multilayer eigenvector centrality (3D projection)

E. Community Detection

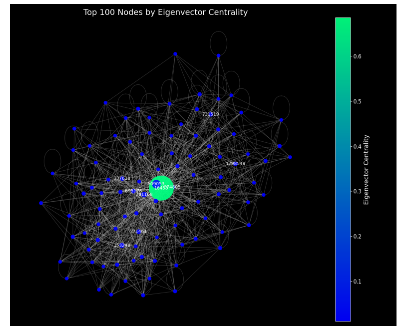
To identify node communities in the multilayer setting, we applied a spectral clustering method based on the normalized graph Laplacian Dickison et al. (2016). The result is obtained through five steps: (1) constructing a binary symmetric adjacency matrix per layer; (2) computing the symmetric normalized Laplacian for each layer; (3) averaging all layer-wise Laplacians to obtain a unified view;



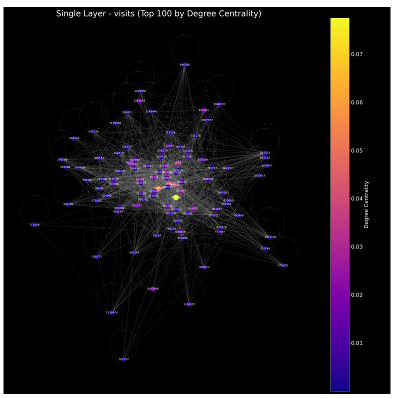
(a) Degree - Layer 4 - Transactions



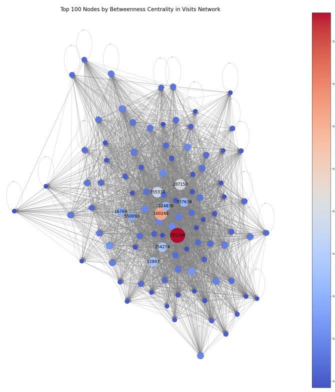
(b) Betweenness - Layer 4 - Transactions



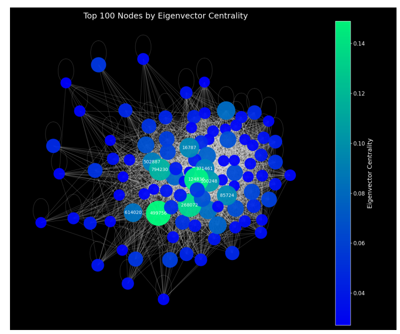
(c) Eigenvector - Layer 4 - Transactions



(d) Degree - Layer 5 - visits



(e) Betweenness - Layer 5 - visits



(f) Eigenvector - Layer 5 - visits

Figure 6: Top 100 central nodes under degree, betweenness, and eigenvector centrality in Layers 4–5.

(4) performing eigendecomposition to extract k-dimensional spectral embeddings; and (5) applying KMeans clustering to assign community labels Tudisco et al. (2018).

This method integrates connectivity signals from all layers while preserving node identity. The final embedding captures both local and global structure, allowing the clustering to reflect multilayer coherence rather than layer-specific artifacts.

We first present a 2D projection of the node embeddings using Principal Component Analysis (PCA). This provides an intuitive overview of the global separation among communities.

As shown in Figure 10, each point represents a sampled user, colored by its community label derived from spectral clustering. The clusters exhibit clear spatial segregation, with relatively dense cores and distinguishable boundaries. This validates the effectiveness of the multilayer Laplacian embedding in encoding modular structure even under dimensionality reduction.

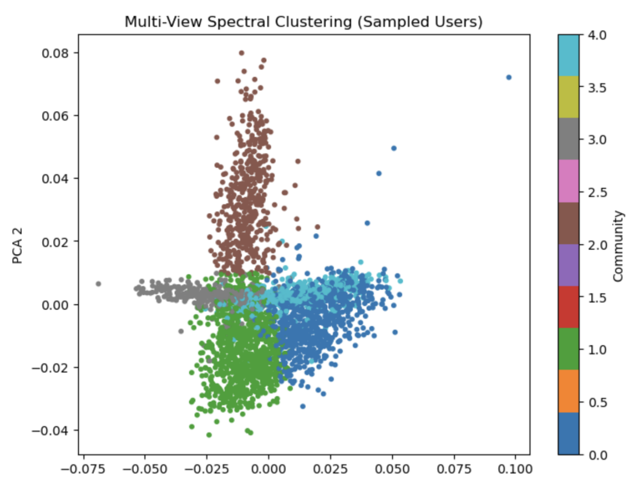


Figure 10: 2D PCA projection of sampled users colored by spectral clustering community

To visualize the resulting communities better, we also plotted the clustered nodes in 3D space from multiple perspectives. Each point represents a node, colored by its assigned cluster label. These three perspectives, shown in Figure 11, help reveal community separation and the geometric structure of the embedding.

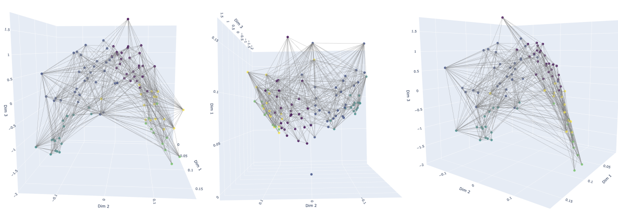


Figure 11: 3D visualization of detected communities from multiple angles

The clustering result shows that certain communities are clearly separated in the embedding space, while others are loosely distributed but still cohesive. Nodes at the interface of clusters tend to have moderate centrality and participate in bridging multiple groups. These insights reinforce the effectiveness of the multilayer embedding in uncovering latent community structure across interaction modalities.

VI. Conclusion and Future works

This study examined node centrality and community structure in a multilayer social network constructed from user interactions on the Timik.pl platform. By combining five types of user activity—friendships, messages, campaigns, transactions, and visits—we constructed a multiplex graph and applied both per-layer and integrated analysis methods.

For centrality analysis, we explored degree, betweenness, and eigenvector metrics in both single-layer and multilayer settings. Our results reveal strong intralayer coherence between centrality measures, but substantial divergence across layers, indicating that each interaction type highlights different aspects of user influence. The multilayer visualizations further revealed nodes with hybrid structural roles that would be overlooked in any individual layer.

For community detection, we employed a spectral clustering approach based on the averaged normalized Laplacian across layers. The resulting node embeddings enabled effective identification of latent community structure. Both 2D and 3D visualizations of the clustering results suggest that multilayer integration improves separation between functional groups, while preserving continuity across interaction types.

Looking ahead, several extensions are worth exploring. One direction is to incorporate edge weights and temporal information into the multilayer model, which would allow us to track dynamic changes in influence and community structure over time. Another is to experiment with alternative multilayer embedding techniques, such as graph neural networks or random-walk-based models, which may better capture nonlinear interactions. Lastly, applying the methodology to other types of real-world networks—such as mobility or citation graphs—would test its generalizability and potential for broader application.

VII. Source Code and Data Set

Github:
https://github.com/MeditatorE/Timik-Multi-layer-Network-Analysis

Data Set:
https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/V6AJRV

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