

# ASTR 415

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December 8, 2022

## Problem Set #6

### 1

1. Check your code by solving the two-body problem for equal masses ( $m_1 = m_2 = 1$ ) without softening: plot phase diagrams of  $r$  vs.  $v_r$ , where  $r$  is the separation and  $v_r = (\mathbf{v} \cdot \mathbf{r})/r$  is the radial component of the relative velocity, for 100 loops around orbits of eccentricity  $e = 0.5$  and  $e = 0.9$ , using both leapfrog and Runge-Kutta. Start at unit apoapse:  $r = r_a = 1$  (you will need to derive the starting  $v$  for yourself). By symmetry your initial conditions are then  $\mathbf{r}_1 = (-1/2, 0, 0)$ ,  $\mathbf{r}_2 = (1/2, 0, 0)$ ,  $\mathbf{v}_1 = (0, -v/2, 0)$ ,  $\mathbf{v}_2 = (0, v/2, 0)$ , where we have chosen to start on the  $x$ -axis with the orbital angular momentum aligned with the positive  $z$ -axis. Use a timestep of 0.05 for  $e = 0.5$  and 0.003 for  $e = 0.9$  (plot approximately 1000 points for each case). Also plot the energy  $E = \frac{1}{2}\mathbf{v}_1^2 + \frac{1}{2}\mathbf{v}_2^2 - 1/r$  vs. time and comment.

You may find the following relations helpful for this problem:

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{GM}; \quad r_a \equiv (1 + e)a.$$

Here  $a$  is the semi-major axis and  $M = m_1 + m_2 = 2$  is the total mass. Assume  $G \equiv 1$ . You will also need Newton's version of Kepler's Third Law to get the orbital period.

For this problem you may want to output the results to a single file, with one line per output step and each line listing data for all particles in order.

BONUS 1 (extra 10%): Generate random initial conditions of your choosing for between 100 and 1000 particles, e.g. particles in a spherical region, particles in a disk, whatever you want. Use a range of masses. Integrate this system long enough for something “interesting” to happen. What should the timestep be? Do you need softening? Is energy conserved? Does the code scale with  $N$  as you expect? Summarize the evolution of your system in a plot of some kind to hand in.

For this problem is a good idea to have an output format that is the same as the input format, with one file per output (use `snprintf()` to embed the current step number to create unique output filenames) – this has the advantage that you can restart from any output step.

BONUS 2 (extra 2%): Make a movie of the results.

Below are some of the animations that are created compiling and running my code using make. My reasoning for testing various softening parameters is to ultimately simulate an N-body problem. Because this assignment is just for binaries I do not make any conclusion. However, in my research I found that for N-body simulations such as the Plummer Sphere model:

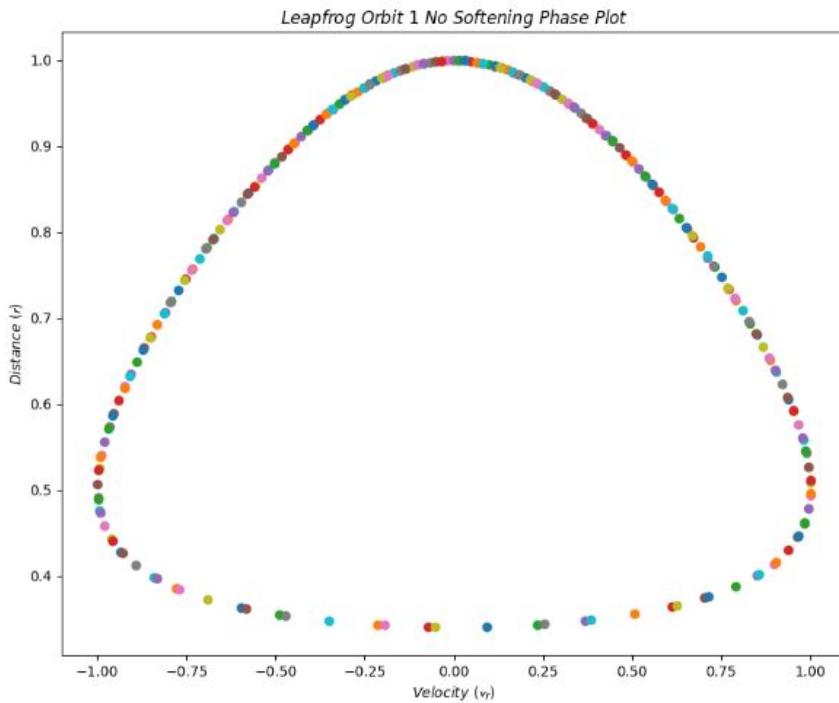
”For small values of the softening the noise dominates the error. For this reason the MASE, for such values of the softening, decreases steeply with N, the number of particles in the configuration. Conversely for large values of the softening it is the bias which dominates. For a sufficiently large value of the softening the MASE does not show any dependence, either on the softening, or on the number of particles.” [1]

**Leapfrog Orbit 1: Eccentricity=0.5, Stepsize=0.05, No Softening**

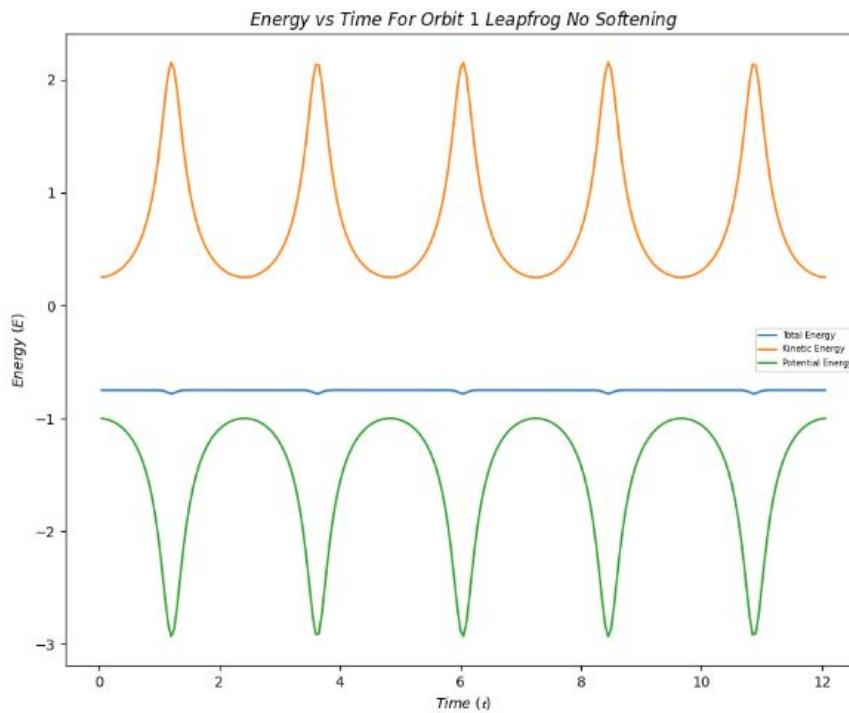
[Embeded Animation 1](#)

Animations only work in Adobe Acrobat or possibly something else fancy with the backend to run animations in a pdf.

Leapfrog Orbit 1 Phase Plot: Eccentricity=0.5, Stepsize=0.05, No Softening



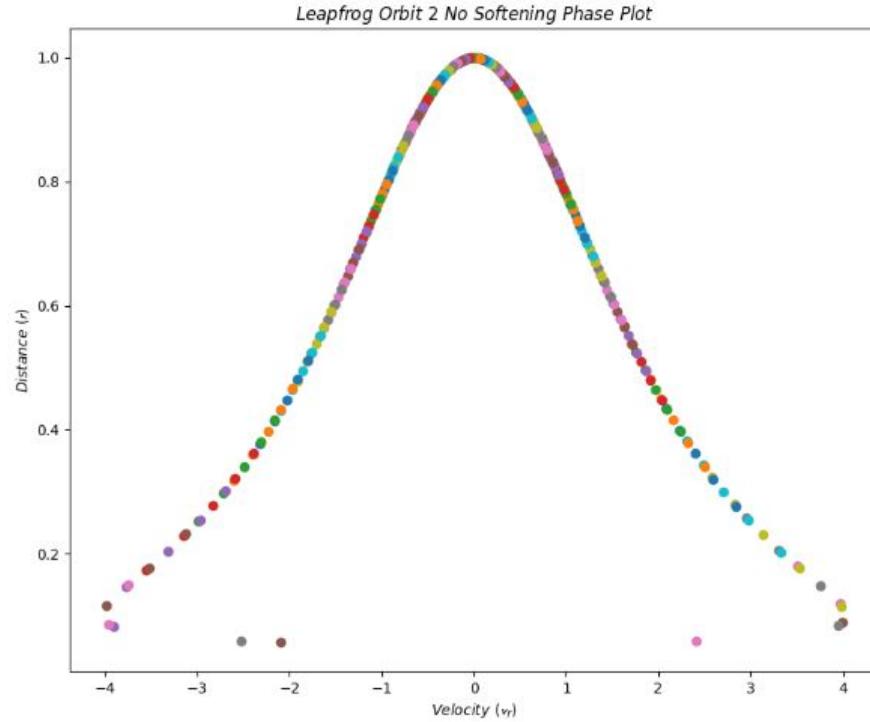
Leapfrog Orbit 1 Energy: Eccentricity=0.5, Stepsize=0.05, No Softening



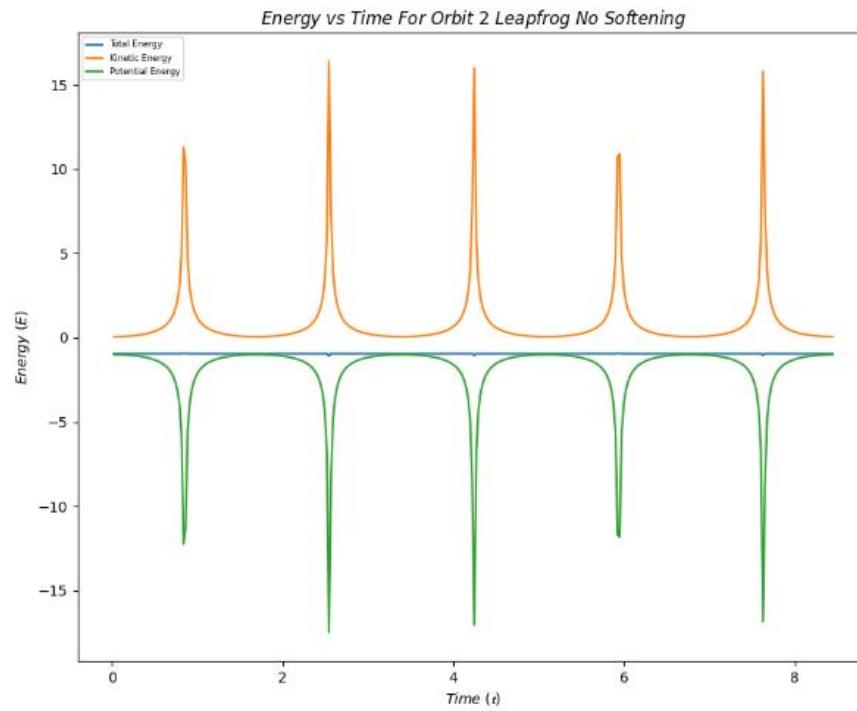
**Leapfrog Orbit 2: Eccentricity=0.9, Stepsize=0.003, No Softening**

[Embeded Animation 2](#)

**Leapfrog Orbit 2 Phase Plot: Eccentricity=0.9, Stepsize=0.003, No Softening**



**Leapfrog Orbit 2 Energy: Eccentricity=0.9, Stepsize=0.003, No Softening**

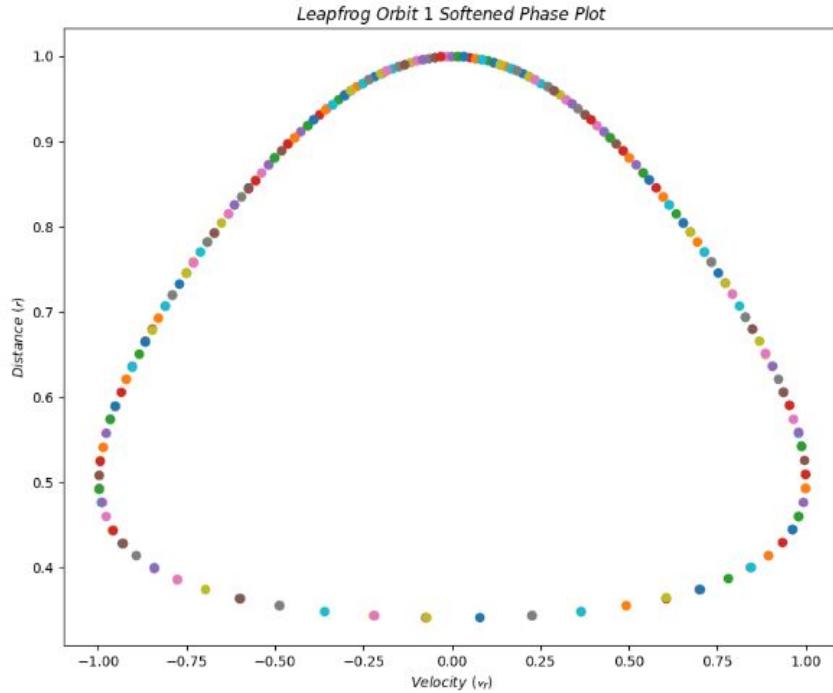


Small changes in the orbital paths can be seen after testing a softening parameter I found in an article [2] by Joshua Barnes of the Institute of Astronomy at the University of Hawaii where he discusses: "How much softening is too much?" He concludes a resolution somewhat better than 0.1a is required to see the inner cusps of the Jaffe, Hernquist, and NFW models. This implies the softening parameter must be several times smaller than 0.1a.

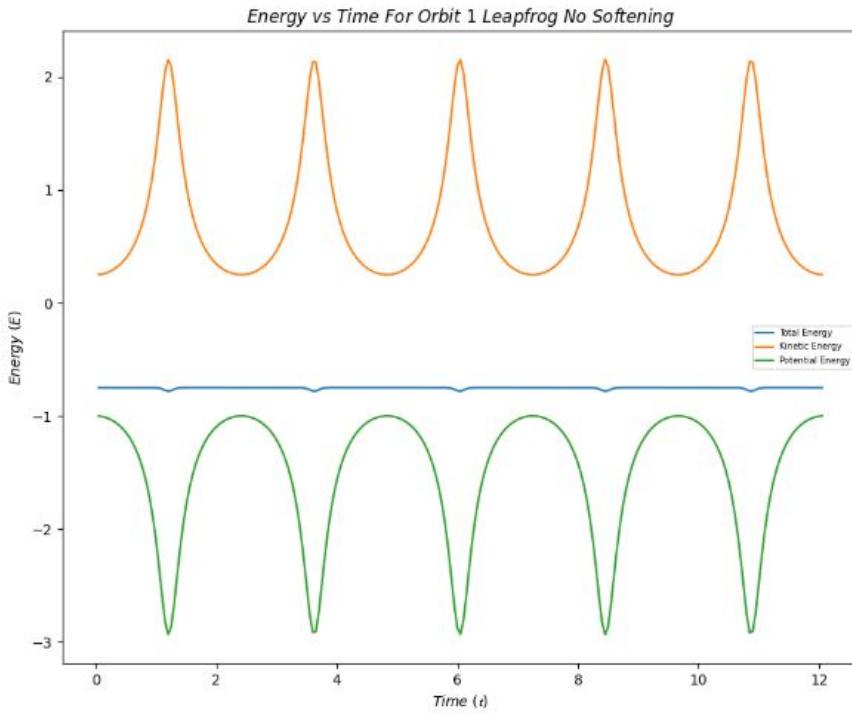
**Leapfrog Orbit 1 Soft: Eccentricity=0.5, StepSize=0.05, Softening=0.1×a/3**

[Embeded Animation 3](#)

Leapfrog Orbit 1 Phase Plot Soft: Eccentricity=0.5, Stepsize=0.05, Softening=0.1×a/3



Leapfrog Orbit 1 Energy: Eccentricity=0.5, Stepsize=0.05, Softening=0.1×a/3



In the case above  $\epsilon=0.1*0.47/3=0.0157$ . The semi major axis  $a=0.47$  for orbit case 1. This follows the wording in Barnes' paper as several times smaller than  $0.1 \times a$ . I have left the latest softening parameter implemented when the code is run as this value. Below is the softening parameter changed for Orbit 2 where  $a=0.26$ .

**Leapfrog Orbit 2: Eccentricity=0.9, Stepsize=0.003, Softening=0.05×a**

[Embeded Animation 4](#)

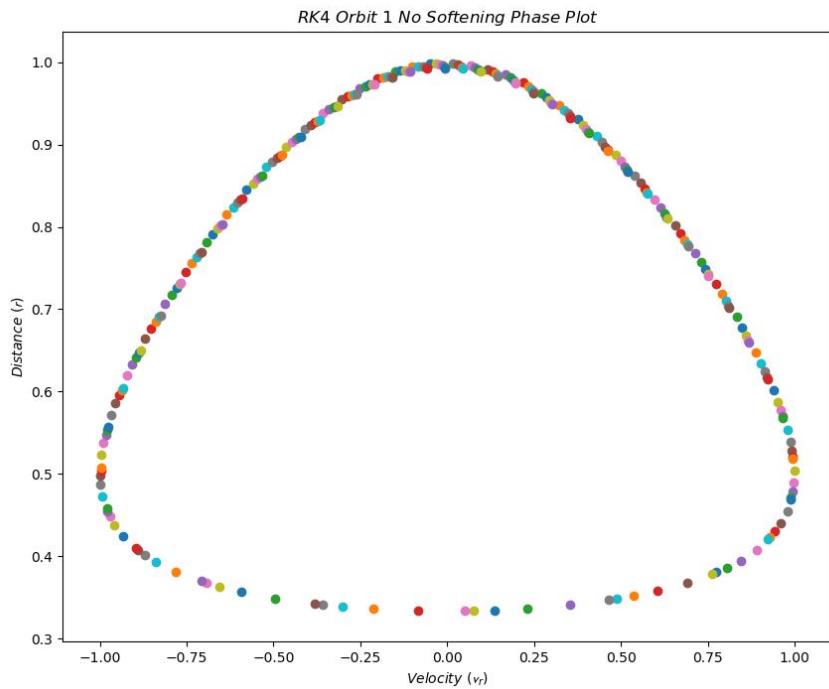
In the event that running the code uses the softening parameter meant for Orbit 1 for calculating Orbit 2 the resulting change in the orbital path for Orbit 2 is significant. This greater error due to the larger softening parameter then should be used for the smaller semi-major axis produces much more precession as shown in Embeded Animation 4.

Another optimal softening parameter I found was for N-body simulations. This varied based on the number of masses in the system. According to the study in Hawaii [1], given a small number of masses in a Plummer Sphere N-body simulation  $\epsilon$  should be equal to  $0.98 \times N^{-0.28}$ . They used a GRAPE system to determine optimal softening for collisionless force calculations. For a large number of masses the parameter is only slightly changed to  $0.98 \times N^{-0.26}$ .

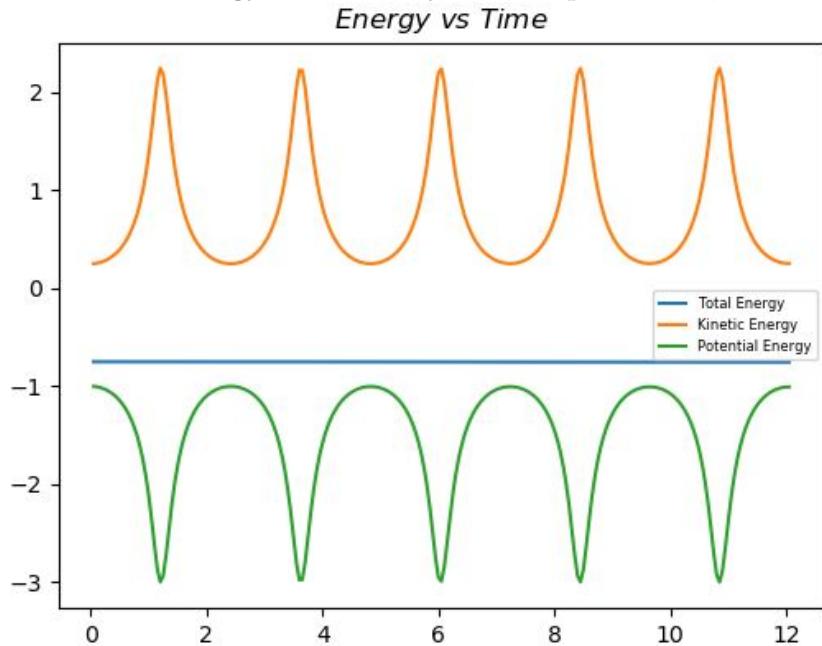
**RK4 Orbit 2: Eccentricity=0.9, StepSize=0.003, No Softening**

[Embeded Animation 5](#)

**RK4 Orbit 1 Phase Plot: Eccentricity=0.5, Stepsize=0.05, No Softening**

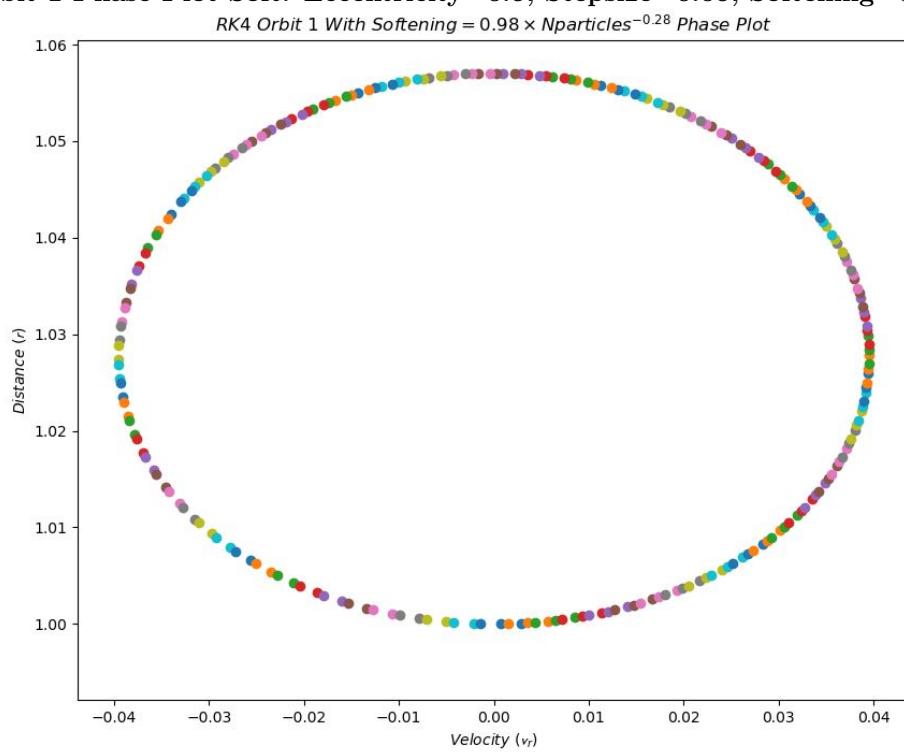


**RK4 Orbit 1 Energy: Eccentricity=0.5, Stepsize=0.05, No Softening**

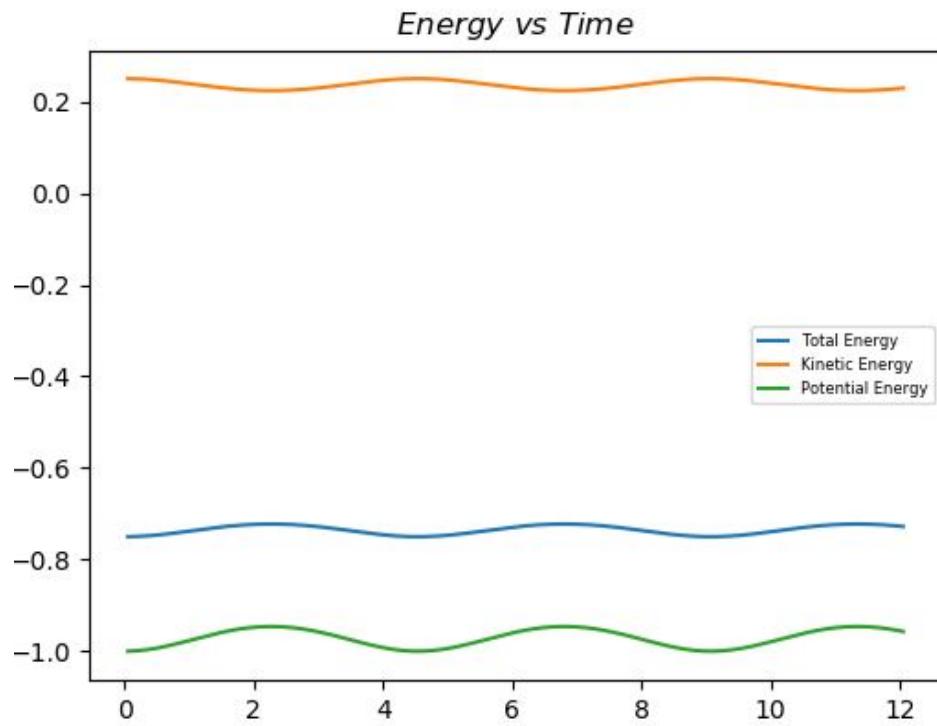


**RK4 Orbit 2 Soft 2:** Eccentricity=0.9, Stepsize=0.003, Softening= $0.98 \times N^{-0.28}$   
[Embeded Animation 6](#)

**RK4 Orbit 1 Phase Plot Soft: Eccentricity=0.5, Stepsize=0.05, Softening=0.98 ×  $N^{-0.28}$**



**RK4 Orbit 1 Energy Soft: Eccentricity=0.5, Stepsize=0.05, Softening=0.98 ×  $N^{-0.28}$**



In total I used python to plot 8 phase plots and 8 energy plots: This included one plot for each orbit case 1 & 2 x Leapfrog vs RK4 x With Softening vs Without Softening. The softening parameter can be accessed and easily changed to the Barnes parameters for either one of the orbit cases (currently set to the semi-major axis of orbit 1) or to the version for N-Body problems based on the number of masses in my problem1\_softening.c file inside the function deriv which is at the beginning of the code.

**Leapfrog Orbit 2 Soft 2: Eccentricity=0.9, Stepsize=0.003, Softening=** $0.98 \times N^{-0.28}$

[Embeded Animation 7](#)

Here is my 5 particle Supernova.

**N-Body Beginnings of a Super Nova?**

[Embeded Animation 8](#)

Creating these gifs require uploading over 100 PNGs, and overleaf is timing me out when I upload a lot, but also not allowing me to compile the pdf because of a time out due to it taking too long. Please see files or my report when I finish for some better examples I have.

**References**

- [1] Athanassoula, E, Fady, E, Lambert, J. C, Bosma, A, Optimal softening for force calculations in collisionless N-body simulations, <https://home.ifa.hawaii.edu/users/barnes/research/smoothing/soft.pdf>
- [2] Barnes, Joshua, Gravitational softening as a smoothing operation, <https://arxiv.org/abs/astro-ph/9912467>