Photonic Implementation of Two-Qubit Gates for Quantum Computation

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1 Introduction

Quantum computing harnesses the principles of quantum mechanics, such as superposition and entanglement, to perform computations, enabling solutions to problems beyond the reach of classical computers. While many physical platforms for quantum computing have been explored, photonic quantum computing distinguishes itself with its low decoherence rates and resilience to environmental noise, making photons ideal carriers of quantum information.

Central to the scalability of quantum computers is the implementation of two-qubit gates. One approach to this is the KLM protocol, which utilizes only linear optical devices in addition to a probabilistic single-photon source, as well as post-selection measurement.

This paper reviews the development of two-qubit gates in photonic quantum computing, focusing on one key experimental demonstration of a non-deterministic CNOT gate by J. L. O'Brien, G. J. Pryde, A. G. White, T. C. Ralph, and D. Branning. We will also show the impact of this realization.

2 Photonic Two Qubit Gate and the KLM protocol

In 1999, Adam and Cerf showed that one-qubit gates could be done efficiently using only linear optical devices.[1] However, the other piece for obtaining universal set of gates, the two-qubit gate, remained a challenge. This is due to the non-interacting nature of photons, while offers low decoherence rate, but also difficult to interact with each other. The most representative two-qubit gate is the CNOT gate, which is essential to implementation of many quantum algorithms and easy to assess its fidelity.

The Knill-Laflamme-Milburn (KLM) protocol introduced in 2001 demonstrated that universal quantum computation is possible using only linear optics, single-photon sources, and photon detectors, provided one accepts a probabilistic implementation of two-qubit gates.[2] The KLM protocol relies on auxiliary photons and measurement-induced entanglement to implement gates like the CNOT and C-sign. Next we will take a closer look at its first experimental implementation.

3 Probabilistic CNOT Gate Realization

In 2003, J. L. O'Brien and collaborators experimentally realized a probabilistic CNOT gate based on the KLM proposal.[3]

3.1 Setting of Experiment

The overall experiment setting is following: Figure 1a is the conceptual drawing of the CNOT gate in this experiment; Figure 1b is the encoding conversion process used; figure 1c is the actual experiment setup divided into five stages.

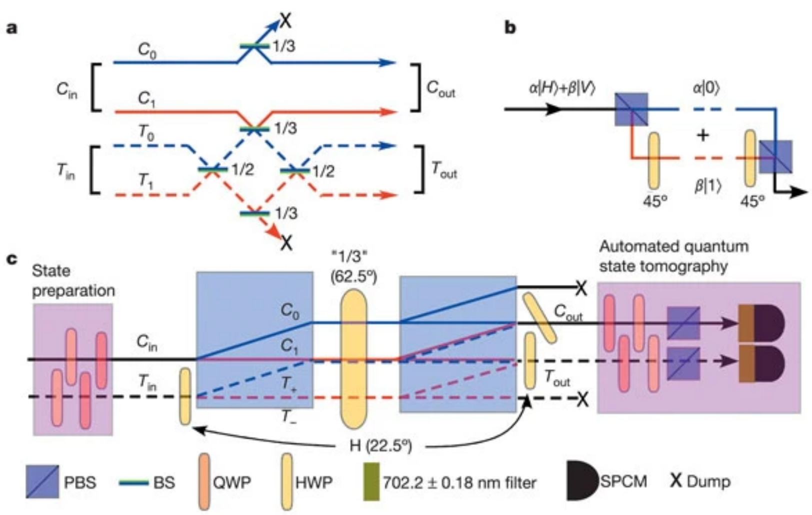

Figure 1: Setting of the experiment.

- **1. Initialization:** Single pair of photons are produced by the spontaneous parametric down-conversion (SPDC) approach, which generates entangled pairs of photons. The experiment will take one photon in each pairs as the control and target qubit. The qubits are initialized in polarization states (H as 0, and V as 1) for the ease of measurement in mature optical devices like polarization beam splitter (PBS).

- **2. Conversion:** Polarization-only gates cannot create the necessary photon-photon interference for a two-qubit operation because two qubits encoded in polarization and traveling in the same spatial mode do not naturally interact. Therefore, the experiment makes a conversion to spatially encoded qubits via the procedure shown in Figure 1b.

- **3. CNOT Gate Operation:** After conversion to spatial encoding, the target modes are combined and then split using two 50% beam splitters (1/2BSs), forming an interferometer. Each arm of the interferometer includes a 33% beam splitter (1/3BS). When the control qubit is in the state $|0\rangle$, this condition holds because there is no interaction between the control and target qubits.

  However, when the control qubit is in the state $|1\rangle$, quantum interference occurs between the control and target photons at the central 1/3BS due to the indistinguishability of their paths. This two-photon quantum interference introduces a π-phase shift in the upper arm of the target interferometer, flipping the target qubit state as follows:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \beta|0\rangle + \alpha|1\rangle.$$

The logical value of the control qubit remains unchanged.

- **4. Conversion:** After passing through the interferometer, the spatially encoded qubits were converted back to polarization encoding for measurement. Single-photon counting modules (SPCMs) were used to detect both the control and target photons. A successful detection of both photons, known as a coincidence event, indicates that the gate operation was performed correctly. The detection window for these events was set to 5 nanoseconds.

- **5. Analysis** This step includes post-selection results from the measurement, as well as state tomography to obtain the final output states.

4 Characterization

In the original KLM CNOT gate, nonlinearity is achieved using ancilla qubit.[2] In this experiment, this is implemented using control and target qubits as their own ancilla, which is valid because a successful gate operation would require concidence event in both qubits.

During the experiment, the initial polarization encoding is converted to spatial encoding and then back to polarization encoding. To ensure accurate operation, it is crucial to preserve the phase relationship between the two basis components throughout this process. Experimentally, this requires the path lengths to remain stable to within a fraction of the wavelength (subwavelength stability). Achieving this involves satisfying two classical interference conditions and one conditional non-classical interference condition, which occurs when the control qubit is in the state 1.

Due to the properties of the 1/3 beam splitters (1/3BSs), single-photon detection does not always occur simultaneously in both the control and target outputs. In the absence of a control photon, the target qubit exits the interferometer in the same state as it entered. However, when both photons are successfully detected—a coincidence event, which occurs with a probability of $P = \frac{1}{9}$—the CNOT gate operation is confirmed to have been correctly implemented.

5 Results and Error Analysis

5.1 Fidelity of the CNOT Gate

Overall, the experiment achieved an overall 84 percent accuracy with logical basis input states, the details of the probabilities can be found here. Figure 2a is the ideal distribution while 2b is the measured distribution. These results indicate the gate has a great performance when the control qubit is in state 0, indicating the two classical interferences successfully preserves the phase.

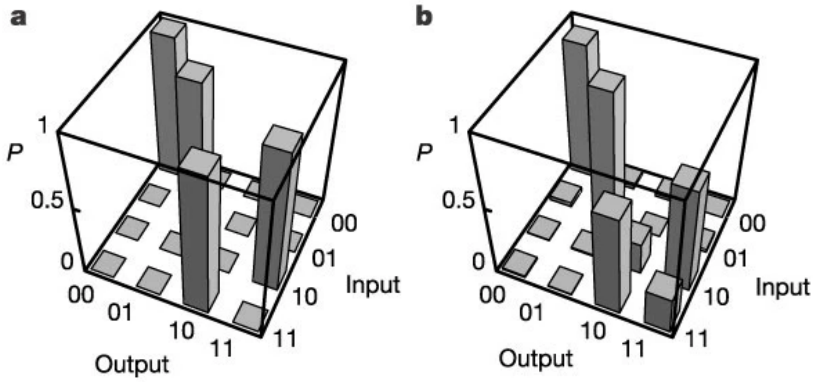

Figure 2: Ideal and measured basis operations of CNOT gate.

Next through quantum state tomography, the experiment is also able to reconstruct the density matrix of the output state, and assess the ability of the gate in creating entanglement. Figure 3 shows the gate could produce maximally entangled state (Bell States) with an average fidelity over 75%, with conditional fringe visibilities (a measure of entanglement given by the state's concurrence squared) exceeding 90% in non-orthogonal bases. Demonstrating strong non-classical interference of the gate.

Table 2 **Characterization of the four Bell states**

State	Fidelity	Tangle	Linear entropy
$\Psi^- = \|01\rangle - \|10\rangle$	0.87(8)	0.65(6)	0.27(6)
$\Psi^+ = \|01\rangle + \|10\rangle$	0.75(9)	0.55(7)	0.31(5)
$\Phi^- = \|00\rangle - \|11\rangle$	0.76(9)	0.46(5)	0.45(3)
$\Phi^+ = \|00\rangle + \|11\rangle$	0.77(9)	0.49(9)	0.45(5)

The degree of entanglement of any state can be measured by calculating the tangle $T = C^2$, where C is the concurrence[26,27]. Similarly, the degree of mixture can be measured by calculating the linear entropy[26,27]. These values, along with fidelities, are given for all four experimentally produced Bell states.

Figure 3: Characterization of the four bell states.

6 Challenges

While the CNOT gate demonstrated high fidelity through both classical and quantum interferences, several challenges persist in the experiment. The primary source of error was identified as decoherence arising from imperfect mode matching between the C_1 and T_+ modes during the non-classical interference. This mismatch directly led to lower fidelities when the control qubit was in the state 1 and the target qubit failed to flip as expected.

Another issue was the fidelity variation among the four Bell states, which could be attributed to minor beam steering effects introduced by the state preparation waveplates. These subtle misalignments during preparation contributed to inconsistent outcomes.

Finally, false coincidence event counting introduced additional errors. These false events inflated the counts where the control qubit was in the state 0 while the target qubit was erroneously flipped, further impacting the overall fidelity.

7 Conclusion

In my opinion, this experiment serves as a significant proof of concept, as it is the first all-optical implementation of the KLM CNOT gate with arbitrary input states. While the experiment in Pittman et al. (2001)[4] demonstrated the gate for logical basis input states, this experiment extends the implementation to arbitrary input states. This enhancement suggests that the CNOT gate could be integrated into teleportation protocols, which would ensure scalability in future quantum computing systems.

The sources of error observed in the experiment, such as imperfect mode matching and minor beam steering effects, can be addressed through advancements in hardware. For instance, these issues could be mitigated by using better materials and more precise control mechanisms. Additionally, false event counting could be reduced by narrowing the detection time window as measurement devices improve.

Furthermore, the high fidelity observed in this experiment is particularly impressive, especially considering that no error correction or extra ancilla qubits were used. This suggests that the experiment is a promising and successful demonstration of the all-optical CNOT gate, with significant potential for scalability and further refinement.

References

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