

# CSim<sup>2</sup>: Compositional Top-down Verification of Concurrent Systems using Rely-Guarantee

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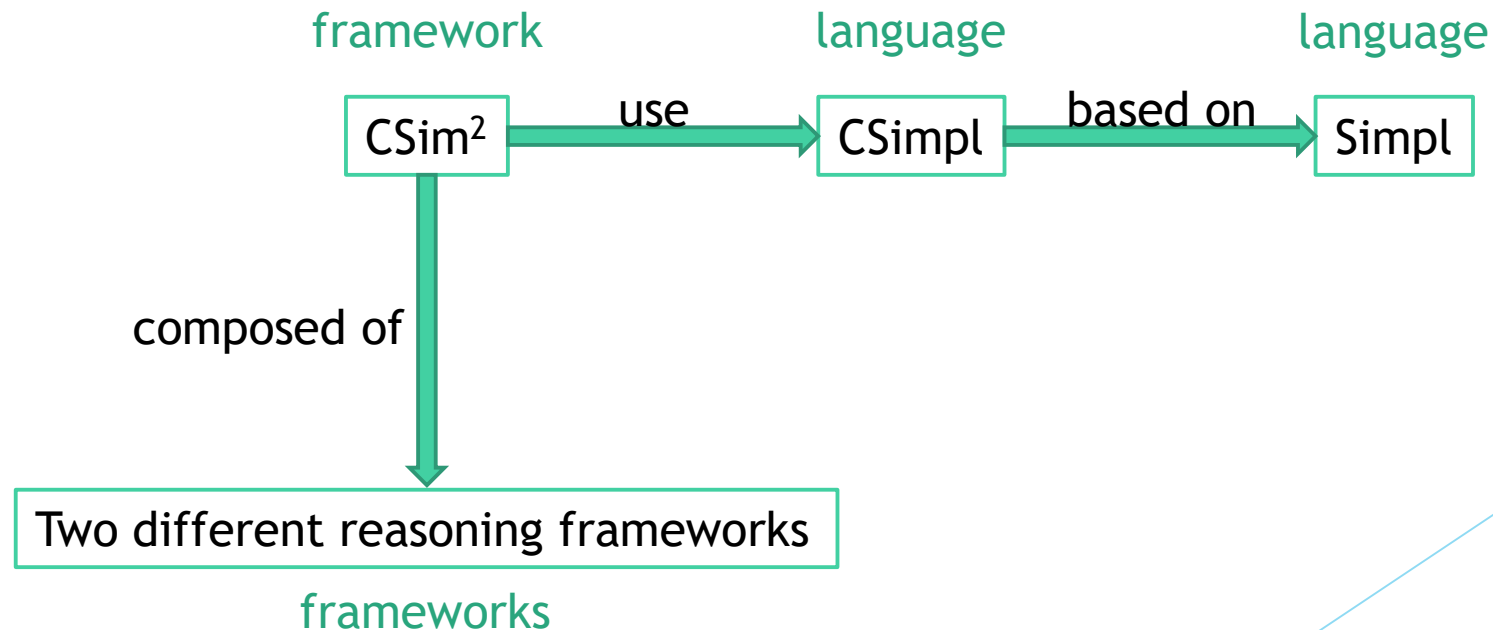
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# 1. Introduction

## main concepts

- ▶ 1. **Top-down verification** uses step-wise verification between different layers of **refinement**, where verification of properties is conducted on the **top** layers representing **high** specification levels. Then, those verified properties are propagated down to the **bottom** layers representing **low** specification levels.
- ▶ 2.



## 2. CSIM<sup>2</sup> Overview

### CSim<sup>2</sup> Architecture

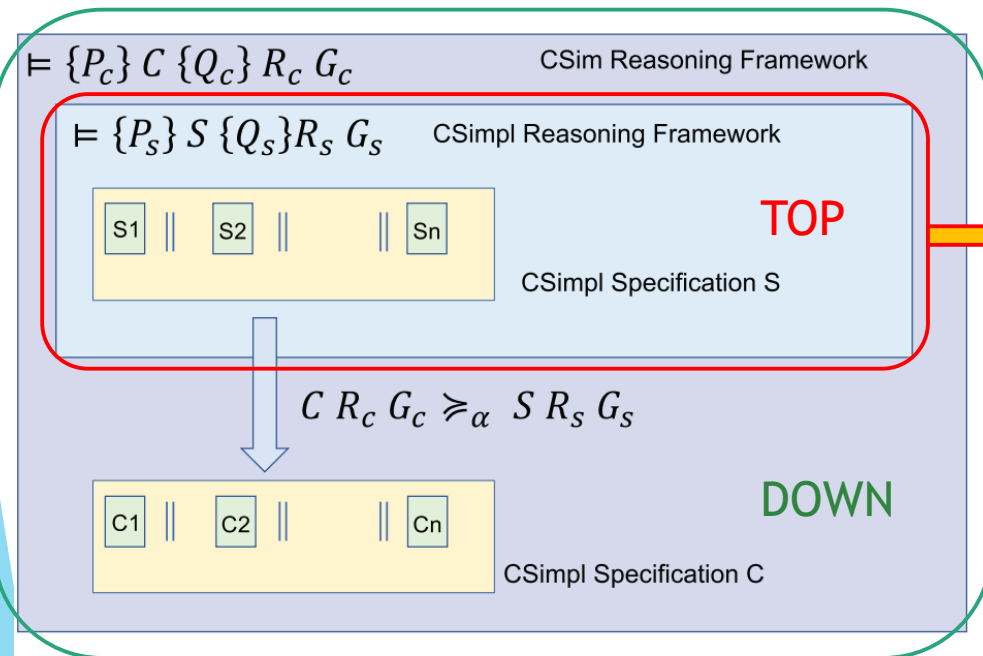


Fig. 1. CSim<sup>2</sup> Architecture.

► CSim<sup>2</sup> allows a **top-down** verification approach to verify properties on concurrent specifications by means of **two different reasoning frameworks**:

► a **rely-guarantee reasoning framework** for the verification of properties in concurrent CSimpl specifications;

► and a **simulation-based rely-guarantee property preservation framework**, to show that properties proven in an abstraction layer are preserved on lower layers.

## 2. CSIM<sup>2</sup> Overview

### A Rely-Guarantee Reasoning Framework

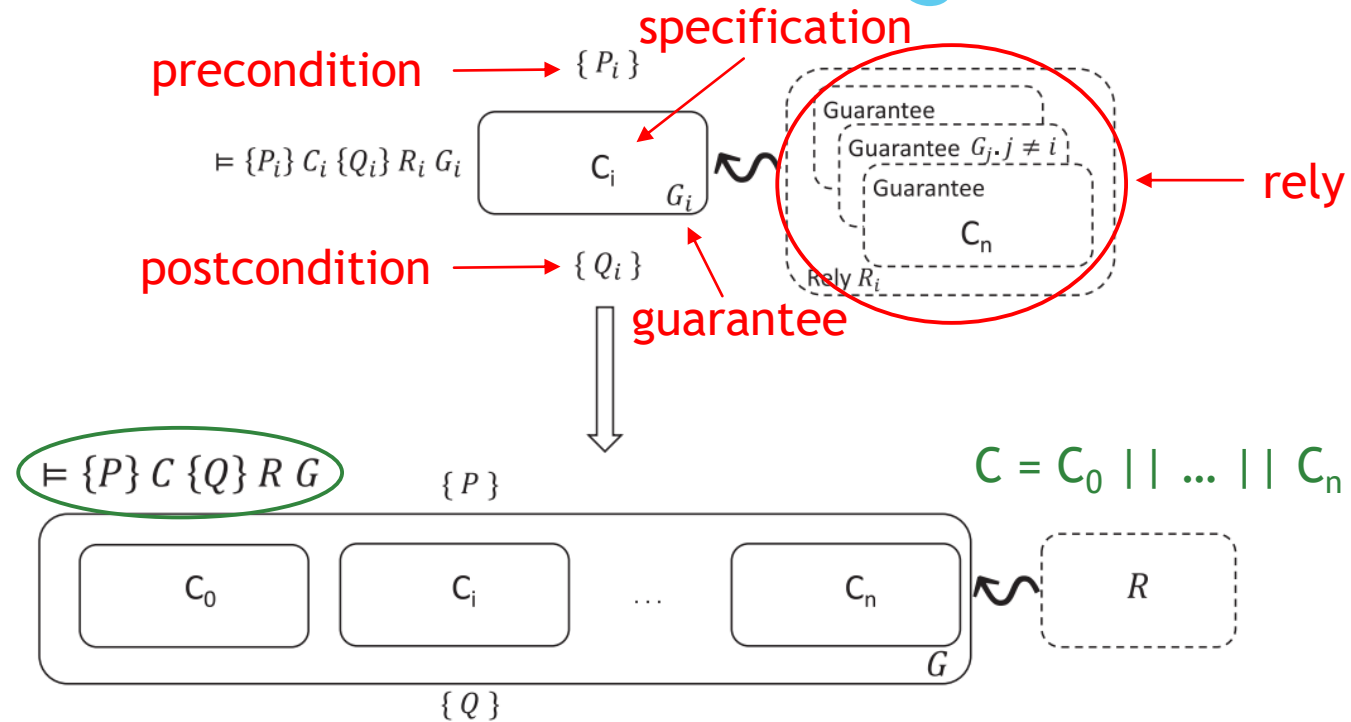


Fig. 2. CSim<sup>2</sup> Property Verification Compositionality.

- CSim<sup>2</sup> provides **sound reasoning rules** for all the constructs of the specification language and a **parallel compositional rule** for the verification of parallel systems.

## 2. CSIM<sup>2</sup> Overview

### A Simulation-Based Rely-Guarantee Property Preservation Framework

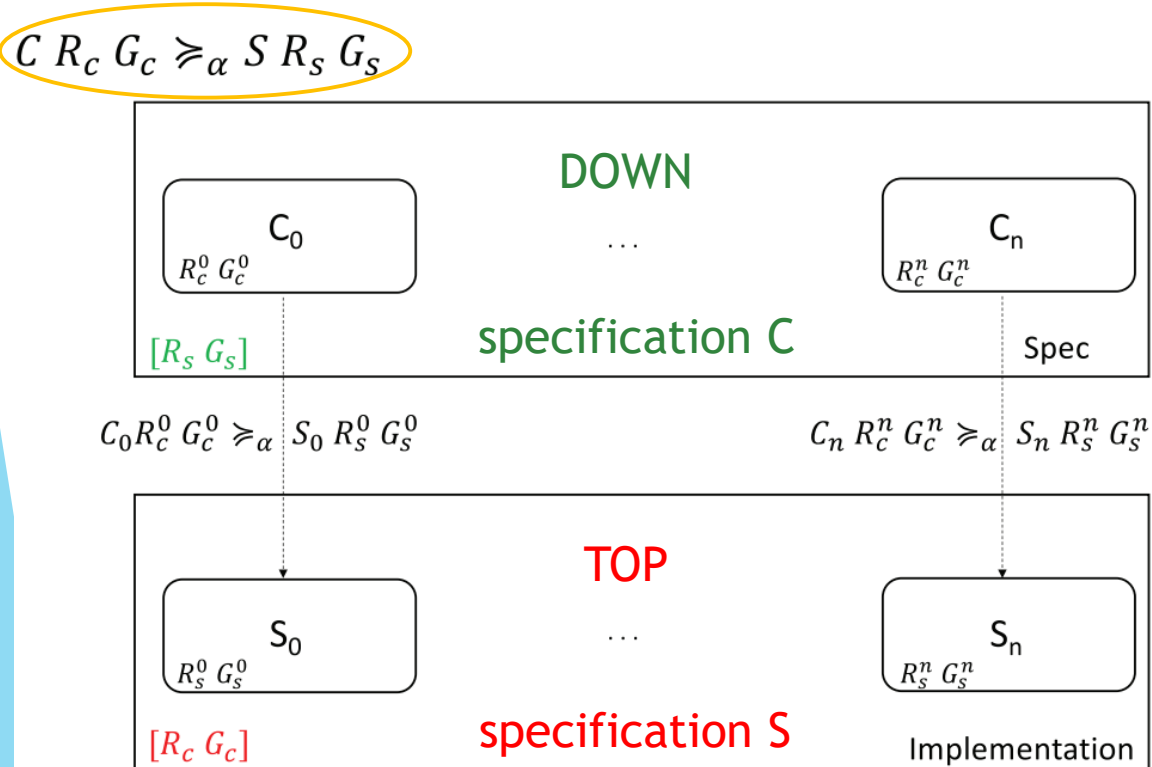


Fig. 3. CSim<sup>2</sup> Simulation Compositionality.

$$|=_{\text{p}} \{ P_s \} S \{ Q_s \} R_s G_s$$

► Prove  $|=_{\text{p}} \{ P_c \} C \{ Q_c \} R_c G_c$  by showing that:

- (1) given a simulation relation  $\alpha$ , and the parallel specification  $S$  with rely and guarantee relations  $R_s$  and  $G_s$  then  $C$ , with the relations  $R_c$  and  $G_c$ , is an implementation of  $S$  in  $\alpha$ , denoted as

$$C R_c G_c \succcurlyeq_\alpha S R_s G_s$$

- (2)  $P_c \subseteq \alpha' P_s$  and  $Q_c \subseteq \alpha' Q_s$ , where  $\alpha'$  is the image operation over a relation and a set.

# 3. Syntax and Semantics of CSimpl Language

## CSimpl Syntax

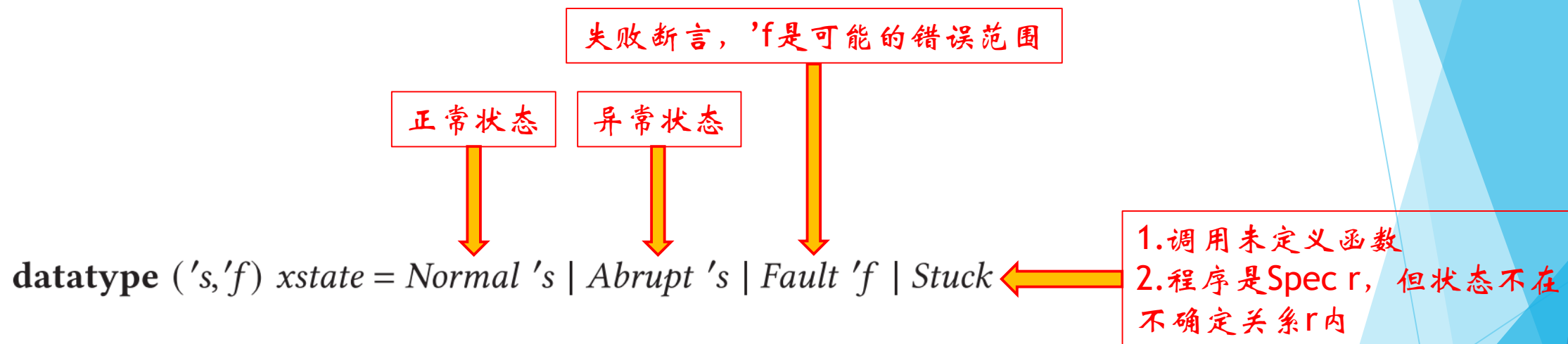
状态      程序名      失败状态

**type-synonym**  $'s \text{ bexp} = 's \text{ set}$   
**datatype**  $('s, 'p, 'f) \text{ com} =$   
 $\text{Skip} \mid \text{Throw} \mid \text{Basic } 's \Rightarrow 's \mid \text{Spec } ('s \times 's) \text{ set} \mid \text{Seq } ('s, 'p, 'f) \text{ com } ('s, 'p, 'f) \text{ com}$   
 $\mid \text{Cond } 's \text{ bexp } ('s, 'p, 'f) \text{ com } ('s, 'p, 'f) \text{ com} \mid \text{While } 's \text{ bexp } ('s, 'p, 'f) \text{ com} \mid \text{Call } 'p$   
 $\mid \text{DynCom } 's \Rightarrow ('s, 'p, 'f) \text{ com} \mid \text{Guard } 'f 's \text{ bexp } ('s, 'p, 'f) \text{ com}$   
 $\mid \text{Catch } ('s, 'p, 'f) \text{ com } ('s, 'p, 'f) \text{ com} \mid \text{Await } 's \text{ bexp } ('s, 'p, 'f) \text{ scom}$   
**datatype**  $('s, 'f) \text{ xstate} = \text{Normal } 's \mid \text{Abrupt } 's \mid \text{Fault } 'f \mid \text{Stuck}$   
**type-synonym**  $('s, 'p, 'f) \text{ config} = ('s, 'p, 'f) \text{ com} \times ('s, 'f) \text{ xstate}$   
**type-synonym**  $('s, 'p, 'f) \text{ body} = 'p \Rightarrow ('s, 'p, 'f) \text{ com option}$   
**type-synonym**  $('s, 'p, 'f) \text{ par-Simpl} = ('s, 'p, 'f) \text{ com list}$

Fig. 4. Syntax and state definition of the CSimpl Language.

### 3. Syntax and Semantics of CSimpl Language

#### CSimpl Syntax

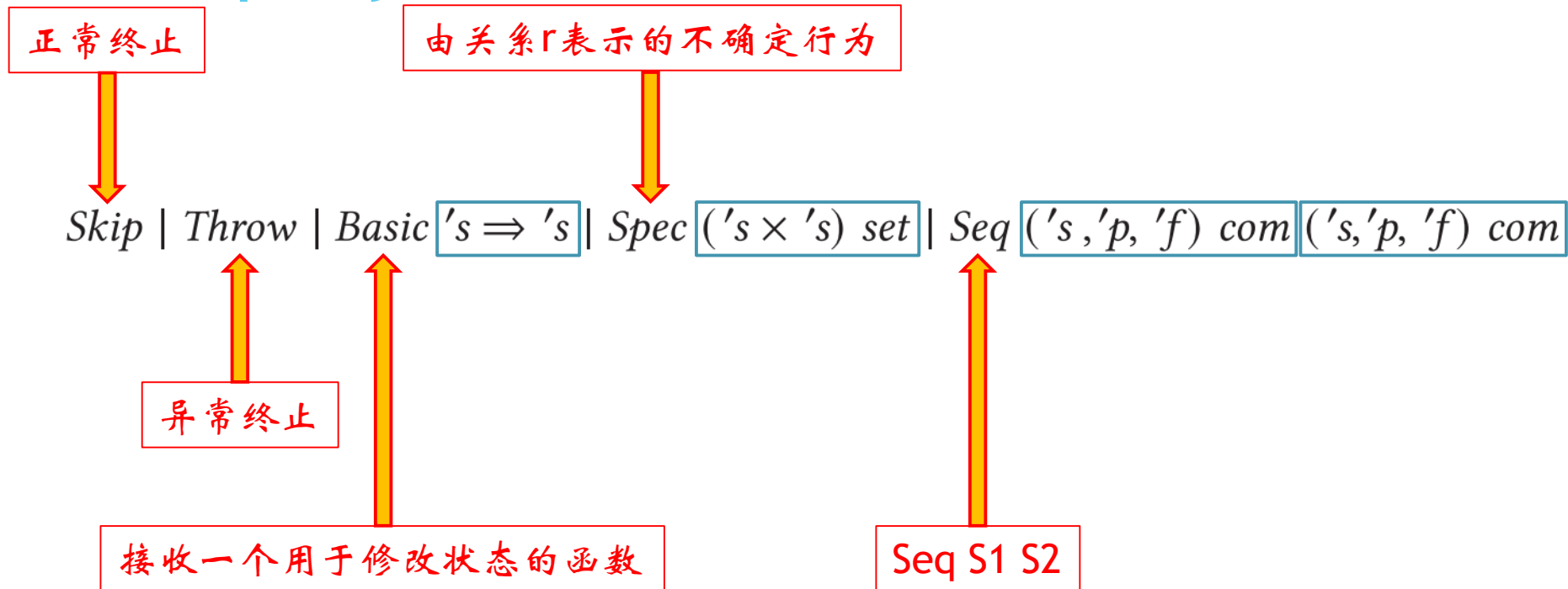


For simplicity on the notation, we represent the xstates `Normal  $\sigma$` , `Abrupt  $\sigma$` , and `Fault  $F$`  with  $\sigma \uparrow_N$ ,  $\sigma \uparrow_A$ , and  $\sigma \uparrow_F$ . We use  $\uparrow^N \sigma$  to represent the predicate  $\exists \sigma_n. \sigma = \text{Normal } \sigma_n$ ,  $\uparrow^A \sigma$  to represent the predicate  $\exists \sigma_a. \sigma = \text{Abrupt } \sigma_a$ , and  $\uparrow^F \sigma$  to represent the predicate  $\exists f. \sigma = \text{Fault } f$ . Throughout this article, we will refer to the type `(' $\sigma$ , 'f)xstate` as ' $\sigma$ ' when is clear in the context.



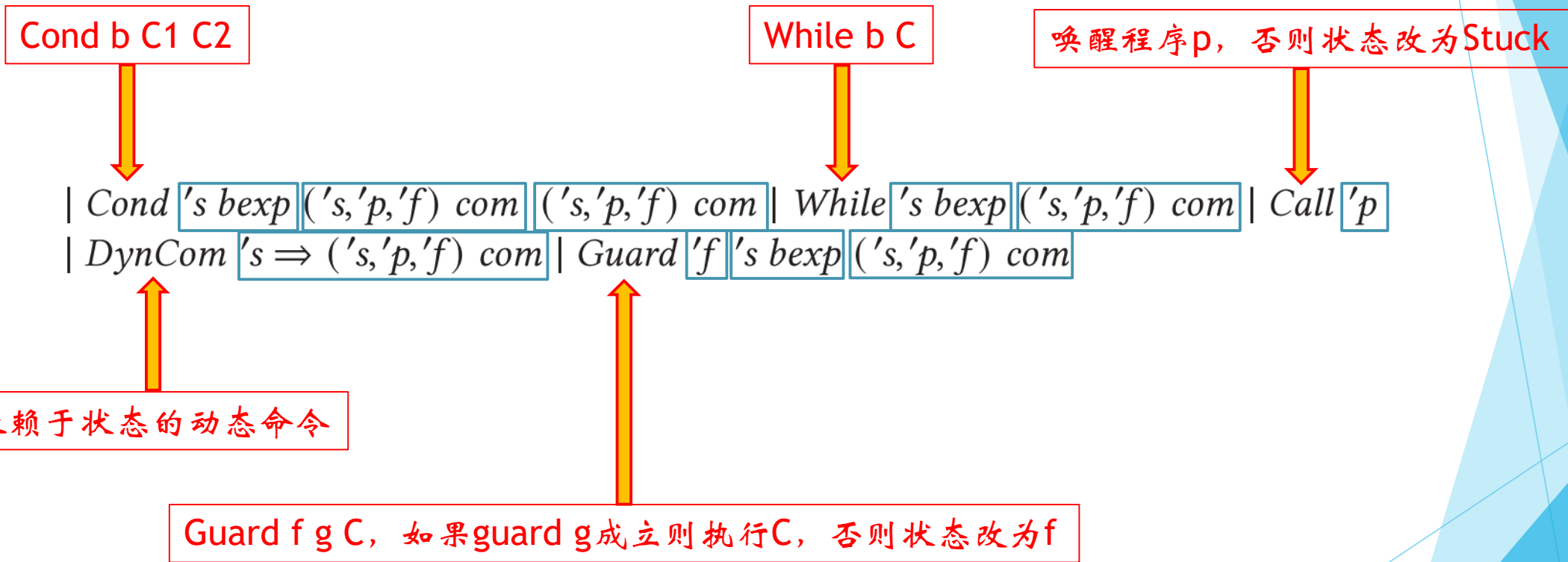
# 3. Syntax and Semantics of CSimpl Language

## CSimpl Syntax



# 3. Syntax and Semantics of CSimpl Language

## CSimpl Syntax



# 3. Syntax and Semantics of CSimpl Language

## CSimpl Syntax

Catch C1 C2, 执行C1, 若C1 Throw, 则执行C2

| Catch ('s,'p,'f) com ('s,'p,'f) com | Await 's bexp ('s,'p,'f) scom

Await b body

- ▶ Await allows synchronization of processes on a predicate **b** over shared variables and it atomically executes **body**.

# 3. Syntax and Semantics of CSimpl Language

## Simpl Syntax

CSimpl extends Simpl by adding two constructs for concurrency: Await and Parallel Composition.

```
type-synonym 's bexp = 's set
datatype ('s, 'p, 'f) com =
  Skip | Throw | Basic 's  $\Rightarrow$  's | Spec ('s  $\times$  's) set | Seq ('s, 'p, 'f) com ('s, 'p, 'f) com
  | Cond 's bexp ('s, 'p, 'f) com ('s, 'p, 'f) com | While 's bexp ('s, 'p, 'f) com | Call 'p
  | DynCom 's  $\Rightarrow$  ('s, 'p, 'f) com | Guard 'f 's bexp ('s, 'p, 'f) com
  | Catch ('s, 'p, 'f) com ('s, 'p, 'f) com | Await 's bexp ('s, 'p, 'f) scom
datatype ('s, 'f) xstate = Normal 's | Abrupt 's | Fault 'f | Stuck
type-synonym ('s, 'p, 'f) config = ('s, 'p, 'f) com  $\times$  ('s, 'f) xstate
type-synonym ('s, 'p, 'f) body = 'p  $\Rightarrow$  ('s, 'p, 'f) com option
type-synonym ('s, 'p, 'f) par-Simpl = ('s, 'p, 'f) com list
```

Fig. 4. Syntax and state definition of the CSimpl Language.

# 3. Syntax and Semantics of CSimpl Language

## Simpl Syntax

► Some examples of the concrete syntax of Simpl are given by:

- $C1; ;_s C2$  represents  $\text{Seq } C1 \ C2$ .
- $\text{IF}_s \ b \ \text{THEN } C1 \ \text{ELSE } C2 \ \text{FI}$  represents  $\text{Cond } b \ C1 \ C2$ .
- $\text{WHIL } E_s \ b \ \text{DO } C \ \text{OD}$  represents  $\text{While } b \ C$ . A variance  $\text{WHILE}_s b \ \text{DO } \text{INV } I \ C \ \text{OD}$  represents the loop annotated with the invariant  $I$ .
- $f :=_s v$  represents  $\text{Basic}(\lambda s. \ s(f\_ := v))$ .  $f$  is used to represent the selects and updates of fields of the state when the state is represented by a record.

# 3. Syntax and Semantics of CSimpl Language

## big step and small step semantics

- ▶ The behavior of Simpl programs is defined in terms of big step and small step semantics.
- ▶ In the big step semantics,

$$\Gamma \vdash \langle C, \sigma \rangle \Rightarrow \sigma'$$

represents that in an **environment** for procedures  $\Gamma$ , the program  $C$  starting from the state  $\sigma$  reaches a final state  $\sigma'$ .

- ▶ The small step semantics uses a fine grain transition to carry out a single step.

$$\Gamma \vdash \langle C, \sigma \rangle \rightarrow \langle C', \sigma' \rangle$$

transitions in a step from the configuration  $\langle C, \sigma \rangle$  to  $\langle C', \sigma' \rangle$ .

### 3. Syntax and Semantics of CSimpl Language

#### partial and total correctness

初始状态是Normal状态

To reason about programs, Simpl includes a sound and partially complete reasoning framework for **partial and total correctness** of specifications based on a Floyd/Hoare-like logic. A Simpl specification is given by a **program  $C$**  in an **environment** for procedures  $\Gamma$ , and a **precondition  $P$**  and **postconditions  $Q, A$**  (for normal and abrupt postconditions). A specification is partially correct, which is denoted by  $\Gamma \models \{P\} C \{Q\}, \{A\}$ , when if  $\Gamma \vdash \langle C, \sigma_n \uparrow_N \rangle \Rightarrow \sigma'$  and  $\sigma_n \in P$ , then  $\sigma' = \sigma'_n \uparrow_N$  ( $\sigma'$  is a normal state) and  $\sigma'_n \in Q$ , or  $\sigma' = \sigma'_a \uparrow_A$  ( $\sigma'$  is an abrupt state) and  $\sigma'_a \in A$ .  $\Gamma \models_t \{P\} C \{Q\}, \{A\}$  represents total correctness and it is satisfiable when in addition to partial correctness,  $C$  finishes for all possible executions starting from the precondition  $P$ . Note that although the postcondition often are expressed as relational postconditions relating the initial and final states, postconditions in Simpl and CSimpl only operate over final states.

终止状态是Abrupt状态

# 3. Syntax and Semantics of CSimpl Language

## CSimpl Semantics

[BASIC]

$$\frac{}{\Gamma \vdash_c (\text{Basic } f, \sigma_{\uparrow N}) \rightarrow (\text{Skip}, f \sigma_{\uparrow N})}$$

[SPECCEC]

$$\frac{(\sigma, \sigma') \in r}{\Gamma \vdash_c (\text{Spec } r, \sigma_{\uparrow N}) \rightarrow (\text{Skip}, \sigma'_{\uparrow N})}$$

[GUARD]

$$\frac{\sigma \in g}{\Gamma \vdash_c (\text{Guard } f \ g \ p, \sigma_{\uparrow N}) \rightarrow (p, \sigma_{\uparrow N})}$$

[DYNCOM]

$$\frac{}{\Gamma \vdash_c (\text{DynCom } c, \sigma_{\uparrow N}) \rightarrow (c \ \sigma, \sigma_{\uparrow N})}$$

[SPECSTUCK]

$$\frac{\forall \sigma'. (\sigma, \sigma') \notin r}{\Gamma \vdash_c (\text{Spec } r, \sigma_{\uparrow N}) \rightarrow (\text{Skip}, \text{Stuck})}$$

[GUARDFault]

$$\frac{\sigma \notin g}{\Gamma \vdash_c (\text{Guard } f \ g \ p, \sigma_{\uparrow N}) \rightarrow (\text{Skip}, f_{\uparrow F})}$$



# 3. Syntax and Semantics of CSimpl Language

## CSimpl Semantics

$$\frac{[\text{SEQ}] \quad \Gamma \vdash_c (c_1, \sigma) \rightarrow (c'_1, \sigma')}{\Gamma \vdash_c (\text{Seq } c_1 \ c_2, \sigma) \rightarrow (\text{Seq } c'_1 \ c_2, \sigma')}$$

$$\frac{[\text{SEQTHROW}] \quad -}{\Gamma \vdash_c (\text{Seq Throw } c_2, \sigma_{\uparrow N}) \rightarrow (\text{Throw}, \sigma_{\uparrow N})}$$

$$\frac{[\text{CATCHTHROW}] \quad -}{\Gamma \vdash_c (\text{Catch Throw } c_2, \sigma_{\uparrow N}) \rightarrow (c_2, \sigma_{\uparrow N})}$$

$$\frac{[\text{SEQSKIP}] \quad -}{\Gamma \vdash_c (\text{Seq Skip } c_2, \sigma) \rightarrow (c_2, \sigma)}$$

$$\frac{[\text{CATCH}] \quad \Gamma \vdash_c (c_1, \sigma) \rightarrow (c'_1, \sigma')}{\Gamma \vdash_c (\text{Catch } c_1 \ c_2, \sigma) \rightarrow (\text{Catch } c'_1 \ c_2, \sigma')}$$

$$\frac{[\text{CATCHSKIP}] \quad -}{\Gamma \vdash_c (\text{Catch Skip } c_2, \sigma) \rightarrow (\text{Skip}, \sigma)}$$

# 3. Syntax and Semantics of CSimpl Language

## CSimpl Semantics

[CONDITIONALTRUE]

$$\frac{\sigma \in b}{\Gamma \vdash_c (\text{Cond } b \ c_1 \ c_2, \sigma \uparrow_N) \rightarrow (c_1, \sigma \uparrow_N)}$$

[CONDITIONALFALSE]

$$\frac{\sigma \notin b}{\Gamma \vdash_c (\text{Cond } b \ c_1 \ c_2, \sigma \uparrow_N) \rightarrow (c_2, \sigma \uparrow_N)}$$

[WHILE]

$$\frac{\sigma \in b}{\Gamma \vdash_c (\text{While } b \ c, \sigma \uparrow_N) \rightarrow (\text{Seq } c \ (\text{While } b \ c), \sigma \uparrow_N)}$$

[WHILEEND]

$$\frac{\sigma \notin b}{\Gamma \vdash_c (\text{While } b \ c, \sigma \uparrow_N) \rightarrow (\text{Skip}, \sigma \uparrow_N)}$$

Some c表示环境中定义了该程序

禁止套娃

[CALL]

$$\Gamma \ p = \text{Some } c \quad c \neq \text{Call } p$$

$$\Gamma \vdash_c (\text{Call } p, \sigma \uparrow_N) \rightarrow (c, \sigma \uparrow_N)$$

p是被唤醒程序的名字

None表示环境中没有定义该程序

[CALLUNDEFINED]

$$\Gamma \ p = \text{None}$$

$$\Gamma \vdash_c (\text{Call } p, \sigma \uparrow_N) \rightarrow (\text{Skip}, \text{Stuck})$$

# 3. Syntax and Semantics of CSimpl Language

## CSimpl Semantics

由于大语义是顺序执行的，所以在环境中去掉使用Await的程序

$$\begin{array}{c}
 \text{[AWAIT]} \\
 \frac{s \in b \quad \boxed{\Gamma_{\neg a} \vdash \langle c, \sigma_{\uparrow N} \rangle \Rightarrow \sigma'} \quad \boxed{\neg(\uparrow^A \sigma')}}{\Gamma \vdash_c (\text{Await } b \ p, \sigma_{\uparrow N}) \rightarrow (\text{Skip}, \sigma')}
 \end{array}$$

$\sigma'$  是Abrupt外的 $\sigma'$

$$\begin{array}{c}
 \text{[AWAITABRUPT]} \\
 \frac{s \in b \quad \Gamma_{\neg a} \vdash \langle c, \sigma_{\uparrow N} \rangle \Rightarrow \boxed{\sigma'_{\uparrow A}}}{\Gamma \vdash_c (\text{Await } b \ p, \sigma_{\uparrow N}) \rightarrow \boxed{(\text{Throw}, \sigma'_{\uparrow N})}}
 \end{array}$$

$\sigma'$  是Abrupt

$$\begin{array}{c}
 \text{[PAR]} \\
 \frac{i \leq n \quad \Gamma \vdash_c (C_i, \sigma) \rightarrow (C'_i, \sigma')}{\Gamma \vdash_p ([C_1 || \dots \boxed{C_i} \dots || C_n], \sigma) \rightarrow ([C_1 || \dots \boxed{C'_i} \dots || C_n], \sigma')}
 \end{array}$$

不用Abrupt，而用  
<Throw, Normal state>  
表示异常终止

### 3. Syntax and Semantics of CSimpl Language

#### CSimpl Semantics

$$\begin{array}{c}
 \text{[ENV]} \quad \frac{}{\Gamma \vdash_c (P, \sigma_{\uparrow N}) \boxed{\rightarrow_e} (P, \sigma')} \\
 \text{[ENV\_N]} \quad \frac{\neg(\uparrow^N \sigma)}{\Gamma \vdash_c (P, \sigma) \rightarrow_e (P, \sigma)} \\
 \text{[P\_ENV]} \quad \frac{}{\Gamma \vdash_p ([C_1 || \dots || C_n], \sigma_{\uparrow N}) \rightarrow_e ([C_1 || \dots || C_n], \sigma'_{\uparrow N})}
 \end{array}$$

对于并行程序，只能从一个Normal状态迁移到另一个Normal状态，  
环境不能产生Fault状态给并行程序

# 3. Syntax and Semantics of CSimpl Language

## CSimpl Semantics

- ▶ In the current version of CSimpl, the parallel construct is carried out in the top level similar to a multi-core architecture in which a number of programs is static, rather than providing a threadlike concurrency where it is possible to create new threads. There is **not** any technical issue in the CSimpl architecture to **support a nested concurrency** other than a more complex proof for soundness.

# 4. Rely-Guarantee for CSimpl

## Rely and Guarantee

- ▶ **Rely**: how the environment interferes with the program
- ▶ **Guarantee**: how the program modifies the environment
- ▶ Therefore a property specification for the verification of parallel systems by using rely-guarantee is composed of five elements: the parallel system itself, a precondition, a postcondition, and a rely and guarantee relations.



## 4. Rely-Guarantee for CSimpl Computation

**Definition 4.1** (*Sequential Component Computation*). A computation is a tuple  $(\Gamma, \text{confs})$  where  $\Gamma$  is an environment for procedures and  $\text{confs} = [(C_0, \sigma_1), (C_2, \sigma_2), \dots, (C_n, \sigma_n)]$  is a list of sequential configurations. The set of possible computations  $\text{cptn}$  is inductively defined as follows:

归纳定义:

- $(\Gamma, [(C, \sigma)]) \in \text{cptn}$
- if  $\Gamma \vdash_c (C, \sigma) \rightarrow_e (C, \sigma')$  and  $(\Gamma, (C, \sigma') \# xs) \in \text{cptn}$  then  $(\Gamma, (C, \sigma) \# (C, \sigma') \# xs) \in \text{cptn}$
- if  $\Gamma \vdash_c (C, \sigma) \rightarrow (C', \sigma')$  and  $(\Gamma, (C', \sigma') \# xs) \in \text{cptn}$  then  $(\Gamma, (C, \sigma) \# (C', \sigma') \# xs) \in \text{cptn}$

**Definition 4.2** (*Computations of an Initial Configuration*). The set of possible computations of an initial configuration  $(C, \sigma)$  with an environment for procedures  $\Gamma$ , denoted as  $\text{cp}(C, \sigma) \Gamma$ , is the set of tuples  $(\Gamma, l)$  such that  $l!0 = (C, \sigma)$  and  $(\Gamma, l) \in \text{cptn}$ .

The set of parallel computations  $\text{par}_{cp}$  is defined similar to  $\text{cp}$  by using parallel configurations and the semantic rules for parallel and environment step transitions defined by rules PAR and P\_ENV in Figures 5 and 6.

## 4. Rely-Guarantee for CSimpl

### Validity of Formulas for Rely-Guarantee in CSimpl

- ▶ By using the notion of computation, we define **validity of a rely-guarantee tuple** from the set of all possible computations from an initial configuration.
- ▶ It also uses the notions of **assumption** of the precondition and the environment, and **commitment** of the component and the postcondition.



## 4. Rely-Guarantee for CSimpl

### Validity of Formulas for Rely-Guarantee in CSimpl

precondition  $p$   
rely  $R$   
environment  $\Gamma$

*Definition 4.3 (Validity Assumption).* The assumption of a predicate  $p$  and an environment relation  $R$ , for an environment of procedures  $\Gamma$ , represented by  $\text{assum } \Gamma \ p \ R$ , is the set of component computations  $(\Gamma, \text{cptns})$  such that for any  $[(C_0, \sigma_0), \dots, (C_n, \sigma_n)] \in \text{cptns}$ , with  $n \geq 0$ , then: (1) there exists a  $\sigma$  where  $\sigma_0 = \sigma \upharpoonright_N$  and  $\sigma \in p$ , and (2) given a  $k < n$  if there is an environment step transition  $\Gamma \vdash_c (C_k, \sigma_k) \rightarrow_e (C_{k+1}, \sigma_{k+1})$ , then  $(\sigma_k, \sigma_{k+1}) \in R$ .

## 4. Rely-Guarantee for CSimpl

### Validity of Formulas for Rely-Guarantee in CSimpl

postcondition  $(q, a)$   
guarantee  $G$   
environment  $\Gamma$   
fault states  $F$

*Definition 4.4 (Validity Commitment).* The commitment of a relation  $G$ , a pair of predicates  $(q, a)$ , and a set of `Fault` states  $F$ , for an environment of procedures  $\Gamma$ , denoted as  $\text{comm } \Gamma \ G \ (q, a) \ F$ , is the set of component computations  $(\Gamma, \text{cptns})$  such that for any  $[(C_0, \sigma_0), \dots, (C_n, \sigma_n)] \in \text{cptns}$ , where  $n \geq 0$  and there is not any  $f$  such that  $\sigma_n = \text{Fault } f$  and  $f \in F$ , then: (1) for any  $k < n$ , if  $\Gamma \vdash_c (C_k, \sigma_k) \rightarrow (C_{k+1}, \sigma_{k+1})$ , then  $(\sigma_k, \sigma_{k+1}) \in G$ ; (2) if  $(C_n, \sigma_n)$  is a final configuration, then there is a  $\sigma'$  such that  $\sigma_n = \sigma'_{\uparrow N}$  (2.1) and if  $C_n = \text{Skip}$  then  $\sigma' \in q$  and if  $C_n = \text{Throw}$  then  $\sigma' \in a$  (2.2).

## 4. Rely-Guarantee for CSimpl

### Validity of Formulas for Rely-Guarantee in CSimpl

*Definition 4.5 (Rely-Guarantee Validity).* A specification of a component  $C$  w.r.t. a precondition  $p$ , a postcondition  $(q, a)$ , a rely and guarantee relations  $R$ , and  $G$ , an environment of procedures  $\Gamma$ , and a set  $F$  of Faults, denoted as  $\Gamma \models_{/F} P \text{ sat}[p, R, G, q, a]$ , is valid iff for all  $\sigma$ ,  $cp \Gamma C \sigma \cap \text{assum}(p, R) \subseteq \text{comm}(G, (q, a)) F$ .

*Definition 4.6 (Rely-Guarantee CValidity).* CValidity of a specification of a component  $C$  w.r.t. a precondition  $p$ , postcondition  $(q, a)$ , a rely relation  $R$ , a guarantee relation  $G$ , an environment of procedures  $\Gamma$ , a specification of procedures  $\Theta$ , and a set  $F$  of Faults, represented by  $\Gamma, \Theta \models_{/F} P \text{ sat}[p, R, G, q, a]$  iff if for any tuple  $(c, p', R', G', q', a') \in \Theta$   $\Gamma \models_{/F} (\text{Call } c) \text{ sat}[p', R', G', q', a']$  is valid, then  $\Gamma \models_{/F} P \text{ sat}[p, R, G, q, a]$ .

根据[28],  $\Theta$ 是一组在验证过程中被认为是理所当然的  
程序规范的假设集合, 它被用来处理递归程序

## 4. Rely-Guarantee for CSimpl

### Validity of Formulas for Rely-Guarantee in CSimpl

**THEOREM 4.7 (VALIDITY\_COMPOSITIONALITY).** *Given an environment for procedures  $\Gamma$ , a set of specifications for recursive procedures  $\Theta$ , a rely-guarantee parallel specification is valid,*

$$\Gamma, \Theta \models_{/F} [C_0 || \dots || C_n] SAT [p, R, G, q, a],$$

if for each  $i \leq n$  there exist  $p_i, q_i, a_i, R_i, G_i$ , representing a rely-guarantee specification for the  $i$  component such that

- (1)  $\Gamma, \Theta \models_{/F} C_i sat [p_i, R_i, G_i, Q_i, A_i];$
- (2)  $R \cup (\bigcup k \in \{k. k \leq n \wedge k \neq i\}. G_k) \subseteq R_i;$
- (3)  $\bigcup j \leq n. G_j \subseteq G;$
- (4)  $p \subseteq (\bigcap j \leq n. p_j);$
- (5)  $(\bigcap j \leq n. q_j) \subseteq q;$
- (6)  $(\bigcup j \leq n. a_j) \subseteq a.$

## 4. Rely-Guarantee for CSimpl

### Inference Rules of the Proof System

简单来说就是前/后置条件P去环境里逛了一圈回来还满足

- ▶ The intuition is that a precondition/postcondition  $P$  is **valid** before/after the execution of a sequential component  $C$  in a concurrent environment **if** the environment preserves  $P$  before/after the execution of  $C$ . Formally, with the relation  $R$  representing the concurrent environment:

$P$  is stable when  $R$  holds.

*Definition 4.8 (Stability).* A set of states  $P$  is stable w.r.t. a relation  $R$ , represented by  $Sta\ P\ R$ , if given two states  $\sigma, \sigma'$ , such that  $\sigma \in P$  and  $(\sigma, \sigma') \in R$ , then  $\sigma' \in P$ .

# 4. Rely-Guarantee for CSimpl

## Rely-Guarantee proof rules for CSimpl

换一种写法:  $p \subseteq \{\sigma \mid f(\sigma) \in q\}$

[BASIC]

$$\frac{\begin{array}{l} Sta\ p\ R \quad Sta\ q\ R \quad p \subseteq \{\sigma. q(f\ \sigma)\} \\ \forall \sigma\ \sigma'. \sigma \in p \wedge (\sigma' = f\ \sigma) \longrightarrow (\sigma \uparrow_N, \sigma' \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Basic\ f\ sat\ [p, R, G, q, a]}$$

[THROW]

$$\frac{Sta\ a\ R \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G}{\Gamma, \Theta \vdash_{/F} Throw\ sat\ [a, R, G, q, a]}$$

[SPEC] 由关系  $r$  表示的不确定行为

$$\frac{\begin{array}{l} Sta\ p\ R \quad Sta\ q\ R \\ p \subseteq \{\sigma. (\forall \sigma'. (\sigma, \sigma') \in r \longrightarrow q\ \sigma') \wedge (\exists \sigma'. (\sigma, \sigma') \in r)\} \\ \forall \sigma\ \sigma'. \sigma \in p \wedge (\sigma, \sigma') \in r \longrightarrow (\sigma \uparrow_N, \sigma' \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Spec\ r\ sat\ [p, R, G, q, a]}$$

[SKIP]

$$\frac{Sta\ q\ R \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G}{\Gamma, \Theta \vdash_{/F} Skip\ sat\ [q, R, G, q, a]}$$



# 4. Rely-Guarantee for CSimpl

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[COND]

$$\frac{\begin{array}{l} Sta\ p\ R \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \\ \Gamma, \Theta \vdash_{/F} c_1 \text{ sat } [p \cap b, R, G, q, a] \\ \Gamma, \Theta \vdash_{/F} c_2 \text{ sat } [p \cap \neg b, R, G, q, a] \end{array}}{\Gamma, \Theta \vdash_{/F} \text{Cond } b\ c_1 c_2 \text{ sat } [p, R, G, q, a]}$$

While程序的后置条件是p且非b

[WHILE]

$$\frac{\begin{array}{l} Sta\ p\ R \quad Sta\ (p \cap \neg b)\ R \quad Sta\ a\ R \\ \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \\ \Gamma, \Theta \vdash_{/F} c \text{ sat } [p \cap b, R, G, p, a] \end{array}}{\Gamma, \Theta \vdash_{/F} \text{While } b\ c \text{ sat } [p, R, G, p \cap \neg b, a]}$$

[AWAIT]

$$\frac{\begin{array}{l} Sta\ p\ R \quad Sta\ q\ R \quad Sta\ a\ R \\ \forall \sigma. \Gamma_{\neg a}, \{\} \vdash_{/F} (p \cap b \cap \{\sigma\})c \\ \quad \{\sigma'. (\sigma \uparrow_N, \sigma' \uparrow_N) \in G\} \cap \{\sigma. q\ \sigma\}, \\ \quad \{\sigma'. (\sigma \uparrow_N, \sigma' \uparrow_N) \in G\} \cap \{\sigma. a\ \sigma\} \end{array}}{\Gamma, \Theta \vdash_{/F} \text{Await } b\ c \text{ sat } [p, R, G, q, a]}$$

?

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### Rely-Guarantee proof rules for CSimpl

[CATCH]

$$\frac{\begin{array}{l} \Gamma, \Theta \vdash_{/F} c1 \text{ sat } [p, R, G, q, \boxed{r}] \\ \Gamma, \Theta \vdash_{/F} c2 \text{ sat } [\boxed{r}, R, G, q, a] \\ Sta\ p\ R \quad Sta\ a\ R \quad \forall \sigma. (\sigma \upharpoonright_N, \sigma \upharpoonright_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Catch\ c1\ c2 \text{ sat } [p, R, G, q, a]}$$

[SEQ]

$$\frac{\begin{array}{l} \Gamma, \Theta \vdash_{/F} c1 \text{ sat } [p, R, G, \boxed{r}, a] \\ \Gamma, \Theta \vdash_{/F} c2 \text{ sat } [\boxed{r}, R, G, q, a] \\ Sta\ p\ R \quad Sta\ a\ R \quad \forall \sigma. (\sigma \upharpoonright_N, \sigma \upharpoonright_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Seq\ c1\ c2 \text{ sat } [p, R, G, q, a]}$$



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[GUARD]

$$\frac{\begin{array}{l} \Gamma, \Theta \vdash_{/F} c \text{ sat } [p \cap g, R, G, q, a] \\ \boxed{Sta (p \cap g) R} \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Guard f g c \text{ sat } [p \cap g, R, G, q, a]}$$

[GUARD FAULT]

$$\frac{\begin{array}{l} \Gamma, \Theta \vdash_{/F} c \text{ sat } [p, R, G, q, a] \\ \boxed{Sta p R \quad f \in F} \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} Guard f g c \text{ sat } [p, R, G, q, a]}$$

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## Rely-Guarantee proof rules for CSimpl

[CALL]

$$\frac{\begin{array}{l} \Gamma, \Theta \vdash_{/F} \text{the}(\Gamma c) \text{ sat } [p, R, G, q, a] \\ \text{Sta } p R \quad c \in \text{dom } \Gamma \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} \text{Call } c \text{ sat } [p, R, G, q, a]}$$

[CALLREC]

$$\frac{\begin{array}{l} (c, p, R, G, q, a) \in \text{Specs} \\ \forall (c, p, R, G, q, a) \in \text{Specs}. \\ \quad c \in \text{dom } \Gamma \wedge \text{Sta } p R \wedge \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \\ \quad \Gamma, \Theta \cup \text{Specs} \vdash_{/F} \text{The}(\Gamma c) \text{ sat } [p, R, G, q, a] \\ \text{Sta } p R \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} \text{Call } c \text{ sat } [p, R, G, q, a]}$$

[ASM]

$$\frac{(c, p, R, G, q, a) \in \Theta}{\Gamma, \Theta \vdash_{/F} \text{Call } c \text{ sat } [p, R, G, q, a]}$$

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[DYNCOM]

$$\frac{\begin{array}{l} \forall \sigma \in p. \Gamma, \Theta \vdash_{/F} c \sigma \text{ sat } [p, R, G, q, a] \\ \text{Sta } p R \quad \forall \sigma. (\sigma \uparrow_N, \sigma \uparrow_N) \in G \end{array}}{\Gamma, \Theta \vdash_{/F} \text{DynCom } c \text{ sat } [p, R, G, q, a]}$$

[PAR]

$$\frac{\begin{array}{l} \forall i \leq n. \Gamma, \Theta \vdash_{/F} C_i \text{ sat } [p_i, R_i, G_i, q_i, a_i] \\ \forall i \leq n. R \cup (\bigcup j \in \{j. j \leq n \wedge j \neq i\}. G_i) \subseteq R_i \\ (\bigcup j \leq n. G_j) \subseteq G \quad p \subseteq (\bigcap i < n. p_i) \\ (\bigcap j \leq n. q_j) \subseteq q \quad (\bigcup j \leq n. a_j) \subseteq a \end{array}}{\Gamma, \Theta \vdash_{/F} [C_0 || \dots || C_n] \text{ SAT } [p, R, G, q, a]}$$

## 4. Rely-Guarantee for CSimpl

### Soundness of the Proof System

The background features abstract, overlapping geometric shapes in various shades of blue, ranging from light sky blue to deep navy blue. These shapes are primarily located on the right side of the image, creating a modern, layered effect. The rest of the background is a solid, very light blue-grey color.

Thanks!