研究生课程

程序验证方法
Rely-Guarantee Method

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Related Paper:

- Qiwen Xu, Willem P. de Roever, Jifeng He:
  The Rely-Guarantee Method for Verifying Shared Variable Concurrent Programs. Formal Asp. Comput. 9(2): 149-174 (1997)

Owicki and Gries Method:

an interference freedom test

formulated in such a way that the knowledge of the complete system code is assumed

Rely-Guarantee Method:

The key point to achieve compositionality is to reformulate this interference freedom test.

The Language Syntax

- $\mathrm{P} ::= \bar{x} := \bar{\mathrm{e}} \mid P_1; P_2 \mid \boldsymbol{if}\ b_1 \rightarrow P_1 \square \ldots \square\ b_{\mathrm{n}} \rightarrow Pn\ \boldsymbol{fi} \mid \boldsymbol{while}\ b\ \boldsymbol{do}\ P\ \boldsymbol{od} \mid \boldsymbol{await}\ b\ \boldsymbol{then}\ P\ \boldsymbol{end} \mid P_1 \| P_2$

A Proof System for Partial Correctness

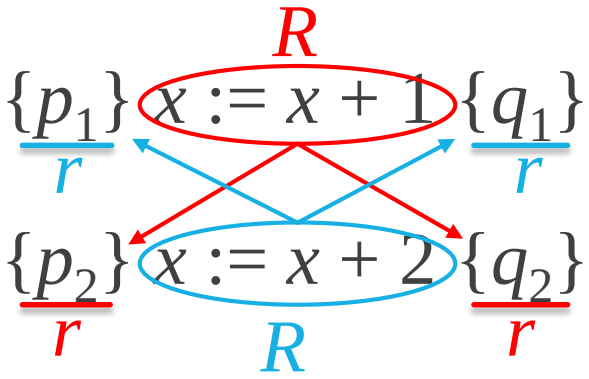
- **Example (Owicki & Gries Method)**

$$P_1 :: x := x + 1 \parallel P_2 :: x := x + 2$$

Let $p_1 \equiv x = 0 \vee x = 2$ $\quad q_1 \equiv x = 1 \vee x = 3$

$p_2 \equiv x = 0 \vee x = 1$ $\quad q_2 \equiv x = 2 \vee x = 3$

The local verification:



A Proof System for Partial Correctness

Interference freedom test: four cases.

$$\{p_1 \wedge p_2\}\ x := x + 2\ \{p_1\}$$

$$\{q_1 \wedge p_2\}\ x := x + 2\ \{q_1\}$$

$$\{p_1 \wedge p_2\}\ x := x + 1\ \{p_2\}$$

$$\{p_1 \wedge q_2\}\ x := x + 1\ \{q_2\}$$

Specification

(pre, rely, guar, post)

- **The assumption is composed of *pre* and *rely*.**
- **The commitment is composed of *guar* and *post*.**
- ***P* sat (*pre*, *rely*, *guar*, *post*), if**

  1) *P* is invoked in a state which satisfies *pre*, and

  2) any environment transition satisfies *rely*,

  then

  3) any component transition satisfies *guar*,

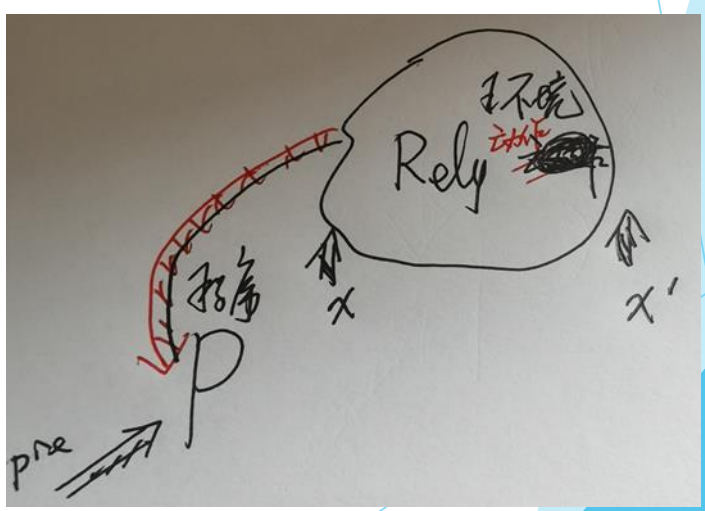
  4) if a computation terminates, the final state satisfies *post*.

$$x := 10 \ \underline{sat} \ (true, x > 0 \rightarrow x' \geq x, true, x \geq 10)$$

Proof System

- **Some notations:**

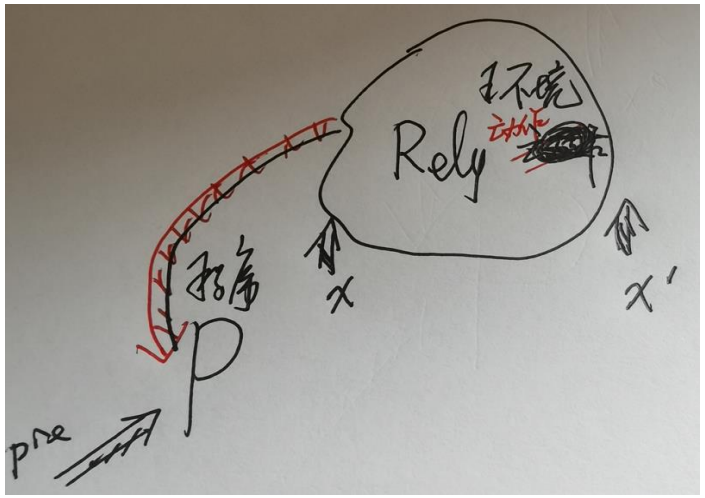
For two predicates $f(y, y_0)$ and $g(y, y', y_0)$, let *f stable when g* be a shorthand for $\forall y, y', y_0.\ f(y, y_0) \wedge g(y, y', y_0) \rightarrow f(y', y_0)$ and $f'(y, y_0)$ denote $f(y', y_0)$.



pre stable when rely

Proof System

- Assignment axiom



$$\frac{pre \rightarrow post[\bar{e}/\bar{x}] \quad (pre \wedge \lceil \bar{x}' = \bar{e} \rceil) \rightarrow guar \quad pre \ \underline{stable\ when}\ rely \quad post \ \underline{stable\ when}\ rely}{\bar{x} := \bar{e}\ \underline{sat}\ (pre, rely, guar, post)}$$

- $\lceil \bar{x}' = \bar{e} \rceil \stackrel{\text{def}}{=} (\bar{x}' = \bar{e} \vee x' = x) \wedge \forall z \in (y - \bar{x}).\ z' = z.$
- In a typical computation, there are a number of environment transitions before and after the component transition. Since *pre* holds initially, it follows from ***pre stable when rely*** that *pre* still holds immediately before the component transition.
- Due to ***post stable when rely***, ***post holds*** ***in any states after a number of environment transitions***.
- $x := 10\ \underline{sat}\ (true, x > 0 \rightarrow x' \geq x, true, x \geq 10)$

Proof System

- Consequence rule

$$\frac{pre \rightarrow pre_1, rely \rightarrow rely_1, guar_1 \rightarrow guar, post_1 \rightarrow post \quad P\ sat\ (pre_1, rely_1, guar_1, post_1)}{P\ \underline{sat}\ (pre, rely, guar, post)}$$

- $\boldsymbol{x := 10\ \underline{sat}\ (x = -2, x > 0 \rightarrow x' \geq x, true, x \geq 10 \vee x = -6)}$

$$\boldsymbol{x := 10\ \underline{sat}\ (true, x > 0 \rightarrow x' \geq x, true, x \geq 10)}$$

Proof System

- Sequential composition rule

$$\frac{P \ sat \ (pre, rely, guar, mid) \qquad Q \ \underline{sat} \ (mid, rely, guar, post)}{P;Q \ \underline{sat} \ (pre, rely, guar, post)}$$

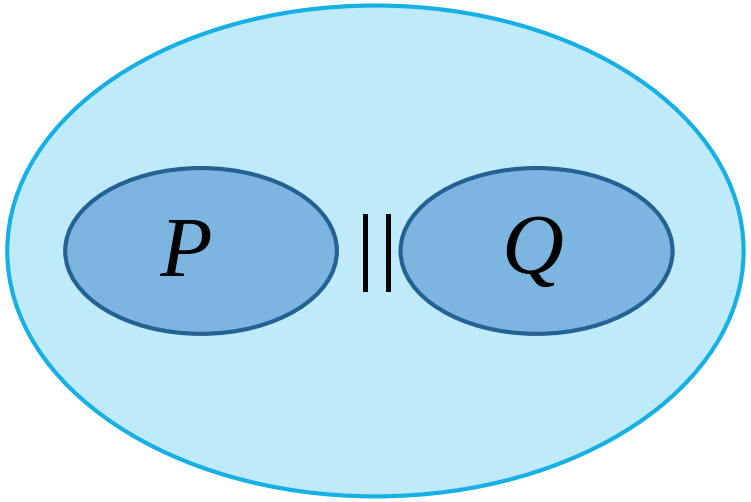
- $$\frac{x := x + 1 \ \underline{sat} \ (x \geq x_0, x_0 \leq x \rightarrow x \leq x', x' \geq x, x \geq x_0 + 1) \qquad x := x + 1 \ \underline{sat} \ (x \geq x_0 + 1, x_0 \leq x \rightarrow x \leq x', x' \geq x, x \geq x_0 + 2)}{x := x + 1; x := x + 1 \ \underline{sat} \ (x \geq x_0, x_0 \leq x \rightarrow x \leq x', x' \geq x, x \geq x_0 + 2)}$$

Proof System

- Parallel rule

$$\frac{\begin{array}{c}(rely \vee guar_1) \rightarrow rely_2 \\ (rely \vee guar_2) \rightarrow rely_1 \\ (guar_1 \vee guar_2) \rightarrow guar \\ P \ \underline{sat}\ (pre, rely_1, guar_1, post_1) \\ Q \ \underline{sat}\ (pre, rely_2, guar_2, post_2)\end{array}}{P \parallel Q \ \underline{sat}\ (pre, rely, guar, post_1 \wedge post_2)}$$

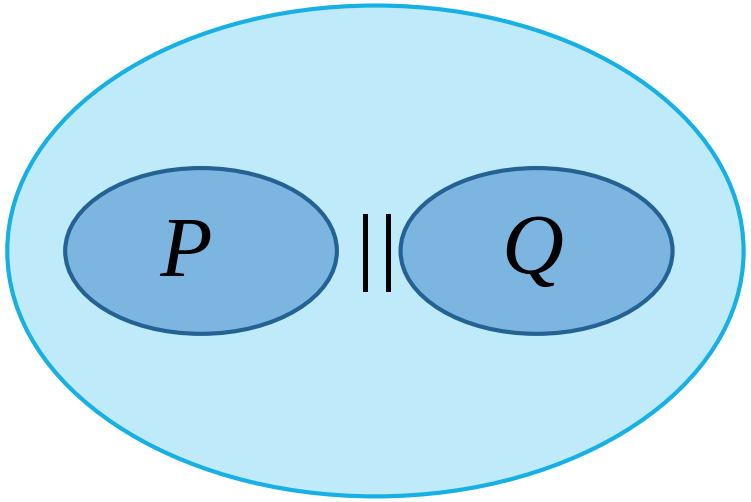
Proof System



$$\frac{\begin{array}{c}(rely \vee guar_1) \rightarrow rely_2 \\ (rely \vee guar_2) \rightarrow rely_1 \\ (guar_1 \vee guar_2) \rightarrow guar \\ P \ \underline{sat}\ (pre, rely_1, guar_1, post_1) \\ Q \ \underline{sat}\ (pre, rely_2, guar_2, post_2)\end{array}}{P \ || \ Q \ \underline{sat}\ (pre, rely, guar, post_1 \wedge post_2)}$$

- **Assume the overall environment is *R*. The environment of process *P* consists of *Q* and *R*, and the environment of process *Q* consists of *P* and *R*.**
- **The strongest rely-condition that *P* can assume is $rely \vee guar_2$ and the strongest rely-condition for *Q* is $rely \vee guar_1$.**

Proof System



$$\frac{\begin{array}{c}(rely \vee guar_1) \rightarrow rely_2 \\ (rely \vee guar_2) \rightarrow rely_1 \\ (guar_1 \vee guar_2) \rightarrow guar \\ P \ \underline{sat}\ (pre, rely_1, guar_1, post_1) \\ Q \ \underline{sat}\ (pre, rely_2, guar_2, post_2)\end{array}}{P \ || \ Q \ \underline{sat}\ (pre, rely, guar, post_1 \wedge post_2)}$$

- **A component transition of $P \ || \ Q$ is either from P or from Q, and hence it satisfies $guar_1 \vee guar_2$.**
- **When both P and Q have terminated, from the two sub-specifications it follows that both $post_1$ and $post_2$ are satisfied.**

Thanks!