

IS893: Advanced Software Security

6. Introduction to Static Analysis

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Static Analysis

- **Over-approximate** (not exact) all possible behavior of a program
- In general, **sound and automatic, but incomplete**
 - May have spurious results
- Based on a foundational theory : Abstract interpretation
- Variants:
 - Under-approximating static analysis: automatic, **complete**, but **unsound**
 - Bug finder: automatic, **unsound, incomplete**, and heuristics

Example

```
1: static char *curfinal = "HDACB FE";
2:
3: keysym = read_from_input();
4:
5: if ((KeySym)(keysym) >= 0xFF9987)
6: {
7:     unparseputc((char)(keysym - 0xFF91 + 'P'), pty);
8:     key = 1;
9: }
10: else if (keysym >= 0)
11: {
12:     if (keysym < 16)
13:     {
14:         if (read_from_input())
15:         {
16:             if (keysym >= 10) return;
17:             curfinal[keysym] = 1;
18:         }
19:         else
20:         {
21:             curfinal[keysym] = 2;
22:         }
23:     }
24:     if (keysym < 10)
25:         unparseputc(curfinal[keysym], pty);
26: }
```

Annotations:

- curfinal: buffer of size 10
- keysym : any integer
- keysym: [0, 15]
- keysym: [0, 9]
- size of curfinal: [10, 10]
keysym: [0, 15]
- keysym: [0, 9]
- SAFE
- Buffer-overflow
- SAFE

Success Stories

Domain-specific
Verification



Windows Device Driver
Microsoft

Astrée

Airbus Controller
ENS / AbsInt

General-purpose
Bug-finding



Stanford / Synopsys



Facebook



SNU / Fasoo.com



Mathworks



GrammaTech



Semmle / Github



JuliaSoft



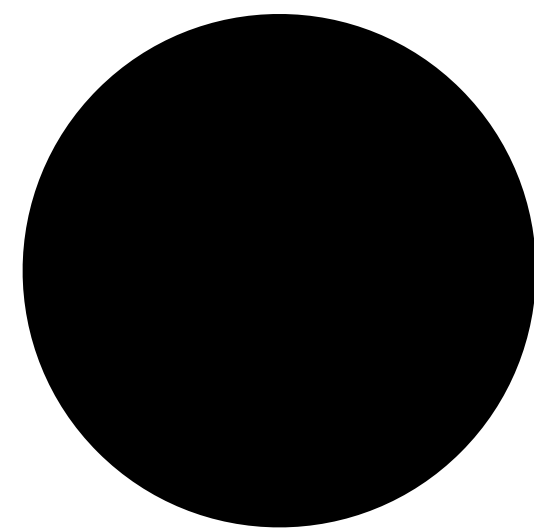
GCC



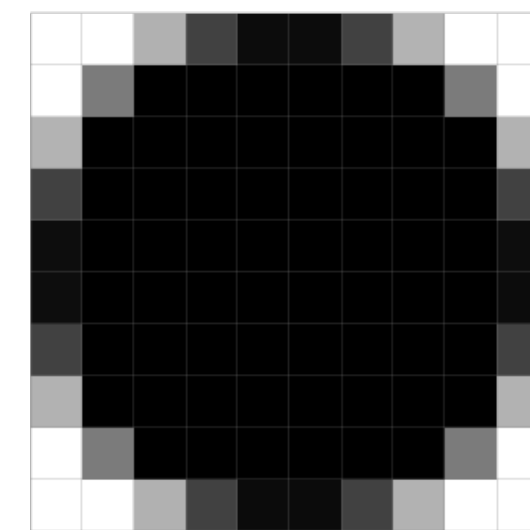
LLVM/Clang

Abstraction

- Concrete (execution, dynamic) vs Abstract (analysis, static)
- Without abstraction, it is undecidable to subsume all possible behavior of SW
- Recall the Rice's theorem and the Halting problem



Concrete



Abstract

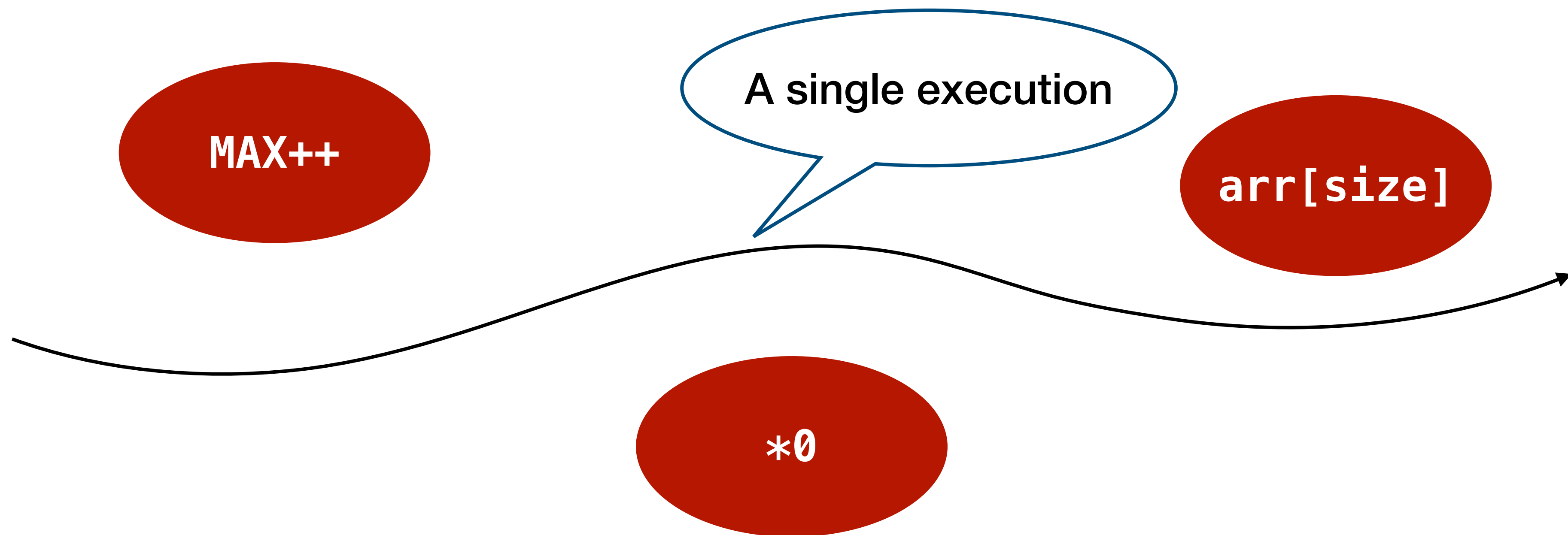
Example

```
x = 3;  
while (*) {  
    x += 2;  
}  
x -= 1;  
print(x);
```

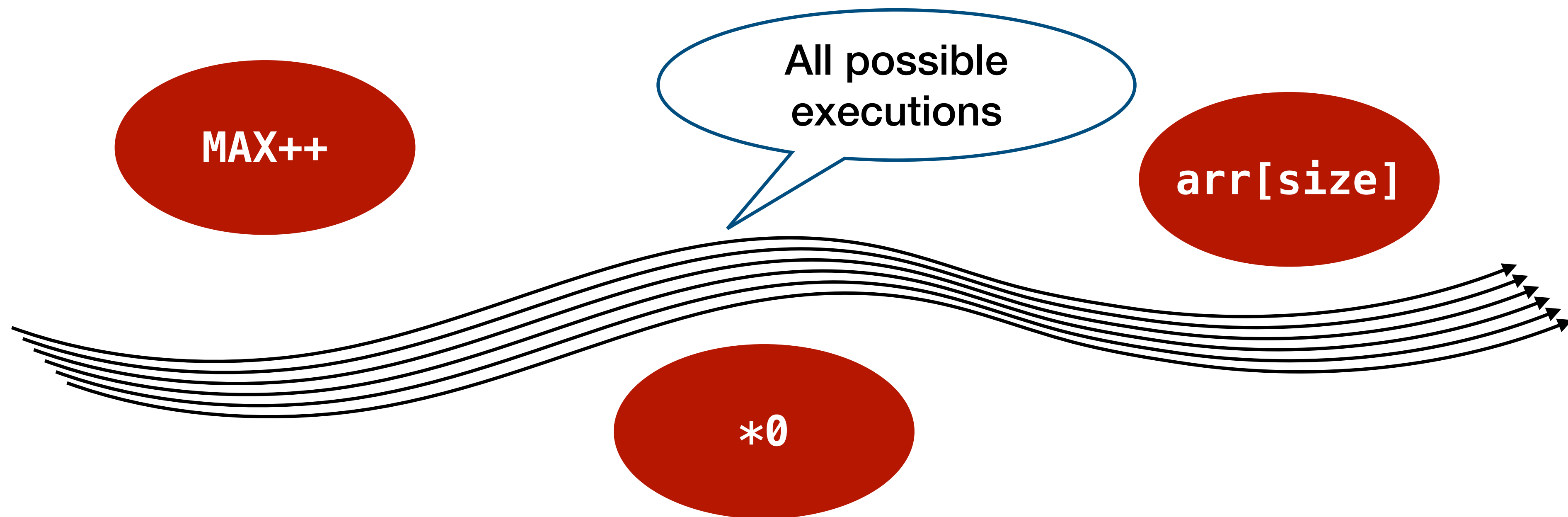
Q: What are the possible output values?

- Concrete interpretation : 2, 4, ..., uncomputable (infinitely many possibilities)
- Abstract interpretation 1 : “integers” (good)
- Abstract interpretation 2 : “positive integers” (better)
- Abstract interpretation 3 : “positive even integers” (best)

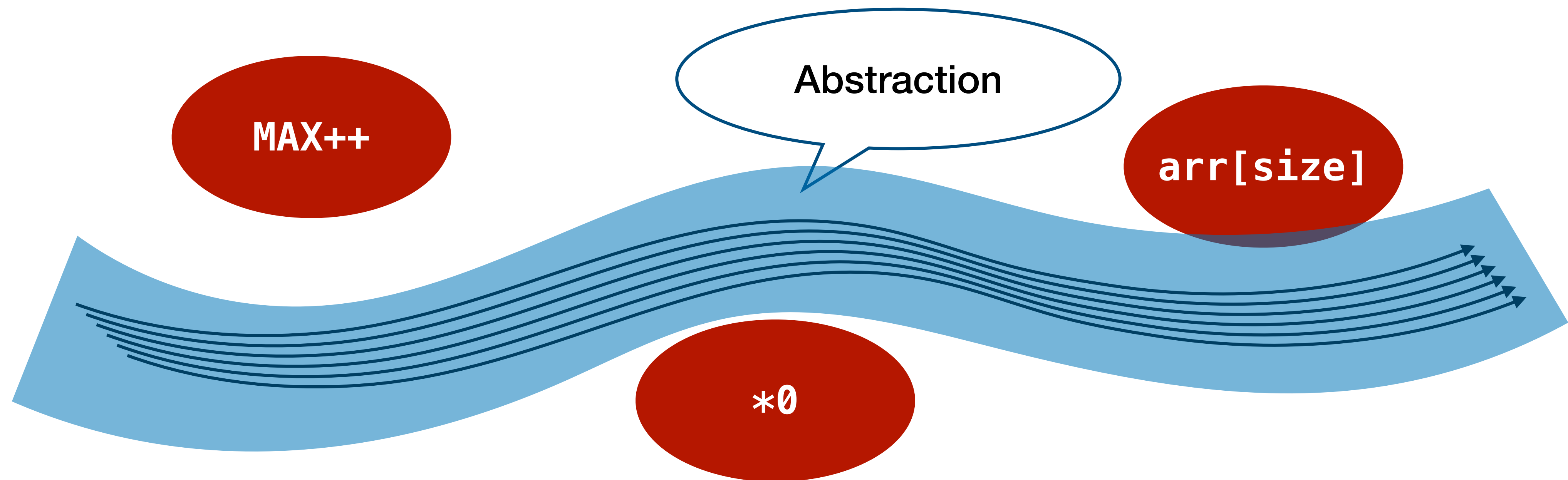
Abstraction of Executions



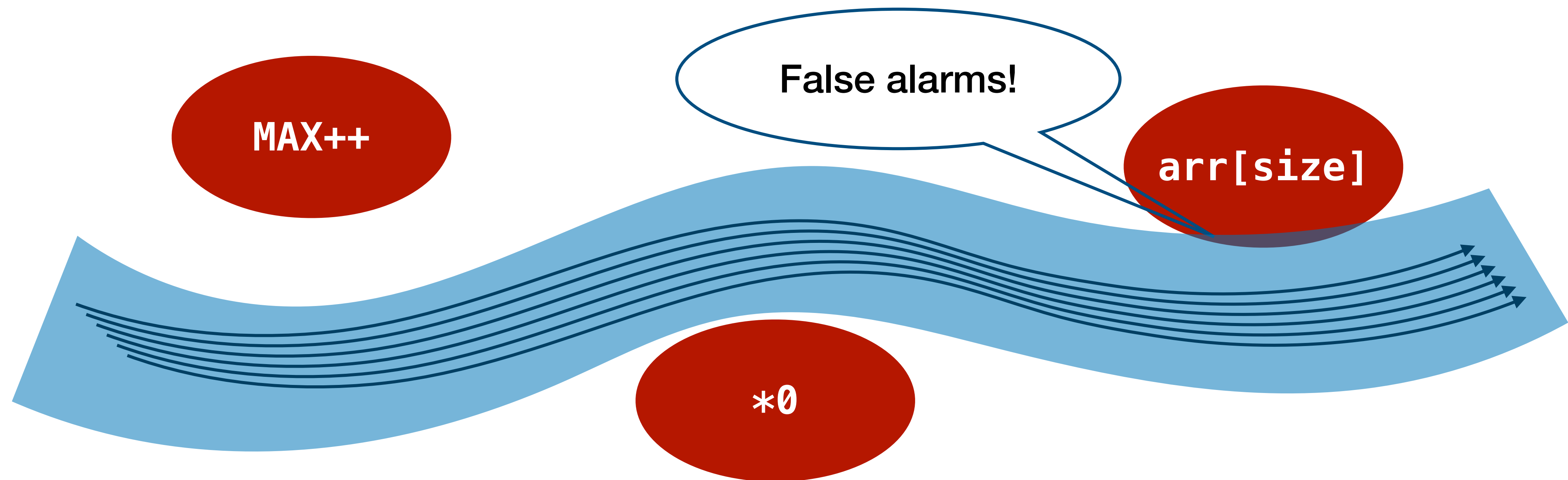
Abstraction of Executions



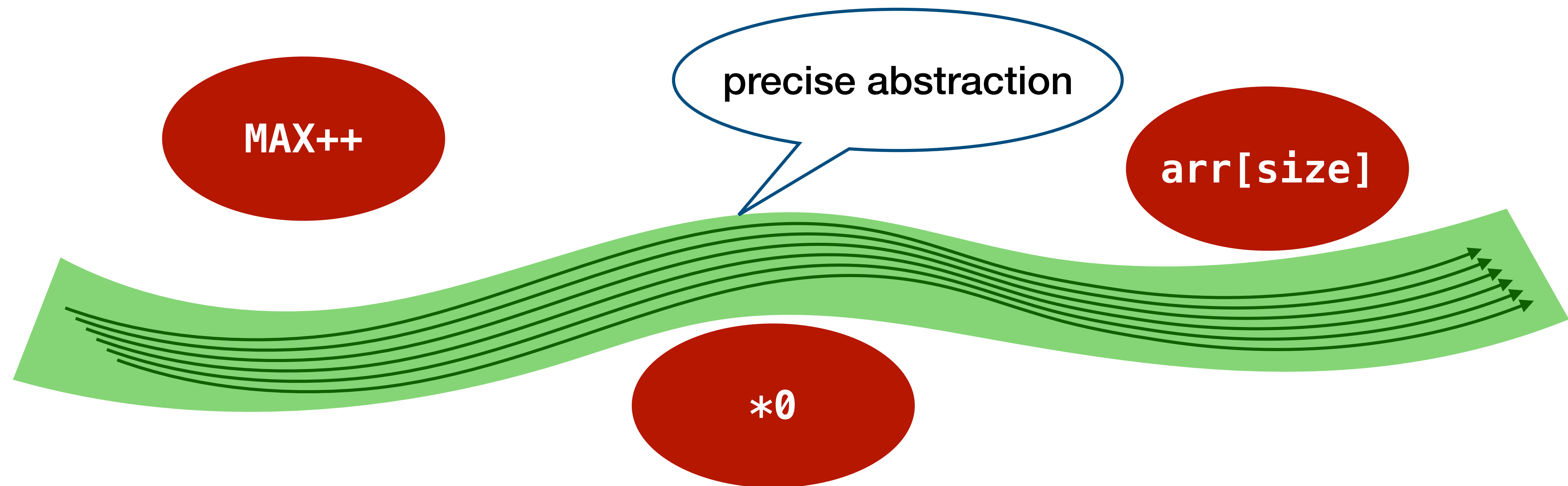
Abstraction of Executions



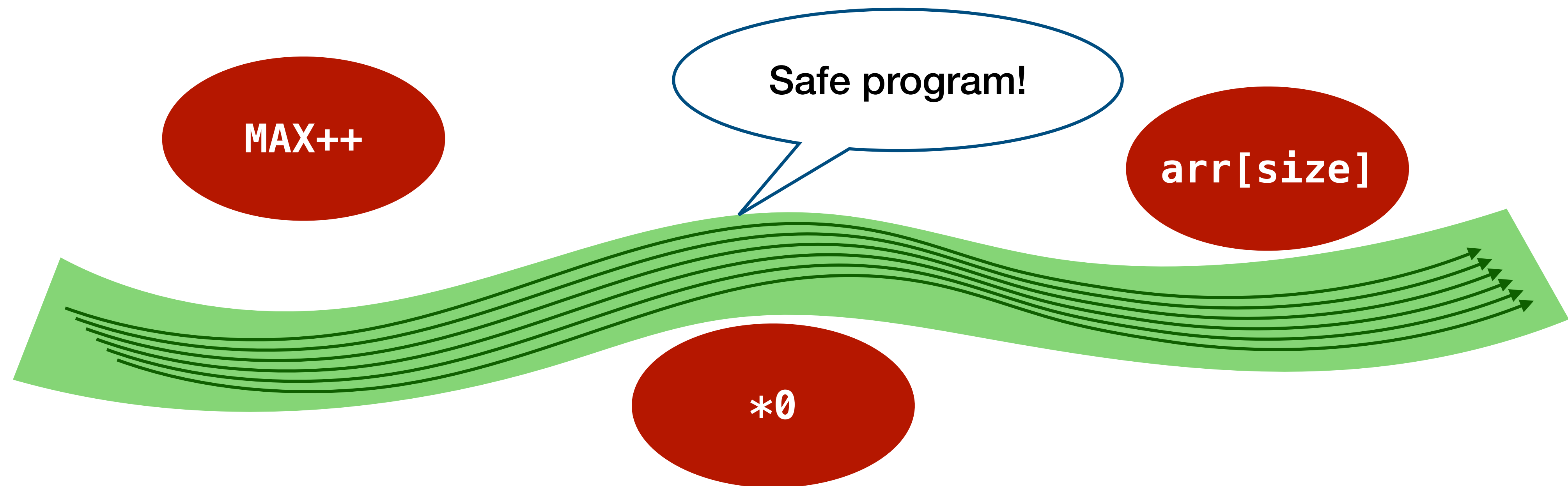
Abstraction of Executions



Abstraction of Executions



Abstraction of Executions



How to analyze?

- Interpret the target program
 - with abstract domains / semantics (= analyzer's concern)
 - not concrete domains / semantics (= interpreter's and compiler's concern)

- Example

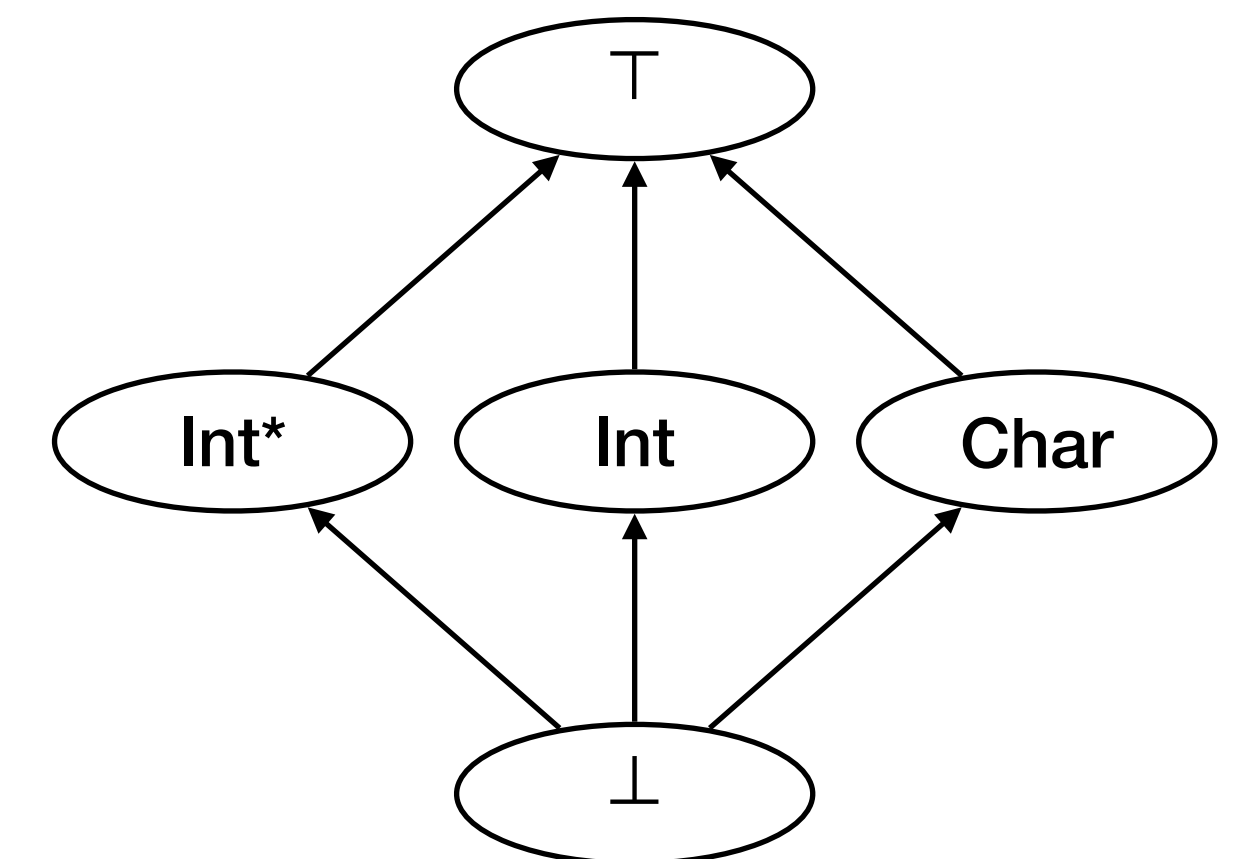
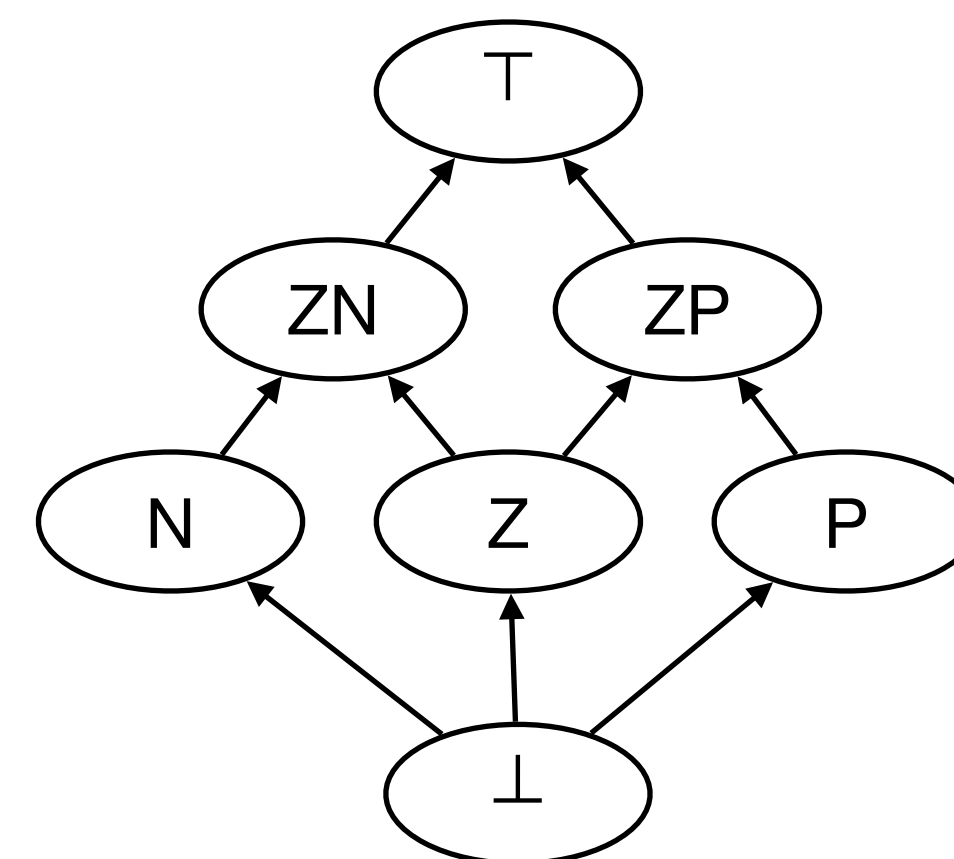
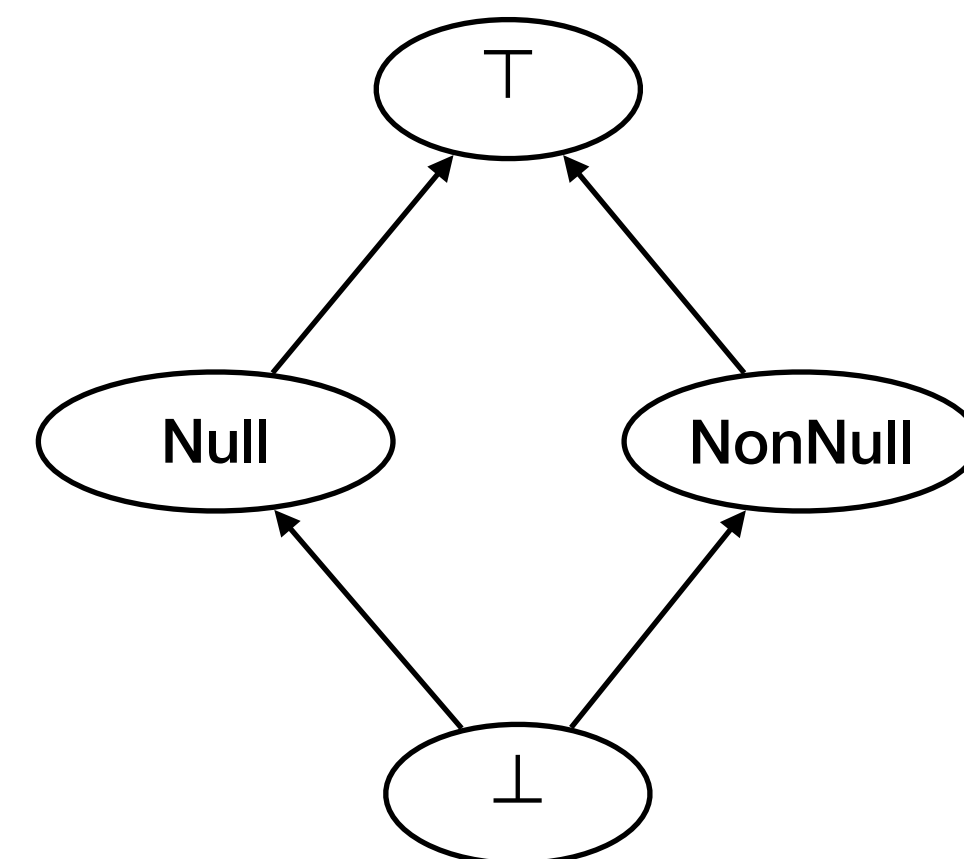
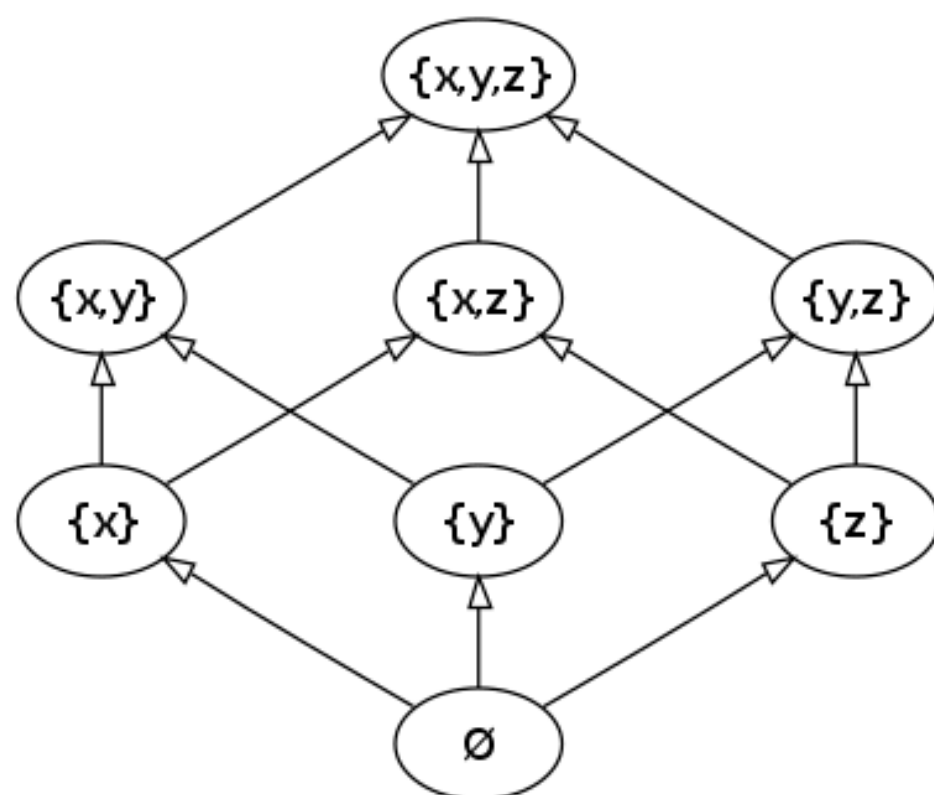
	Concrete	Abstract 1	Abstract 2	Abstract 3
<code>x = 3;</code>	<code>{3}</code>	<code>Int</code>	<code>Pos</code>	<code>PosOdd</code>
<code>while (*) {</code>				
<code>x += 2;</code>				
<code>}</code>	<code>{3, 5, 7, ...}</code>	<code>Int</code>	<code>Pos</code>	<code>PosOdd</code>
<code>x -= 1;</code>	<code>{2, 4, 6, ...}</code>	<code>Int</code>	<code>ZeroOrPos</code>	<code>PosEven</code>
<code>print(x);</code>				

Abstract Domain

- A set of abstract properties: a crucial design choice for static analysis
- (Semantic) Property: points of interest in programs
 - E.g., “`p == NULL?`”, “`idx < size?`”, “fp can be only f, g, or h?”, etc
- Abstract property: approximated semantic properties
 - E.g., Int, Pos, [0,10], NonNull, {f,g,h}, DontKnow, etc

Design of Abstract Domain

- Abstract domain: CPO (complete partial order)
 - “larger” elements have “more” (“imprecise”) information
 - C.f, domain theory
- Example:



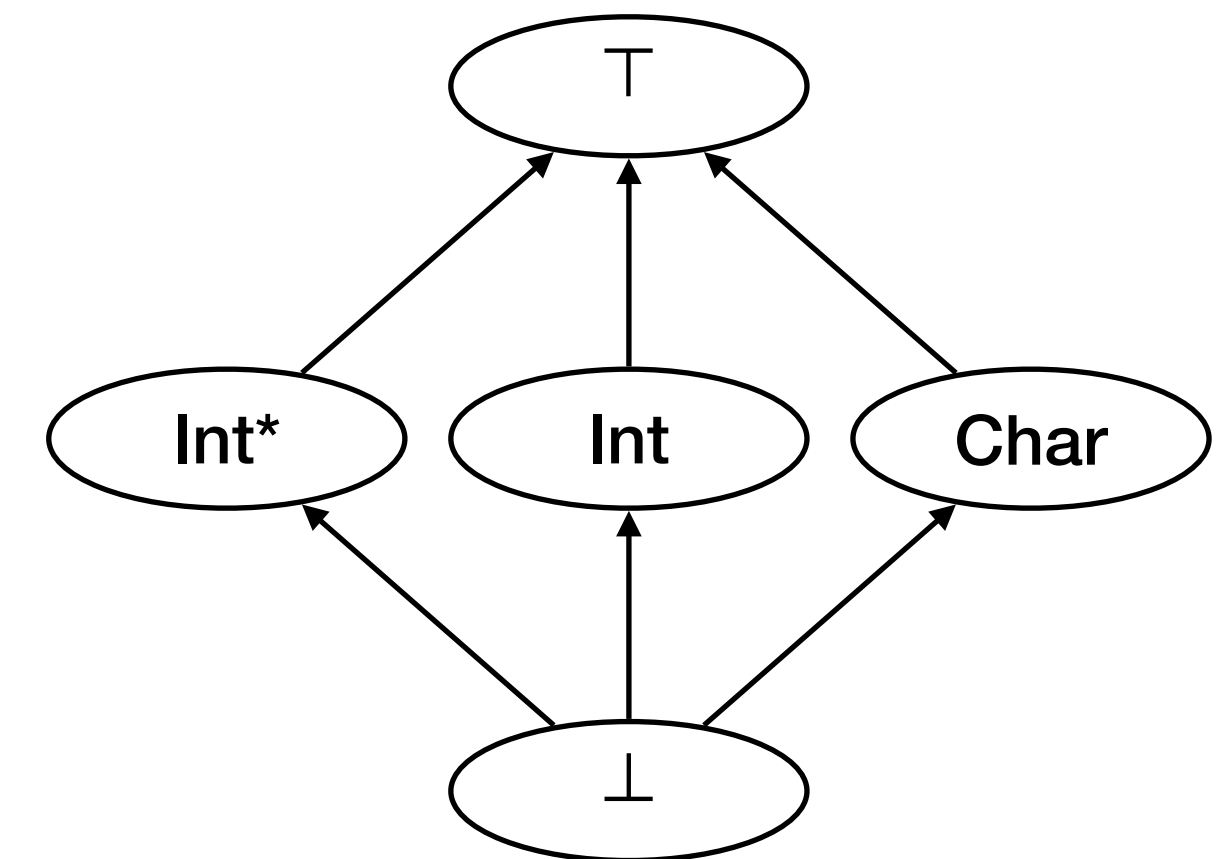
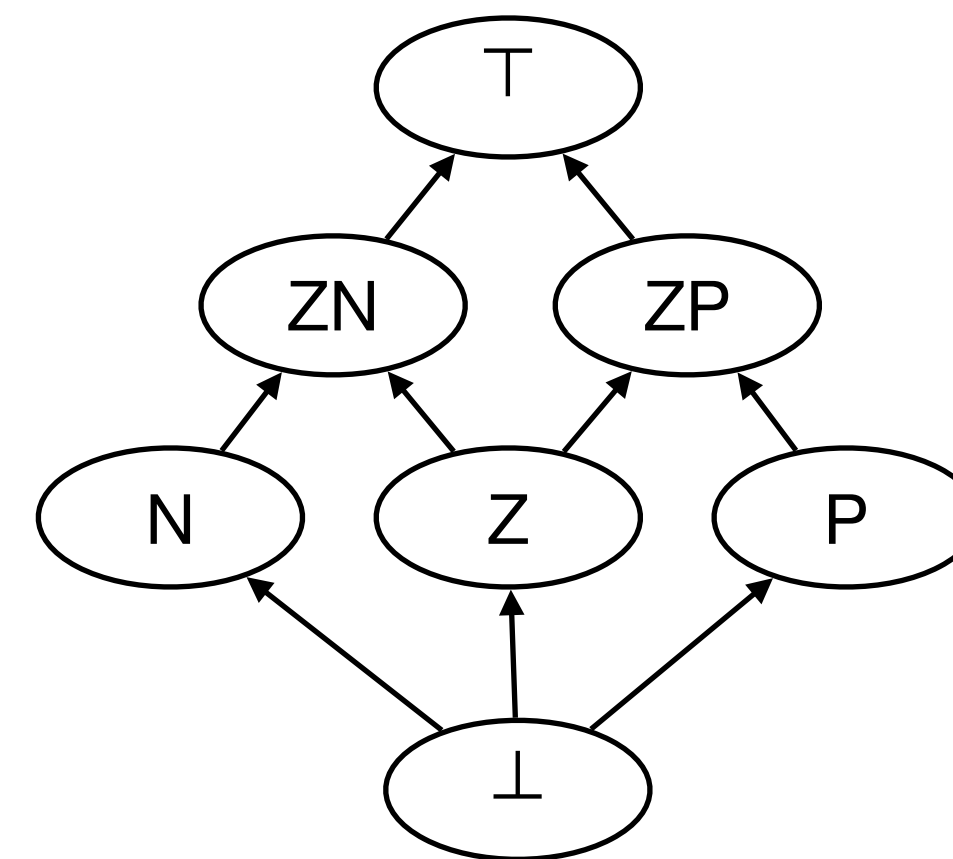
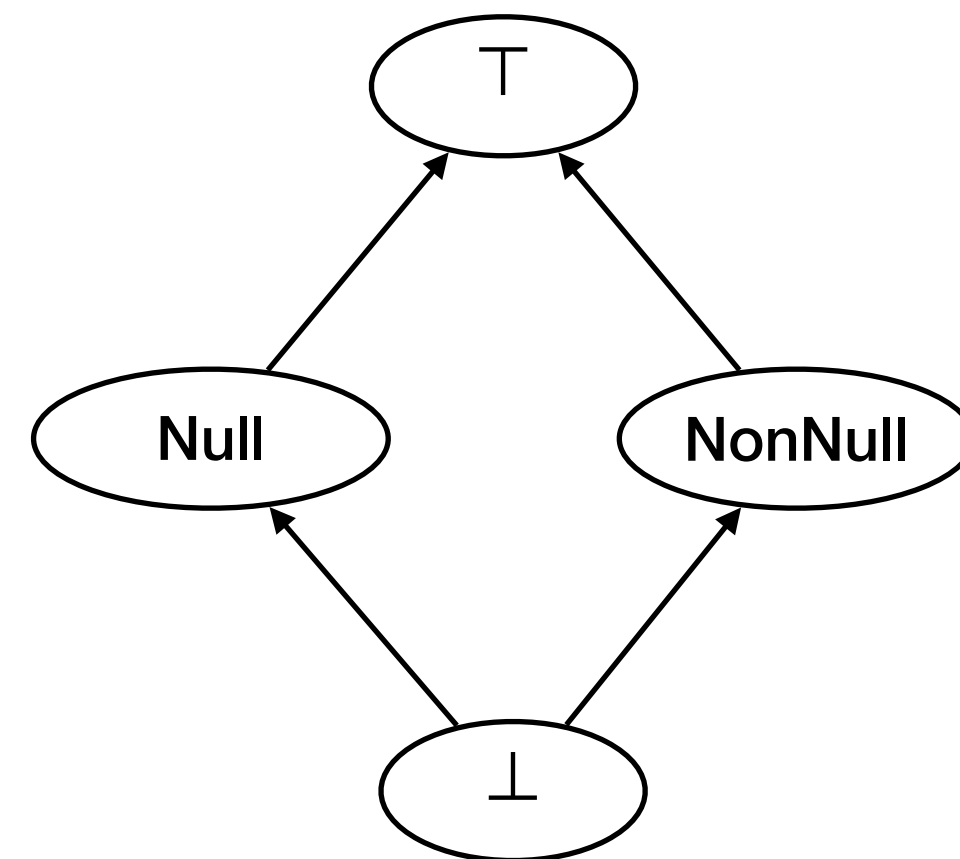
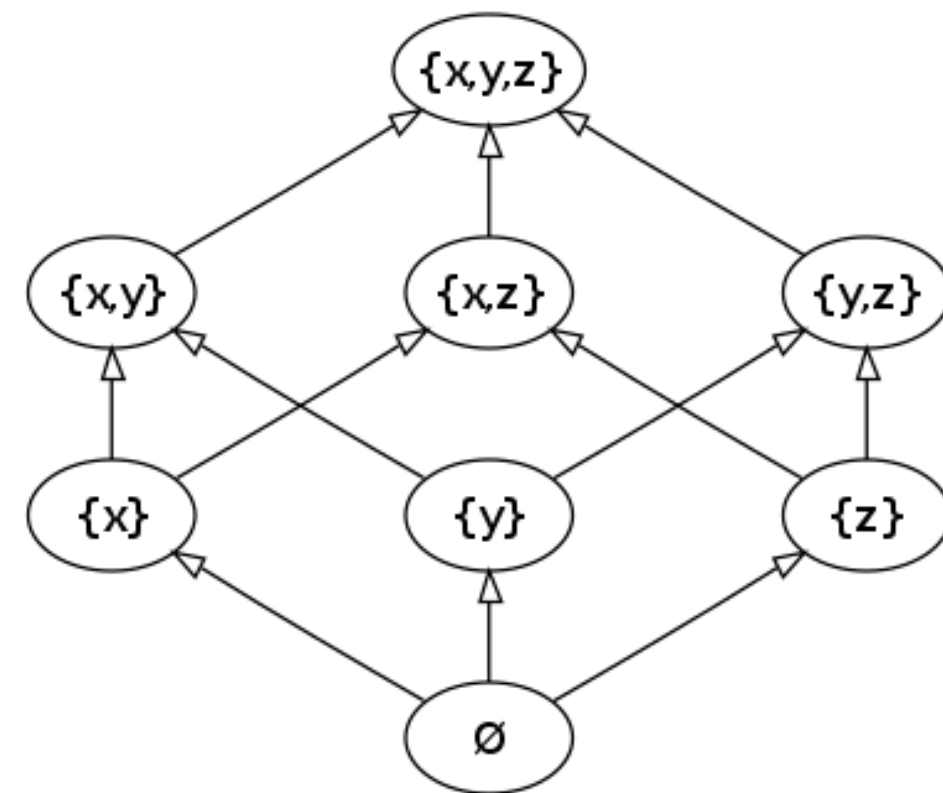
Partial Order

Definition (Partial Order). A binary relation \sqsubseteq is a **partial order** on a set D if it has:

1. reflexivity: $a \sqsubseteq a$ for all $a \in D$
2. Antisymmetry: $a \sqsubseteq b$ and $b \sqsubseteq a$ implies $a = b$
3. Transitivity: $a \sqsubseteq b$ and $b \sqsubseteq c$ implies $a \sqsubseteq c$

A set D with a partial order \sqsubseteq is called a **partially ordered set** (D, \sqsubseteq) , or simply **poset**.

Example



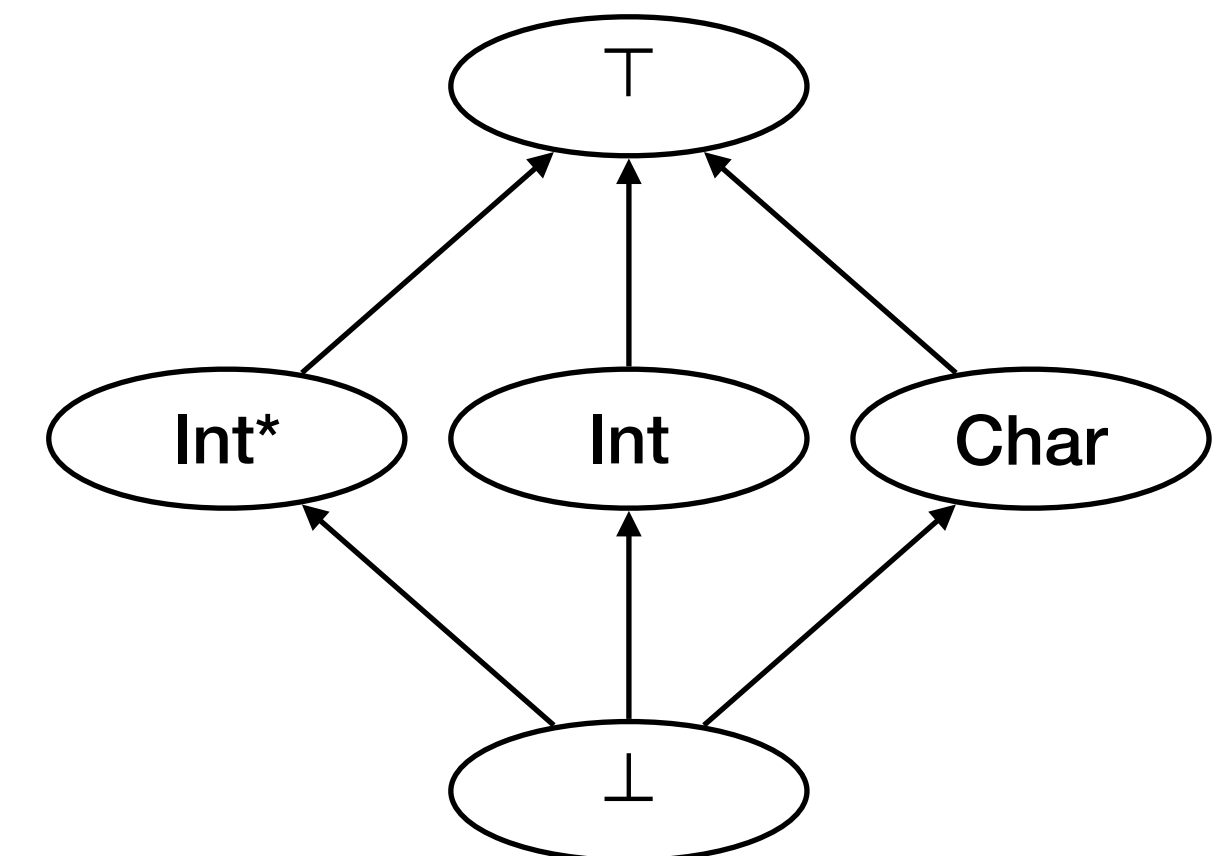
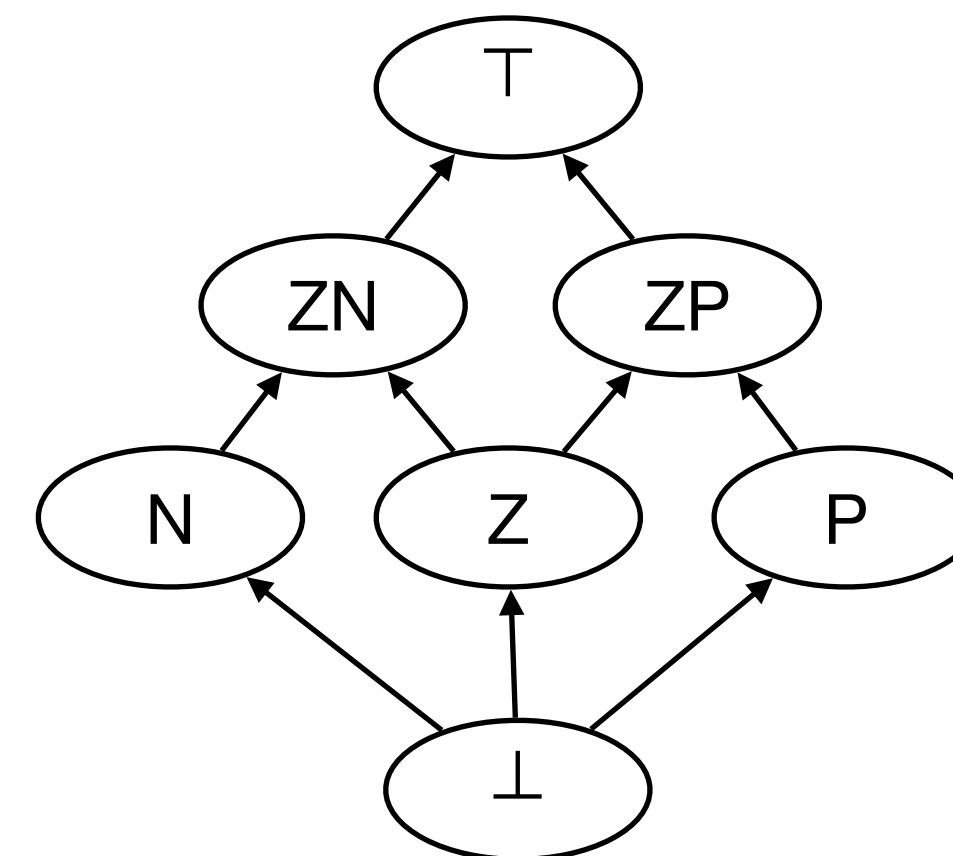
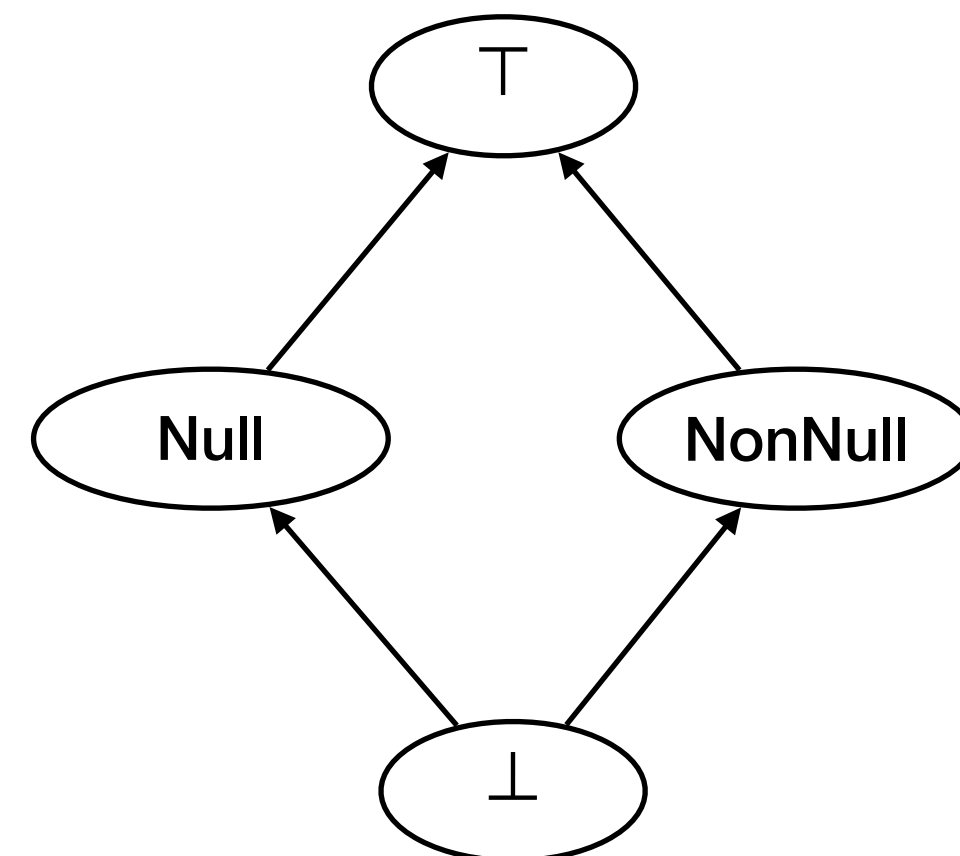
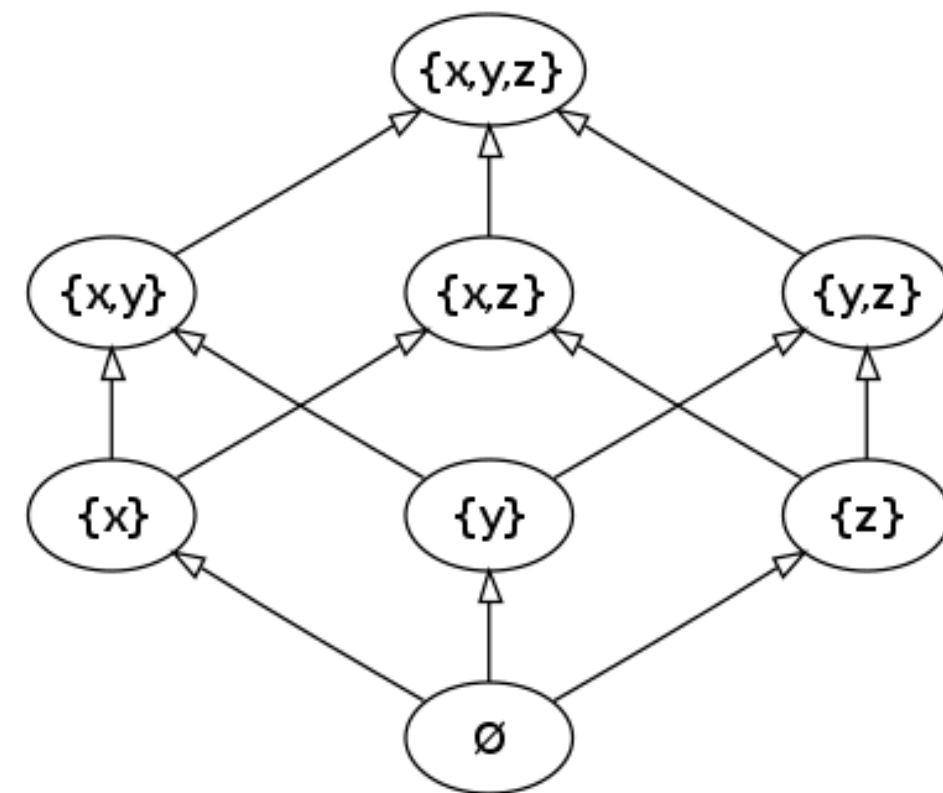
Least Upper Bound (Join)

Definition (Least Upper Bound). For a partial ordered set (D, \sqsubseteq) and subset $X \subseteq D$, $d \in D$ is an **upper bound** of X iff

$$\forall x \in X. x \sqsubseteq d.$$

An upper bound d is the **least upper bound** of X iff for all upper bounds y of X , $d \sqsubseteq y$. The least upper bound of X is denoted by $\bigsqcup X$.

Example

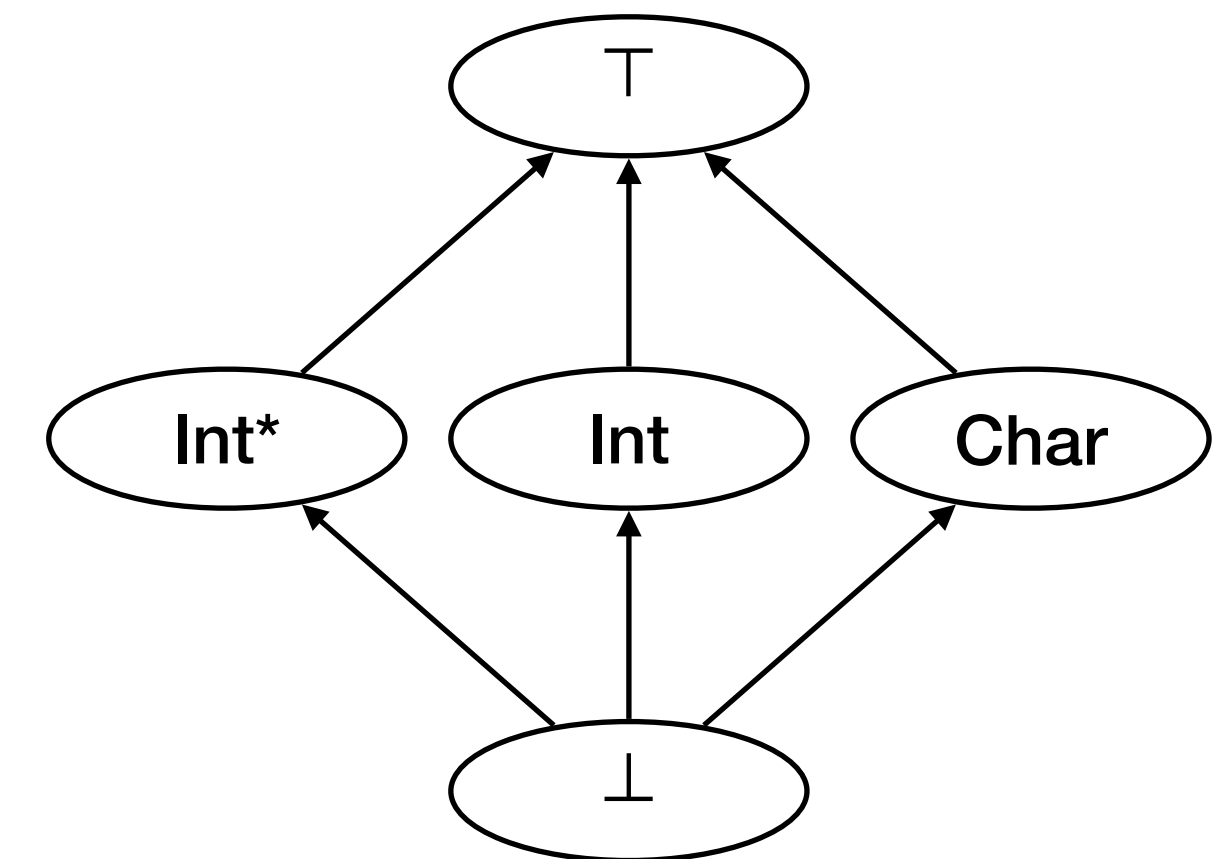
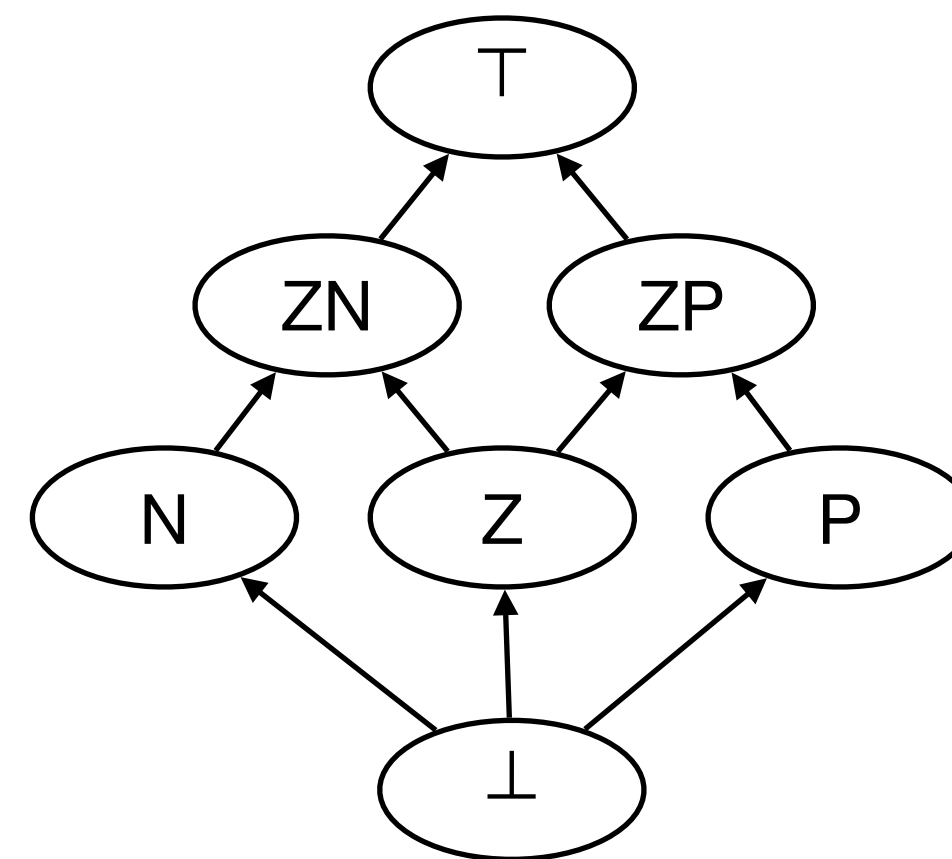
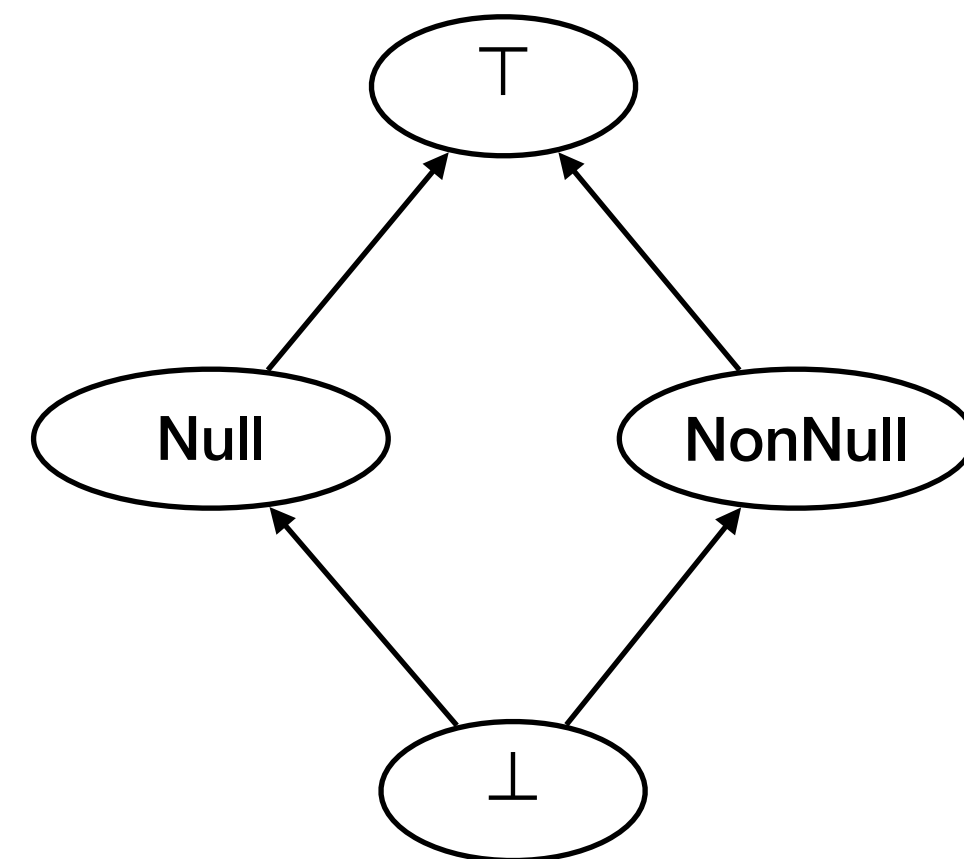
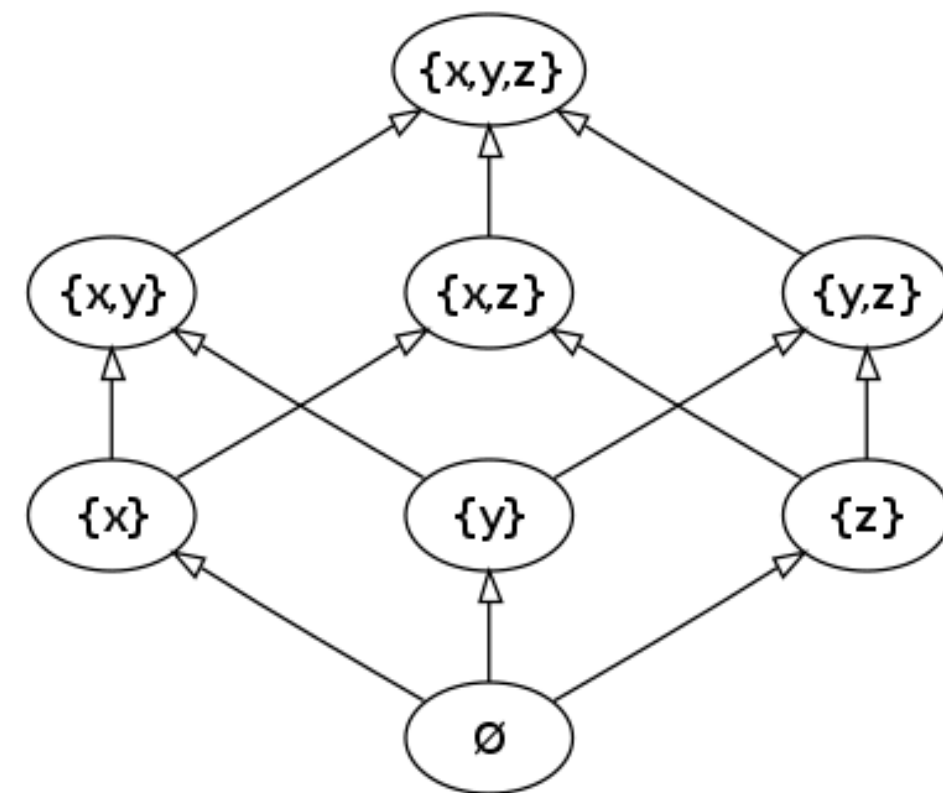


Chain

Definition (Chain). Let (D, \sqsubseteq) be a partial ordered set. A subset $X \subseteq D$ is called **chain** if X is totally ordered:

$$\forall x_1, x_2 \in X. x_1 \sqsubseteq x_2 \text{ or } x_2 \sqsubseteq x_1 .$$

Example

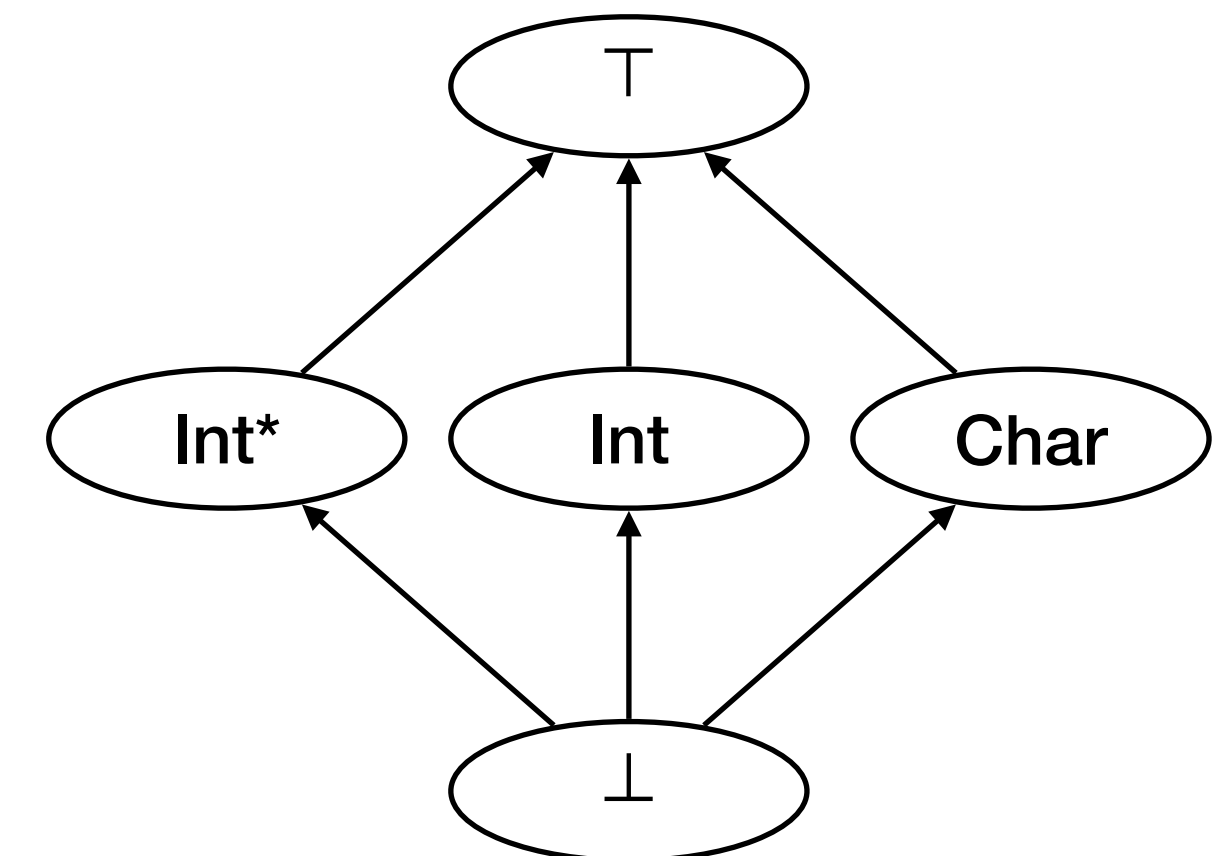
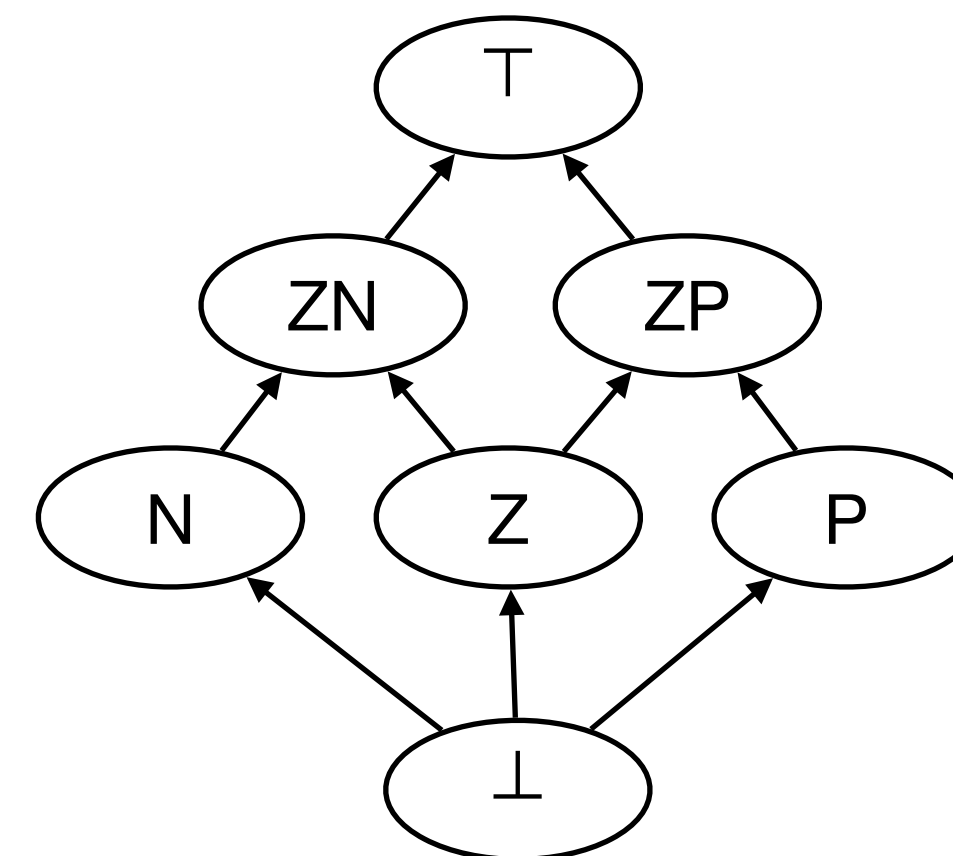
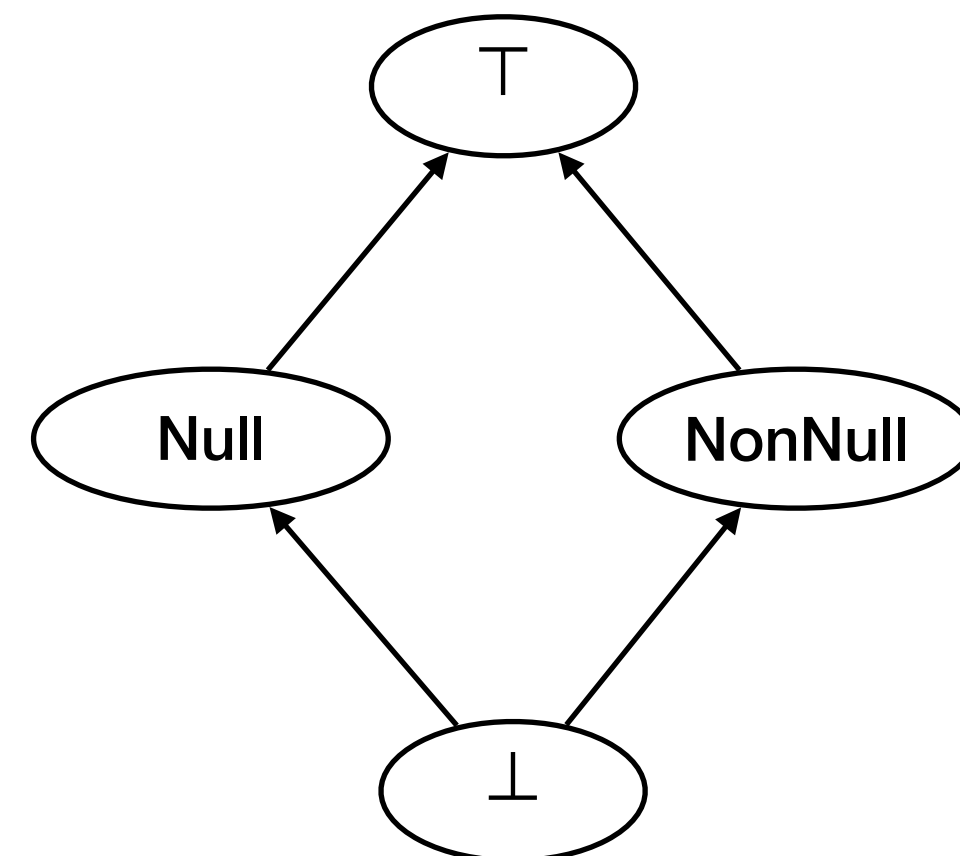
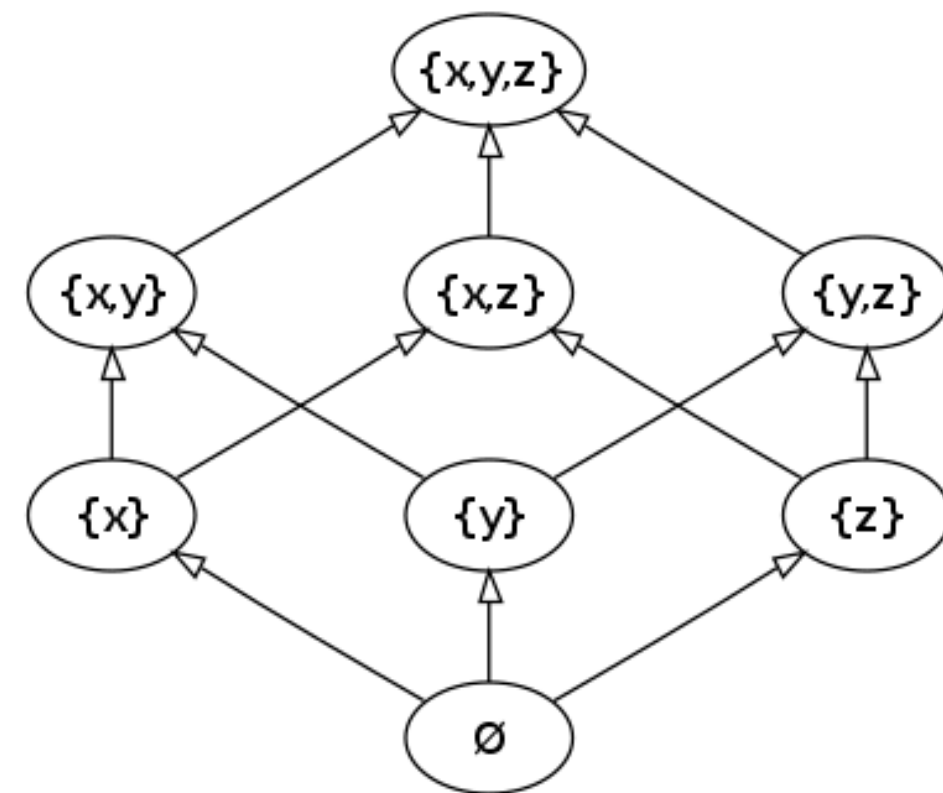


CPO

Definition (CPO). A poset (D, \sqsubseteq) is a **CPO** (complete partial order) if every chain X of D has $\bigsqcup X \in D$.

Lemma. If poset (D, \sqsubseteq) is a CPO, it has the **least element** $\perp = \bigsqcup \emptyset$

Example



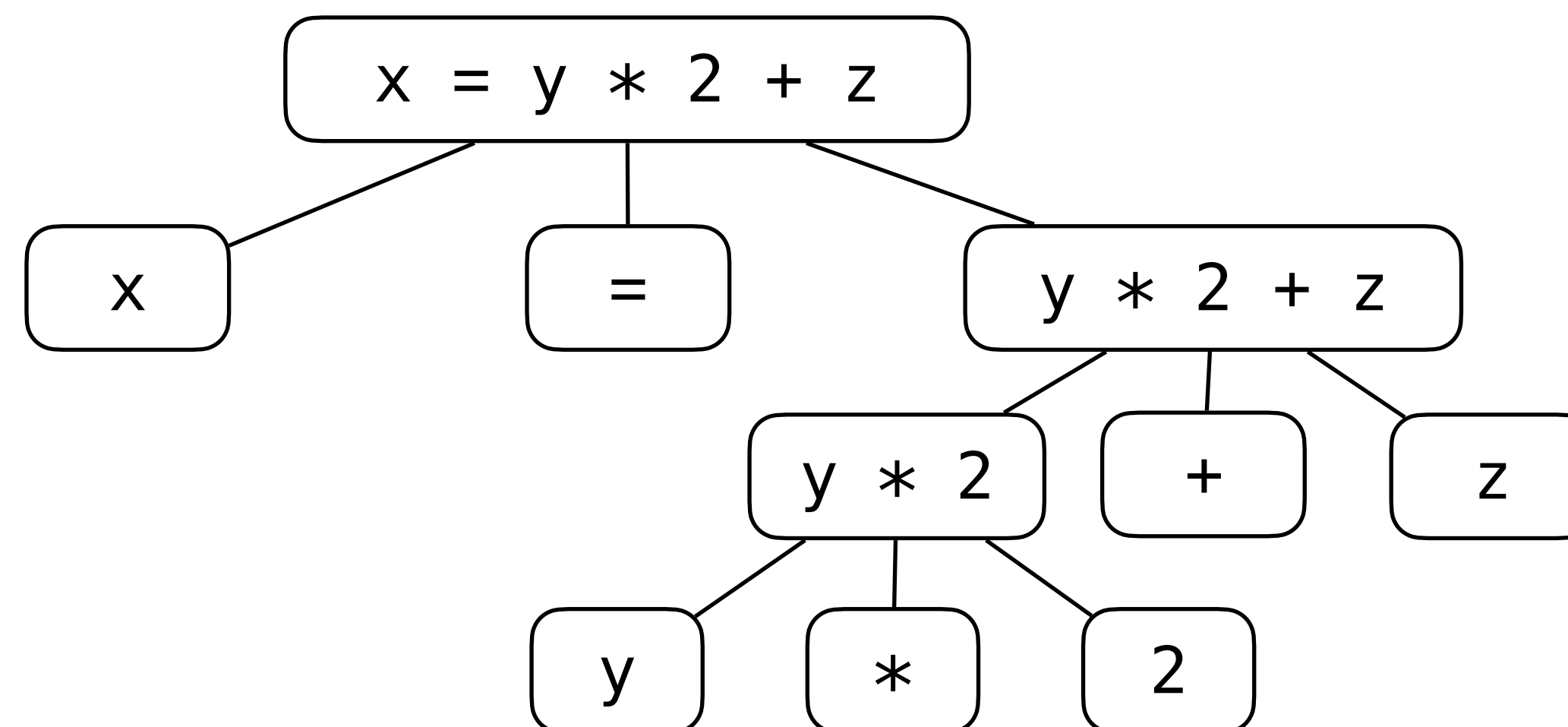
Abstraction & Concretization

- Concretization of an abstract element: estimation of concrete elements
 - Concretization of bottom: non-termination, unreachable, or error
 - Concretization of top: “Unknown”
 - Concretization of X: all possibilities subsumed by X if reachable
- Example:

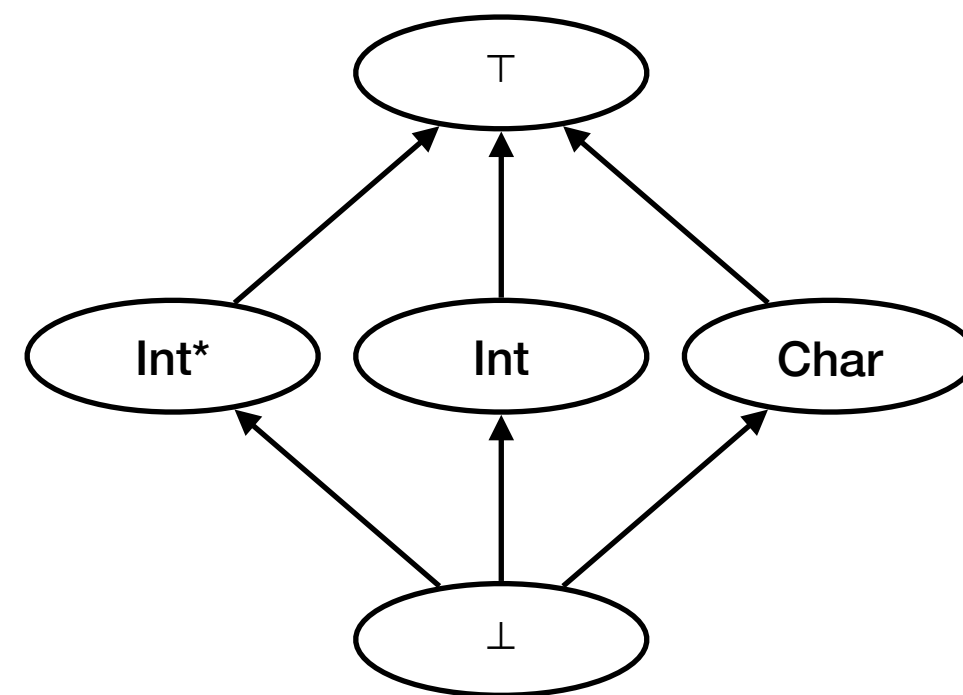
```
x = input();    // Top: any integer (if reachable)
y = 1 / x;      // Top: any integer (if reachable)
z = 1;          // Pos: a positive integer (if reachable)
if (x > 0 && x < 0) {
    // Bot: unreachable
}
```

Abstract Semantics

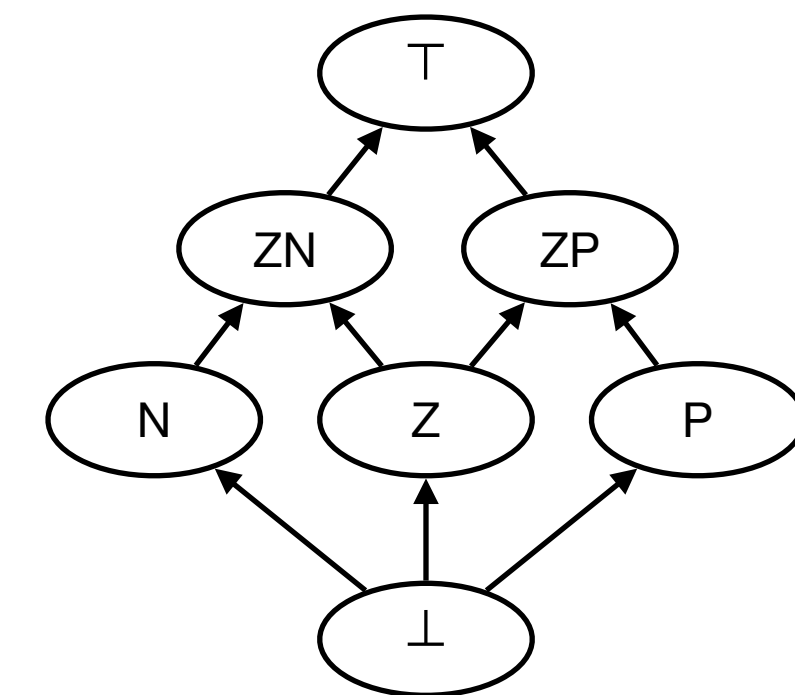
- Abstraction of actual program behavior (i.e., concrete semantics)
- Defined with respect to the underlying abstract domains
- Usually, defined compositionally
 - E.g., abstract semantics of “ $x = y * 2 + z$ ”



Example



```
x = 1;           // x: int
y = 2;           // y: int
if(*)
    r = x * y;    // r: int
else
    r = x - y;    // r: int
                  // r: int
```



```
x = 1;           // x: P
y = 2;           // y: P
if(*)
    r = x * y;    // r: P
else
    r = x - 1;    // r: ZP
                  // r: ZP
```

Design of Static Analysis

- How to design a correct static analysis?
 - General principle & framework: abstract interpretation
 - Specialized frameworks (limited but powerful enough)
 - Static analysis by equations (e.g., data-flow analysis)
 - Static analysis by monotonic closure (e.g., constraint-based analysis)
 - Static analysis by proof construction (e.g., type system)

Conclusion

- Static analysis: a method to estimate SW behavior in advance
- Key: abstraction (approximation)
 - Abstract domain: CPO
 - Abstract semantics: operations on the underlying abstract domains
- Pros: can subsume all possible behavior of SW
- Cons: may have spurious results