

# IS893: Advanced Software Security

## 11. Symbolic Execution

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# Symbolic Execution

- Run programs with symbolic inputs rather than specific inputs
  - Symbol: a class of input
- Compute symbolic memory (mapping from variables to symbolic expressions)
- Symbolic expressions: expressions involving operators over symbols and concrete values
  - E.g.,  $\alpha \times 4 + \beta$
- Many practical tools: KLEE, Clang static analyzer, etc

# Example

```
// Goal: find assert-failing x and y
void foobar(int x, int y) {
    if (x - 1 == 12345) {
        assert(x - y != 0);
    }
}
```

## Random testing (concrete execution)

x	y	x - y != 0
125	353	TRUE
45	242156	TRUE
...	...	...

## Symbolic execution

$\mathbf{x} = \alpha_x \wedge \mathbf{y} = \alpha_y \wedge \alpha_x - 1 = 12345 \wedge \alpha_x - \alpha_y = 0$   
: **SAT** when  $\alpha_x = \alpha_y = 12346$

# State

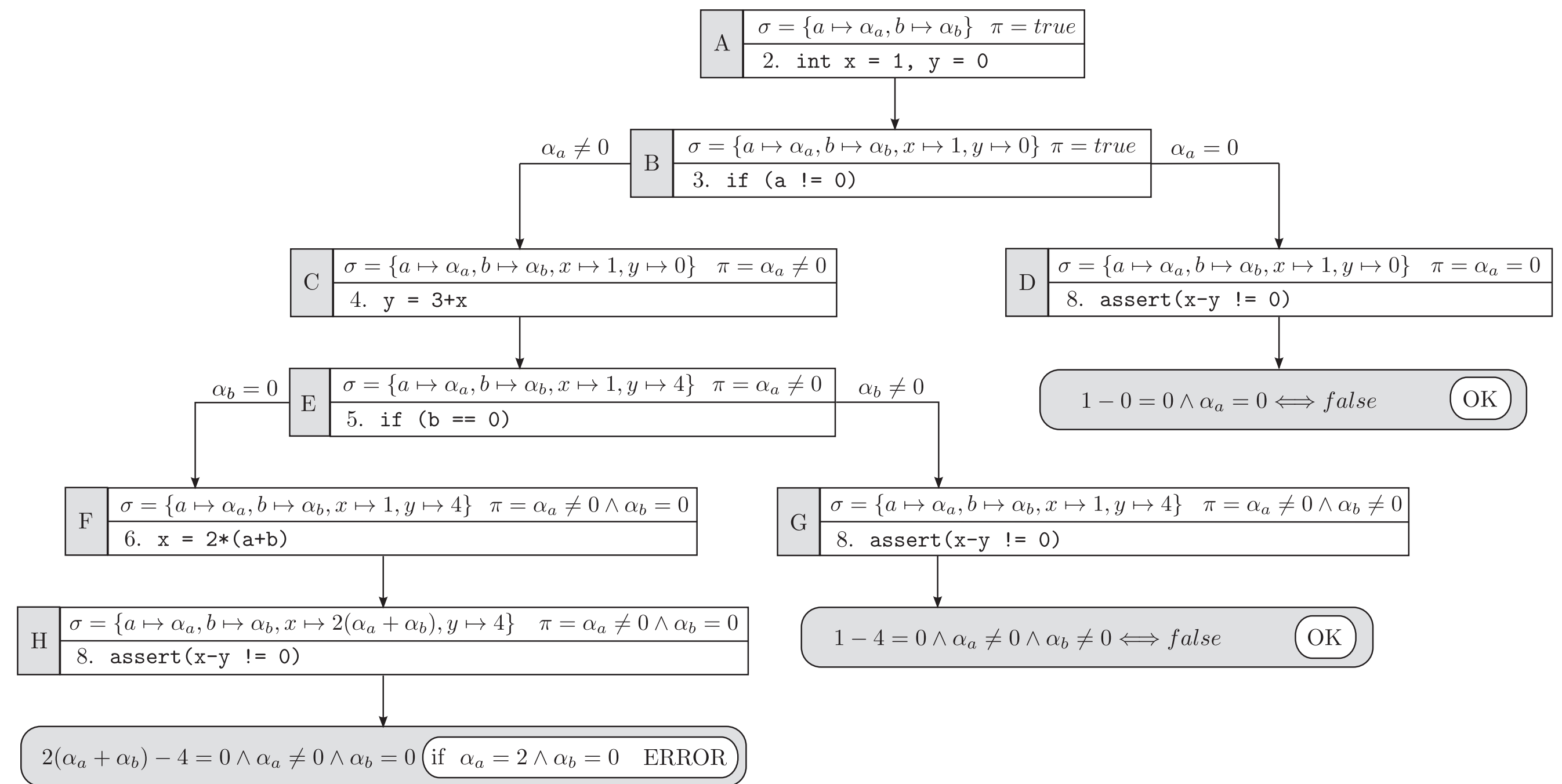
- Symbolic execution maintains a state  $(stmt, \sigma, \pi)$  where:
  - $stmt$  : the next statement to evaluate
  - $\sigma$  : a symbolic store (a mapping from program variables to symbolic expressions)
  - $\pi$  : the path constraint (assumptions on the symbols to reach  $stmt$ )

# State Transition

- Give state  $(stmt, \sigma, \pi)$ , the symbolic engine changes the state depending on  $stmt$ :
  - $x = e$  : update the symbolic store  $\sigma[x \mapsto e_s]$  where  $e_s$  is the symbolic expression obtained by evaluating  $e$
  - $\text{if } e \text{ then } s_{true} \text{ else } s_{false}$  : proceed to each branch with path constraint  $\pi_{true} = \pi \wedge e_s$ , and  $\pi_{false} = \pi \wedge \neg e_s$ , respectively
  - $\text{goto } s$  : advance the symbolic execution to statement  $s$

# Example

```
void foobar(int a, int b) {
  int x = 1, y = 0;
  if (a != 0) {
    y = 3 + x;
    if (b == 0)
      x = 2 * (a + b);
  }
  assert(x - y != 0);
}
```



\*Baldoni et al., A Survey of Symbolic Execution Techniques, ACM Computing Survey, 2018

# Path Feasibility

```
void f(int a, int b) {  
    if (a == 12345) {  
        assert(b == 0); // satisfiable?  
    }  
}
```

```
void f(int a, int b) {  
    if (a == 12345) {  
        if(b != 0) {  
            error(); // reachable?  
        }  
    }  
}
```

- Is a path executed by the symbolic execution engine feasible?
- The feasibility can be determined by an SMT solver
  - If feasible, crashing inputs are found
  - If not, it is a spurious path

# Theoretical Aspects

- Symbols soundly subsumes all possibilities of concrete values
- SMT solvers find failing inputs
- May not terminate in the presence of loops
  - Some stopping criteria needed (e.g., bounded loop iterations)



# Practical Limitation: Scalability

- State space explosion:  $N$  branches  $\rightarrow 2^N$  paths
  - Many of them are infeasible paths
- Constraint solving: limited scalability and expressiveness
  - E.g., large constraints, non-linear constraints, etc

```
void foobar(int x, int y, int z) {  
    if (pow(x, 3) + pow(y, 3) == pow(z, 3)) {  
        error();  
    } else {  
        ok();  
    }  
}
```

# Dynamic Symbolic Execution

- Combine concrete execution with symbolic execution
  - testing (scalability) + symbolic execution (constraint solving)
- Execute the program with random concrete inputs
- Keep track of both concrete values and symbolic constraints
- Use concrete values to simplify symbolic constraints

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 22$   
 $y = 7$

**Symbolic  
State**

$X = \alpha_x$   
 $y = \alpha_y$

**Path  
Constraint**

true

1st iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 22$   
 $y = 7$   
 $z = 14$

**Symbolic  
State**

$x = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$


**Path  
Constraint**

true

1st iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}  
  
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



Concrete State	Symbolic State	Path Constraint
<b>How to get in to the true branch?</b>  <b>Solve:</b> $2 * \alpha_y = \alpha_x$ <b>Solution:</b> $\alpha_x = 2, \alpha_y = 1$		
$x = 22$ $y = 7$ $z = 14$	$x = \alpha_x$ $y = \alpha_y$ $z = 2 * \alpha_y$	$2 * \alpha_y \neq \alpha_x$

1st iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 2$   
 $y = 1$

**Symbolic  
State**

$X = \alpha_x$   
 $y = \alpha_y$

**Path  
Constraint**

true

2nd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 2$   
 $y = 1$   
 $z = 2$

**Symbolic  
State**

$X = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$


**Path  
Constraint**

true

2nd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)   
            error();  
}
```

**Concrete  
State**

$x = 2$   
 $y = 1$   
 $z = 2$

**Symbolic  
State**

$x = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$

**Path  
Constraint**


$2 * \alpha_y = \alpha_x$

2nd iteration



# Example

```
int double(int v) {  
    return 2 * v;  
}  
  
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



Concrete State	Symbolic State	Path Constraint
<b>How to get in to the true branch?</b>  <b>Solve:</b> $2 * \alpha_y = \alpha_x \wedge \alpha_x > \alpha_y + 10$ <b>Solution:</b> $\alpha_x = 30, \alpha_y = 15$		
$x = 2$ $y = 1$ $z = 2$	$x = \alpha_x$ $y = \alpha_y$ $z = 2 * \alpha_y$	$2 * \alpha_y = \alpha_x$ $\wedge \alpha_x \leq \alpha_y + 10$

2nd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 30$   
 $y = 15$

**Symbolic  
State**

$X = \alpha_x$   
 $y = \alpha_y$

**Path  
Constraint**

$\pi = \text{true}$

3rd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



**Concrete  
State**

$x = 30$   
 $y = 15$   
 $z = 30$

**Symbolic  
State**

$x = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$


**Path  
Constraint**

$\pi = \text{true}$

3rd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}
```

```
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)   
            error();  
}
```

**Concrete  
State**

$x = 30$   
 $y = 15$   
 $z = 30$

**Symbolic  
State**

$x = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$

**Path  
Constraint**

$2 * \alpha_y = \alpha_x$

3rd iteration

# Example

```
int double(int v) {  
    return 2 * v;  
}  
  
void test_me(int x, int y) {  
    int z = double(y);  
    if (z == x)  
        if (x > y + 10)  
            error();  
}
```



x = 30  
y = 15  
z = 30

**Symbolic  
State**


$x = \alpha_x$   
 $y = \alpha_y$   
 $z = 2 * \alpha_y$

**Path  
Constraint**

$2 * \alpha_y = \alpha_x$   
 $\wedge \alpha_x > \alpha_y + 10$

3rd iteration

# More Complex Example

	Concrete State	Symbolic State	Path Constraint
<pre>void test_me(int x, int y) {     int z = secure_hash(y);     if (z == x)         if (x &gt; y + 10)             error(); }</pre>	 $x = 22$ $y = 7$	$x = \alpha_x$ $y = \alpha_y$	true

1st iteration

# More Complex Example

```
void test_me(int x, int y) {  
  int z = secure_hash(y);  
  if (z == x)  
    if (x > y + 10)  
      error();  
}
```



Concrete State	Symbolic State	Path Constraint
$x = 22$ $y = 7$ $z = 912490858127853$	$x = \alpha_x$ $y = \alpha_y$ $z = \text{secure\_hash}(\alpha_y)$	true

1st iteration

# More Complex Example

```
void test_me(int x, int y) {  
  int z = secure_hash(y);  
  if (z == x)  
    if (x > y + 10)  
      error();  
}
```

Idea:  
Replace  $\alpha_y$  by  
concrete value 7

Concrete State	Symbolic State	Path Constraint
How to get in to the true branch?		
Solve: $\text{secure\_hash}(\alpha_y) = \alpha_x$ <b>Extremely hard to solve!</b>		
x = 22 y = 7 z = 912490858127853	X = $\alpha_x$ y = $\alpha_y$ z = $\text{secure\_hash}(\alpha_y)$	$\text{secure\_hash}(\alpha_y) \neq \alpha_x$

1st iteration



# More Complex Example


```
void test_me(int x, int y) {  
  int z = secure_hash(y);  
  if (z == x)  
    if (x > y + 10)  
      error();  
}
```



Concrete State	Symbolic State	Path Constraint
<div>How to get in to the true branch?  Solve: <math>912490858127853 = \alpha_x</math> Solution: <math>\alpha_x = 912490858127853, \alpha_y = 7</math></div>		
$x = 22$ $y = 7$ $z = 912490858127853$	$X = \alpha_x$ $y = \alpha_y$ $z = \text{secure\_hash}(\alpha_y)$	$\text{secure\_hash}(\alpha_y) \neq \alpha_x$

1st iteration

# More Complex Example

	Concrete State	Symbolic State	Path Constraint
<pre>void test_me(int x, int y) {   int z = secure_hash(y);   if (z == x)     if (x &gt; y + 10)       error(); }</pre>	<p> <math>x = 912490858127853</math> <math>y = 7</math></p>	$x = \alpha_x$ $y = \alpha_y$	true


2nd iteration

# More Complex Example

	Concrete State	Symbolic State	Path Constraint
<pre>void test_me(int x, int y) {   int z = secure_hash(y);   if (z == x)     if (x &gt; y + 10)       error(); }</pre>	<pre>x = 912490858127853 y = 7 z = 912490858127853</pre>	<pre>X = <math>\alpha_x</math> y = <math>\alpha_y</math> z = secure_hash(<math>\alpha_y</math>)</pre>	true

2nd iteration


# More Complex Example

```
void test_me(int x, int y) {
    int z = secure_hash(y);
    if (z == x)
        if (x > y + 10) 
            error();
}
```

Concrete State	Symbolic State	Path Constraint
x = 912490858127853 y = 7 z = 912490858127853	x = $\alpha_x$ y = $\alpha_y$ z = secure_hash( $\alpha_y$ )	secure_hash( $\alpha_y$ ) = $\alpha_x$

2nd iteration

# More Complex Example


	Concrete State	Symbolic State	Path Constraint
<pre>void test_me(int x, int y, int z = secure_hash(y); if (z == x)     if (x &gt; y + 10)         error(); }</pre>	<div><p>Error!</p></div> <p>x = 912490858127853 y = 7 z = 912490858127853</p>	<p>x = <math>\alpha_x</math> y = <math>\alpha_y</math> z = secure_hash(<math>\alpha_y</math>)</p>	<p>secure_hash(<math>\alpha_y</math>) = <math>\alpha_x</math> <math>\wedge</math> <math>\alpha_x &gt; \alpha_y + 10</math></p>

2nd iteration

# Characteristics

- Reduced number of calls to SMT solvers
  - If reachable by the concrete execution, then no need for constraint solving
- Simplified path constraints by replacing symbols with concrete values
- Caveat: may miss real bugs due to the concretization
  - Unsafely declare a reachable branch as unreachable

# Catastrophic Example

	Concrete State	Symbolic State	Path Constraint
<pre>void test_me(int x, int y) {   if (x != y) {     int z = secure_hash(x);     int w = secure_hash(y);     if (z == w)       error();   } }</pre>	 $x = 22$ $y = 7$	$x = \alpha_x$ $y = \alpha_y$	true

1st iteration

# Catastrophic Example

```
void test_me(int x, int y) {  
  if (x != y) {  
    int z = secure_hash(x);  
    int w = secure_hash(y);  
    if (z == w)  
      error();  
  }  
}
```



**Concrete  
State**

x = 22  
y = 7  
z = 124912759192859  
w = 912490858127853

**Symbolic  
State**

$x = \alpha_x$   
 $y = \alpha_y$

**Path  
Constraint**


$\alpha_x \neq \alpha_y$

1st iteration



# Catastrophic Example

```
void test_me(int x, int y) {  
  if (x != y) {  
    int z = secure_hash(x);  
    int w = secure_hash(y);  
    if (z == w)  
      error();  
  }  
}
```



Concrete State	Symbolic State	Path Constraint
<div>How to get in to the true branch?  Solve: <math>\alpha_x \neq \alpha_y \wedge \text{hash}(\alpha_x) = \text{hash}(\alpha_y)</math> <b>Use concrete vales!</b></div>		
x = 22 y = 7 z = 124912759192859 w = 912490858127853	x = $\alpha_x$ y = $\alpha_y$	$\alpha_x \neq \alpha_y$ $\wedge \text{hash}(\alpha_x) \neq \text{hash}(\alpha_y)$

1st iteration

# Catastrophic Example

```
void test_me(int x, int y) {  
  if (x != y) {  
    int z = secure_hash(x);  
    int w = secure_hash(y);  
    if (z == w)  
      error();  
  }  
}
```



Concrete State	Symbolic State	Path Constraint
<b>How to get in to the true branch?</b> <b>Solve:</b> $22 \neq 7 \wedge 124\dots859 = 912\dots853$ <b>Unsatisfiable</b>		
$x = 22$ $y = 7$ $z = 124912759192859$ $w = 912490858127853$	$x = \alpha_x$ $y = \alpha_y$	$\alpha_x \neq \alpha_y$ $\wedge \text{hash}(\alpha_x) \neq \text{hash}(\alpha_y)$

1st iteration

# Conclusion

- Symbolic execution: run programs with symbolic inputs
  - Effective exploration compared to random testing (i.e., white box)
  - Less scalable because of constraint solving
- Dynamic symbolic execution: combine concrete and symbolic execution
  - Efficient: less number of calls to SMT solver, simplified constraints
  - False negative: may miss real bugs