

# Praktikum z ekonometrie

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- 1 The nature of missing data
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# The nature of missing data

## Missing completely at random (*MCAR*)

- The probability that an observation  $X_i$  is missing is unrelated to the value of  $X_i$  or to the value of any other variables.
- Any piece of data is equally likely to be missing.
- Analyses based on data with *MCAR* observations remain unbiased. We may lose power (increased standard errors), but the estimated parameters are not biased by the absence of data.

## Missing at random (*MAR*)

- Data meets the requirement that missingness does not depend on the value of  $X_i$  after controlling for another variable in our analysis.
- For example, data are *MCAR* in a specific (demographic) subgroup.

## Missing Not at Random (*MNAR*)

- Missingness of  $X_i$  depends on its value (e.g. income in surveys)
- The only way to obtain unbiased estimates of (regression) parameters is to model the missingness.

# Traditional treatment of missing data

## Listwise deletion (complete cases analysis)

- We omit all rows with missing data – missing information for at least one variable in the  $i$ -th individual observation. Then, we run our analyses on the observations that remain. This often results in a substantial decrease in sample size. Under the assumption that data are missing completely at random, LRMs lead to unbiased parameter estimates – still, we lose power due to exclusion of (potentially large number of) observations.

### R code

```
newData <- data[complete.cases(data)==T, ]  
# data is a data.frame  
# or  
newData <- na.omit(data)
```

## Hot deck imputation

- Historically used by the US Census Bureau (since 1950's). Respondent's missing data were replaced by observed replacement data – drawn at random from a group of similar participants. Suitable, given only a few missing observations need to be replaced and given the draw is random.

# Traditional treatment of missing data

## Mean substitution

- ✓ Simple
- ✗ In simple linear regression models (SLRMs), this adds no new information but increases sample size – that leads to underestimated standard errors only.

**Example:** Data on salary and citation level of publications. 62 cases with complete data and 7 cases for which the citation index was missing. Correlations and regression coefficients were compared as follows:

Analysis	$n$	$corr$	$\hat{\beta}_1$	$s.e.(\hat{\beta}_1)$
Complete cases only	62	.55	310.747	60.95
With mean substitution	69	.54	310.747	59.12

## Mean substitution, contnd.

- Mean imputation can be valid especially when data are missing as MCAR.
- It is fast, simple, easy to implement, and no cases are excluded.
- But even under MCAR, this method still leads to underestimation of the population variance
- Bias is proportional to  $(\text{nobs} - 1)/(\text{nobs} + \text{nmis} - 1)$ .  
Smaller standard errors increase the possibility of Type I errors.

# Traditional treatment of missing data

## Regression substitution

- Uses linear regression (auxiliary LRM) to predict what the missing values of regressors should be, on the basis of other variables that are present.
- For SLRMs, the same problem of error variance as in mean substitution remains. We do not add more information but we increase the sample size and (spuriously) reduce the standard error.
- May be useful for MLRMs.

## Stochastic regression substitution

- This approach adds a randomly sampled residual term from the normal (or other) distribution to each value estimated by regression substitution. Adding a bit of random error to each substitution reduces, but does not eliminate, the problem of spurious reduction of the standard errors.



## Maximum Likelihood Expectation-Maximization

- Computationally complex, maximum likelihood approach to the estimation of missing values Many approaches exist (e.g. the Expectation-Maximization algorithm)

[https://www.uvm.edu/~dhowell/StatPages/Missing\\_Data/Missing-Part-Two.html](https://www.uvm.edu/~dhowell/StatPages/Missing_Data/Missing-Part-Two.html)

# Multiple imputation (MI)

## Multiple Imputation (MI)

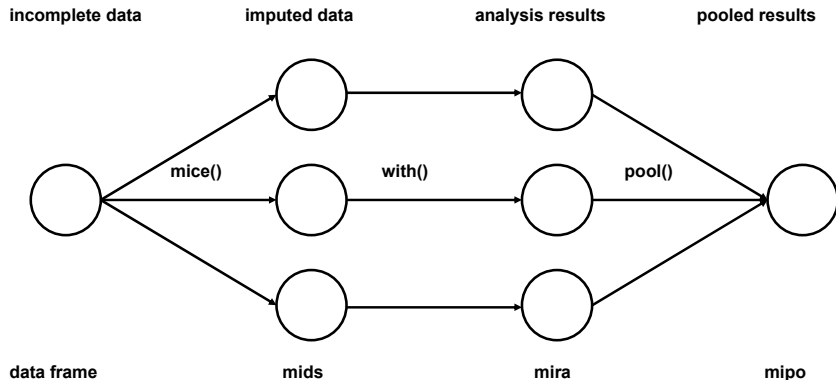
R: {mice}, {mi}, {Amelia}, ...

### MI motivation and algorithm

- Create several (say, 5) imputed values (versions) for each missing item – regressor  $x_{ij}$ .
- Each of the (5) versions of imputed data is used for estimation (using OLS, ML or other adequate approach)
- Information obtained from all (e.g. 5) estimates is conveniently summarized.

# Multiple imputation (MI)

Multiple imputation scheme (example with  $m = 3$  imputations):



# Multiple imputation (MI)

## Multiple imputation - 7 choices of model specification

- 1 Decide on MAR assumption plausibility (MAR/MNAR).
- 2 Imputation model choice (univariate, multivariate, data type).
- 3 Choice of predictors for MI.
- 4 Should we impute variables that are functions of incomplete variables (e.g. interaction terms)?
- 5 Choice of order of imputation (can affect results).
- 6 MI is (generally) based on numerical algorithm: we need to choose starting setup and control (limit) the number of iterations.
- 7 We need to choose  $m$  – the number of imputed datasets.

In R (`{mice}`), most of the choices have generally valid default setting. However – all the choices are always made in MI and they affect the resulting imputations.

## **Predictive mean matching (pmm) in R** – general description

- Implemented in {mice} and other packages.
- General purpose semi-parametric imputation method.
- Suitable especially for imputing quantitative variables that are not normally distributed.
- Imputations are restricted to observed values.
- Can preserve non-linear relations even if the structural part of the imputation model is wrong.

# Multiple imputation (MI)

## Predictive mean matching (pmm) in R – algorithm

Suppose there is a single variable  $x$  that has some cases with missing data, and a set of variables  $\mathbf{z}$  (with no missing data, used to impute  $x$ ).

- 1 For cases with no missing data, estimate LRM  $x \leftarrow \mathbf{z}$ , producing  $\hat{\beta}$  and  $\text{var}(\hat{\beta})$  estimates.
- 2 Make a random draw from the “posterior predictive distribution” of  $\hat{\beta}$ , producing a new set of coefficients  $\hat{\beta}^*$ .

Typically, this would be a random draw from a multivariate normal distribution with mean  $\hat{\beta}$  and covariance  $\text{var}(\hat{\beta})$ .

This step is necessary to produce sufficient variability in the imputed values, and is common to all “proper” methods of MI.

## Predictive mean matching (pmm) in R – algorithm contnd.

- 3 Using  $\hat{\beta}^*$ , generate predicted values for  $x$  for **all cases**, both with data missing on  $x$  and with data present.
- 4 For each case with missing  $x$ , identify a set of cases with observed  $x$  whose **predicted** values are close to the predicted value for the case with missing data.
- 5 From among those close cases, randomly choose one and assign its observed value to substitute for the missing value.
- 6 Repeat steps 2 through 5 for each of the  $m$  imputed datasets.

# Multiple imputation (MI)

## Predictive mean matching (**pmm**) in R – recap.

- Compared with regression-based methods, **pmm** produces imputed values that are much more like real values.
  - If the original variable is skewed, the imputed values will also be skewed.
  - If the original variable is bounded by 0 and 100, the imputed values will also be bounded by 0 and 100.
  - And if the real values are discrete (number of children), the imputed values will also be discrete.
- Unfortunately, for **pmm**, there's no mathematical theory to justify it (only Monte Carlo simulations support its use).
- However (based on MC simulations), **pmm** does almost as well as parametric methods for a correctly specified model, and a little better than parametric methods in certain misspecified models.



# Multiple imputation (MI)

## Multiple Imputation (contnd.)

Regression coefficients from five imputed data sets

Data set	Estimated parameter	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
1	Coefficient	-11.535	-2.780	1.029	-.031	-0.359	0.572
	Variance	43.204	3.323	0.013	0.013	0.013	0.012
2	Coefficient	-11.501	-4.149	1.040	-0.093	-0.583	0.876
	Variance	40.488	2.680	0.010	0.009	0.009	0.007
3	Coefficient	-10.141	-5.038	0.766	0.123	-0.252	0.625
	Variance	42.055	3.301	0.010	0.010	0.010	0.009
4	Coefficient	-11.533	-6.920	0.870	0.084	-0.458	0.815
	Variance	28.751	1.796	0.081	0.007	0.007	0.007
5	Coefficient	-14.586	-1.115	0.718	0.050	-0.373	0.814
	Variance	32.856	2.362	0.009	0.009	0.009	0.008
Mean $b_i$		-11.859	-4.000	0.885	0.027	-0.405	0.740
Mean Var. ( $\bar{W}$ )		37.471	2.692	0.025	0.010	0.010	0.009
Var. of $b_i$ (B)		2.682	4.859	0.022	0.008	0.015	0.018
T							
$\sqrt{T}$		40.69	8.523	0.051	0.020	0.028	0.031
$t$		6.379	2.919	0.226	0.141	0.167	0.176
		-1.859	-1.370	3.916*	0.191	2.425*	4.204*

\*  $p < .05$  "Var." refers to the squared standard error of the coefficient.

[https://www.uvm.edu/~dhowell/StatPages/Missing\\_Data/Missing-Part-Two.html](https://www.uvm.edu/~dhowell/StatPages/Missing_Data/Missing-Part-Two.html)

## Special considerations apply to missing dependent variable data

- If we can assume that data are missing completely at random (*MCAR*), we will lose power because of smaller sample sizes, but we will not have problems with biased estimates.
- If data are missing not at random (*MNAR*), the **only way to obtain an unbiased estimate of parameters is to model missingness**. In other words we need to use a model that accounts for the missing data.
- Broadly speaking, such models are:
  - Censored Regression Models (e.g. duration analysis)
  - Truncated Regression Models
  - Sample Selection Correction models (Heckit)
  - ...