# Praktikum z ekonometrie

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# Block 1 – Missing data – Outline

# The nature of missing data

Missing completely at random ($MCAR$)

- The probability that an observation $X_i$ is missing is unrelated to the value of $X_i$ or to the value of any other variables.
- Any piece of data is equally likely to be missing.
- Analyses based on data with $MCAR$ observations remain unbiased. We may lose power (increased standard errors), but the estimated parameters are not biased by the absence of data.

Missing at random ($MAR$)

- Data meets the requirement that missingness does not depend on the value of $X_i$ after controlling for another variable in our analysis.
- For example, data are MCAR in a specific (demographic) subgroup.

Missing Not at Random ($MNAR$)

- Missigness of $X_i$ depends on its value (e.g. income in surveys)
- The only way to obtain an unbiased estimates of (regression) parameters is to model the missingness.

# Traditional treatment of missing data

**Listwise deletion (complete cases analysis)**

- We omit all rows with missing data – missing information for at least one variable in the $i$-th individual observation. Then, we run our analyses on the observations that remain. This often results in a substantial decrease in sample size. Under the assumption that data are missing completely at random, LRMs lead to unbiased parameter estimates – still, we lose power due to exclusion of (potentially large number of) observations.

R code

```
newData <- data[complete.cases(data)==T, ]
# data is a data.frame
# or
newData <- na.omit(data)
```

# Traditional treatment of missing data

**Hot deck imputation**

- Historically used by the US Census Bureau (since 1950's). Respondent's missing data were replaced by observed replacement data – drawn at random from a group of similar participants. Suitable, given only a few missing observations need to be replaced and given the draw is random.

# Traditional treatment of missing data

**Mean substitution**

- ✓ Simple
- ✗ In simple linear regression models (SLRMs), this adds no new information but increases sample size – that leads to underestimated standard errors only.

**Example:** Data on salary and citation level of publications. 62 cases with complete data and 7 cases for which the citation index was missing. Correlations and regression coefficients were compared as follows:

| Analysis | $n$ | $corr$ | $\widehat{\beta}_1$ | $s.e.(\widehat{\beta}_1)$ |
|---|---|---|---|---|
| Complete cases only | 62 | .55 | 310.747 | 60.95 |
| With mean substitution | 69 | .54 | 310.747 | 59.12 |

**Mean substitution, contnd.**

- Mean imputation can be usefull for multiple linear regression models, especially when data are missing as MCAR.
- It is fast, simple, easy to implement, and no cases are excluded.
- Even under MCAR, this method still leads to underestimation of coefficient variance.
- Bias in variance estimation is proportional to $(\text{nobs} - 1)/(\text{nobs} + \text{nmis} - 1)$.

  Smaller standard errors increase the possibility of Type I error (rejecting true null hypothesis).

# More advanced treatment of missing data

**Regression substitution**

- Uses linear regression (auxiliary LRM) to predict what the missing values of regressors should be – on the basis of other variables that are present.
- May be useful for MLRMs.
- For SLRMs, this approach would be equivalent to mean substitution. We do not add more information but we increase the sample size and (spuriously) reduce standard errors.

**Stochastic regression substitution**

- Build on regression substitution: this approach adds a randomly sampled residual term from the normal (or other) distribution to each value estimated by regression substitution. Adding a bit of random error to each substitution reduces, but does not eliminate, the problem of spurious reduction of the standard errors.

# More advanced treatment of missing data

**Maximum Likelihood Expectation-Maximization**

- Maximum likelihood approach – alternative to OLS – for the estimation of missing values.

  Many approaches exist (e.g. the Expectation-Maximization algorithm)

  `https://www.uvm.edu/~dhowell/StatPages/Missing_Data/Missing-Part-Two.html`

**Predictive mean matching (PMM)**

- Discussed within the Multiple imputation section (next)

# Multiple imputation for CS data

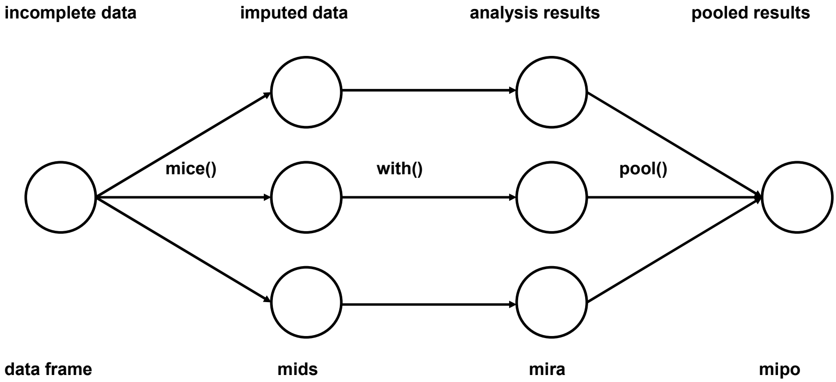
**Multiple Imputation (MI)**
`R`: {`mice`}, {`mi`}, {`Amelia`}, ...

MI motivation and algorithm

- Use PMM to create several (say, 5) – imputed values for each missing item – regressor $x_{ij}$.
- Each version of imputed data is organized into a separate data set and used for estimation (OLS, ML or other adequate approach).
- Information obtained from all estimates is conveniently summarized.

# Multiple imputation (MI)

Multiple imputation scheme (example with $m = 3$ imputations):




{`mice`} object–types used for MI.

# Multiple imputation (MI)

**Multiple imputation - 7 choices to be made**

1. Decide on MAR assumption plausibility (MAR/MNAR).
2. Imputation model choice (univariate, multivariate, data type).
3. Choice of predictors for MI.
4. Should we impute variables that are functions of incomplete variables (e.g. interaction terms)?
5. If data is missing in more than one variable, ordering for imputation can affect results.
6. MI is based on a numerical algorithm (say, PMM): we need to choose starting setup ("proximity" conditions and possibly other hyper-parameters, say number of iterations).
7. We need to choose $m$ – the number of imputed datasets.

In `R` ({mice}), most of the choices have generally valid default setting.

However, the choices are made & affect the resulting imputations.

**Predictive mean matching (`pmm`) in `R`** – general description

- Implemented in {mice} and other packages.
- General purpose semi-parametric imputation method.
- Suitable especially for imputing quantitative variables that are not normally distributed.
- Imputations are restricted to previously observed values.
- Can preserve non-linear relations even if the structural part of the imputation model is wrong.

# Multiple imputation (MI)

**Predictive mean matching (`pmm`) in `R`** – algorithm

Suppose there is a single variable $x$ that has some cases with missing data, and a set of relevant variables $\boldsymbol{z}$ with no missing data:

$$\{x_i, z_{i1}, \ldots, z_{iL}\};\ i = 1, \ldots, n$$

1. For cases with no missing $x_i$ data, estimate LRM $x_i \leftarrow \boldsymbol{z}_i$, producing $\hat{\boldsymbol{\beta}}$ and $\mathrm{var}(\hat{\boldsymbol{\beta}})$ estimates.
2. Make a random draw from the "posterior predictive distribution" of $\hat{\boldsymbol{\beta}}$, producing a new set of coefficients $\hat{\boldsymbol{\beta}}^*$.
    - Random draw from a multivariate normal distribution with mean $\hat{\boldsymbol{\beta}}$ and cov. matrix $\mathrm{var}(\hat{\boldsymbol{\beta}})$.

      Other distributions can be used, upon data and model used.
    - This step is necessary to produce sufficient variability in the imputed values, and is common to all "proper" methods of MI.

# Multiple imputation (MI)

**Predictive mean matching (`pmm`) in `R`** – algorithm contnd.

3. Using $(\boldsymbol{z}_i \hat{\boldsymbol{\beta}}^*)$ , generate predicted values $\hat{x}_i$ for **all cases**, both with data missing on $x_i$ and with data observations present.
4. For each case with missing $x_i$, take the fitted $\hat{x}_i$ and search for a set of similar "closely matching" $\hat{x}_j$ predictions (we are only interested in $j$ cases where $x_j$ is observed).
   - For "close" values, proximity rules are defined separately.
5. From each missing $x_i$, randomly choose one $x_j$ (observation) from the set of "close" observations and use its observed value to substitute for the missing value ($x_j$ observation is imputed, NOT the $\hat{x}_j$ value).
6. For MI, repeat steps 2 through 5 $m$-times to produce $m$ imputed datasets.

**Predictive mean matching (`pmm`) in `R`** – recap.

- Compared with regression-based methods, PMM produces imputed values that are much more like real values.
    - If the original variable is skewed, imputed values will also be skewed.
    - If the original variable is bounded by 0 and 100, imputed values will also be bounded by 0 and 100.
    - If the real values are discrete (say, number of children), imputed values will also be discrete.
- Generally speaking, there's no mathematical proof/theory to "justify" PMM (efficiency).
- PMM efficiency can be demonstrated by Monte Carlo simulations.

# Multiple imputation (MI)

## Multiple Imputation (empirical output example)

**Regression coefficients from five imputed data sets**

| Data set | Estimated parameter | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|---|---|
| 1 | Coefficient | -11.535 | -2.780 | 1.029 | -.031 | -0.359 | 0.572 |
| | Variance | 43.204 | 3.323 | 0.013 | 0.013 | 0.013 | 0.012 |
| 2 | Coefficient | -11.501 | -4.149 | 1.040 | -0.093 | -0.583 | 0.876 |
| | Variance | 40.488 | 2.680 | 0.010 | 0.009 | 0.009 | 0.007 |
| 3 | Coefficient | -10.141 | -5.038 | 0.766 | 0.123 | -0.252 | 0.625 |
| | Variance. | 42.055 | 3.301 | 0.010 | 0.010 | 0.010 | 0.009 |
| 4 | Coefficient | -11.533 | -6.920 | 0.870 | 0.084 | -0.458 | 0.815 |
| | Variance | 28.751 | 1.796 | 0.081 | 0.007 | 0.007 | 0.007 |
| 5 | Coefficient | -14.586 | -1.115 | 0.718 | 0.050 | -0.373 | 0.814 |
| | Variance | 32.856 | 2.362 | 0.009 | 0.009 | 0.009 | 0.008 |
| | Mean $b_i$ | -11.859 | -4.000 | 0.885 | 0.027 | -0.405 | 0.740 |
| | Mean Var.($\bar{W}$) | 37.471 | 2.692 | 0.025 | 0.010 | 0.010 | 0.009 |
| | Var. of $b_i$ (B) | 2.682 | 4.859 | 0.022 | 0.008 | 0.015 | 0.018 |
| | T | 40.69 | 8.523 | 0.051 | 0.020 | 0.028 | 0.031 |
| | $\sqrt{T}$ | 6.379 | 2.919 | 0.226 | 0.141 | 0.167 | 0.176 |
| | $t$ | -1.859 | -1.370 | 3.916* | 0.191 | 2.425* | 4.204* |

* $p < .05$ "Var." refers to the squared standard error of the coefficient.

https://www.uvm.edu/~dhowell/StatPages/Missing_Data/Missing-Part-Two.html

# Imputation for TS data

- Univariate TS imputation
  - `R` packages `imputeTS`, `zoo`, etc.
  - LOCF, linear & spline interpolation, Kalman filter, ...
- Multivariate TS imputation
  - `R` package `Amelia`
  - Use time trend (and polynomes), leads, lags, priors, ...

# Missing dependent variable data

**Special considerations apply to missing dependent variable data**

- If we can assume that data are missing completely at random *(MCAR)*, we will lose power because of smaller sample sizes, but we will not have problems with biased estimates.
- If data are missing not at random *(MNAR)*, the **only way to obtain an unbiased estimate of parameters is to model missingness**. In other words, we need to use a model that accounts for the missing data.
- Broadly speaking, such models are:
    - Censored Regression Models (e.g. duration analysis)
    - Truncated Regression Models
    - Sample Selection Correction models (Heckit)
    - ...