Assignment 3

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Download all python codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment3/codes

and latex-tikz codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment3

0.1 Problem

(Section 3.10) 60. Solve the system of linear equations using matrix method.

$$4x - 3y = 3 \tag{0.1.1}$$
$$3x - 5y = 7 \tag{0.1.2}$$

0.2 Explanation

The solution of linear equations can be obtained using matrix method as follow:

1) Write both the equations in matrix form.
$$\begin{pmatrix} a & b \end{pmatrix} \mathbf{x} = c \tag{0.2.1}$$
2) Form the Augmented matrix $(A|B)$.
3) Reduce the augmented matrix to row echelon form.
4) If $Rank(A) = Rank(A|B)$, then the system is said to be consistent. Further, there exist a unique solution if $Rank(A) = n$ (number of unknown) or infinite number of solutions if $Rank(A) < n$.
5) If $Rank(A) \neq Rank(A|B)$, then the system is said to be inconsistent and no solution exists for the linear equations.

0.3 Solution

Writing both euations in matrix form:

$$\begin{pmatrix} 4 & -3 \end{pmatrix} \mathbf{x} = 3 \tag{0.3.1}$$
$$\begin{pmatrix} 3 & -5 \end{pmatrix} \mathbf{x} = 7 \tag{0.3.2}$$

Forming the augmented matrix and reducing the matrix to row echelon form:

$$\begin{pmatrix} 4 & -3 & 3 \\ 3 & -5 & 7 \end{pmatrix} \tag{0.3.3}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/4} \begin{pmatrix} 1 & -3/4 & 3/4 \\ 3 & -5 & 7 \end{pmatrix} \tag{0.3.4}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -3/4 & 3/4 \\ 0 & -11/4 & 19/4 \end{pmatrix} \tag{0.3.5}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 \times -4/11} \begin{pmatrix} 1 & -3/4 & 3/4 \\ 0 & 1 & -19/11 \end{pmatrix} \tag{0.3.6}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 + 3/4 \times R_2} \begin{pmatrix} 1 & 0 & -6/11 \\ 0 & 1 & -19/11 \end{pmatrix} \tag{0.3.7}$$

Here, $Rank(A) = Rank(A|B)$. Therefore, the system is consistent. Also, there exist a unique solution as $Rank(A) = n$ (number of unknown).
From equation 0.3.7, we get:

$$x = \frac{-6}{11} \tag{0.3.8}$$
$$y = \frac{-19}{11} \tag{0.3.9}$$

Plotting the lines and the intersection point:

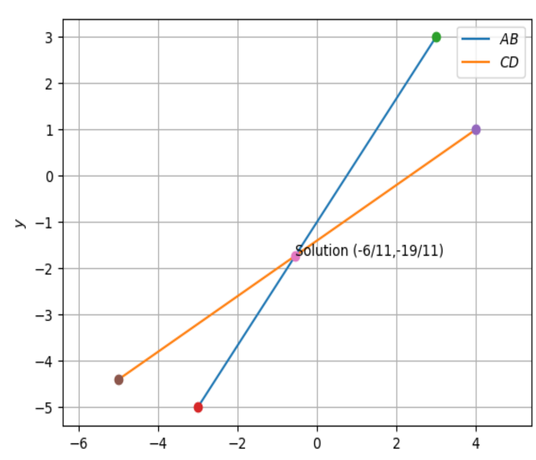

Fig. 5: Lines and their intersection denoting the solution