Probability and Statistics

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Contents

***Abstract*—This book provides a computational approach to probability and statistics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/probability/codes
```

1 Probability

1.1 Examples

1. A coin is tossed 1000 times with the following frequencies:
   Head : 455, Tail : 545
   Compute the probability for each event.
   Solution: Let $X \in \{0, 1\}$ represent the random variable, where 0 represents head and 1 represents tail. From the given information,

$$\Pr(X = 0) = \frac{455}{1000} \tag{1.1.1.1}$$
$$= 0.45 \tag{1.1.1.2}$$
$$\Pr(X = 1) = 1 - \Pr(X = 0) \tag{1.1.1.3}$$
$$= 0.545 \tag{1.1.1.4}$$

   Codes for the above are available in

```
solutions/1−10/codes/probexm/probexm1.py
```

2. Two coins are tossed simultaneously 500 times, and we get
   Two heads : 105 times
   One head : 275 times
   No head : 120 times
   Find the probability of occurrence of each of these events.
   Solution: Let $X_1 \{0, 1\}$ represent the first coin and $X_2 \{0, 1\}$ represent the second coin, where 0 represents tail and 1 represents head. Define

$$X = X_1 + X_2, \tag{1.1.2.1}$$

   Hence $X \in \{0, 1, 2\}$. From the given informa-

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tion,

$$\Pr(X=1) = \frac{105}{500} \tag{1.1.2.2}$$
$$= 0.21 \tag{1.1.2.3}$$
$$\Pr(X=2) = \frac{275}{500} \tag{1.1.2.4}$$
$$= 0.55 \tag{1.1.2.5}$$
$$\Pr(X=0) = \frac{120}{500} \tag{1.1.2.6}$$
$$= 0.24 \tag{1.1.2.7}$$

3. A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following Table 1.1.3. Find the probability of getting each outcome.

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

TABLE 1.1.3

Solution: Let $X \in \{i\}_{i=1}^{6}$ and f_i be the correspnding frequency. Then,

$$\Pr(X=i) = \frac{f_i}{1000} \tag{1.1.3.1}$$

The following code computes the probabilities

solutions/1−10/codes/probexm/probexm3.py

4. On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table 1.1.4 below

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

TABLE 1.1.4

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at random. What is the probability that the digit in its unit place is 6?
Solution:

$$P_r(X=i) = \frac{f_i}{200} \tag{1.1.4.1}$$

From table **??**

$$P_r(X=6) = \frac{14}{200} \tag{1.1.4.2}$$
$$= 0.07 \tag{1.1.4.3}$$

The outputs of Python program are attached below: The **Law Of Large Numbers** is a

TABLE 2: For 200 randomly generated numbers

Digit	**Frequency**	**Probability**
0	21	0.105
1	13	0.065
2	20	0.1
3	21	0.105
4	20	0.1
5	25	0.125
6	15	0.075
7	24	0.12
8	20	0.1
9	21	0.105

TABLE 3: For 10000 randomly generated numbers

Digit	**Frequency**	**Probability**
0	1007	0.1007
1	988	0.0988
2	997	0.0997
3	1010	0.101
4	1005	0.1005
5	1018	0.1018
6	1000	0.1
7	984	0.0984
8	1019	0.1019
9	972	0.0972

fundamental concept for probability and statistics. It states that that as the number of trials increase, the experimental probability will get closer and closer to the theoretical probability. From the output tables 2 and 3, we can deduce that as the number of trials increase, the ratio of the number of successful occurrences to the number of trials will tend to approach the theoretical probability of the outcome for an individual trial. Since all the digits are equiprobable, ideally each probability should be 1/10=0.1 In Table 3, when number of trials are 10,000, probability of each digit is approximately 0.1 with very little deviation. eg. 0.1005.
With 200 samples, Tables 2 and 3 are slightly different because the number of simulations is

not sufficient for convergence in the probabilities.

5. The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.
(i) What is the probability that on a given day it was correct?
(ii) What is the probability that it was not correct on a given day?
Solution: Let $X \in \{0, 1\}$ be the random variable with 1 denoting correct forecast. From the given information,

$$\Pr(X = 1) = \frac{175}{250} \tag{1.1.5.1}$$
$$= 0.7 \tag{1.1.5.2}$$
$$\Pr(X = 0) = 1 - \Pr(X = 1) \tag{1.1.5.3}$$
$$= 0.3 \tag{1.1.5.4}$$

6. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. Table 1.1.6 shows the results of 1000 cases. If you buy a tyre of this company,

Distance(in km)	> 4000	4000-9000	9001-14000	<14000
Frequency	20	210	325	445

TABLE 1.1.6

what is the probability that :
(i) it will need to be replaced before it has covered 4000 km?
(ii) it will last more than 9000 km?
(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?
Solution: From the given information,

a)

$$\Pr(X > 9000) = \frac{325 + 445}{1000} \tag{1.1.6.1}$$
$$= 0.77 \tag{1.1.6.2}$$

b)

$$\Pr(4000 < X < 14000) = \frac{20 + 210 + 325}{1000} \tag{1.1.6.3}$$
$$= 0.0.555 \tag{1.1.6.4}$$

c)

$$\Pr(X < 4000) = \frac{20}{1000} \tag{1.1.6.5}$$
$$= 0.02 \tag{1.1.6.6}$$

Related codes are available in

solutions/1−10/codes/probexm/probexm6.py

7. The percentage of marks obtained by a student in the monthly unit tests are given in Table 1.1.7 below. Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Unit test	I	II	III	IV	V
Frequency	69	71	73	68	74

TABLE 1.1.7

Solution: From the given information,

$$\Pr(X > 70) = \frac{3}{5} \tag{1.1.7.1}$$
$$= 0.6 \tag{1.1.7.2}$$

8. An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the Table 1.1.8. Find the probabilities of the following events for a driver chosen at random from the city:
(i) being 18-29 years of age *and* having exactly 3 accidents in one year.
(ii) being 30-50 years of age *and* having one or more accidents in a year.
(iii) having no accidents in one year.

Age of drivers	**Accidents in one year**				
(in years)	**0**	**1**	**2**	**3**	**over 3**
18-29	440	160	110	61	35
30-50	505	125	60	22	18
Above 50	360	45	35	15	9

TABLE 1.1.8

Solution: Let $X \in 1, 2, 3$ represent the random variable representing the age groups of the drivers. Let Y represent the accidents

a) Then,

$$\Pr(X = 1, Y = 3) = \frac{61}{2000} \tag{1.1.8.1}$$
$$= 0.03 \tag{1.1.8.2}$$

b)

$$\Pr(X = 2, Y \geq 1) = \frac{125 + 60 + 22 + 18}{2000} \tag{1.1.8.3}$$

c)

$$\Pr(Y = 0) = \frac{440 + 505 + 360}{2000} \tag{1.1.8.4}$$
$$= 0.65 \tag{1.1.8.5}$$

Related code is available in

solutions/1−10/codes/probexm/probexm8.py

9. Consider the frequency distribution in Table **??** below which gives the weights of 38 students of a class. (i) Find the probability that the weight of a student in the class lies in the interval 46-50 kg.
(ii) Give two events in this context, one having probability 0 and the other having probability 1.

Weights (in kg)	**Number of students**
31-35	9
36-40	5
41-45	14
46-50	3
51-55	1
56-60	2
61-65	2
66-70	1
71-75	1
Total	38

TABLE 1.1.9

Solution:

a) From the given information,

$$\Pr(46 < X < 50) = \frac{3}{38} \tag{1.1.9.1}$$
$$= 0.079 \tag{1.1.9.2}$$

b) There is no student whose weight is less than 31 kg thus the probability of a student to have the weight less than 31 kg = 0

All of the student in this context have the weight between 31-75 so we can say that the probability of the students to have the weight in the range 31-75 = 1

10. Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded in Table 1.1.10
What is the probability of germination of
(i)more than 40 seeds in a bag?
(ii) 49 seeds in a bag?
(iii) more that 35 seeds in a bag?

Bag	1	2	3	4	5
No.of seeds germinated	40	48	42	39	41

TABLE 1.1.10

Solution: Let X represent the seeds and Y represent the bags.

a)

$$\Pr(X > 40) = \frac{3}{5} \tag{1.1.10.1}$$
$$= 0.6 \tag{1.1.10.2}$$

b)

$$\Pr(X = 49) = \frac{0}{5} \tag{1.1.10.3}$$
$$= 0 \tag{1.1.10.4}$$

c)

$$\Pr(X > 35) = \frac{5}{5} \tag{1.1.10.5}$$
$$= 1 \tag{1.1.10.6}$$

Related code is available in

solutions/1−10/codes/probexm/probexm10.py

11. If P(A)=$\frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, Evaluate P(A/B)?

12. A family has two children. What is the probability that both the children are boys

given that at least one of them is a boy?

13. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

14. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10 percentage of the girls study in class XII. What is the probability that a student chosen randomly studies in Class XII given that the chosen student is a girl?

15. A die is thrown three times. Events A and B are defined as below:
A : 4 on the third throw.
B : 6 on the first and 5 on the second throw.
Find the probability of A given that B has already occurred?

16. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

17. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that ”the die shows a number greater than 4” given that ”there is at least one tail”.

18. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

19. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

20. A die is thrown. If E is the event ”the number appearing is a multiple of 3” and F be the event ”the number appearing is even” then find whether E and F are independent ?

21. An unbiased die is thrown twice. Let the event A be ”odd number on the first throw” and B the event ”odd number on the second throw”. Check the independence of the events A and B.
Solution: Events A and B are independent.
22. Three coins are tossed simultaneously. Consider the event E ”three heads or three tails”, F ”at least two heads” and G ”at most two heads”. Of the pairs (E,F), (E,G) and (F,G), which are independent? which are dependent?
Solution: Let $X_i \in \{0, 1\}$ represent the toss of each coin, with 1 being a head Let

$$X = X_1 + X_2 + X_3 \tag{1.1.22.1}$$

Then,

$$\Pr(E) = \Pr(\{X = 3\} + \{X = 0\}) \tag{1.1.22.2}$$
$$= \Pr(X = 3) + \Pr(X = 0) \tag{1.1.22.3}$$
$$= {}^3C_3\left(\frac{1}{2}\right)^3 + {}^3C_0\left(\frac{1}{2}\right)^3 \tag{1.1.22.4}$$
$$= \frac{1}{4} \tag{1.1.22.5}$$

$$\Pr(F) = \Pr(X \geq 2) \tag{1.1.22.6}$$
$$= {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_3\left(\frac{1}{2}\right)^3 \tag{1.1.22.7}$$
$$= \frac{1}{2} \tag{1.1.22.8}$$

$$\Pr(G) = \Pr(X \leq 2) \tag{1.1.22.9}$$
$$= 1 - \Pr(X > 2) \tag{1.1.22.10}$$
$$= 1 - {}^3C_3\left(\frac{1}{2}\right)^3 \tag{1.1.22.11}$$
$$= \frac{7}{8} \tag{1.1.22.12}$$

Now,

$$\Pr(EF) = \Pr([\{X = 3\} + \{X = 0\}]\{X \geq 2\}) \tag{1.1.22.13}$$
$$= \Pr(\{X = 3\}\{X \geq 2\} \tag{1.1.22.14}$$
$$+ \{X = 0\}\{X \geq 2\}) \tag{1.1.22.15}$$
$$= \Pr(X = 3) = \frac{1}{8} \tag{1.1.22.16}$$

Similarly,

$$\Pr(EG) = \Pr([\{X=3\}+\{X=0\}]\{X \le 2\}) \tag{1.1.22.17}$$
$$= \Pr(\{X=3\}\{X \le 2\} \tag{1.1.22.18}$$
$$+ \{X=0\}\{X \le 2\}) \tag{1.1.22.19}$$
$$= \Pr(X=0) = \frac{1}{8} \tag{1.1.22.20}$$

and

$$\Pr(FG) = \Pr(\{X \ge 2\}\{X \le 2\}) \tag{1.1.22.21}$$
$$= \Pr(\{X=2\}) \tag{1.1.22.22}$$
$$= {}^3C_2\left(\frac{1}{2}\right)^3 = \frac{3}{8} \tag{1.1.22.23}$$

From the above equations we see that

$$P(EF) = P(E)P(F) \tag{1.1.22.24}$$
$$P(GF) \neq P(G)P(F) \tag{1.1.22.25}$$
$$P(EG) \neq P(E)P(G) \tag{1.1.22.26}$$

Hence only the pair (E,F) are independent events. The pairs (F,G) and (G,E) are dependent events.

23. Prove that if E and F are independent events, then so are the events E and $F^{'}$.
Solution: From the given information,

$$\Pr(EF) = \Pr(E)\Pr(F) \tag{1.1.23.1}$$

Then,

$$\Pr(EF') = \Pr(E(1-F)) = \Pr(E-EF) \tag{1.1.23.2}$$
$$= \Pr(E) - \Pr(E \cap F) \tag{1.1.23.3}$$
$$= \Pr(E) - \Pr(E)\Pr(F) \tag{1.1.23.4}$$
$$= \Pr(E)(1-\Pr(F)) \tag{1.1.23.5}$$
$$= \Pr(E)\Pr(1-F) \tag{1.1.23.6}$$
$$= \Pr(E)\Pr(F') \tag{1.1.23.7}$$

$\therefore E$ and F' are independent events.

24. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1- $P(A^{'})P(B^{'})$
Solution: From the given information, using the fact that A, B are independent,

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{1.1.24.1}$$
$$= \Pr(A) + \Pr(B - AB) \tag{1.1.24.2}$$
$$= \Pr(A) + \Pr(A'B) \tag{1.1.24.3}$$
$$= \Pr(A) + \Pr(A')\Pr(B) \tag{1.1.24.4}$$
$$= \Pr(A) + \Pr(A')(1-\Pr(B')) \tag{1.1.24.5}$$
$$= \Pr(A) + \Pr(A') - \Pr(A')\Pr(B') \tag{1.1.24.6}$$
$$= 1 - \Pr(A')\Pr(B') \tag{1.1.24.7}$$

25. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.
Solution: Let S denote strike and J denote job. From the given information,

$$\Pr(S) = 0.65, \Pr(J|S') = 0.8, \Pr(J|S) = 0.32 \tag{1.1.25.1}$$

Then,

$$\Pr(J) = \Pr(JS) + \Pr(JS') \tag{1.1.25.2}$$
$$= \Pr(J|S)\Pr(S) + \Pr(J|S')\Pr(S') \tag{1.1.25.3}$$
$$= \Pr(J|S)\Pr(S) + \Pr(J|S')(1-\Pr(S)) \tag{1.1.25.4}$$
$$= (0.65)(0.32) + (0.35)(0.80) = 0.488 \tag{1.1.25.5}$$

26. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.
Solution: Let $X \in 1, 2$ represent the Bag and $Y \in \{0, 1\}$ represent the colour, where 1 denotes

red. From the given information,

$$\Pr(X = 1) = \Pr(X = 2) = \frac{1}{2} \quad (1.1.26.1)$$

$$\Pr(Y = 1|X = 1) = \frac{3}{7} \quad (1.1.26.2)$$

$$\Pr(Y = 1|X = 2) = \frac{5}{11} \quad (1.1.26.3)$$

Thus,

$$\Pr(X = 2|Y = 1) = \frac{\Pr(X = 2, Y = 1)}{\Pr(Y = 1)} \quad (1.1.26.4)$$

$$= \frac{\Pr(Y = 1|X = 2)\Pr(X = 2)}{\Pr(Y = 1|X = 1)\Pr(X = 1) + \Pr(Y = 1|X = 2)\Pr(X = 2)} \quad (1.1.26.5)$$

$$= \frac{\frac{5}{11} \times \frac{1}{2}}{\frac{3}{7} \times \frac{1}{2} + \frac{5}{11} \times \frac{1}{2}} \quad (1.1.26.6)$$

$$= \frac{35}{68} \quad (1.1.26.7)$$

27. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?
Solution: Let $X \in \{1, 2, 3\}$ represent the box and $Y_1, Y_2 \in \{0, 1\}$ represent the coins, 1 representing gold. Then,

$$\Pr(X = 1) = \Pr(X = 2) = \Pr(X = 3) = \frac{1}{3} \quad (1.1.27.1)$$

$$\Pr(Y_1 = 1, Y_2 = 1|X = 1) = 1, \quad (1.1.27.2)$$

$$\Pr(Y_1 = 1, Y_2 = 1|X = 2) = 0 \quad (1.1.27.3)$$

$$\Pr(Y = 1, Y_2 = 0|X = 3)$$
$$= \Pr(Y_1 = 1, Y_2 = 0|X = 3)$$
$$= \frac{1}{2} \quad (1.1.27.4)$$

Then

$$\Pr(Y_1 = 1|Y_2 = 1) = \frac{\Pr(Y_1 = 1, Y_2 = 1)}{\Pr(Y_2 = 1)} \quad (1.1.27.5)$$

Now,

$$\Pr(Y_1 = 1, Y_2 = 1)$$
$$= \sum_i \Pr(Y_1 = 1, Y_2 = 1, X = i)$$
$$= \sum_i \Pr(Y_1 = 1, Y_2 = 1|X = i)\Pr(X = i) = \frac{1}{3} \quad (1.1.27.6)$$

and

$$\Pr(Y_2 = 1)$$
$$= \Pr(Y_1 = 1, Y_2 = 1) + \Pr(Y_1 = 0, Y_2 = 1)$$
$$= \sum_i \Pr(Y_1 = 1, Y_2 = 1|X = i)\Pr(X = i)$$
$$+ \sum_i \Pr(Y_1 = 0, Y_2 = 1|X = i)\Pr(X = i)$$
$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \quad (1.1.27.7)$$

Substituting from (1.1.27.6) and (1.1.27.7) in (1.1.27.5),

$$\Pr(Y_1 = 1|Y_2 = 1) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \quad (1.1.27.8)$$

28. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?
Solution: Let $X, Y \in \{0, 1\}$ represent HIV with 1 being positive. From the given information,

$$\Pr(X = 1|Y = 1) = \frac{9}{10}, \Pr(X = 0|Y = 1) = \frac{1}{10} \quad (1.1.28.1)$$

$$\Pr(X = 1|Y = 0) = \frac{1}{100}, \Pr(X = 0|Y = 0) = \frac{99}{100} \quad (1.1.28.2)$$

$$\Pr(Y = 1) = \frac{1}{1000}, \Pr(Y = 0) = \frac{999}{1000} \quad (1.1.28.3)$$

Then,

$$\Pr(Y = 1|X = 1)$$
$$= \frac{\Pr(X = 1|Y = 1)\Pr(Y = 1)}{\Pr(X = 1|Y = 1)\Pr(Y = 1) + \Pr(X = 1|Y = 0)\Pr(Y = 0)}$$
$$= \frac{\frac{9}{10} \times \frac{1}{1000}}{\frac{9}{10} \times \frac{1}{1000} + \frac{1}{100} \times \frac{999}{1000}} = \frac{10}{121} \quad (1.1.28.4)$$

29. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?
Solution: Let $X \in \{1, 2, 3\}$ represent the machines and $Y \in \{0, 1\}$ represent the bolt quality, 0 denoting defective bolt. From the given information,

$$\Pr(X = 1) = \frac{25}{100} \quad (1.1.29.1)$$
$$\Pr(X = 2) = \frac{35}{100} \quad (1.1.29.2)$$
$$\Pr(X = 3) = \frac{40}{100} \quad (1.1.29.3)$$

and

$$\Pr(Y = 0|X = 1) = \frac{5}{100} \quad (1.1.29.4)$$
$$\Pr(Y = 0|X = 2) = \frac{4}{100} \quad (1.1.29.5)$$
$$\Pr(Y = 0|X = 3) = \frac{2}{100} \quad (1.1.29.6)$$

Then,

$$\Pr(X = 2|Y = 0)$$
$$= \frac{\Pr(Y = 0|X = 2)\Pr(X = 2)}{\Pr(Y = 0|X = 1)\Pr(X = 1) + \Pr(Y = 0|X = 2)\Pr(X = 2) + \Pr(Y = 0|X = 3)\Pr(X = 3)}$$
$$= \frac{\frac{4}{100} \times \frac{35}{100}}{\frac{5}{100} \times \frac{25}{100} + \frac{4}{100} \times \frac{35}{100} + \frac{2}{100} \times \frac{40}{100}}$$
$$= \frac{28}{69} \quad (1.1.29.7)$$

30. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?
Solution: Let $X \in \{0, 1, 2, 3\}$ represent the mode of travel and $Y \in \{0, 1\}$ represent the time, where 0 denotes being late. From the given information,

$$\Pr(X = 1) = \frac{3}{10} \quad (1.1.30.1)$$
$$\Pr(X = 2) = \frac{1}{5} \quad (1.1.30.2)$$
$$\Pr(X = 3) = \frac{1}{10} \quad (1.1.30.3)$$
$$\Pr(X = 4) = \frac{2}{5} \quad (1.1.30.4)$$

and

$$\Pr(Y = 0|X = 1) = \frac{1}{4} \quad (1.1.30.5)$$
$$\Pr(Y = 0|X = 2) = \frac{1}{3} \quad (1.1.30.6)$$
$$\Pr(Y = 0|X = 3) = \frac{1}{12} \quad (1.1.30.7)$$
$$\Pr(Y = 0|X = 4) = 0 \quad (1.1.30.8)$$

Then,

$$\Pr(X = 1|Y = 0)$$
$$= \frac{\Pr(Y = 0|X = 1)\Pr(X = 1)}{\sum_{i=1}^{4}\Pr(Y = 0|X = i)\Pr(X = i)}$$
$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{1}{4} \times \frac{3}{10} + \frac{1}{3} \times \frac{1}{5} + \frac{1}{12} \times \frac{1}{10}} = \frac{1}{2} \quad (1.1.30.9)$$

31. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

32. A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

33. A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

34. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

35. Find the probability distribution of number of doublets in three throws of a pair of dice?

36. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.
P(X=x)= $\begin{pmatrix} 0.1, if x = 0 \\ kx, if x = 1 or 2 \\ k(5-x), if x = 3 or 4 \\ 0, otherwise \end{pmatrix}$
 a) Find the value of k.
 b) What is the probability that you study at least two hours ? Exactly two hours? At most two hours?

37. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

38. Find the variance of the number obtained on a throw of an unbiased die.

39. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

40. Six balls are drawn successively from an urn containing 7 red and 9 black balls. Tell whether or not the trials of drawing balls are Bernoulli trials when after each draw the ball drawn is
(i) replaced
(ii) not replaced in the urn.

41. If a fair coin is tossed 10 times, find the probability of
 a) exactly six heads
 b) at least six heads
 c) at most six heads

Solution: Let X be the random variable denoting the number of times head is obtained when a coin is tossed n times. Then by Binomial distribution,

$$\Pr(X = 1) = p \quad (1.1.41.1)$$
$$\Pr(X = k) = {}^{n}C_{k} p^{k} (1 - p)^{n-k} \quad (1.1.41.2)$$
$$k = 0, \dots, n \quad (1.1.41.3)$$

For the given problem, $n = 10$ and $p = 1 - p = \frac{1}{2}$ for a fair coin
 a) From (1.1.41.3),

$$\Pr(X = 6) = {}^{10}C_{6} \left(\frac{1}{2}\right)^{10} = \frac{105}{512} \quad (1.1.41.4)$$

 b) Similarly,

$$\Pr(X \geq 6) = \sum_{k=6}^{10} \Pr(X = k) \quad (1.1.41.5)$$
$$= \sum_{k=6}^{10} {}^{10}C_{k} \left(\frac{1}{2}\right)^{10} \quad (1.1.41.6)$$
$$= \frac{193}{512} \quad (1.1.41.7)$$

 c)

$$\Pr(X \leq 6) = 1 - \Pr(X \geq 6) + \Pr(X = 6) \quad (1.1.41.8)$$
$$= 1 - \frac{193}{512} + \frac{105}{512} \quad (1.1.41.9)$$
$$= \frac{53}{64} \quad (1.1.41.10)$$

upon substituting (1.1.41.4) and (1.1.41.4),

The python code for the above problem is,

```
./solutions/20−10/prob/codes/exam41.py
```

Experimental probability is calculated using the number of heads obtained in each of the 1,000,000 random experiments of tossing of 10 coins. The code compares the experimental probability to the theoretical probability. As number of experiments increase, the experimental probability approaches the theoritical probability.

42. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

Solution: Let X be the random variable representing the number of defective eggs from the ten eggs picked. X follows a binomial distribution. Since the probability of an egg being defective is 10%, substituting n=10, p= 0.1 and k=0 in equation (1.1.41.3), probability that there is atleast one defective egg is

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - (0.9)^{10} \quad (1.1.42.1)$$
$$= 0.6513215599 \quad (1.1.42.2)$$

The python code for the above problem is,

.solutions/20−10/prob/codes/exam42.py

43. Coloured balls are distributed in four boxes as shown in Table 1.1.43

Box	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

TABLE 1.1.43: Distribution of the balls in the boxes

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III?
Solution: Let $B \in \{1, 2, 3, 4\}$ denote the box number in sequence and $C \in \{1, 2, 3, 4\}$ denote the colours Black, White, Red and Blue respectively. Given that a black ball is selected, the probability that it is picked from box III is

$$\Pr(B = 3|C = 1)$$
$$= \frac{\Pr(C = 1|B = 3)\Pr(B = 3)}{\sum_{j=1}^{4}\Pr(C = 1|B = j)\Pr(B = j)} \quad (1.1.43.1)$$

From Table 1.1.43,

$$\Pr(C = 1|B = 1) = \frac{1}{6} \quad (1.1.43.2)$$
$$\Pr(C = 1|B = 2) = \frac{1}{4} \quad (1.1.43.3)$$
$$\Pr(C = 1|B = 3) = \frac{1}{7} \quad (1.1.43.4)$$
$$\Pr(C = 1|B = 4) = \frac{4}{13} \quad (1.1.43.5)$$

and

$$\Pr(B = 1) = \Pr(B = 2)$$
$$= \Pr(B = 3) = \Pr(B = 4) = \frac{1}{4} \quad (1.1.43.6)$$

Substituting from (1.1.43.5) and (1.1.43.6) in (1.1.43.1),

$$\Pr(B = 3|C = 1) = \frac{156}{947} \quad (1.1.43.7)$$

The python code for the above problem is,

solutions/20−10/prob/codes/exam43.py

44. Find the mean of the Binomial distribution B(4,$\frac{1}{3}$).
Solution: For a Binomial distribution $X \sim B(n, p)$

$$\Pr(X = 1) = p, \quad (1.1.44.1)$$
$$\Pr(X = k) = {}^{n}C_{k}p^{k}(1 - p)^{n-k}, \quad k = 0, \ldots, n \quad (1.1.44.2)$$

The mean is given by

$$E[X] = \sum_{k=0}^{n} k \Pr(X = k) \quad (1.1.44.3)$$
$$= \sum_{k=0}^{n} k^{n}C_{k}p^{k}(1 - p)^{n-k} \quad (1.1.44.4)$$
$$= np \quad (1.1.44.5)$$

Here $p = \frac{1}{3}$ and $n = 4$. Hence

$$E[X] = \frac{4}{3} \quad (1.1.44.6)$$

The python code for the above problem is,

solutions/20−10/prob/codes/exam44.py

45. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?
Solution: Let X be the random variable representing the number of times the shooter hits the target. Let n be the total number of times that the shoter fires. Then from the given information,

$$X \sim B(n, p), p = \frac{3}{4} \quad (1.1.45.1)$$
$$\Pr(X \geq 1) \geq 0.99 \quad (1.1.45.2)$$

Then from (1.1.41.3) probability of hitting target atleast once is

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - {}^{n}C_0 \left(\frac{1}{4}\right)^n \tag{1.1.45.3}$$

$$\geq 0.99 \tag{1.1.45.4}$$

$$\implies 1 - \left(\frac{1}{4}\right)^n \geq 0.99 \tag{1.1.45.5}$$

$$\implies \left(\frac{1}{4}\right)^n \leq 0.01 \tag{1.1.45.6}$$

$$\text{or, } n = 4 \tag{1.1.45.7}$$

The python code for the above problem is,

```
solutions/20−10/prob/codes/exam45.py
```

46. *: Random Process*A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.
Solution: Let $X_k \in \{1, 2, 3, 4, 5, 6\}$ be the discrete random process representing the trials. Then, the odd trials belong to A and the even trials belong to B. Then, the probability that someone wins at the nth trial is

$$\Pr(X_n = 6 | X_k \neq 6, k = 1, 2, \ldots, n-1) = \frac{1}{6}\left(\frac{5}{6}\right)^{n-1}. \tag{1.1.46.1}$$

The probability that A wins is obtained by summing up over the even probabilities

$$\sum_{m=0}^{\infty} \Pr(X_{2m+1} = 6 | X_k \neq 6, k = 1, 2, \ldots, n-1) = \frac{1}{6}\sum_{m=0}^{\infty}\left(\frac{5}{6}\right)^{2m} = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11} \tag{1.1.46.2}$$

The probabilty that B wins is then given by

$$1 - \frac{6}{11} = \frac{5}{11} \tag{1.1.46.3}$$

The python code for the above problem is,

```
solutions/20−10/prob/codes/exam46.py
```

In the above code 1000000 random outputs of a die are generated for A and B each.The probabilities are calculated using the total number of times A gets a six first and the total number of times B get a six first.

47. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.
Solution: Let $X \in \{0, 1\}$ denote the setup. Let $Y_1, Y_2 \in \{0, 1\}$ denote the item production such that Y_1 and Y_2 are independent. Then, from the given information,

$$\Pr(Y_1 = 1 | X = 1) = \Pr(Y_2 = 1 | X = 1) \tag{1.1.47.1}$$

$$= \frac{90}{100} = \frac{9}{10} \tag{1.1.47.2}$$

$$\Pr(Y_1 = 1 | X = 0) = \Pr(Y_2 = 1 | X = 0) \tag{1.1.47.3}$$

$$= \frac{40}{10} = \frac{2}{5} \tag{1.1.47.4}$$

$$\Pr(X = 1) = \frac{80}{100} = \frac{4}{5} \tag{1.1.47.5}$$

Then

$$\Pr(X = 1 | Y_1 = 1, Y_2 = 1) = \frac{\Pr(X = 1, Y_1 = 1, Y_2 = 1)}{\Pr(Y_1 = 1, Y_2 = 1)}$$

$$= \frac{\Pr(Y_1 = 1, Y_2 = 1 | X = 1)\Pr(X = 1)}{\Pr(Y_1 = 1, Y_2 = 1 | X = 1)\Pr(X = 1) + \Pr(Y_1 = 1, Y_2 = 1 | X = 0)\Pr(X = 0)} \tag{1.1.47.6}$$

which can be expressed as

$$\frac{\prod_{k=1}^{2}\Pr(Y_k = 1 | X = 1)\Pr(X = 1)}{\sum_{i=0}^{1}\prod_{k=1}^{2}\Pr(Y_k = 1 | X = i)\Pr(X = i)} = \frac{\left(\frac{9}{10}\right)^2\left(\frac{4}{5}\right)}{\left(\frac{9}{10}\right)^2\left(\frac{4}{5}\right) + \left(\frac{2}{5}\right)^2\left(\frac{1}{5}\right)} = \frac{81}{85} \tag{1.1.47.7}$$

The python code for the above problem is,

```
solutions/20−10/prob/codes/exam47.py
```

48. Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.
Solution: Let the random variable be $X \in \{0, 1\}$. Then

$$\Pr(X = 0) = \Pr(X = 1) = \frac{1}{2} \tag{1.1.48.1}$$

solutions/20−10/prob/codes/exam48.py

49. A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size.Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the (i) yellow ball?
(ii) red ball?
(iii) blue ball?
Solution: Let the random variable representing the events be $X \in \{0, 1, 2\}$ Then

$$\Pr(X = i) = \frac{1}{3}, \quad i = 0, 1, 2. \tag{1.1.49.1}$$

The python code for the distribution is

solution/20−10/prob/codes/exam49.py

50. Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?
Solution: Let

$$X\epsilon\{1, 2, 3, 4, 5, 6\} \tag{1.1.50.1}$$

For a fair dice,

$$\Pr(X = k) = \begin{cases} \frac{1}{6} & k = 1, 2, 3, 4, 5, 6 \\ 0 & otherwise \end{cases} \tag{1.1.50.2}$$

a)

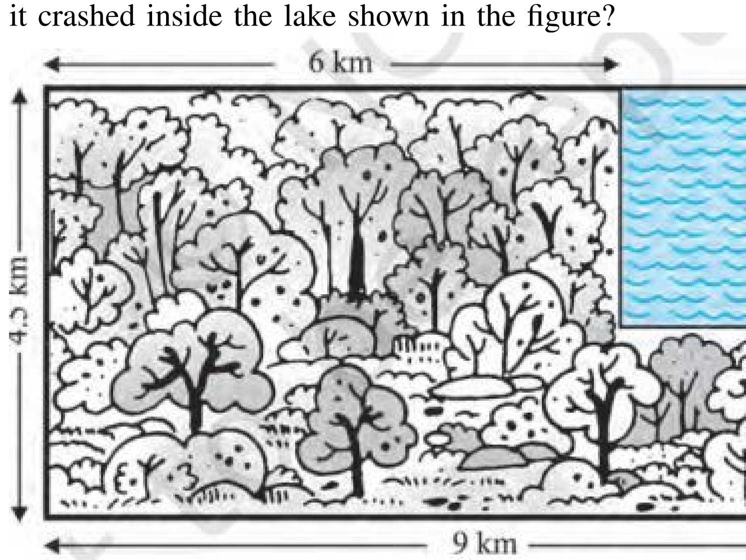
$$\Pr(X > 4) = \Pr(X = 5) + \Pr(X = 6) = \frac{1}{3} \tag{1.1.50.3}$$

b)

$$\Pr(X \leq 4) = 1 - \Pr(X > 4 =) = \frac{2}{3} \tag{1.1.50.4}$$

solutions/20−10/prob/codes/exam50.py

51. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will
(i) be an ace,
(ii) not be an ace.
52. Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?
53. Savita and Hamida are friends. What is the probability that both will have
(i) different birthdays?
(ii) the same birthday? (ignoring a leap year).
54. There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical.Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of
(i) a girl?
(ii) a boy?
55. A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be
(i) white? (ii) blue? (iii) red?
56. Harpreet tosses two different coins simultaneously (say, one is of rupee 1 and other of rupee 2). What is the probability that she gets at least one head?
57. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing.What is the probability that the music will stop within the first half-minute after starting?
58. A missing helicopter is reported to have crashed somewhere in the rectangular region shown in Fig. 15.2. What is the probability that it crashed inside the lake shown in the figure?



59. A carton consists of 100 shirts of which 88

are good, 8 have minor defects and 4 have major defects.Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects.One shirt is drawn at random from the carton. What is the probability that
(i) it is acceptable to Jimmy?
(ii) it is acceptable to Sujatha?

60. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes.What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) 8?
(ii) 13?
(iii) less than or equal to 12?

1.2 *Exercises*

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.
Solution: Let the sample space be $X \in \{0, 1\}$. From the given information, the probability of hitting a boundary is

$$\Pr(X = 1) = \frac{6}{30} \tag{1.2.1.1}$$
$$= \frac{1}{5} \tag{1.2.1.2}$$

Hence, the probability of not hitting the boundary is

$$\Pr(X = 0) = 1 - \Pr(X = 1) = 1 - \frac{1}{5} \tag{1.2.1.3}$$
$$= \frac{4}{5} \tag{1.2.1.4}$$

2. 1500 families with 2 children were selected randomly, and the following data in Table 1.2.2 were recorded. Compute the probability of a family, chosen at random, having
a) 2 girls
b) 1 girl
c) No girl
Also check whether the sum of these probabilities is 1.
Solution: Let X be the random variable representing the number of girls.

No.of girls in a family	2	1	0
No. of families	475	814	211

TABLE 1.2.2

a)

$$\Pr(X = 2) = \frac{475}{1500} \tag{1.2.2.1}$$
$$= 0.316 \tag{1.2.2.2}$$

b)

$$\Pr(X = 1) = \frac{814}{1500} \tag{1.2.2.3}$$
$$= 0.5427 \tag{1.2.2.4}$$

c)

$$\Pr(X = 0) = \frac{211}{1500} \tag{1.2.2.5}$$
$$= 0.1407 \tag{1.2.2.6}$$

It is easy to verify that

$$\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) = 1 \tag{1.2.2.7}$$

3. In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph in Fig. 1.2.3 was prepared for the data so obtained. Find the probability that a student of the class was born in August.

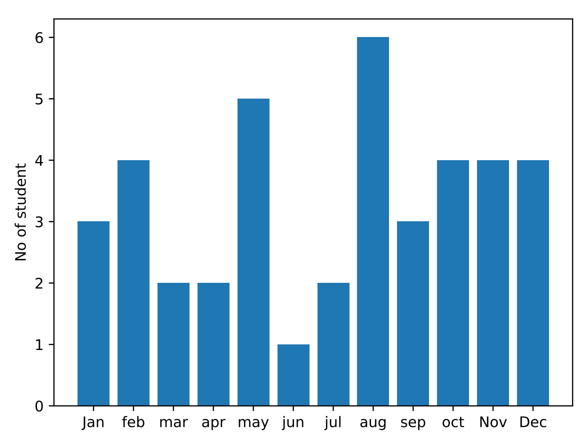


Fig. 1.2.3: student birth figure

Solution: Total no of the studend in a year = 40 no of student of class August = 6 let probabilaty of a student to be of august class be P(A)

$$P(A) = \frac{6}{40} = 0.15 \tag{1.2.3.1}$$

codes for the above equation can be get from here

```
solutions/1−10/codes/prob/prob3.py
```

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomeslisted in Table 1.2.4. If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

TABLE 1.2.4

Solution:

a) From the given information,

$$\Pr(X < 20) = \frac{7}{90} \tag{1.2.4.1}$$
$$= 0.07 \tag{1.2.4.2}$$

b)

$$\Pr(X \geq 60) = \frac{15+8}{90} \tag{1.2.4.3}$$
$$= 0.256 \tag{1.2.4.4}$$

5. Refer to Table 1.2.5.
   a) Find the probability that a student obtained less than 20% in the mathematics test.
   b) Find the probability that a student obtained marks 60 or above.

Marks	**Number of students**
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

TABLE 1.2.5

Solution:

a) From the given information,

$$\Pr(X < 20) = \frac{7}{90} \tag{1.2.5.1}$$
$$= 0.07 \tag{1.2.5.2}$$

b)

$$\Pr(X \geq 60) = \frac{15+8}{90} \tag{1.2.5.3}$$
$$= 0.256 \tag{1.2.5.4}$$

6. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in Table 1.2.6 Find the probability that a student chosen at random
   a) likes statistics,
   b) does not like it.

Opinion	**Number of students**
like	135
dislike	65

TABLE 1.2.6

Solution: Let $X \in \{0, 1\}$ be the random variable denoting dislikes and likes.

a)

$$\Pr(X = 1) = \frac{135}{200} \tag{1.2.6.1}$$
$$= 0.675 \tag{1.2.6.2}$$

b)

$$\Pr(X = 0) = \frac{65}{200} = 0.325 \tag{1.2.6.3}$$

7. The distance (in kms) of 40 engineers from their residence to their place of work were found as follows in Table 1.2.7. What is the empirical probability that an engineer lives
   a) less than 7 km from her place of work?
   b) more than or equal to 7 km from her place of work?
   c) within $\frac{1}{2}$km from her place of work?

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

TABLE 1.2.7

Solution:

a) total no of people working at the work place = 40 no of people live less than 7km from the work place = 9 let probability of a emgineer livinf less than 7 km from workplace = P(A)

$$P(A) = \frac{9}{40} \tag{1.2.7.1}$$

b) no of people live more than or equal 7km from the work place = 31 let probability of a emgineer livinf less than 7 km from workplace = P(B)

$$P(B) = \frac{31}{40} \tag{1.2.7.2}$$

c) there is no one who live within $\frac{1}{2}$ km from the work place so the probability will be 0.

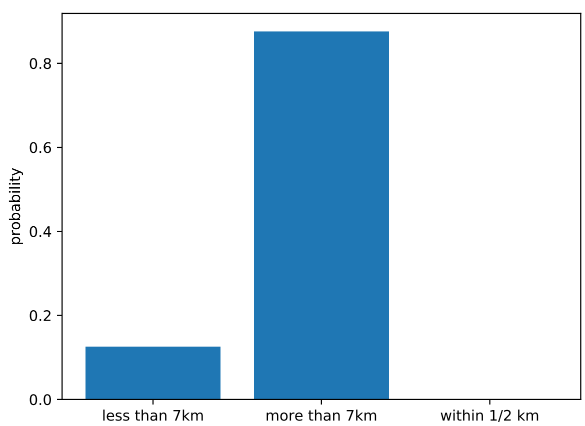


Fig. 1.2.7: probabilities a man to be near from work place

solutions/1−10/figs/prob/prob7.py

8. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the Table 1.2.8 Suppose a family is chosen. Find the probability that the family chosen is

a) earning ₹10000 – ₹13000 per month and owning exactly 2 vehicles.

b) earning ₹16000 or more per month and owning exactly 1 vehicle.

c) earning less than ₹7000 per month and does not own any vehicle.

d) earning ₹13000 – ₹16000 per month and owning more than 2 vehicles.

e) owning not more than 1 vehicle.

Monthly income (in ₹)	**vehicles per family**			
	0	**1**	**2**	**Above 2**
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

TABLE 1.2.8

Solution: Let X be the random variable denoting the number of vehicles and Y be the income.

a) no of total families chosen for survey = 2400

$$\Pr(X = 2, 10000 < Y < 13000) = \frac{29}{2400} \tag{1.2.8.1}$$
$$= 0.012 \tag{1.2.8.2}$$

b)

$$\Pr(X = 1, Y > 16000) = \frac{579}{2400} \tag{1.2.8.3}$$
$$= 0.241 \tag{1.2.8.4}$$

c)

$$\Pr(X = 0, Y < 7000) = \frac{10}{2400} \tag{1.2.8.5}$$
$$= 0.0042 \tag{1.2.8.6}$$

d)

$$\Pr(X > 2, 13000 < Y < 16000) = \frac{25}{2400} \tag{1.2.8.7}$$
$$= 0.0104 \tag{1.2.8.8}$$

e) The number of families is given by the sum of columns 0 and 1 in Table 1.2.8. Hence,

$$\Pr(X < 2) = \frac{1892}{2400} \tag{1.2.8.9}$$
$$= 0.78833 \tag{1.2.8.10}$$

9. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg)
4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00
Find the probability that any of these bags chosen at random contains more than 5 kg of flour.
Solution:
 a) No of bags having weight more than 5 Kg = 7
 total no of bags = 11

$$P(A) = \frac{7}{11} \tag{1.2.9.1}$$
$$= 0.636 \tag{1.2.9.2}$$

 codes for the above equation can be get from here

```
solutions/1−10/codes/prob/prob9.py
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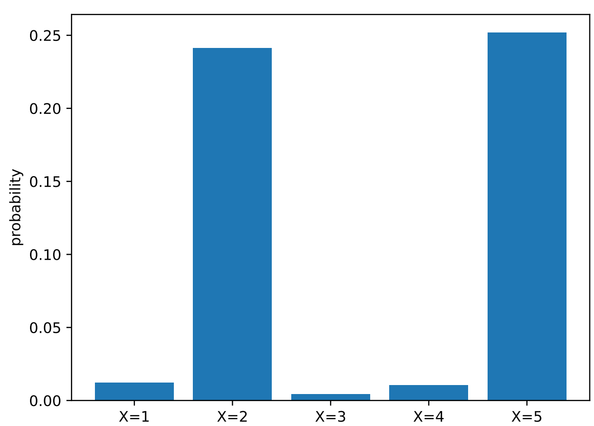

Fig. 1.2.9: probability of bag to be more than 5 Kg

10. From Table 1.2.10, prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days. **Solution:**
 a) P(A) be the prbability of concentration of sulpher

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	10.04

TABLE 1.2.10

concentration of sulphur	friquency
0.01	2
0.02	1
0.03	1
0.04	2
0.05	2
0.06	2
0.07	3
0.08	4
0.09	2
0.10	1
0.11	2
0.12	1
0.13	1
0.16	1
0.17	1
0.18	2
0.20	1
0.22	1

TABLE 1.2.10

$$p(A) = \frac{1+1+1}{30} \tag{1.2.10.1}$$
$$= 0.1 \tag{1.2.10.2}$$

 codes for the above equation can be get from here

```
solutions/1−10/codes/prob/prob10.py
```

11. A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O, A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.
prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

12. Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find

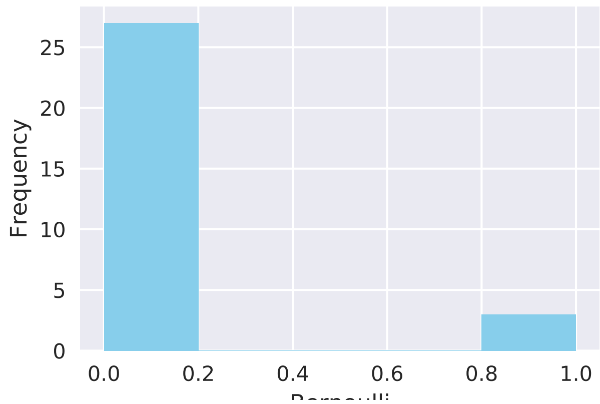

Fig. 1.2.10: probability of SO_2 0.12 to 0.16

P(E/F) and P(F/E)?

13. Compute P(A/B), if P(B) = 0.5 and P (A ∩ B) = 0.32.

14. If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, find
(i) P(A ∩ B)
(ii) P(A/B)
(iii) P(A ∪ B)

15. Evaluate P(A ∪ B), if 2P(A) = P(B) = $\frac{5}{13}$ and P(A/B) = $\frac{2}{5}$.

16. If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{11}{7}$ find
(i) P(A ∩ B)
(ii) P(A/B)
(iii) P(B/A)

17. Determine P(E/F), if a coin is tossed three times
(i) E : head on third toss , F : heads on first two tosses
(ii) E : at least two heads , F : at most two heads
(iii) E : at most two tails , F : at least one tail

18. Determine P(E/F), if two coins are tossed once, where
(i) E : tail appears on one coin, F : one coin shows head
(ii) E : no tail appears, F : no head appears

19. Determine P(E/F), if a die is thrown three times,
E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses

20. Determine P(E/F), if mother, father and son line up at random for a family picture
E : son on one end, F : father in middle

21. A black and a red dice are rolled.
(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.
Solution: Let $Y_i \in \{1,2,3,4,5,6\}$represent the outcome where Y_1 denotes black dice.

a)

$$\begin{aligned}
&\Pr(Y_1 + Y_2 > 9 | Y_1 = 5) \\
&= \frac{\Pr(Y_1 + Y_2 > 9, Y_1 = 5)}{\Pr(Y_1 = 5)} \\
&= \frac{\Pr(Y_2 > 4, Y_1 = 5)}{\Pr(Y_1 = 5)} \\
&= \Pr(Y_2 > 4) = \frac{1}{3}
\end{aligned} \tag{1.2.21.1}$$

b)

$$\begin{aligned}
&\Pr(Y_1 + Y_2 = 8 | Y_2 < 4) \\
&= \frac{\Pr(Y_1 > 4, Y_2 < 4)}{\Pr(Y_2 < 4)} \\
&= \Pr(Y_1 > 4) = \frac{1}{3}
\end{aligned} \tag{1.2.21.2}$$

22. A fair die is rolled. Consider the events E = (1, 3, 5), F = (2, 3) and G = (2, 3, 4, 5) Find
(i) P(E/F) and P(F/E)
(ii) P(E/G) and P(G/E)
(iii) P((E ∪ F)/G) and P((E ∩ F)/G)

Solution: From the given information,

$$\Pr(E) = \frac{3}{6} = \frac{1}{2} \tag{1.2.22.1}$$
$$\Pr(F) = \frac{2}{6} = \frac{1}{3} \tag{1.2.22.2}$$
$$\Pr(G) = \frac{4}{6} = \frac{2}{3} \tag{1.2.22.3}$$
$$\Pr(EF) = \frac{1}{6} \tag{1.2.22.4}$$
$$\Pr(EG) = \frac{2}{6} = \frac{1}{3} \tag{1.2.22.5}$$
$$\Pr(FG) = \frac{2}{6} = \frac{1}{3} \tag{1.2.22.6}$$
$$\Pr(EFG) = \frac{1}{6} \tag{1.2.22.7}$$

a)

$$\Pr(E|F) = \frac{\Pr(EF)}{\Pr(F)} \tag{1.2.22.8}$$
$$\Pr(E|F) = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2} \tag{1.2.22.9}$$
$$\Pr(F|E) = \frac{\Pr(FE)}{\Pr(E)} \tag{1.2.22.10}$$
$$\Pr(F|E) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3} \tag{1.2.22.11}$$

b)

$$\Pr(E|G) = \frac{\Pr(EG)}{\Pr(G)} \tag{1.2.22.12}$$
$$\Pr(E|G) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \tag{1.2.22.13}$$
$$\Pr(G|E) = \frac{\Pr(GE)}{\Pr(G)} \tag{1.2.22.14}$$
$$\Pr(G|E) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \tag{1.2.22.15}$$
$$\tag{1.2.22.16}$$

c)

$$\begin{aligned}\Pr(E+F|G) &= \frac{\Pr(\{E+F\}G)}{\Pr(G)} \\ &= \frac{\Pr(EG+FG)}{\Pr(G)} \\ &= \frac{\Pr(EG)+\Pr(FG)-\Pr(EFG)}{\Pr(G)} \\ &= \frac{3}{4}\end{aligned} \tag{1.2.22.17}$$

and

$$\Pr(EF|G) = \frac{\Pr(EFG)}{\Pr(G)} = \frac{1}{4} \tag{1.2.22.18}$$

23. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(i) the youngest is a girl,
(ii) at least one is a girl?
Solution: Let $X \in \{0,1\}$ represent the gender where 1 represents a girl. Let $Y_1, Y_2 \in \{0,1\}$ represent the child in the family, where Y_1 denotes the older child.
  a) Since Y_1, Y_2 are independent,
  $$\Pr(Y_1 = 1, Y_2 = 1|Y_2 = 1) = \frac{1}{2} \tag{1.2.23.1}$$
  b)
  $$\begin{aligned}&\Pr(Y_1 = 1, Y_2 = 1|1 - \{Y_2 = 0, Y_1 = 0\}) \\ &= \frac{\Pr(\{Y_1 = 1\}\{Y_2 = 1\}[1 - \{Y_2 = 0\}\{Y_1 = 0\}])}{1 - \Pr(Y_2 = 0, Y_1 = 0)} \\ &= \frac{\Pr(\{Y_1 = 1\}\{Y_2 = 1\}) - \Pr(\{Y_1 = 1\}\{Y_2 = 1\}\{Y_1 = 0\}\{Y_2 = 0\})}{1 - \Pr(Y_2 = 0, Y_1 = 0)} \\ &= \frac{\Pr(\{Y_1 = 1\}\{Y_2 = 1\})}{1 - \Pr(Y_2 = 0, Y_1 = 0)} = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}\end{aligned} \tag{1.2.23.2}$$

24. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?
Solution: Let $X \in \{0,1\}$ where 0 represents an easy question. Let $Y \in \{0,1\}$ where 1 denotes multiple choice questions. From the given information,
$$\Pr(X = 0, Y = 0) = \frac{300}{1400} = \frac{3}{14} \tag{1.2.24.1}$$
$$\Pr(X = 1, Y = 0) = \frac{200}{1400} = \frac{2}{14} \tag{1.2.24.2}$$
$$\Pr(X = 0, Y = 1) = \frac{500}{1400} = \frac{5}{14} \tag{1.2.24.3}$$
$$\Pr(X = 1, Y = 1) = \frac{400}{1400} = \frac{4}{14} \tag{1.2.24.4}$$

Then,

$$\Pr(X = 0|Y = 1) = \frac{\Pr(X = 0, Y = 1)}{\Pr(Y = 1)} \tag{1.2.24.5}$$

$$= \frac{\Pr(X = 0, Y = 1)}{\sum_i \Pr(X = i, Y = 1)} \tag{1.2.24.6}$$

$$= \frac{\frac{5}{14}}{\frac{5}{14} + \frac{4}{14}} \tag{1.2.24.7}$$

$$= \frac{5}{9} \tag{1.2.24.8}$$

25. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.
Solution: Let $X_1, X_2 \in \{1, 2, 3, 4, 5, 6\}$ represent the two dice.

$$\Pr(X_1 \neq X_2) = \frac{6 \times 5}{6 \times 6} = \frac{5}{6} \tag{1.2.25.1}$$

Then,

$$\Pr(X_1 + X_2 = 4|X_1 \neq X_2) = \frac{\Pr(X_1 + X_2 = 4, X_1 \neq X_2)}{\Pr(X_1 \neq X_2)} = \frac{\frac{2}{36}}{\frac{5}{6}} = \frac{1}{15} \tag{1.2.25.2}$$

26. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.
Solution: Let $X_k \in \{-1, 0, 3, 6, r\}, k = 1, 2, \dots$ represent the described process, where $r, 3, 6$ denote the outcome of the die and -1,0 denote the outcome of the coin, 0 representing a tail. In general, the transition probabilities for the Markov Chain are

$$\Pr(X_n = 0|X_{n-1} = r) = \Pr(X_n = -1|X_{n-1} = r) \tag{1.2.26.1}$$

$$= \frac{1}{2} \tag{1.2.26.2}$$

$$\Pr(X_n = 0|X_{n-1} = 3) = \Pr(X_n = -1|X_{n-1} = 3) \tag{1.2.26.3}$$

$$= 0 \tag{1.2.26.4}$$

$$\Pr(X_n = 0|X_{n-1} = 6) = \Pr(X_n = -1|X_{n-1} = 6) \tag{1.2.26.5}$$

$$= 0 \tag{1.2.26.6}$$

$$\Pr(X_n = 3|X_{n-1} = r) = 0 \tag{1.2.26.7}$$

$$\Pr(X_n = 6|X_{n-1} = r) = 0 \tag{1.2.26.8}$$

$$\Pr(X_n = r|X_{n-1} = r) = 0 \tag{1.2.26.9}$$

$$\Pr(X_n = 3|X_{n-1} = 6) = \Pr(X_n = 6|X_{n-1} = 3) \tag{1.2.26.10}$$

$$\Pr(X_n = 3|X_{n-1} = 3) = \Pr(X_n = 6|X_{n-1} = 6) \tag{1.2.26.11}$$

$$= \frac{1}{6} \tag{1.2.26.12}$$

$$\Pr(X_n = r|X_{n-1} = 3) = \Pr(X_n = r|X_{n-1} = 6) \tag{1.2.26.13}$$

$$= \frac{4}{6} \tag{1.2.26.14}$$

Thus,

$$\Pr(X_2 = 0|X_1 = 3) = 0 \tag{1.2.26.15}$$

27. Choose the correct answer, if P(A) = $\frac{1}{2}$, P(B) = 0, then P(A/B) is
    a) 0
    b) $\frac{1}{2}$
    c) not defined
    d) 1

Solution:

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{1.2.27.1}$$

$$\because \Pr(B) = 0, B = 0, \implies AB = 0 \tag{1.2.27.2}$$

$$\text{or, } \Pr(AB) = 0 \tag{1.2.27.3}$$

$$\implies \Pr(A|B) = 0 \tag{1.2.27.4}$$

28. If A and B are events such that P(A/B) = P(B/A), then
    a) A ⊂ B but A ≠ B

b) A = B
c) A ∩ B = ϕ
d) P(A) = P(B)

Solution:

$$\Pr(A|B) = \Pr(B|A) \tag{1.2.28.1}$$
$$\implies \frac{\Pr(AB)}{\Pr(A)} = \frac{\Pr(AB)}{\Pr(B)} \tag{1.2.28.2}$$
$$\implies \Pr(AB) = 0 \implies AB = 0 \tag{1.2.28.3}$$
$$\text{or, } \Pr(A) = \Pr(B) \tag{1.2.28.4}$$

29. If P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$, find P (A ∩ B) if A and B are independent events.
Solution:
$$\Pr(AB) = \Pr(A)\Pr(B) = \frac{3}{25} \tag{1.2.29.1}$$

30. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.
Solution: Let $X_1, X_2 \in \{0,1\}$ represent the colour, where 0 denotes black and 1 denotes red. From the given information,
$$\Pr(X_1 = 0) = \frac{26}{52} = \frac{1}{2} \tag{1.2.30.1}$$
$$\Pr(X_2 = 0|X_1 = 0) = \frac{25}{51} \tag{1.2.30.2}$$
Then,
$$\Pr(X_1 = 0, X_2 = 0)$$
$$= \Pr(X_2 = 0|X_1 = 0)\Pr(X_1 = 0) = \frac{25}{102} \tag{1.2.30.3}$$

31. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

32. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

33. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

34. Let E and F be events with P(E) = $\frac{3}{5}$, P(F) = $\frac{3}{10}$ and P(E ∩ F) = $\frac{1}{5}$. Are E and F independent?

35. Given that the events A and B are such that P(A) = $\frac{1}{2}$, P(A ∪B)= $\frac{3}{5}$ and P(B) = p. Find p if they are
(i) mutually exclusive
(ii) independent.

36. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find
(i) P(A ∩ B)
(ii) P(A ∪ B)
(iii) P(A/B)
(iv) P(B/A)

37. If A and B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and P(A ∩ B) = $\frac{1}{8}$. find P (not A and not B).

38. Events A and B are such that P (A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B are independent ?

39. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find
(i) P(A and B)
(ii) P(A and not B)
(iii) P(A or B)
(iv) P(neither A nor B)

40. A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution: $X_i \in \{0,1\}$, where 0 represents an even number. The 3 trials are represented by
$$X = X_1 + X_2 + X_3 \tag{1.2.40.1}$$
Then $X \sim B\left(3, \frac{1}{2}\right)$ The probability of getting only an even number is
$$\Pr(X = 0) = {}^nC_0\left(\frac{1}{2}\right)^3 \tag{1.2.40.2}$$
Thus, the probability of getting at least one odd

number is

$$1 - \Pr(X = 0) = 1 - \frac{1}{8} \tag{1.2.40.3}$$
$$= \frac{7}{8} \tag{1.2.40.4}$$

solutions/40−50/probability/codes/Q40binom.py

41. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red.
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.
Solution: Let $X \in \{0, 1\}$ where 0 represents black.
a) Probability of picking a black ball
$$\Pr(X = 0) = \frac{10}{18} = \frac{5}{9} \tag{1.2.41.1}$$
b) Probability of picking a red ball
$$\Pr(X = 1) = 1 - \Pr(X = 0) = \frac{4}{9} \tag{1.2.41.2}$$
c) Two balls are drawn with replacement. So each event is independent of each other. Probability that both balls are red
$$\Pr(X_1 = 1, X_2 = 1) = \left(\frac{4}{9}\right)^2 \tag{1.2.41.3}$$
$$= \frac{16}{81} \tag{1.2.41.4}$$
d) Probability that first ball is black and second is red
$$\Pr(X_1 = 0, X_2 = 1) = \frac{5}{9} \times \frac{4}{9} \tag{1.2.41.5}$$
$$= \frac{20}{81} \tag{1.2.41.6}$$
e) Probability that one ball is black and other is red
$$\Pr(X_1 = 0, X_2 = 1) + \Pr(X_1 = 1, X_2 = 0)$$
$$= \frac{16}{81} + \frac{20}{81} = \frac{4}{9} \tag{1.2.41.7}$$
f) The python code for finding probability using a sample size of 10000 can be downloaded from

solutions/40−50/probability/codes/Q41.py

42. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved
(ii) exactly one of them solves the problem.
Solution: Let $A, B \in \{0, 1\}$ where 1 indicates solving a problem. Given that
$$\Pr(A = 1) = \frac{1}{2}, \Pr(B = 1) = \frac{1}{3} \tag{1.2.42.1}$$
a) A problem is solved when either A or B solves the problem or both solve the problem. So the probability that problem is solved
$$\Pr(A = 1, B = 0) + \Pr(A = 0, B = 1)$$
$$+ \Pr(A = 1, B = 1)$$
$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3}$$
$$= \frac{2}{3} \tag{1.2.42.2}$$
b) Probability that exactly one of them solves the problem is
$$\Pr(A = 1, B = 0) + \Pr(A = 0, B = 1)$$
$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$$
$$= \frac{1}{2} \tag{1.2.42.3}$$

solutions/40−50/probability/codes/Q42.py

43. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?
(i) E : 'the card drawn is a spade' F : 'the card drawn is an ace'
(ii) E : 'the card drawn is black' F : 'the card drawn is a king'
(iii) E : 'the card drawn is a king or queen' F : 'the card drawn is a queen or jack'.
Solution: Two events E and F are said to be independent if they satisfy the criterion:
$$P(E \cap F) = P(E)P(F) \tag{1.2.43.1}$$
a) There are 13 cards of spades, 4 cards of aces

and 1 card of ace of spades.

$$P(E) = \frac{13}{52} \quad (1.2.43.2)$$
$$P(F) = \frac{4}{52} \quad (1.2.43.3)$$
$$P(E \cap F) = \frac{1}{52} \quad (1.2.43.4)$$

Clearly, $P(E \cap F) = P(E)P(F)$. Therefore E and F are independent events.

b) There are 26 black cards, 4 king cards and 2 black and king cards.

$$P(E) = \frac{26}{52} \quad (1.2.43.5)$$
$$P(F) = \frac{4}{52} \quad (1.2.43.6)$$
$$P(E \cap F) = \frac{2}{52} \quad (1.2.43.7)$$

Clearly, $P(E \cap F) = P(E)P(F)$. Therefore E and F are independent events.

c) There are 8 kings or queens, 8 queens or jacks. In both of these, common is the quuen cards.

$$P(E) = \frac{8}{52} \quad (1.2.43.8)$$
$$P(F) = \frac{8}{52} \quad (1.2.43.9)$$
$$P(E \cap F) = \frac{4}{52} \quad (1.2.43.10)$$

Clearly, $P(E \cap F) \neq P(E)P(F)$. Therefore E and F are not independent events.

44. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Solution: Let $X \in \{0, 1\}$ where 0 represents Hindi. From the given information,

$$\Pr(X = 0) = \frac{60}{100} = \frac{3}{5} \quad (1.2.44.1)$$
$$\Pr(X = 1) = \frac{40}{100} = \frac{2}{5} \quad (1.2.44.2)$$
$$\Pr(X = 0, X = 1) = \frac{20}{100} = \frac{1}{5} \quad (1.2.44.3)$$

Note that the events are dependent.

a) Probability that the student reads neither Hindi nor English

$$\Pr(X \neq 0, X \neq 1) = 1 - [\Pr(X = 0) + \Pr(X = 1) - \Pr(X = 0, X = 1)] = \frac{1}{5} \quad (1.2.44.4)$$

b) Probability that she reads English newspaper if it is known that she reads Hindi newspaper

$$\Pr(X = 1|X = 0) = \frac{\Pr(X = 1, X = 0)}{\Pr(X = 0)} \quad (1.2.44.5)$$
$$= \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3} \quad (1.2.44.6)$$

c) Probability that she reads Hindi newspaper if it is known that she reads English newspaper

$$\Pr(X = 0|X = 1) = \frac{\Pr(X = 1, X = 0)}{\Pr(X = 1)} \quad (1.2.44.7)$$
$$= \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2} \quad (1.2.44.8)$$

Choose the correct answer:

45. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

a) 0
b) $\frac{1}{3}$
c) $\frac{1}{12}$
d) $\frac{1}{36}$

Solution: Let $X_1, X_2 \in \{1, 2, 3, 4, 5, 6\}$ represent the two dice. The desired probability is

$$\Pr(X_1 = 2, X_2 = 2) = \Pr(X_1 = 2)\Pr(X_2 = 2) = \frac{1}{36} \quad (1.2.45.1)$$

46. Two events A and B will be independent, if

a) A and B are mutually exclusive
b) $P(A'B') = [1 - \text{P(A)}]\ [1 - \text{P(B)}]$
c) P(A) = P(B)
d) P(A) + P(B) = 1

Solution:

a) A and B are not mutually exclusive because $P(A \cap B) = P(A) \times P(B)$ and it is not zero.
b) Also $P(A) = P(B)$ is not necessariy true.
c) $P(A) + P(B)$ is not always equal to 1.
d) If A and B are independent,

$$\begin{aligned} P(A'B') &= P(A')P(B') \\ &= (1 - P(A))(1 - P(B)) \end{aligned} \tag{1.2.46.1}$$

e) Answer= option(b)

47. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?
Solution: Let $X \in 0, 1$ where 0 represents black. Let X_1 represent the event representing drawing the first ball. X_2 represent the event of drawing the second ball. Then probability of the second ball being red is

$$\begin{aligned} &\Pr(X_2 = 1) \\ &= \Pr(X_2 = 1, X_1 = 1) + \Pr(X_2 = 1, X_1 = 0) \\ &= \Pr(X_2 = 1|X_1 = 1)\Pr(X_1 = 1) \\ &+ \Pr(X_2 = 1|X_1 = 0)\Pr(X_1 = 0) \end{aligned} \tag{1.2.47.1}$$

From the given information,

$$\Pr(X_1 = 0) = \Pr(X_1 = 1) = \frac{5}{10} = \frac{1}{2}. \tag{1.2.47.2}$$

Also,

$$\Pr(X_2 = 1|X_1 = 0) = \frac{5}{5+2+7} = \frac{5}{12} \tag{1.2.47.3}$$

$$\Pr(X_2 = 1|X_1 = 1) = \frac{5+2}{5+2+5} = \frac{7}{12} \tag{1.2.47.4}$$

Thus,

$$\begin{aligned} \Pr(X_2 = 1) &= \frac{7}{12} \times \frac{1}{2} \\ &+ \frac{5}{12} \times \frac{1}{2} = \frac{1}{2} \end{aligned} \tag{1.2.47.5}$$

The python code for finding probability using a sample size of 10000 can be downloaded from

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solutions/40−50/probability/codes/Q47.py
```

48. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.
Solution: Let $X \in \{0, 1\}$ represent the bags, where 0 represents the first bag and $Y \in \{0, 1\}$ represent the colour, 0 being black. The desired probability is

$$\begin{aligned} &\Pr(X = 0|Y = 1) \\ &= \frac{\Pr(Y = 1|X = 0)\Pr(X = 0)}{\Pr(Y = 1|X = 0)\Pr(X = 0) + \Pr(Y = 1|X = 0)\Pr(X = 0)} \end{aligned} \tag{1.2.48.1}$$

From the given information,

$$\Pr(X = 0) = \Pr(X = 1) = \frac{1}{2} \tag{1.2.48.2}$$

$$\Pr(Y = 1|X = 0) = \frac{1}{2} \tag{1.2.48.3}$$

$$\Pr(Y = 1|X = 1) = \frac{1}{4} \tag{1.2.48.4}$$

Hence,

$$\Pr(X = 0|Y = 1) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} \tag{1.2.48.5}$$

$$= \frac{2}{3} \tag{1.2.48.6}$$

```
solutions/40−50/probability/codes/Q48.py
```

49. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostelier?
Solution: Let $X \in \{0, 1\}$ represent student residence, 0 being a hostel residence. Let $Y\{0, 1\}$ represent the grade, 0 being A grade. The

objective is to find

$$\Pr(X=0|Y=0) = \frac{\Pr(Y=0|X=0)\Pr(X=0)}{\Pr(Y=0|X=0)\Pr(X=0)+\Pr(Y=0|X=1)\Pr(X=1)} \tag{1.2.49.1}$$

From the given information,

$$\Pr(Y=0|X=0) = \frac{3}{10} \tag{1.2.49.2}$$
$$\Pr(Y=0|X=1) = \frac{2}{10} \tag{1.2.49.3}$$
$$\Pr(X=0) = \frac{6}{10} \tag{1.2.49.4}$$
$$\Pr(X=1) = \frac{4}{10} \tag{1.2.49.5}$$

Hence,

$$\Pr(X=0|Y=0) = \frac{\frac{3}{10}\times\frac{6}{10}}{\frac{3}{10}\times\frac{6}{10}+\frac{2}{10}\times\frac{4}{10}} = \frac{9}{13} \tag{1.2.49.6}$$

The python code for finding probability using a sample size of 10000 can be downloaded from

probability/codes/Q49.py

50. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?
Solution: Let $X \in \{0,1\}$ represent student knowledge where 0 denotes a guess. Let $Y = \{0,1\}$ represent the correctness of the answer, with 0 being the case when the answer is incorrect. Then, we need to find

$$\Pr(X=1|Y=1) = \frac{\Pr(Y=1|X=1)\Pr(X=1)}{\Pr(Y=1|X=1)\Pr(X=1)+\Pr(Y=1|X=0)\Pr(X=0)} \tag{1.2.50.1}$$

From the given information,

$$\Pr(Y=1|X=0) = \frac{1}{4} \tag{1.2.50.2}$$
$$\Pr(Y=1|X=1) = 1 \tag{1.2.50.3}$$
$$\Pr(X=0) = \frac{3}{4} \tag{1.2.50.4}$$
$$\Pr(X=1) = \frac{1}{4} \tag{1.2.50.5}$$

$\because$ if the student knows the answer, she will definitely be correct. Hence,

$$\Pr(X=1|Y=1) = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}} = \frac{4}{7} \tag{1.2.50.6}$$

solutions/40−50/probability/codes/Q50.py

51. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

52. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?

53. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

54. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

55. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability

of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

56. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

57. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

58. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Choose a correct answer

59. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

a) $\frac{4}{5}$
b) $\frac{1}{2}$
c) $\frac{1}{5}$
d) $\frac{2}{5}$

60. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

a) $P(A/B) = \frac{P(B)}{P(A)}$
b) $P(A/B) < P(A)$
c) $P(A/B) \geq P(A)$
d) None of these

61. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P(X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

(iii)

X	-1	0	1
P(X)	0.6	0.1	0.2

(iv)

X	3	2	1	0	-1
P(X)	0.3	0.2	0.4	0.1	0.05

62. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable ?

63. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

64. Find the probability distribution of
(i) number of heads in two tosses of a coin.
(ii) number of tails in the simultaneous tosses of three coins.
(iii) number of heads in four tosses of a coin.

65. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die

66. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

67. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

68. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

Determine
(i) k
(ii) P(X ¡ 3)
(iii) P(X ¿ 6)
(iv) P(0 ¡ X ¡ 3)

69. Find the mean number of heads in three tosses of a fair coin.

70. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

71. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

72. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

73. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

74. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

Choose the correct answer in each of the following:

75. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

a) 1
b) 2
c) 5
d) $\frac{8}{3}$

76. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

a) $\frac{37}{221}$
b) $\frac{5}{13}$
c) $\frac{1}{13}$
d) $\frac{2}{13}$

77. A die is thrown 6 times. If ‘getting an odd number’ is a success, what is the probability of
(i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes?

78. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

79. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

80. Five cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that
(i) all the five cards are spades?
(ii) only 3 cards are spades?
(iii) none is a spade?

81. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs
(i) none
(ii) not more than one
(iii) more than one
(iv) at least one
will fuse after 150 days of use.

82. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

83. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a

fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

84. Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome.
(Hint : P(X = 3) is the maximum among all P(x_i), x_i= 0,1,2,3,4,5,6)

85. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

86. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$.What is the probability that he will win a prize
(a) at least once
(b) exactly once
(c) at least twice?

87. Find the probability of getting 5 exactly twice in 7 throws of a die.

88. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

89. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

In each of the following, choose the correct answer:

90. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is
a) 10^{-1}
b) $(\frac{1}{2})^5$
c) $(\frac{9}{10})^5$
d) $\frac{9}{10}$

91. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is
a) ${}^5C_4(\frac{4}{5})^4\frac{1}{5}$
b) $(\frac{4}{5})^4\frac{1}{5}$
c) ${}^5C_1(\frac{4}{5})^4\frac{1}{5}$
d) None of these

92. A and B are two events such that P (A) $\neq$ 0. Find P(B/A), if
(i) A is a subset of B
(ii) A $\cap$ B = ϕ

93. A couple has two children,
(i) Find the probability that both children are males, if it is known that at least one of the children is male.
(ii) Find the probability that both children are females, if it is known that the elder child is a female.

94. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

95. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

96. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that
(i) all will bear 'X' mark.
(ii) not more than 2 will bear 'Y' mark.
(iii) at least one ball will bear 'Y' mark.
(iv) the number of balls with 'X' mark and 'Y' mark will be equal.

97. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

98. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

99. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

100. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

101. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

102. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

103. Suppose we have four boxes A,B,C and D containing coloured marbles as given below:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

104. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

105. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

106. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:
P(A fails) = 0.2
P(B fails alone) = 0.15
P(A and B fail) = 0.15

Evaluate the following probabilities
(i) P(A fails—B has failed)
(ii) P(A fails alone)

107. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Choose the correct answer in each of the following:

108. If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then (A) $A \subset B$
(B) $B \subset A$
(C) $B = \phi$
(D) $A = \phi$

109. If $P(A/B) > P(A)$, then which of the following is correct : (A) $P(B/A) < P(B)$
(B) $P(A \cap B) < P(A) . P(B)$
(C) $P(B/A) > P(B)$
(D) $P(B/A) = P(B)$

110. If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then
(A) $P(B/A) = 1$
(B) $P(A/B) = 1$
(C) $P(B/A) = 0$
(D) $P(A/B) = 0$

111. Complete the following statements:
(i) Probability of an event E + Probability of the event ‘not E’ =———— .
(ii) The probability of an event that cannot happen is———- . Such an event is called——— .
(iii) The probability of an event that is certain to happen is ———.
(iv) The sum of the probabilities of all the

elementary events of an experiment is———-.
(v) The probability of an event is greater than or equal to and less than or equal to ————.

112. Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car. The car starts or does not start.
(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
(iii) A trial is made to answer a true-false question. The answer is right or wrong.
(iv) A baby is born. It is a boy or a girl.

113. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?
114. Which of the following cannot be the probability of an event?
(A)$\frac{2}{3}$(B) −1.5 (C) 15
115. If P(E) = 0.05, what is the probability of 'not E'?
116. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
(i) an orange flavoured candy?
(ii) a lemon flavoured candy?
117. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?
118. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is
(i) red ?
(ii) not red?
119. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be
(i) red ?
(ii) white ?
(iii) not green?
120. A piggy bank contains hundred 50p coins, fifty rupee 1 coins, twenty rupee 2 coins and ten rupee 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin
(i) will be a 50 p coin ?
(ii) will not be a rupee5 coin?
121. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 1.2.121). What is the probability that the fish taken out is a male fish?

Fig. 1.2.121

Solution: Let $X \in \{0, 1\}$ represent the male and female fish respectively. Then the desired probability is

$$\Pr(X = 0) = \frac{5}{5+8} = \frac{5}{13} \qquad (1.2.121.1)$$

The python code for the distribution is

```
solutions/20−10/prob/codes/fish.py
```

The code checks how many times a male fish is picked out of the total times (taken as 100,000 in the given code) a fish is picked up from the tank with replacement.
122. A game of chance consists of spinning an arrow which comes to rest pointing at one

of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 1.2.122), and these are equally likely outcomes. What is the probability that it will point at
(i) 8 ?
(ii) an odd number?
(iii) a number greater than 2?
(iv) a number less than 9?

Fig. 1.2.122

Solution: Let

$$X \epsilon \{1,2,3,4,5,6,7,8\} \tag{1.2.122.1}$$

Since all events are equally likely,

$$\Pr(X = i) = \frac{1}{8} \quad i = 1,2,\ldots,8. \tag{1.2.122.2}$$

a)

$$\Pr(X = 8) = \frac{1}{8} \tag{1.2.122.3}$$

b) Probability of occurance of odd numbers is

$$\Pr(X = 1) + \Pr(X = 3) + \Pr(X = 5) + \Pr(X = 7) = \frac{1}{2} \tag{1.2.122.4}$$

c)

$$\Pr(X > 2) = 1 - \Pr(X < 2) = \frac{3}{4} \tag{1.2.122.5}$$

d)

$$\Pr(X < 9) = \frac{8}{8} = 1 \tag{1.2.122.6}$$

The python code for the distribution is

```
solutions/chance/prob/codes/chance.py
```

The above code checks occurance of each of these events when the arrow is spinned 100,000 times.

123. A die is thrown once. Find the probability of getting
(i) a prime number;
(ii) a number lying between 2 and 6;
(iii) an odd number.
Solution: Let X

$$X \epsilon \{1,2,3,4,5,6\} \tag{1.2.123.1}$$

Since all events are equally likely,

$$\Pr(X = i) = \begin{cases} \frac{1}{6} & 1 \le i \le 6 \\ 0 & otherwise \end{cases} \tag{1.2.123.2}$$

a) The probability that the outcome is a prime number is

$$\Pr(X = 2) + \Pr(X = 3) + \Pr(X = 5) = \frac{1}{2} \tag{1.2.123.3}$$

b) Probability of occurance of number between 2 and 6 is

$$\Pr(2 < X < 6) = \frac{1}{2} \tag{1.2.123.4}$$

c) Probability of occurance of odd number is

$$\Pr(X = 1) + \Pr(X = 3) + \Pr(X = 5) = \frac{1}{2} \tag{1.2.123.5}$$

The python code for the distribution is

```
solutions/20−10/prob/codes/dice123.py
```

The above code checks number of times each of the above events occur when the dice is thrown 100,000 times.

124. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour
(ii) a face card
(iii) a red face card
(iv) the jack of hearts
(v) a spade
(vi) the queen of diamonds
Solution: Let $X \in \{0, 1, 2, 3\}$ be the card type, $Y \in 0, 1$ be the colour and $\mathbf{Z} \in \{0, 1, \ldots, 12\}$ be the card number. The sample size = total number of cards ina deck

a) The probability of drawing a king of red colour

$$\Pr(Y = 0, Z = 11) = \Pr(Y = 0)\Pr(Z = 11) \tag{1.2.124.1}$$
$$= \frac{1}{2} \times \frac{4}{52} = \frac{1}{23} \tag{1.2.124.2}$$

b) The probability of drawing a face card is

$$\Pr(8 \le Z \le 11) = \frac{12}{52} = \frac{3}{13} \tag{1.2.124.3}$$

c) The probability of drawing a red face card from the deck is

$$\Pr(Y = 0, 8 \le Z \le 11)$$
$$= \Pr(Y = 0)\Pr(8 \le Z \le 11)$$
$$= \frac{1}{2} \times \frac{3}{13} = \frac{3}{23} \tag{1.2.124.4}$$

d) The probability of drawing a jack of hearts is

$$\Pr(Z = 9, X = 0) = \frac{1}{13} \times \frac{1}{4} = \frac{1}{52} \tag{1.2.124.5}$$

e) The probability of drawing a spade is

$$\Pr(X = 1) = \frac{13}{52} = \frac{1}{4} \tag{1.2.124.6}$$

f) The probability of drawing a queen of diamond is

$$\Pr(X = 2, Z = 10) = \Pr(X = 2)\Pr(Z = 10) \tag{1.2.124.7}$$
$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{1.2.124.8}$$

The python code for the distribution is

```
solutions/20−10/prob/codes/cards125.py
```

125. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is the queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?
Solution:

a) The probability that a queen is picked is

$$\Pr(Z = 10 | Z \in \{10, 11, 12, 13, 14\}) = \frac{1}{5} \tag{1.2.125.1}$$

b) After a queen is drawn and put aside The probability that an ace is picked is

$$\Pr(Z = 14 | Z \in 10, 11, 13, 14) = \frac{1}{4} \tag{1.2.125.2}$$

The probability that a queen is picked from the remaining cards is

$$\Pr(Z = 12 | Z \in 10, 11, 13, 14) = 0 \tag{1.2.125.3}$$

The python code below calculates the above probabilities for 100000 picks

```
solutions/20−10/prob/codes/cards126.py
```

126. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
Solution: Let $X\{0, 1\}$ represent the good and bad pens respectively. The probability of taking

out a good pen is

$$\Pr(X = 0) = \frac{132}{144} = \frac{11}{12} \tag{1.2.126.1}$$

The python code for the above solution is

solutions/20−10/prob/codes/pens127.py

127. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?
Solution: Let $X \in \{0, 1\}$.
a) The probability of drawing a defective bulb is

$$\Pr(X = 0) = \frac{4}{20} = \frac{1}{5} \tag{1.2.127.1}$$

b) After drawing a non defective bulb, The probability of drawing a non-defective bulb is

$$\Pr(X = 1) = \frac{15}{19} \tag{1.2.127.2}$$

The python code for the above solution is

solutions/20−10/prob/codes/exer128.py

128. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.
Solution: (i)The sample size

$$S = 90 \tag{1.2.128.1}$$

(i)number of discs bearing a two digit number is

$$T = 81 \tag{1.2.128.2}$$

The probability of drawing a disc bearing two digit number is

$$\Pr(T) = \frac{T}{S} = \frac{81}{90} \tag{1.2.128.3}$$
$$= \frac{9}{10} \tag{1.2.128.4}$$

(ii)number of discs bearing a perfect square is

$$Sq = 9 \tag{1.2.128.5}$$

The probability of drawing a disc bearing perfect square is

$$\Pr(Sq) = \frac{Sq}{S} = \frac{9}{90} \tag{1.2.128.6}$$
$$= \frac{1}{10} \tag{1.2.128.7}$$

(iii)number of discs bearing number divisible by 5 is

$$F = 18 \tag{1.2.128.8}$$

The probability of drawing a disc bearing number divisible by 5 is

$$\Pr(F) = \frac{F}{S} = \frac{18}{90} \tag{1.2.128.9}$$
$$= \frac{1}{5} \tag{1.2.128.10}$$

The python code for the above solution is

solutions/20−10/prob/codes/exer129.py

129. A child has a die whose six faces show the letters as given in Fig. 1.2.129 .

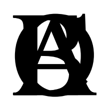

Fig. 1.2.129

The die is thrown once. What is the probability of getting (i) A? (ii) D?
Solution: The sample size= total faces of a die

$$S = 6 \tag{1.2.129.1}$$

(i)number of faces on which letter A appears

$$A = 2 \tag{1.2.129.2}$$

The probability of getting an A

$$\Pr(A) = \frac{A}{S} = \frac{2}{6} \tag{1.2.129.3}$$
$$= \frac{1}{3} \tag{1.2.129.4}$$

(ii)number of faces on which letter D appears

$$D = 1 \tag{1.2.129.5}$$

The probability of getting an A

$$\Pr(D) = \frac{D}{S} = \frac{1}{6} \tag{1.2.129.6}$$

The python code for the above solution is

```
./prob/codes/exer130.py
```

130. Suppose you drop a die at random on the rectangular region shown in Fig. 1.2.130. What is the probability that it will land inside the circle with diameter 1m?

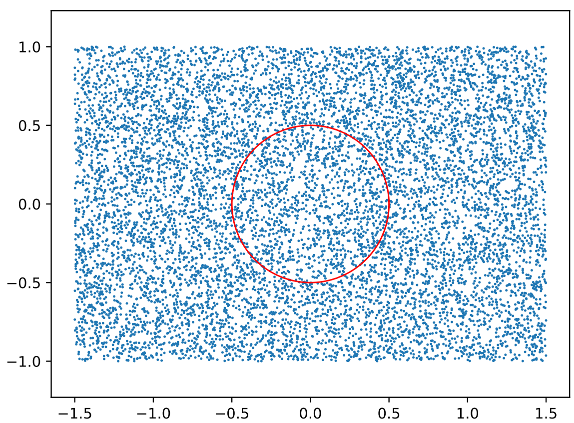

Fig. 1.2.130

Solution: In Fig. 1.2.130, the sample size S is the area of the rectangle given by

$$S = 3x2 = 6m^2 \tag{1.2.130.1}$$

The event size is the area of the circle given by

$$E = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}m^2 \tag{1.2.130.2}$$

The probabilty of the dice landing in the circle is

$$\Pr(E) = \frac{E}{S} = \frac{\pi}{24} \tag{1.2.130.3}$$

The python code is available in

```
/codes/rect.py
```

The python code generates 10,000 points uniformly within the rectangle of dimensions 3×2 and checks for the number of points within the circle of radius 0.5. The ratio of these is close to $\frac{\pi}{24}$. Note that each time the code is run, the ratio will change, but will still be close to $\frac{\pi}{24}$.

131. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it ?
(ii) She will not buy it ?
Solution: The sample size

$$S = 144 \tag{1.2.131.1}$$

The number of bad pens is

$$B = 20 \tag{1.2.131.2}$$

The probability that she doesn't buy a pen is

$$\Pr(B) = \frac{B}{S} = \frac{20}{144} \tag{1.2.131.3}$$
$$= \frac{5}{36} \tag{1.2.131.4}$$

The probability that she buys a pen is

$$\Pr(G) = 1 - \Pr(B) = \frac{31}{36} \tag{1.2.131.5}$$

The python code for the distribution is

```
solutions/10−1/prob/codes/prob2_a.py
```

132. Two dice, one blue and one grey, are thrown at the same time.
   a) Complete Table 1.2.132.
   b) A student argues that there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Solution:

   a) The following code simulates the experiment and prints the output in Table **??**

```
solutions/10−1/prob/codes/prob3_excel.py
```

Event	Value
2	1/36
3	-
4	-
5	-
6	-
7	-
8	5/36
9	-
10	-
11	-
12	1/36

TABLE 1.2.132: Input Values

Event	Value
2	1/36
3	0.055556
4	0.083333
5	0.1111111
6	0.138889
7	0.166667
8	5/36
9	0.1111111
10	0.083333
11	0.055556
12	1/36

TABLE 1.2.132: Output Values

Table is completed as follows 1.2.137

b) Let X_1 and X_2 represent the two dice and

$$X = X_1 + X_2 \tag{1.2.132.1}$$

where

$$p_{X_1}(n) = p_{X_2}(n) = \frac{1}{6}\{u(n) - u(n-6)\} \tag{1.2.132.2}$$

Then, $\because X_1$ and X_2 are independent,

$$p_X(n) = p_{X_1}(n) * p_{X_2}(n) \tag{1.2.132.3}$$

$$= \begin{cases} \frac{k-1}{36} & k < 8 \\ \frac{13-k}{36} & 8 \le k \le 12 \end{cases} \neq \frac{1}{11} \tag{1.2.132.4}$$

It is obvious that the argument mentioned by the student is incorrect.

133. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.
Solution: Let $X_i \in \{0, 1\}, i = 1, 2, 3$ represent a coin toss, or, the Bernoulli random variable. Then the outcome of the game is

$$X = X_1 + X_2 + X_3 \tag{1.2.133.1}$$

If

$$\Pr(X_i = 1) = p, \tag{1.2.133.2}$$

$$\Pr(X = k) = {}^nC_k p^k (1-p)^k, \quad k = 0, \dots, n \tag{1.2.133.3}$$

X is known as a Binomial random variable. For the given problem, $n = 3, p = \frac{1}{2}$ and the probability of a win is

$$\Pr(X = 3) + \Pr(X = 0) = \frac{1}{8} + \frac{1}{8} \tag{1.2.133.4}$$

$$= \frac{1}{4} \tag{1.2.133.5}$$

The loss probability is then

$$1 - \frac{1}{4} = \frac{3}{4} \tag{1.2.133.6}$$

The python code for the distribution of data,

```
solutions/10−1/prob/codes/prob4.py
```

134. A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment
Solution:

(i) Let $X_i \in \{1, 2, \ldots, 6\}$.

$$\Pr(X_1 \neq 5, X_2 \neq 5) = \Pr(X_1 \neq 5)\Pr(X_2 \neq 5) \tag{1.2.134.1}$$

$\because$ the two events are independent. Also,

$$\Pr(X_1 = 5) = \Pr(X_2 = 5) = \frac{1}{6} \tag{1.2.134.2}$$

$$\implies \Pr(X_1 \neq 5) = \Pr(X_2 \neq 5) \tag{1.2.134.3}$$

$$= 1 - \frac{1}{6} = \frac{5}{6} \tag{1.2.134.4}$$

From (1.2.134.1),

$$\Pr(X_1 \neq 5, X_2 \neq 5)$$
$$= [1 - \Pr(X_1 = 5)]\,[1 - \Pr(X_2 = 5)]$$
$$= \frac{25}{36} \tag{1.2.134.5}$$

upon substituting from (1.2.134.4)

(ii) The probability that 5 doesn't come at all is

$$1 - \Pr(X_1 \neq 5, X_2 \neq 5) = 1 - \frac{25}{36} = \frac{11}{36} \tag{1.2.134.6}$$

The python code for the problem is

```
solutions/10−1/prob/codes/prob5.py
```

135. Which of the following arguments are correct and which are not correct? Give reasons for your answer.
(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$
(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.
Solution:
1. In the given question,
The sample size = Total number of possibilities(S)=6

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \tag{1.2.135.1}$$

Event size= Odd number =3

$$\begin{pmatrix} 1 & 3 & 5 \end{pmatrix} \tag{1.2.135.2}$$

Probability for this event is = $\frac{1}{2}$
The python code for the distribution of data,

```
prob/codes/prob6_b.py
```

This shows the diagrametic representation of dice with the live update of probability with the role of dice.

136. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?
Solution: In the given question,
a) The sample size = Total number of possibilities(S)=25
The possibilities are shown in the below table 1.2.136 Event size=Both same day=5

Possibilities	
Shyam	Ekta
Tu	Tu,W,Th,F,Sa
W	Tu,W,Th,F,Sa
Th	Tu,W,Th,F,Sa
F	Tu,W,Th,F,Sa
Sa	Tu,W,Th,F,Sa

TABLE 1.2.136: Input Values

Possibilities are given in table 1.2.136 Prob-

Possibilities	
Shyam	Ekta
Tu	Tu
W	W
Th	Th
F	F
Sa	Sa

TABLE 1.2.136: Event Values

ability =

$$P = \frac{1}{5} \tag{1.2.136.1}$$

a) Event size = On consequitive days=8
Possibilities are given in the table 1.2.136
Probability =

Possibilities	
Shyam	Ekta
Tu	W
W	Tu,Th
Th	W,F
F	Th,Sa
Sa	F

TABLE 1.2.136: Event Values

$$P = \frac{8}{25} \tag{1.2.136.2}$$

a) Event size= On different days=20
Possibilities are given in the table 1.2.136
Probability =

Possibilities	
Shyam	Ekta
Tu	W,Th,F,Sa
W	Tu,Th,F,Sa
Th	Tu,W,F,Sa
F	Tu,W,Th,Sa
Sa	Tu,W,Th,F

TABLE 1.2.136: Event Values

$$P = \frac{4}{5} \tag{1.2.136.3}$$

137. A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

Number in first throw

Number in second throw	+	1	2	2	3	3	6
	1	2	3	3	4	4	7
	2	3	4	4	5	5	8
	2					5	
	3						
	3			5			
	6	7	8	8	9	9	12

What is the probability that the total score is (i) even? (ii) 6? (iii) at least 6?
Solution: In the given question,
The total number of possibilities=36
The Table 1.2.137 shows the possibilities

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

TABLE 1.2.137

a) Event size= No. of even numbers= 18
Probability=

$$P = \frac{1}{2} \tag{1.2.137.1}$$

a) Event size= No. of six=4
Probability=

$$P = \frac{1}{9} \tag{1.2.137.2}$$

a) Event size= Atleast six=15
Probability=

$$P = \frac{5}{12} \tag{1.2.137.3}$$

The python code for the calculation and completion of the excel file is at

```
solutions/10−1/prob/codes/prob8.py
```

138. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.
Solution: Let $X \in \{0, 1\}$ where 0 represents red. From the given information, if the number of blue balls is x,

$$\Pr(X = 1) = 2\Pr(X = 0) \tag{1.2.138.1}$$

$$\implies \frac{x}{x+5} = 2 \times \frac{5}{x+5} \tag{1.2.138.2}$$

$$\implies x = 10 \tag{1.2.138.3}$$

139. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box,what is the probability that it will be a black ball?
If 6 more black balls are put in the box, the probability of drawing a black ball is now

double of what it was before. Find x.
Solution: Let $X \in \{0, 1\}$ such that 0 represents black. Then,

$$\Pr(X = 0) = \frac{x}{12} \tag{1.2.139.1}$$

If 6 more black balls are put in the bag,

$$\Pr(X = 0) = \frac{x+6}{12+6} \tag{1.2.139.2}$$

From the given information

$$\frac{x+6}{12+6} = \frac{2x}{12} \tag{1.2.139.3}$$
$$\implies x = 3 \tag{1.2.139.4}$$

140. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$ Find the number of blue balls in the jar.

2 Statistics

2.1 Examples

1. The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in Table 2.1.1 below. Find the mean of the marks obtained by the students.
Solution: Let X represent the marks and

marks obtained	No of students
10	1
20	1
36	3
40	4
50	3
56	2
60	4
70	4
72	1
80	1
88	2
92	3
95	1

TABLE 2.1.1

$p(X = i)$ be the probability of a student obtaining marks i. The mean is then obtained as

$$E[X] = \sum_i (X = i) p(X = i) = \frac{1761}{30} \tag{2.1.1.1}$$
$$= 59.3 \tag{2.1.1.2}$$

The related code is available in

```
solutions/1−10/codes/statexm/statexm1.py
```

2. Table 2.1.2 below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed in this section.
Solution: The following code computes the

Percentage of female teachers	No of states
15-25	6
25-35	11
35-45	7
45-55	4
55-65	4
65-75	2
75-85	1

TABLE 2.1.2
Source : Seventh All India School Education Survey conducted by NCERT

mean using all the methods.

```
solutions/1−10/codes/statexm/statexm2.py
```

3. The distribution below in Table 2.1.3 shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?
Solution: The following code can be used for obtaining the answer.

```
solutions/1−10/codes/statexm/statexm2.py
```

4. The wickets taken by a bowler in 10 cricket matches are as follows:
2 6 4 5 0 2 1 3 2 3
Find the mode of the data.

No of wickets	No of bowlers
20-60	7
60-100	5
100-150	16
150-250	12
250-350	2
350-450	3

TABLE 2.1.3

Solution: From the above table we can see that the friquency of the 2 is 3 which is more than any other no so the mode of the given data is 2.
codes for the above equations can be get from

```
codes/statexm/statexm4.py
```

5. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household: Find the

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

TABLE 2.1.5

mode of this data
Solution: In this table max frequency is 8 and modal class related to it is 3-5.

$$l = 3 \tag{2.1.5.1}$$
$$h = 2 \tag{2.1.5.2}$$
$$f_1 = 8 \tag{2.1.5.3}$$
$$f_0 = 7 \tag{2.1.5.4}$$
$$f_2 = 2 \tag{2.1.5.5}$$
$$Mode = l + \frac{f_1 - f_o}{2f_1 - f_o - f_2} \times h \tag{2.1.5.6}$$
$$= 3 + \frac{8-7}{2\times 8 - 7 - 2} \times 2 \tag{2.1.5.7}$$
$$= 3.28 \tag{2.1.5.8}$$

Related code is available in

```
codes/static5.py
```

6. The marks distribution of 30 students in a mathematics examination are given in Table 2.1.6 Find the mode of this data. Also compare and interpret the mode and the mean.

Class interval	No of student
10-25	2
25-40	3
40-55	7
55-70	6
70-85	6
85-100	6

TABLE 2.1.6

Solution: See Table 2.1.6. In this table max

Class interval	No of student	midpoint (x)	f.x
10-25	2	17.5	35
25-40	3	32.5	97.5
40-55	7	47.5	332.5
55-70	6	62.5	375
70-85	6	77.5	465
85-100	6	92.5	555

TABLE 2.1.6

friquency is 8 and modal class related to it is 3-5.

$$l = 40 \tag{2.1.6.1}$$
$$h = 15 \tag{2.1.6.2}$$
$$f_1 = 3 \tag{2.1.6.3}$$
$$f_0 = 7 \tag{2.1.6.4}$$
$$f_2 = 6 \tag{2.1.6.5}$$
$$Mode = l + \frac{f_1 - f_o}{2f_1 - f_o - f_2} \times h \tag{2.1.6.6}$$
$$= 40 + \frac{7-3}{2\times 7 - 6 - 3} \times 2 \tag{2.1.6.7}$$
$$= 52 \tag{2.1.6.8}$$

Related code is available in

```
./solutions/1−10/codes/statexm/sataexm6.py
```

7. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the data in Table 2.1.7 was obtained. Find the median height.
Solution: See Table 2.1.7.

$$Median = l + \frac{\frac{n}{2} - cf}{f} \times h \tag{2.1.7.1}$$

total no of girls n = 51
n/2 = 25.5
nearest class to the middle comulative friquency 25.5 = 145-150
lower limit l = 145
friquency of preceding class $f_2 = 11$
f = 18
h = 5

$$Median = 145 + \frac{25.5 - 11}{18} \times 5 \quad = 149.03 \tag{2.1.7.2}$$

Related code is available in

```
codes/statexm/statexm7.py
```

Height (in cm)	Number of girls
Less than 140	4
Less than 140	11
Less than 140	29
Less than 140	40
Less than 140	46
Less than 140	51

TABLE 2.1.7

Height(in cm)	No of girls (cf)	frequency (f)
<140	4	4
140-145	11	7
145-150	29	18
150-155	40	11
155-160	46	6
160-165	51	5

TABLE 2.1.7

8. The median of the data in Table 2.1.8 is 525. Find the values of x and y, if the total frequency is 100.
Solution: See Table 2.1.8
n = 100
implies

$$76 + x + y = 100 \tag{2.1.8.1}$$
$$x + y = 24 \tag{2.1.8.2}$$

class related to the Median 525 = 500-600 from anove we can say that

class interval	frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

TABLE 2.1.8

class interval	frequency	cumulitive friquency
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 +x
500-600	20	56 + x
600-700	y	56 +x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y

TABLE 2.1.8

$$l = 500 \tag{2.1.8.3}$$
$$h = 100 \tag{2.1.8.4}$$
$$f = 20 \tag{2.1.8.5}$$
$$cf = 36 + x \tag{2.1.8.6}$$

from eq 3.8.1.2 and 3.8.1.10

$$9 + y = 24 \quad (2.1.8.11)$$
$$y = 15 \quad (2.1.8.12)$$

Related code is available in

```
solutions/1−10/codes/statexm/statexm8.py
```

9. The annual profits earned by 30 shops of a shopping complex in a locality give rise to the distribution in Table 2.1.9. Draw both ogives for the data and obtain the median profit.

Profit (Rs in lakhs)	**Number of shops (frequency)**
≥ 5	30
≥ 10	28
≥ 15	16
≥ 20	14
≥ 25	10
≥ 30	7
≥ 35	3

TABLE 2.1.9

Solution: See Table 2.1.9.

profit	frequency	c.f
5-10	2	2
10-15	12	14
15-20	2	16
20-25	4	20
25-30	3	23
30-35	4	27
35-40	3	30

TABLE 2.1.9

we can get the mdian by using the ogive graphs.Two types of ogive graphs are used in this method one is less than type ogive and other one is more than type ogive.
Less than type ogive graph is drawn by using the coordinates of lower limit of class and corresponding comulative frequency.
More than type ogive graph is drawn by using coordinates of upper limit of class and corresponding cumulative frequency.
the crossing point of the both graph will give 'Median

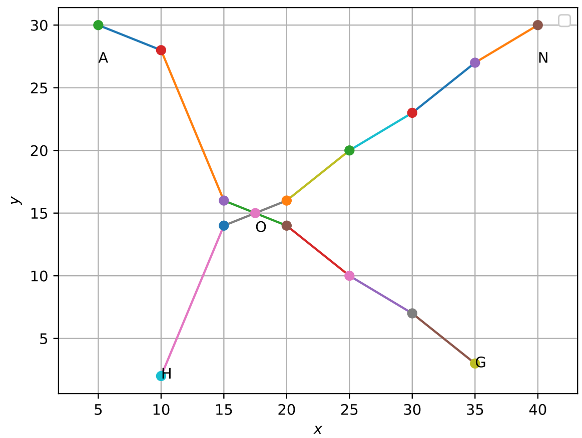

Fig. 2.1.9: less than and more than ogives

Related code is available in

```
solutions/1−10/codes/statexm/statexm9.py
```

10. Consider the marks obtained by 10 students in a mathematics test as given below:
42 25 78 75 62 55 36 95 73 60
Find the highest and the lowest marks.
11. Consider the marks obtained (out of 100 marks) by 30 students of Class IX of a school. Draw the freqeuncy distribution table.

10	20	36	92	95	40	50	56	60	70
92	88	80	70	72	70	36	40	36	40
92	40	50	50	56	60	70	60	60	80

12. 100 plants each were planted in 100 schools during Van Mahotsava. After one month, the number of plants that survived were recorded as follows. Draw the grouped distribution table in intervals 20-29, 30-39 etc...

95	67	28	32	65	65	69	33	98	96
76	42	32	38	42	40	40	69	95	92
75	83	76	83	85	62	37	65	63	42
89	65	73	81	49	52	64	76	83	92
93	68	52	79	81	83	59	82	75	82
86	90	44	62	31	36	38	42	39	83
87	56	58	23	35	76	83	85	30	68
69	83	86	43	45	39	83	75	66	83
92	75	89	66	91	27	88	89	93	42
53	69	90	55	66	49	52	83	34	36

13. Let us now consider the following frequency distribution in Table 2.1.13 which gives the weights of 38 students of a class. If two new students of weights 35.5 kg and 40.5 kg are admitted in the class, revise the distribution

table accordingly.

Weights(in kg)	Number of students
31-35	9
36-40	5
41-45	14
46-50	3
51-55	1
56-60	2
61-65	2
66-70	1
71-75	1
Total	38

TABLE 2.1.13

14. In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:

Observe the bar graph given above and answer the following questions:

(i) Many students were born in the month of November?
(ii)In which month were the maximum number of students born?

15. A family with a monthly income of ₹20,000 had planned the following expenditures per month under various heads:

Heads	Expenditure(in 1000₹)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

16. A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 - 20, 20 - 30, . . ., 60 - 70, 70 - 100. Then she formed the following table:

Marks	Number of students
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

17. Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table.

Marks	No.of Students
0-10	5
10-20	10
20-30	4
30-40	6
40-50	7
50-60	3
60-70	2
70-80	2
80-90	3
90-100	9
Total	51

Draw a frequency polygon corresponding to this frequency distribution table.

18. n a city, the weekly observations made in a study on the cost of living index are given in the following table:

Cost of living index	No. of weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2
Total	52

Draw a frequency polygon for the data above (without constructing a histogram).

19. 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively. Find the mean (or average) time in a week devoted by them for social work.

20. Find the mean of the marks obtained by 30 students of Class IX of a school, given below

10	20	36	92	95	40	50	56	60	70
92	88	80	70	72	70	36	40	36	40
92	40	50	50	56	60	70	60	60	80

21. The heights (in cm) of 9 students of a class are as follows:
155 160 145 149 150 147 152 144 148
Find the median of this data.
Solution: Let N be the no. of observations = 9
Arranging the heights in ascending order we get:
144,145,147,148,149,150,152,155,160

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} term = 5^{th} term \tag{2.1.21.1}$$

$\therefore$ the median is 149.

22. The points scored by a Kabaddi team in a series of matches are as follows:
17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28
Find the median of the points scored by the team.
Solution: Arranging the points scored by the team in ascending order we get:
2,5,7,7,8,8,10,10,14,15,17,18,24,27,28,48

Let N be the no. of observations = 16

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} \text{value} + \left(\frac{N}{2}+1\right)^{th} \text{value}}{2} \tag{2.1.22.1}$$

$$\text{Median} = \frac{(8)^{th} \text{value} + (9)^{th} \text{value}}{2} \tag{2.1.22.2}$$

$$\text{Median} = \frac{10+14}{2} \tag{2.1.22.3}$$

$$\text{Median} = 12 \tag{2.1.22.4}$$

23. Find the mode of the following marks (out of 10) obtained by 20 students:
4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9
Solution:

Mark	Frequency
2	1
3	2
4	3
5	2
6	3
7	3
9	4
10	2

TABLE 2.1.23: Marks obtained by students

As we can see from table 2.1.23, 9 occurs the maximum number of times.
Thus Mode = 9.

24. Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹5,000 per month each while the supervisor gets ₹15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.
Solution:
a) Finding Mean

Mean salary =

$$\frac{\text{Supervisor's salary} + 4 \times \text{Labourer's salary}}{5}$$

$$= \frac{15000 + 5000 + 5000 + 5000 + 5000}{5} \tag{2.1.24.1}$$

$\therefore$ the mean salary is 7000.

b) Finding Median
We need to arrange the salaries in ascending order.Thus we get 5000,5000,5000,5000,15000
Let N = no.of employees = 5

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} term = 3^{rd} value \tag{2.1.24.2}$$

$\therefore$ the median is 5000.

c) Finding Mode
Mode is the highest occurring frequency of the distribution. 5000 is the most repeating salary.
$\therefore$ Modal salary is 5000.

2.2 Exercises

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data in Table 2.2.1 regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

TABLE 2.2.1

Solution:

2. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method.
3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

Daily pocket allowance(in rupees)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	f	5	4

4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

Number of heartbeats for minute	65-68	68-71	71-
Number of women	2	4	3

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50-52	53-55	56-58	59-61
Number of boxes	15	110	135	115

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure(in rupees)	100-150	150-200	20
Number of households	4	5	

Find the mean daily expenditure on food by a suitable method.

7. To find out the concentration of so_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	0.00-0.04	0.04-0.08
Frequency	4	9

Find the mean concentration of SO_2 in the air.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0-6	6-10	10-14	14-20	20-
Number of students	11	10	7	4	4

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate. **EXERCISE 14.2**

10. The following table shows the ages of the patients admitted in a hospital during a year: Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.
11. The following data gives the information on the observed lifetimes (in hours)

of 225 electrical components :

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

12. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in rupees)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000
Number of families	24	40	33	28	30	22	

13. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Number of states / U.T.	3	8	9	10	3	0	0	2

14. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10
Number of batsmen	4	18	9	7	6	3	1

Find the mode of the data.

15. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

EXERCISE 14.3

16. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

17. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

18. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years on wards but less than 60 year.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50
Number of policy holders	2	6	24	45		78	89

19. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	14
Number of leaves	2	6	24	

Find the median length of the leaves. (Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, . . ., 171.5 - 180.5.)

20. The distribution below in Table 2.2.20 gives the weights of 30 students of a class. Find the median weight of the students. **Solution:**

weight	no.of.student	Cum.Freq
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
total	30	

TABLE 2.2.20: Frequency distribution of the weights of students

The following python code computes the mean, median and mode.

```
codes/statistics/exercises/q22.py
```

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \tag{2.2.20.1}$$

$$n = \sum f_i = 100 \implies \frac{n}{2} = 50 \tag{2.2.20.2}$$

(2.2.20.3)

$\therefore$ 55-60 is the median class.

Here l is the lower limit of the median class = 55

h is the classinterval =5

cf is the cumulative frequency of the class before median class = 13

f is the frequency of the median class

$$\text{Median} = 55 + \frac{15-13}{6} \times 5 \tag{2.2.20.4}$$

$$\text{Median} = 55 + 1.67 = 56.67 \tag{2.2.20.5}$$

Hence median weight is 56.67

21. The following distribution in Table 2.2.21 gives the daily income of 50 workers of a factory. Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Dailywages	workers	Cum.Freq
100-120	12	12
120-140	14	12+14=26
140-160	8	26+8=34
160-180	6	34+6=40
180-200	10	40+10=50
Total		50

TABLE 2.2.21: Wages obtained by workers

Solution:
The following python code generates the required ogive.

```
./solutions/20−30/codes/statistics/
exercises/q23.py
```

wages	workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

TABLE 2.2.21: Wages obtained using less than cumulative frequency

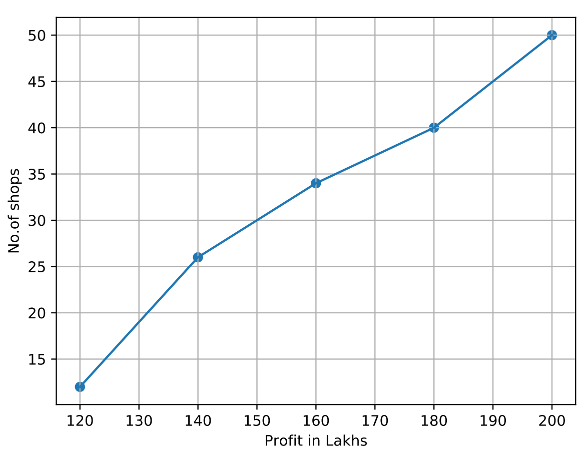

Fig. 2.2.21: Ogive of Q.23

22. During the medical check-up of 35 students of a class, their weights were recorded as in Table 2.2.22. Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Weight	No.of.student
<38	0
<40	3
<42	5
<44	9
<46	14
<48	28
<50	32
<52	35

TABLE 2.2.22: Frequency distribution of the weights of students

Solution: The following python code computes the median .

```
codes/statistics/exercises/q24.py
```

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \tag{2.2.22.1}$$

$$n = \sum f_i = 100 \implies \frac{n}{2} = 50 \tag{2.2.22.2}$$

$$\tag{2.2.22.3}$$

∴ 46-48 is the median class.
Here l is the lower limit of the median class = 46
h is the classinterval =2
cf is the cumulative frequency of the class before median class = 14
f is the frequency of the median class =14

$$\text{Median} = 46 + \frac{17.5 - 14}{14} \times 2 \qquad (2.2.22.4)$$

$$\text{Median} = 46 + 0.5 = 46.5 \qquad (2.2.22.5)$$

Hence median weight is 46.5

weight	No.of.student	Cum.Freq
0-38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

TABLE 2.2.22: Frequency distribution of the weights of students

23. The following table gives production yield per hectare of wheat of 100 farms of a village. Change the distribution to more than type distribution and draw its ogive.

Prodn.Yield	No.of.farms
50-55	2
55-60	8
60-65	12
65-70	24
70-75	38
75-80	16
total	100

TABLE 2.2.23: production yield per hectare of wheat of 100 farms

Solution:
Given the production yield per hectare of wheat of 100 farms of a village. The following python code generates the required ogive.

```
./solutions/20−30/codes/statistics/
exercises/q25.py
```

24. Give five examples of data that you can collect from your day-to-day life.

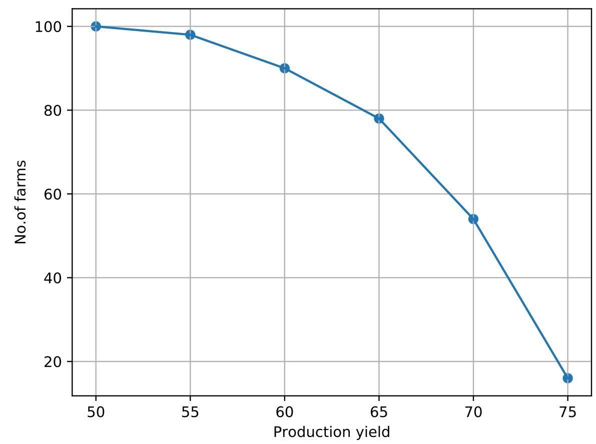

Fig. 2.2.23

Prodn.yield	No.of.farms
More than 50	100
More than 55	100-2=98
More than 60	98-8=90
More than 65	90-12=78
More than 70	78-24=54
More than 75	54-38=16

TABLE 2.2.23: production yield using more than cumulative frequency

25. Classify the data in Q.1 above as primary or secondary data.

26. The blood groups of 30 students of Class VIII are recorded as follows:
A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.
Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?
Solution: As we can see from table 2.2.26, the most common blood group is O and the rarest blood group is AB.

27. The distance (in km) of 40 engineers from their residence to their place of work were found as follows: 5,3,10,20, 25, 11, 13, 7, 12, 31,19, 10, 12, 17, 18, 11, 32, 17, 16, 2,7,9,7,8,3,5,12, 15, 18 ,3,12, 14, 2,9,6, 15, 15, 7,6, 12.Construct

BloodGroup	No.of.Student
A	9
B	6
AB	3
O	12
Total	30

TABLE 2.2.26

a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

Solution: See Table 2.2.27. Since the minimum =2 and maximum = 32, we take the intervals as 0-5,5-10 and so on upto 30-35. It can be observed that there are very few engineers whose homes are at more than or equal to 20km distance from their work place and most engineers have their workplace at distance of 0km to 15km from their homes.

Distance(km)	No.of.engineers
0-5	5
005-10	11
10-15	11
15-20	9
20-25	1
25-30	1
30-35	2
Total	40

TABLE 2.2.27: distance of engineers from their residence to their place of work

28. The relative humidity (in %) of a certain city for a month of 30 days was as follows:
(i) Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc.
(ii) Which month or season do you think this data is about?
(iii) What is the range of this data?

Solution: See Table 2.2.28 . Since the minimum =84.9 and maximum = 99.2, we take the intervals as 84-86,86-88 and so on upto 98-100. As the relative humidity is high,

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89.0

TABLE 2.2.28: Relative Humidity

the data is about rainy season.

$$\text{Range} = \text{Maximum value} - \text{Minimum value} \tag{2.2.28.1}$$

$$\text{Range} = 99.2 - 84.9 \tag{2.2.28.2}$$

$$\text{Range} = 14.3 \tag{2.2.28.3}$$

RelativeHumidity	NO.of.days
84-86	1
86-88	1
88-90	2
90-92	2
92-94	7
94-96	6
96-98	7
98-100	4
Total	30

TABLE 2.2.28: Relative humidity in %

29. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

(i) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.
(ii) What can you conclude about their heights from the table?

30. A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is listed in Table 2.2.30
    a) Make a grouped frequency distribution table for this data with class intervals as 0.00-0.04, 0.04-0.08, and so on.
    b) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

0.03	0.08	0.04
0.16	0.02	0.18
0.11	0.12	0.22
0.08	0.1	0.09
0.11	0.05	0.01
0.08	0.09	0.17
0.05	0.06	0.2
0.08	0.13	0.07
0.01	0.06	0.18
0.07	0.07	0.04

TABLE 2.2.30: Concentrations of sulphur dioxide in air in ppm for 30 days

Solution: Least value=0.01
Greatest value=0.23
class interval=0.04
The grouped frequency distribution Table

Class	frequency
0.0-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.2	4
0.2-0.24	2

TABLE 2.2.30: Grouped frequency distribution table for the data in 2.2.30

2.2.30 is construced using the python code

```
solutions/20−10/stat/codes/exer32.py
```

From the table 2.2.30, The sulphur dioxide concentration was greater than 0.11ppm for 8 days.

31. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down asin Table 2.2.31
Prepare a frequency distribution table for the data given above.
Solution: The possible values are 0,1,2,3.
The frequency distribution table is given in table 2.2.31. The frequency distribution table 2.2.31 is constructed using the python code

```
./solutions/20−10/stat/codes/exer33.py
```

0	1	2	2	1
1	3	1	1	2
3	0	0	1	1
2	3	1	3	0
2	0	1	2	1
2	3	2	2	0

TABLE 2.2.31: Number of heads obtained when 3 coins were tossed 30 times

no of heads	Frequency
0	6
1	10
2	9
3	5
Total	30

TABLE 2.2.31: Frequency distribution table for the data in 2.2.31

32. The value of π upto 50 decimal places is given below: 3.14159265358979323846264338327950288419716939937510
(i)Make a frequency distribution of the digits from 0 to 9 after the decimal point.
(ii) What are the most and the least frequently occurring digits?
Solution: The 50 digits appearing after the decimal point of π are tabulated as follows:
The frequency distribution table for the digits

1	4	1	5	9	2	6	5	3	5
8	9	7	9	3	2	3	8	4	6
2	6	4	3	3	8	3	2	7	9
5	0	2	8	8	4	1	9	7	1
6	9	3	9	9	3	7	5	1	0

TABLE 2.2.32: The 50 decimal places of π

0-9 is given in table 2.2.32
From the table 2.2.32,
0 is the least frequent digit
3,9 are the most frequent digits.
The python code for above problem is

```
./solutions/20−10/stat/codes/exer34.py
```

no of heads	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

TABLE 2.2.32: Frequency distribution table for the numbers in 2.2.32

33. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as in Table 2.2.33

1	6	2	3	5	12
5	8	4	8	10	3
4	12	2	8	15	1
17	6	3	2	8	5
9	6	8	7	14	12

TABLE 2.2.33: Number of hours 30 children spent watching TV in a week

(i) Make a grouped frequency distribution table for this data,taking class width 5 and one of the class intervals as 5-10.
(ii) How many children watched television for 15 or more hours a week?
Solution: Maximum number of hours=17
class interval=5
Since one of the class intervals must be 5-10 the class intervals are taken as 0-5,5-10,10-15,15-20. The grouped frequency distribution table 2.2.33 is construced using the python code

```
./solutions/20−10/stat/codes/exer35.py
```

Class	frequency
0-5	10
5-10	13
10-15	5
15-20	2
Total:	30

TABLE 2.2.33: Grouped frequency distribution table for the data in Table2.2.33

From the table 2.2.33, 2 children watched TV for more than or equal to 15 hours a week.

34. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as in Table 2.2.34

2.6	3	3.7	3.2	2.2
4.1	3.5	4.5	3.5	2.3
3.2	3.4	3.8	3.2	4.6
3.7	2.5	4.4	3.4	3.3
2.9	3	4.3	2.8	3.5
3.2	3.9	3.2	3.2	3.1
3.7	3.4	4.6	3.8	3.2
2.6	3.5	4.2	2.9	3.6

TABLE 2.2.34: Lives of 40 batteries

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.
Solution: Starting interval=2-2.5
class interval=0.5
Greatest value=4.6
Therefor end interval=4.5-5
The grouped frequency distribution table

Class	frequency
2.0-2.5	2
2.5-3.0	6
3.0-3.5	14
3.5-4.0	11
4.0-4.5	4
4.5-5.0	3
Total	40

TABLE 2.2.34: Grouped frequency distribution table for car-battery lives

2.2.34 is construced using the python code

```
./stat/codes/exer36.py
```

35. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %) in Table 2.2.35

Causes	**Fatality rate(%)**
Reproductive health cond.	31.8
Neuropsychiatric cond.	25.4
Injuries	12.4
Cardiovascular conditions	4.3
Respiratory conditions	4.1
Other causes	22

TABLE 2.2.35: Illness and fatality rate amongst women

(i) Represent the information given above graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.
Solution: (i) The bar graph representing the above data is shown in figure 2.2.35
(ii)Reproductive health conditions is the major

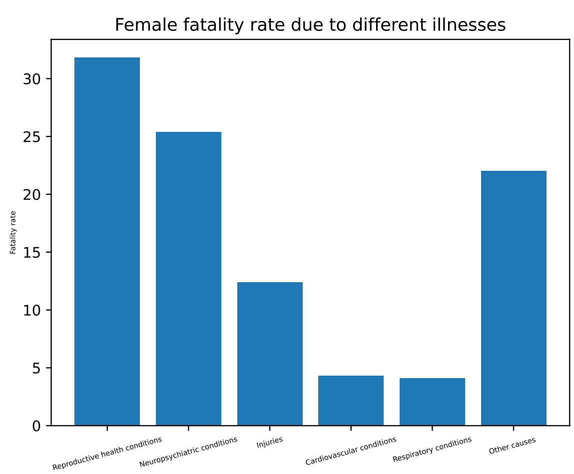

Fig. 2.2.35: Illnesses and respective fatality rates amongst women

cause of women's illness and death.
(iii)Lack of awareness and understanding about reproductive health results in high female fatality rate due to reproductive health conditions.

The python code used to generate the bar graph 2.2.35 is

```
./solutions/20−10/stat/codes/exer37.py
```

36. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below in Table 2.2.36

Section	**girls/1000 boys**
Scheduled caste (SC)	940
Scheduled Tribe (ST)	970
Non-SC/ST	920
Backward districts	950
Non-backward district	920
Rural	930
Urban	910

TABLE 2.2.36: Number of girls per 1000 boys

(i) Represent the information above by a bar graph.
(ii) In the classroom discuss what conclusions can be arrived at from the graph.
Solution: The bar graph representing the above data is shown in figure 2.2.36
The python code for the above problem is

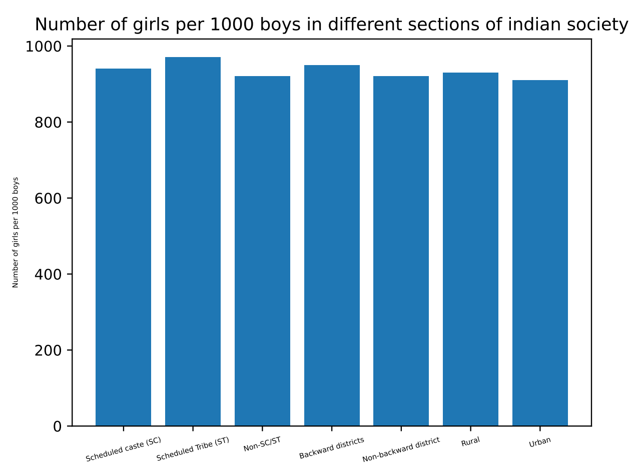

Fig. 2.2.36: Number of girls per 1000 boys

```
./solutions/20−10/stat/codes/exer38.py
```

37. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political party	Seats won
A	75
B	55
C	37
D	29
E	10
F	37

TABLE 2.2.37: Illness and fatality rate amongst women

(i) Draw a bar graph to represent the polling results.
(ii) Which political party won the maximum number of seats?
Solution: (i) The bar graph representing the above data is shown in figure 2.2.37
(ii)From the graph in 2.2.37, Political party

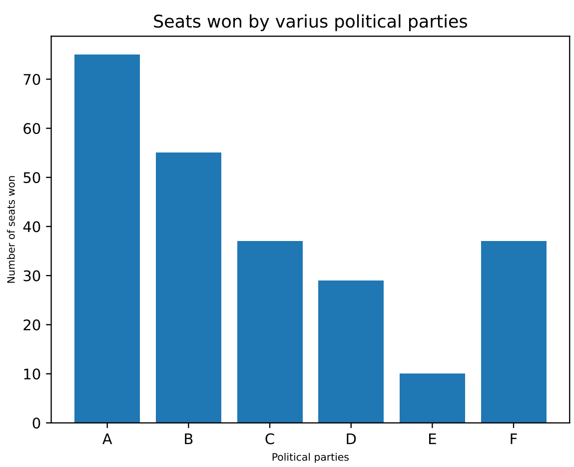

Fig. 2.2.37: Seats won by different political parties

A won the maximum seats The python code used to generate the bar graph for the above problem is

```
./solutions/20−10/stat/codes/exer39.py
```

38. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:
(i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]
(ii) Is there any other suitable graphical representation for the same data?
(iii) Is it correct to conclude that the maximum

Length (mm)	No. of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

TABLE 2.2.38: Lengths of 40 leaves in mm

number of leaves are 153 mm long? why?
Solution: (i)To draw a histogram, the data must be made continuous.

$$Gap/2 = \frac{127 - 126}{2} = \frac{1}{2} = 0.5 \qquad (2.2.38.1)$$
$$(2.2.38.2)$$

So we add 0.5 to every upperclass limit and subtract 0.5 from every lower class limit to obtain continous grouped frequency distribution table as shown in Table2.2.38 The histogram

Length (mm)	No. of leaves
117.5-126.5	3
126.5-135.5	5
135.5-144.5	9
144.5-153.5	12
153.5-162.5	5
162.5-171.5	4
171.5-180.5	2

TABLE 2.2.38: Illness and fatality rate amongst women

is shown in figure 2.2.38 The below python code was used o generate the histogram 2.2.38

```
./solutions/20−10/stat/codes/exer40.py
```

(ii)The same data can also be represented using a frequency polygon.
(iii)No, the maximum leaves have lengths ranging from 144.5mm to 153.5mm and might not be equal to 153mm.

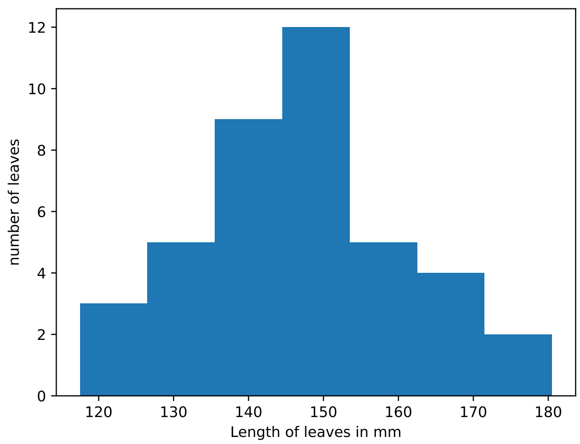

Fig. 2.2.38: Length of leaves in mm

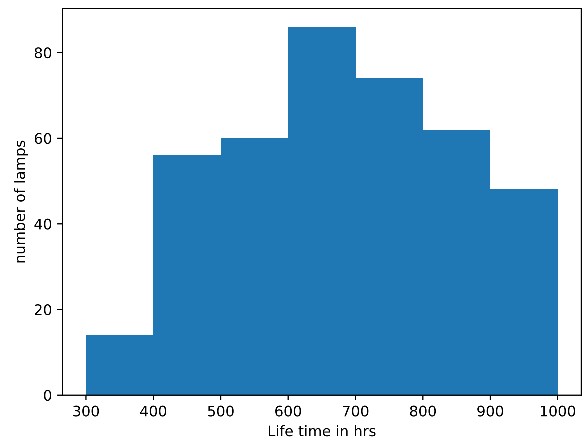

Fig. 2.2.39: Lives of neon lamps

39. The following table gives the life times of 400 neon lamps:
    (i) Represent the given information with the help of a histogram.

Life time in hrs	No. of lamps
300-400	14
400-500	56
500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48
Total	**400**

TABLE 2.2.39: Lives of neon lamps

    (ii) How many lamps have a life time of more than 700 hours?
    Solution: (i)The data is already continuous. The histogram is created using the following python code

```
./solutions/20−10/stat/codes/exer41.py
```

    The histogram is shown in figure 2.2.39 (ii)184 lamps have a life time of more than 700 hours.
40. The following table gives the distribution of students of two sections according to the marks obtained by them: Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

Section A		Section B	
Marks	**Frequency**	**Marks**	**Frequency**
0-10	3	0-10	5
10-20	9	10-20	19
20-30	17	20-30	15
30-40	12	30-40	10
40-50	9	40-50	1

    Solution: The figure 2.2.40 represents the frequency polygon of two sections. From the graph the following can be drawn

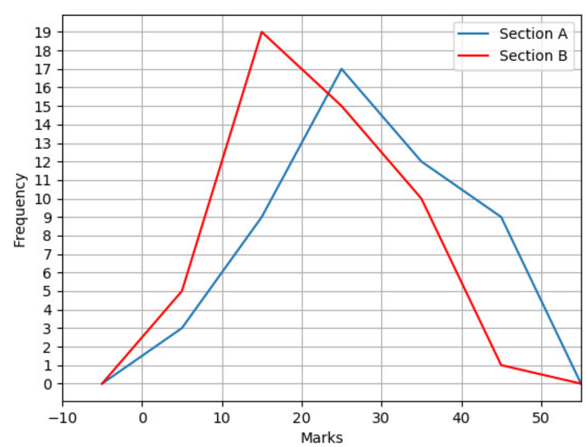

Fig. 2.2.40: Frequency polygon of Section A and Section B

a) Section A has more number of students who got more marks in the range 30-50.
b) Section B has more students in the marks range of 0-20 than Section A.
c) Section A has performed better than Section B.

41. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below: Represent the data of both the teams on

No.of balls	Team A	Team B
1? 6	2	5
7 – 12	1	6
13-18	8	2
19-24	9	10
25-30	4	5
31-36	5	6
37-42	6	3
43-48	10	4
49-54	6	8
55-60	2	10

the same graph by frequency polygons.
Solution: First we need to make the class intervals continuous. For this we need to subtract and add 0.5 to lower class limit and upper class limit respectively. The continuous class intervals are represented in 2.2.41. The frequency

No.of balls	Team A	Team B
0.5- 6.5	2	5
6.5-12.5	1	6
12.5-18.5	8	2
18.5-24.5	9	10
24.5-30.5	4	5
30.5-36.5	5	6
36.5-42.5	6	3
42.5-48.5	10	4
48.5-54.5	6	8
54.5-60.5	2	10

TABLE 2.2.41: Continuous class intervals of given data

polygons of given data set is represented in fig. 2.2.41 The python code for the plot can be downloaded from

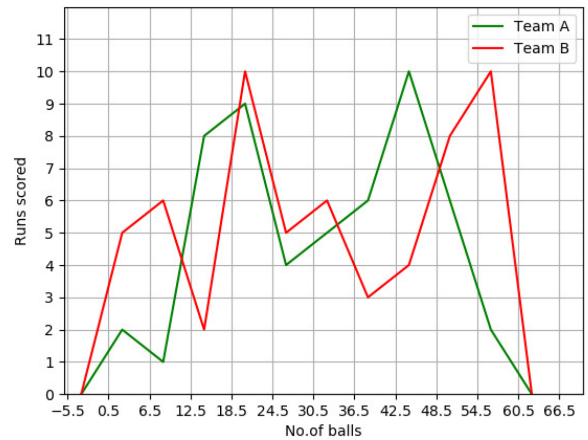

Fig. 2.2.41: Frequency polygon of Team A and Team B

solutions/40−50/statistics/codes/Q43.py

42. A random survey of the number of children of various age groups playing in a park was found as follows: Draw a histogram to plot the above

Age (in years)	No.of children
1- 2	5
2- 3	3
3- 5	6
5- 7	12
7- 10	9
10- 15	10
15- 17	4

data.
Solution: The histogram of given data set is represented in Fig. 2.2.42 The python code for the plot can be downloaded from

solutions/40−50/statistics/codes/Q44.py

43. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:
a) Draw a histogram to depict the given information.
b) Write the class interval in which the maximum number of surnames lie.

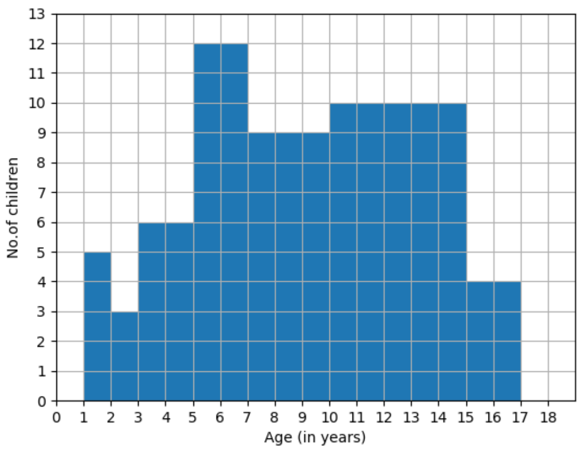

Fig. 2.2.42: Histogram showing No.of children vs. age(in years)

No.of letters	No.of surnames
1- 4	6
4- 6	30
6- 8	44
8- 12	16
12- 20	4

Solution: The histogram for the above data is plotted in the Fig. 2.2.43

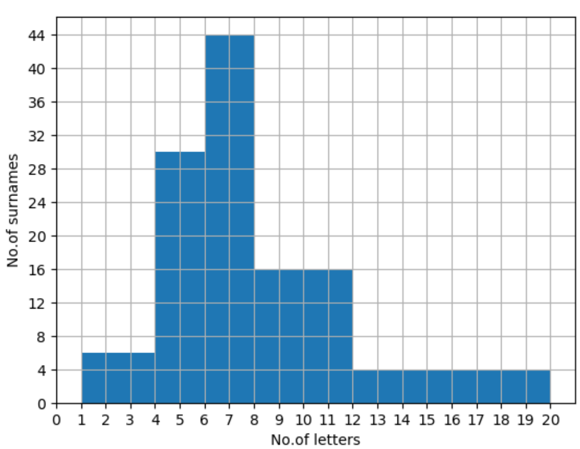

Fig. 2.2.43: Histogram of no.of surnames vs. no.of letters

From the graph, we can easily say that the maximum number of surnames is 44 which lies in the interval of 6 - 8. Download the python code for the figure from

solutions/40−50/statistics/codes/Q45.py

44. The following number of goals were scored by a team in a series of 10 matches:
2, 3, 4, 5, 0, 1, 3, 3, 4, 3
Find the mean, median and mode of these scores. **Solution:** First we will sort the given data in ascending order. 0,1,2,3,3,3,3,4,4,5 Let n= No.of elements in the given data= 10.

$$Mean = \frac{Sum of data values}{n} \quad (2.2.44.1)$$
$$= \frac{0+1+2+3+3+3+3+4+4+5}{10} \quad (2.2.44.2)$$
$$= 2.8 \quad (2.2.44.3)$$
$$Mode = 3 \quad (2.2.44.4)$$

Since n is even , the median is given by the average of $\left(\frac{n}{2}\right)^{th}$ and$\left(\frac{n}{2}+1\right)^{th}$ element.

$$Median = \frac{3+3}{2} \quad (2.2.44.5)$$
$$= 3 \quad (2.2.44.6)$$

The python code to calculate mean, median , mode for the given data can be downloaded from

solutions/40−50/statistics/codes/Q46.py

45. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data. **Solution:** First we will sort the given data in ascending order. 39,40,40,41,42,46,48,52,52,52,54,60,62,96,98
Let n= No.of elements in the given data= 15.

$$Mean = \frac{Sum of data values}{n} \quad (2.2.45.1)$$
$$= 54.8 \quad (2.2.45.2)$$
$$Mode = 52 \quad (2.2.45.3)$$

Since n is odd , the median is the $\left(\frac{n+1}{2}\right)^{th}$ element.

$$Median = 52 \quad (2.2.45.4)$$

The python code to calculate mean, median , mode for the given data can be downloaded from

solutions/40−50/statistics/codes/Q47.py

46. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.
29, 32, 48, 50, x, x + 2, 72, 78, 84, 95.
Solution: The no.of elements in the given data=n= 10 Since n is even , the median is given by the average of $\left(\frac{n}{2}\right)^{th}$ and$\left(\frac{n}{2}+1\right)^{th}$ element.

$$\frac{x+x+2}{2} = 63 \tag{2.2.46.1}$$
$$x = 62 \tag{2.2.46.2}$$

47. Find the mode of
14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.
Solution: Mode is the value which occurs for the maximum time. Here the mode is 14. The python code to find mode can be downloaded from

solutions/40−50/statistics/codes/Q49.py

48. Find the mean salary of 60 workers of a factory from the following table:

Salary (in Rs.)	**No.of Workers**
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

TABLE 2.2.48

Solution: To find the mean, we need to multiply Salary and No.of workers. This can be seen in given table 2.2.48 :

$$Mean = \frac{305000}{60} \tag{2.2.48.1}$$
$$= 5083.33 \tag{2.2.48.2}$$

Salary (in Rs.)	**No.of Workers**	**Salary X No.of workers**
3000	16	48000
4000	12	48000
5000	10	50000
6000	8	48000
7000	6	42000
8000	4	32000
9000	3	27000
10000	1	10000
Total	60	305000

TABLE 2.2.48: Finding mean of the given data

49. Give one example of a situation in which
a) the mean is an appropriate measure of central tendency.
b) the mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.

Solution:

a) When the data set has normal distribution with no skewness, such that the adjacent values dont show bigger deviation from adjacent values then in that case mean is an appropriate measure of central tendency. Example: Birth weight, Height(over a large population)
b) When the skewness increases the difference between mean and median increases. Hence median becomes the appropriate measure of central tendency. Example: Income distribution in a developing country.