Constructions using Python

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Contents

***Abstract*—This book introduces constructions in high school geometry using Python. The content and exercises are based on NCERT textbooks from Class 6-12. A simple introduction to Python and LaTeXfigures is provided in the process.**

Download all python codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/constructions/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/constructions/figs
```

1 Examples

1.1. Draw Fig. 1.1 for $a = 4, c = 3$.
Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{1.1.1}$$

The python code for Fig. 1.1 is

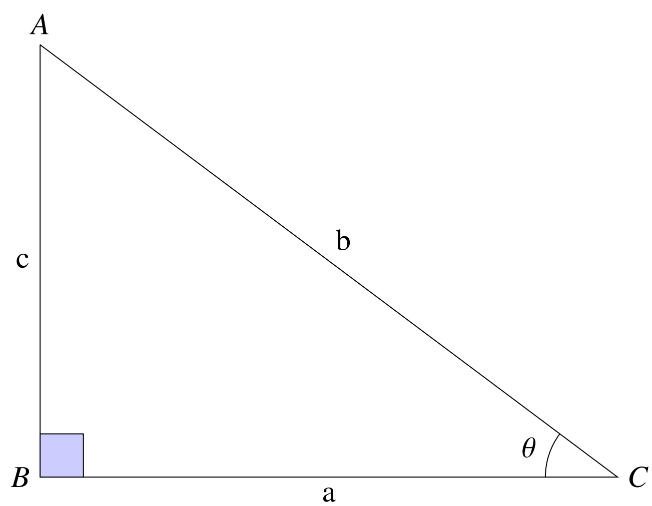

Fig. 1.1: Right Angled Triangle

```
codes/triangle/tri_right_angle.py
```

and the equivalent latex-tikz code is

```
figs/triangle/tri_right_angle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle/tri_right_angle_alone.tex
```

1.2. Draw Fig. 1.2 for $a = 4, c = 3$.
Solution: The vertex $\mathbf{A}$ can be expressed in

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polar coordinate form as

$$\mathbf{A} = b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{1.2.1}$$

where

$$b = \sqrt{a^2+c^2} = 5, \tan\theta = \frac{3}{4} \tag{1.2.2}$$

The python code for Fig. 1.2 is

codes/triangle/tri_polar.py

and the equivalent latex-tikz code is

figs/triangle/tri_polar.tex

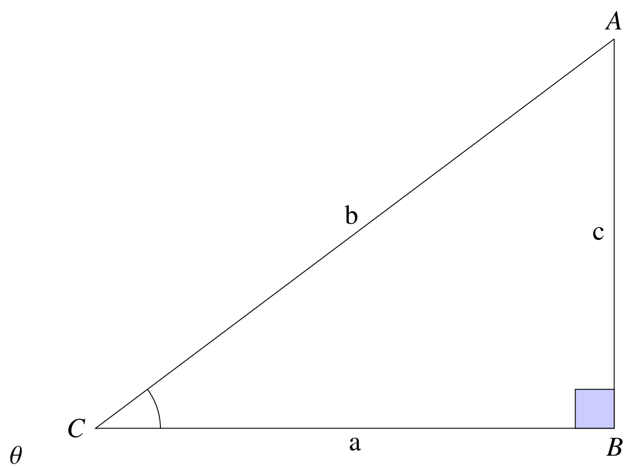

Fig. 1.2: Right Angled Triangle

1.3. Draw Fig. 1.3 with $a = 6$, $b = 5$ and $c = 4$.

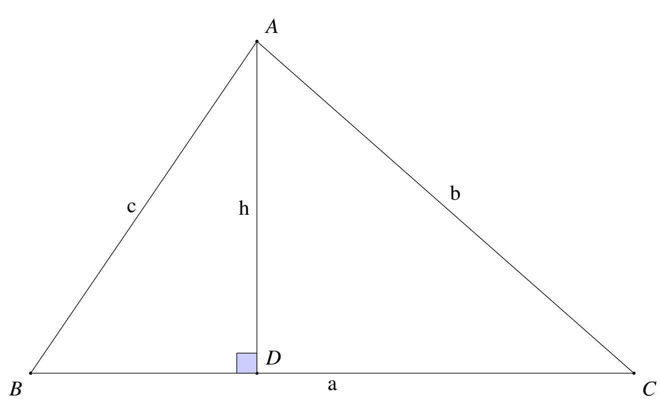

Fig. 1.3

Solution: Let the vertices of $\triangle ABC$ and $\mathbf{D}$ be

$$\mathbf{A} = \begin{pmatrix}p\\ q\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a\\ 0\end{pmatrix}, \mathbf{D} = \begin{pmatrix}p\\ 0\end{pmatrix} \tag{1.3.1}$$

Then

$$AB = \|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \tag{1.3.2}$$

$$BC = \|\mathbf{C}-\mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \tag{1.3.3}$$

$$AC = \|\mathbf{A}-\mathbf{C}\|^2 = b^2 \tag{1.3.4}$$

From (1.3.4),

$$b^2 = \|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{C}\|^T\|\mathbf{A}-\mathbf{C}\| \tag{1.3.5}$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \tag{1.3.6}$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad \left(\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A}\right) \tag{1.3.7}$$

$$= a^2 + c^2 - 2ap \tag{1.3.8}$$

yielding

$$p = \frac{a^2+c^2-b^2}{2a} \tag{1.3.9}$$

From (1.3.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \tag{1.3.10}$$

$$\implies q = \pm\sqrt{c^2-p^2} \tag{1.3.11}$$

The python code for Fig. 1.3 is

codes/triangle/tri_sss.py

and the equivalent latex-tikz code is

figs/triangle/tri_sss.tex

1.4. Construct parallelogram $ABCD$ in Fig. 1.4 given that $BC = 5, AB = 6, \angle C = 85°$.
Solution: BD is found using the cosine formula and $\triangle BDC$ is drawn using the approach in Construction 1.3 with

$$\mathbf{B} = \begin{pmatrix}0\\ 0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}5\\ 0\end{pmatrix}, \tag{1.4.1}$$

Since the diagonals bisect each other,

$$\mathbf{O} = \frac{\mathbf{B}+\mathbf{D}}{2} \tag{1.4.2}$$

$$\mathbf{A} = 2\mathbf{O} - \mathbf{C}. \tag{1.4.3}$$

AB and AD are then joined to complete the ||gm. The python code for Fig. 1.4 is

codes/quad/pgm_sas.py

and The equivalent latex-tikz code is

figs/quad/pgm_sas.tex

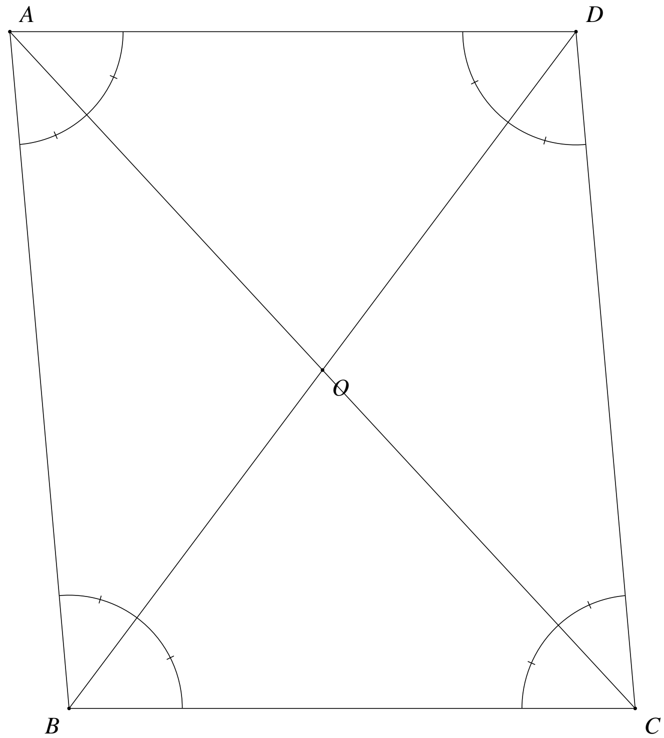

Fig. 1.4: Parallelogram Properties

1.5. Draw the ||gm $ABCD$ in Fig. 1.5 with $BC = 6, CD = 4.5$ and $BD = 7.5$. Show that it is a rectangle.

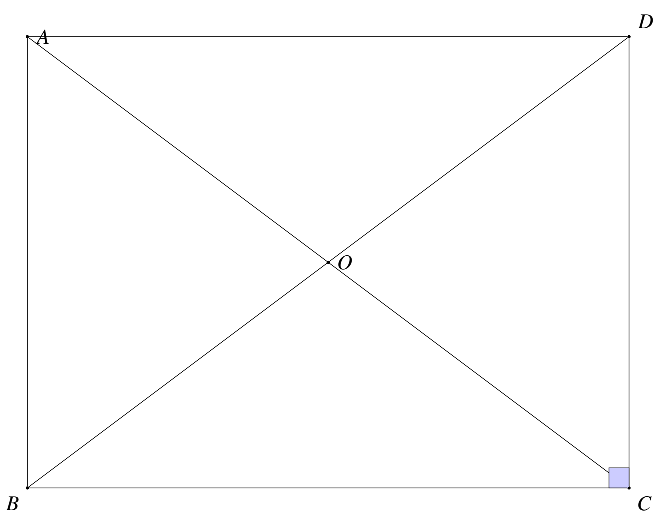

Fig. 1.5: Rectangle

Solution: It is easy to verify that

$$BD^2 = BC^2 + C^2 \tag{1.5.1}$$

Hence, using Baudhayana theorem,

$$\angle BCD = 90^\circ \tag{1.5.2}$$

and $ABCD$ is a rectangle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \tag{1.5.3}$$

The python code for Fig. 1.5 is

```
codes/quad/pgm_sss.py
```

and the equivalent latex-tikz code is

```
figs/quad/pgm_sss.tex
```

1.6. Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.

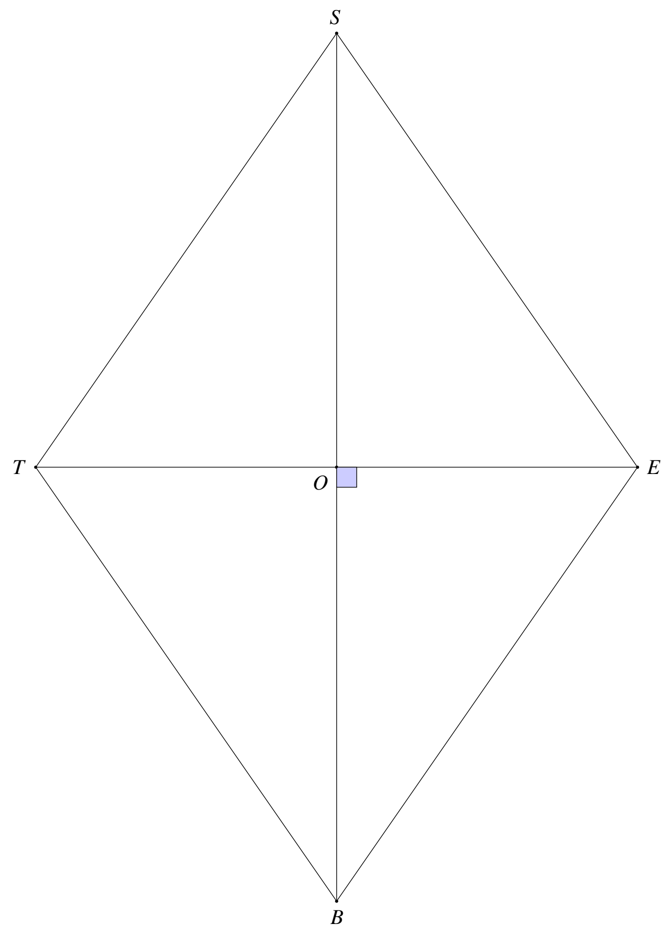

Fig. 1.6: Rhombus

Solution: The coordinates of the various points in Fig. 1.6 are obtained as

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -4.5 \end{pmatrix} \tag{1.6.1}$$

$$\mathbf{E} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{1.6.2}$$

1.7. A square is a rectangle whose sides are equal. Draw a square of side 4.5.

Solution: The coordinates of the various points in Fig. 1.7 are obtained as

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4.5 \end{pmatrix} \tag{1.7.1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix} \mathbf{O} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{1.7.2}$$

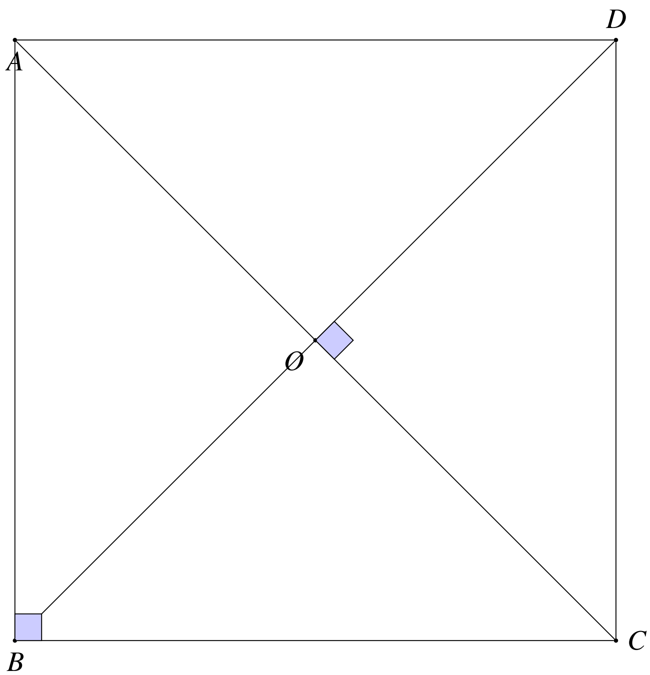

Fig. 1.7: Square

2 Exercises

2.1. Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.

2.2. Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = h = 4$cm

2.3. In $\triangle ABC$, given that $a+b+c = 11, \angle B = 45^{\circ}$ and $\angle C = 45^{\circ}$, find a, b, c and sketch the triangle.

2.4. Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.

2.5. Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.

2.6. $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.

2.7. In $\triangle ABC$, $a = 8, \angle B = 45^{\circ}$ and $c - b = 3.5$. Sketch $\triangle ABC$.

2.8. In $\triangle ABC$, $a = 6, \angle B = 60^{\circ}$ and $b-c = 2$. Sketch $\triangle ABC$.

2.9. Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^{\circ}$ and $\angle C = 90^{\circ}$.

2.10. Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.

2.11. Draw an equilateral triangle of side 5.5.

2.12. Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?

2.13. Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.

2.14. Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60°$.

2.15. Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90°$.

2.16. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°.

2.17. Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60°$.

2.18. Construct $\triangle XYZ$ if $XY = 6, \angle X = 30°$ and $\angle Y = 100°$.

2.19. If $AC = 7, \angle A = 60°$ and $\angle B = 50°$, can you draw the triangle?

2.20. Construct $\triangle ABC$ given that $\angle A = 60°, \angle B = 30°$ and $AB = 5.8$.

2.21. Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105°$ and $\angle R = 40°$.

2.22. Can you construct $\triangle DEF$ such that $EF = 7.2, \angle E = 110°$ and $\angle F = 180°$?

2.23. Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.

2.24. Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.

2.25. Construct right angled $\triangle$ whose hypotenuse is 6 and one of the legs is 4.

2.26. Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.

2.27. Construct the triangles in Table 2.27.

2.28. Construct a quadrilateral $ABCD$ such that $AB = 5, \angle A = 50°, AC = 4, BD = 5$ and $AD = 6$.

2.29. Construct $PQRS$ where $PQ = 4, QR = 6, RS = 5, PS = 5.5$ and $PR = 7$.

2.30. Draw $JUMP$ with $JU = 3.5, UM = 4, MP = 5, PJ = 4.5$ and $PU = 6.5$

2.31. Construct a quadrilateral $ABCD$ such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7$ and $AD = 5.5$.

2.32. Can you construct a quadrilateral $PQRS$ with $PQ = 3, RS = 3, PS = 7.5, PR = 8$ and $SQ = 4$?

2.33. Construct $LIFT$ such that $LI = 4, IF = 3, TL =$

S.No	Triangle	Given Measurements		
1	$\triangle$ABC	$\angle A = 85°$	$\angle B = 115°$	AB = 5
2	$\triangle$PQR	$\angle Q = 30°$	$\angle R = 60°$	QR = 4.7
3	$\triangle$ABC	$\angle A = 70°$	$\angle B = 50°$	AC = 3
4	$\triangle$LMN	$\angle L = 60°$	$\angle N = 120°$	LM = 5
5	$\triangle$ABC	BC = 2	AB = 4	AC = 2
6	$\triangle$PQR	PQ = 2.5	QR = 4	PR = 3.5
7	$\triangle$XYZ	XY = 3	YZ = 4	XZ = 5
8	$\triangle$DEF	DE = 4.5	EF = 5.5	DF = 4

TABLE 2.27

$2.5, LF = 4.5, IT = 4$.

2.34. Draw $GOLD$ such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.

2.35. DRAW rhombus $BEND$ such that $BN = 5.6$, $DE = 6.5$.

2.36. construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2.37. Can you construct the above quadrilateral MIST if $\angle M = 100°$ instead of $75°$.

2.38. Can you constrcut the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75°, \angle L = 150°$ and $\angle A = 140°$?

2.39. Construct $MORE$ where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°, \angle R = 105°$.

2.40. Construct $PLAN$ where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.

2.41. Draw rectangle $OKAY$ with $OK = 7$ and $KA = 5$.

2.42. Construct $ABCD$, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105°$ and $\angle C = 80°$.

2.43. Construct $DEAR$ with $DE = 4, EA = 5, AR = 4.5, \angle E = 60°$ and $\angle A = 90°$.

2.44. Construct $TRUE$ with $TR = 3.5, RU = 3, UE = 4\angle R = 75°$ and $\angle U = 120°$.

2.45. Can you construct a rhombus $ABCD$ with $AC = 6$ and $BD = 7$?

2.46. Draw a square $READ$ with $RE = 5.1$.

2.47. Draw a rhombus who diagonals are 5.2 and 6.4.

2.48. Draw a rectangle with adjacent sides 5 and 4.

2.49. Draw a parallelogram $OKAY$ with $OK = 5.5$ and $KA = 4.2$.

2.50. Construct a kite $EASY$ if $AY = 8, EY = 4$ and $SY = 6$.

2.51. Draw a circle of diameter 6.1

2.52. With the same centre **O**, draw two circles of radii 4 and 2.5

2.53. Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.

2.54. Draw a circle of radius 3 and any two of its diameters. Draw the ends of these diameters. What figure do you get?

2.55. Let **A** and **B** be the centres of two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is $AB \perp CD$?

2.56. Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.
Solution: Take the centre of both circles to be at the origin.

2.57. Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
Solution: Take the diameter to be on the x-axis.

2.58. Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°.
Solution: The tangent is perpendicular to the radius.

2.59. Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
Solution: Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \tag{2.59.1}$$

2.60. Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^{\circ}$. BD is the perpendicular from **B** on AC (altitude). The circle through **B**, **C**, **D** (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from **A** to this circle.

2.61. Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**