Faculty of Engineering
University of Jaffna, Sri Lanka
MC1020 Mathematics

Assignment - 04 **Duration: 90 Minutes**

1. Identify the relationship between set $A = \{1, 2, 3\}$ and set $B = \{2, 3, 4\}$?

   (a) A is a proper subset of B
   (b) A is not a subset of B
   (c) A is a superset of B
   (d) A is a subset of B

2. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$.Find $n(A \cup B)$?

   (a) 5 (b) 6 (c) 7 (d) 8

3. If $A = \{x \mid x$ is a prime number less than 10$\}$ and $B = \{x \mid x$ is an even number less than 10$\}$ then find the set $(A \cap B)$?

   (a) $\{2, 3, 5, 7\}$
   (b) $\{2, 4, 6, 8\}$
   (c) $\{\}$
   (d) $\{2\}$

4. If A, B are two sets then the notation $A \oplus B$ is defined by

   (a) $(A - B) \cup (B - A)$
   (b) $(A - B) \cap (B - A)$
   (c) $(A - B) \cup (B + A)$
   (d) $(A - B) \cap (B + A)$

5. Find the conjugate of $\dfrac{3 + 2i}{2 - i}$

   (a) $\dfrac{4}{5} - \dfrac{7}{5}i$
   (b) $\dfrac{4}{5} + \dfrac{7}{5}i$
   (c) $-\dfrac{4}{5} - \dfrac{7}{5}i$
   (d) $-\dfrac{4}{5} + \dfrac{7}{5}i$

6. $(\sin\theta + i\cos\theta)^n$ is equal to

   (a) $\cos n\theta + i \sin n\theta$
   (b) $\sin n\theta + i \cos n\theta$
   (c) $\cos n(\dfrac{\pi}{2} - \theta) + i \sin n(\dfrac{\pi}{2} - \theta)$
   (d) None of the above

7. If $Z_1 = 1 - i$ and $Z_2 = i$ then find Re $\left(\frac{\bar{Z}_1 . Z_2}{Z_1}\right)$

   (a) 1
   (b) -1
   (c) 0
   (d) 2

8. The locus represented by $|Z-1| = |Z+i|$ is

   (a) a circle of radius 1

   (b) an ellipse with foci at (1,0) & (0,-1)

   (c) a straight line through the origin

   (d) a circle on the line joining (1,0),(0,1) as diameter

9. Let $f(x) = |x|$ is a function, identify the type of critical point at $x = 0$,

   (a) a minimum

   (b) a maximum

   (c) point of inflection

   (d) neither minimum or maximum

10. Let $f : R \to R$, $f(x) = \sqrt{x^2 - 3x + 2}$. Find the domain of $f(x)$

    (a) $(-\infty, 1] \cup [2, \infty)$

    (b) $(-\infty, 1) \cup (2, \infty)$

    (c) $(-\infty, -3] \cup [3, \infty]$

    (d) $(-\infty, -3] \cup (3, \infty)$

11. I) $f(x) = x^2$ where $f : \mathbb{N} \to \mathbb{N}$
    II) $f(x) = x + 1$ where $f : \mathbb{Z} \to \mathbb{Z}$
    Based on statements I and II, select the correct answer,

    (a) I is injective and II is surjective

    (b) both are injective and surjective

    (c) both are injective

    (d) I is surjective and II is injective

12. Find the linear approximation of the parabola $f(x) = x^2$ at the point $x = 1$.

    (a) $x^2 + 1$

    (b) $2x + 1$

    (c) $2x - 1$

    (d) $2x + 2$

13. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate value theorem guarantees that

    (a) $f(0) = 0$

    (b) $f(c) = 1$ for at least one c between -3 and 6.

    (c) $f(c) = 0$ for at least one c between -1 and 3.

    (d) $-1 \leq f(x) \leq 3$ for all x between -3 and 6.

14. Given that $f(x,y) = \dfrac{x+y}{1-xy}; x = 2u - v$ and $y = 4u - 2v$,
find the value of $\dfrac{\partial f}{\partial u}$ at $u = 2$ and $v = 3$

(a) 18 (b) -12 (c) 9 (d) 12

15. Evaluate the $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$.

(a) -1
(b) 0
(c) 1
(d) the limit does not exist

16. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1,1,3)$.

(a) $z = 2x - 4y - 3$
(b) $z = 4x - 2y + 3$
(c) $z = 4x + 2y - 3$
(d) None of the above.

17. Find the shortest distance from the point $(1,0,-2)$ to the plane $x + 2y + z = 4$.

(a) $\dfrac{5\sqrt{6}}{6}$ (b) $\dfrac{5\sqrt{3}}{3}$ (c) $\dfrac{6\sqrt{5}}{6}$ (d) $\dfrac{6\sqrt{5}}{5}$

18. A plane π has equation $ax+by+z = d$. The points $A(2,-1,2), B(4,-4,2), C(5,-6,3)$ lie on π. Find the values of a, b, d ?

(a) $a = 2,\ b = 3,\ d = 6$
(b) $a = 3,\ b = 2,\ d = 8$
(c) $a = 3,\ b = 2,\ d = 6$
(d) $a = 2,\ b = 3,\ d = 4$

19. Find the equation of the plane passing through $A(4,2,0),\ B(3,1,1)$ and $C(4,-1,1)$.

(a) $2x + y + z = 8$
(b) $2x + y + 3z = 10$
(c) $2x + 2y + 3z = 8$
(d) $2x + y + 3z = 10$

20. Find the equation of the line which goes through $(4,-2,-7)$ and which is parallel to both $2x - 5y - 2z = 8,$ and $x + 3y - 3z = 12$.

(a) $l \equiv \begin{pmatrix} 4 \\ -2 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 21 \\ 4 \\ 11 \end{pmatrix}$
(b) $l \equiv \begin{pmatrix} 4 \\ -2 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$
(c) $l \equiv \begin{pmatrix} 4 \\ -2 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$
(d) $l \equiv \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 21 \\ 4 \\ 11 \end{pmatrix}$

21. Find the directional derivative of the function $f(x, y) = xy$ at point $P(0, -2)$ in the direction of $v = \frac{1}{2}i + \frac{\sqrt{3}}{2}j$

(a) 1 (b) -2 (c) -1 (d) 0

22. For what values of x does the series $\sum_{n=0}^{\infty} \frac{1}{4^n}(x-1)^{2n}$ converges

(a) $-2 < x < 2$ (c) $-3 < x < 1$

(b) $-1 < x < 3$ (d) $x < 3$

23. Choose the suitable the condition for the convergent of the series, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

(a) $b_{n+1} < b_n$ & $\lim_{n\to\infty} b_n = 0$ (c) $b_{n+1} = b_n$ & $\lim_{n\to\infty} b_n = 0$

(b) $b_{n+1} > b_n$ & $\lim_{n\to\infty} b_n = 0$ (d) none of these

24. Find the interval of convergence for the function $\frac{x^3}{x+2}$, using power series representation.

(a) (-1,1) (c) (-2,2)

(b) (0,2) (d) (-2,0)

25. Find the coefficient of x^n in the Taylor polynomial of degree n $n \geq 2$ for $f(x) = \sqrt{1+x}$ about $x = 0$

(a) $\frac{(-1)^n.3.5.7.....(2n-1)}{2^n n!}$ (c) $\frac{(-1)^{n+1}.3.5.7.....(2n-3)}{2^n}$

(b) $\frac{(-1)^n.3.5.7.....(2n-1)}{2^n}$ (d) $\frac{(-1)^{n+1}.3.5.7.....(2n-3)}{2^n n!}$