Faculty of Engineering
University of Jaffna, Sri Lanka
MC 1020 Mathematics - April 2023

Assignment Test - 01 **Duration: 90 Minutes**

Part I

Underline the correct answer

1. If $A = \{m, a, t, h\}$ then, How many non-empty subsets are available for A?

   (a) 16 (b) 4 (c) 15 (d) None of the above

2. The cardinality of the set $\{a, b, c, \{a, b, c\}\}$is

   (a) 2 (b) 4 (c) 6 (d) None of the above

3. The power set of $\mathbb{Z} = \{0, 1\}$ is

   (a) $\mathbb{P}(\mathbb{Z}) = \{\{0\}, \{1\}, \{\mathbb{Z}\}\}$
   (b) $\mathbb{P}(\mathbb{Z}) = \{\phi, \{\mathbb{Z}\}\}$
   (c) $\mathbb{P}(\mathbb{Z}) = \{\{\mathbb{Z}\}\}$
   (d) $\mathbb{P}(\mathbb{Z}) = \{\{0\}, \{1\}, \{\mathbb{Z}\}, \phi\})$

4. Let $A = \{x\epsilon\mathbb{R} | -3 < x < 2\}$. and $A = \left\{x\epsilon\mathbb{R} | x^2 + x - 6 < 0\right\}$. Which of the following is true?

   (a) $A = B$ (b) $A \subseteq B$ (c) $A \neq B$ (d) (a) and (b)

5. What is the another way of writing the set $B = \left\{x\epsilon\mathbb{R} : \left|x - 3\right| < 2\right\}$

   (a) $(2, 3]$ (b) $[2, 4]$ (c) $(2, 3)$ (d) $(1, 5)$

6. Find the angle between the vectors $u = \begin{bmatrix} -\cos t \\ \sin t \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix}$.

   (a) $\frac{\pi}{2}$ (b) 0 (c) π (d) $3t$

7. Calculate the area of the parallelogram whose edges are $P = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$.

   (a) 20.88 (b) 41.66 (c) 10.44 (d) 5.22

8. Given vector u, find its projection p in the direction of vector v.

(a) $p = \left[\frac{u.v}{v.v}\right] v$ (b) $p = \left[\frac{u.v}{u.u}\right] v$ (c) $p = \left[\frac{u.v}{v.v}\right] u$ (d) $p = \left[\frac{u.v}{u.u}\right] u$

9. Resolve the vector u, perpendicular $u_{\perp}$, to the vector v where $u = \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

(a) $u_{\perp} = \begin{bmatrix} 0 \\ -4 \\ 4 \end{bmatrix}$ (b) $u_{\perp} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ (c) $u_{\perp} = \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$ (d) $u_{\perp} = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$

10. Find an equation that defines the line that passes through the point p in the direction of a vector u where $p = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $u = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(a) $x = 7 - 3t, y = 9 + 2t, z = 7 + 5t$
(b) $x = 4 + 5t, y = 1 - 3t, z = 3 + 2t$
(c) $x = 3 + 2t, y = 4 + 4t, z = 1 - 3t$
(d) $x = 1 - 3t, y = 3 + 2t, z = 4 + 5t$

11. If $z_1 = 3 + 4i$, $z_2 = 7 - 3i$, then $Im(z_1.z_2)$ is

(a) 33 (b) 19 (c) $33 + 19i$ (d) $19 + 33i$

12. Find the polar form of the complex number $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

(a) $2(\text{Cos}\frac{\pi}{3} + i\text{Sin}\frac{\pi}{3})$
(b) $2(\text{Cos}\frac{2\pi}{3} - i\text{Sin}\frac{2\pi}{3})$
(c) $2(\text{Cos}\frac{2\pi}{3} + i\text{Sin}\frac{2\pi}{3})$
(d) $2(\text{Cos}\frac{\pi}{3} - i\text{Sin}\frac{\pi}{3})$

13. It is known that the polynomial equation $z^4 - 4z^3 + 14z^2 - 36z + 45 = 0$ has $3i$ and $2 - i$ as two of its roots.What are other two roots.

(a) $3i, 2 + i$ (b) $2 - 3i, i$ (c) $-3i, 2 + i$ (d) $1 - 3i, 2 + i$

14. If $z = z^*$, then

(a) z is purely real
(b) z is purely imaginary
(c) Re(z) = Im(z)
(d) z is any complex number

15. The square roots of – 8i are

(a) $2–2i, –2 + 2i$

(b) $2 + 2i, –2 + 2i$

(c) $2–2i, –2–2i$

(d) $-2–2i, 2 + 2i$

Part II

Answer the following questions

1. (a) Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

   i. $A \cup B$

   ii. $B \setminus A$

   (b) A and B are sets, then Prove the following

   i. $A \setminus B = A \cap \bar{B}$.

   ii. $(A \cap B) \cup (A \cap \bar{B}) = A$.

2. The lines l, $r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, and m, $r = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, lie in the same plane π.

   (a) Find the co-ordinates of any two points on each of the lines.

   (b) Show that all the four points you found in part (i) lie on the plane $x - z = 2$.

   (c) Explain why you now have more than sufficient evidence to show that the plane π has equation $x - z = 2$.

   (d) Find the co ordinates of the point where the lines l and m intersect.

3. (a) Find all the solution of the equation $z^3 + n = 0$, where n is positive real number.

   (b) If $u = 3 - 3i$, find u^4 in the form rcisθ

   (c) Given that $w = -1 + 2i$ is a root of the equation $w^3 + 7w^2 + 15w + 25 = 0$, find other two roots of the equation.

   (d) Draw an argand diagram showing the set of points z for which $|z - 3 - 4i| = 5$

Write the number of the question in this column

Part II

Q1)

a) let A - set of students who live within one mile of school

B - set of students who walk to class

(i) $A \cup B$ - set of students who live within one mile or (also) students who walk to class.

(ii) $B \setminus A$ - set of only students who walk to class not the students who live within one mile of school.

b)

(i) $A \setminus B = A \cap \overline{B}$

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$\Leftrightarrow x \in A \cap B^c$

$\Leftrightarrow x \in A \wedge x \in B^c$

$\Leftrightarrow x \in A \wedge x \notin B$

$\Leftrightarrow x \in A \wedge \neg(x \in B)$

$\Leftrightarrow x \in A - B$

$x \in A \setminus B$

$\therefore A \setminus B = A \cap \overline{B}$

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ii) $(A \cap B) \cup (A \cap \bar{B}) = A$

$\Leftrightarrow x \in (A \cap B) \cup (A \cap \bar{B})$

$\Leftrightarrow x \in (A \cap B) \vee x \in (A \cap B^c)$

$\Leftrightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in B^c)$

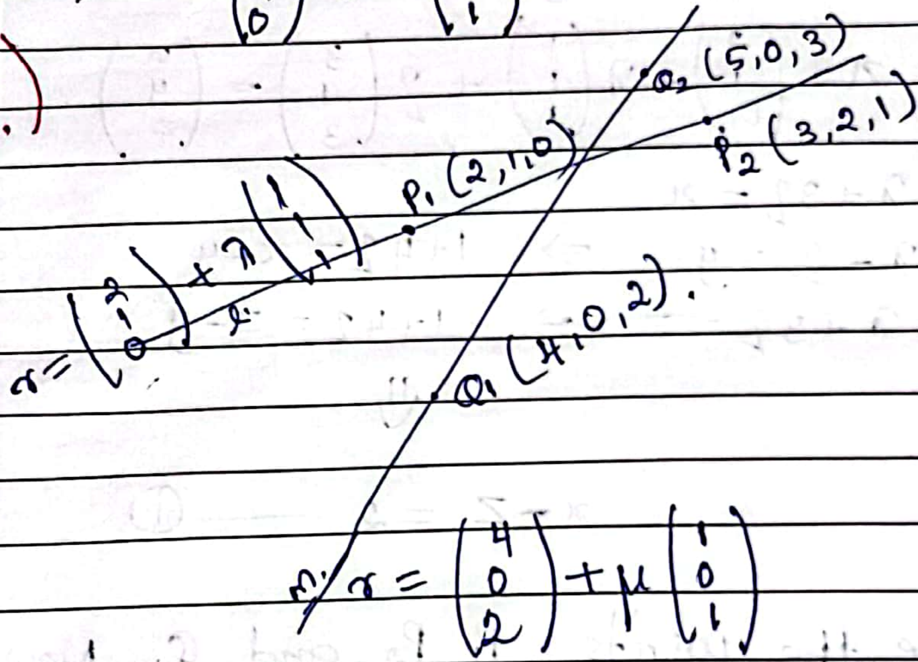
$\Leftrightarrow x \in A \wedge (x \in B \vee x \in B^c)$

$\Leftrightarrow x \in A \wedge T$

$\Leftrightarrow x \in A$

$\therefore (A \cap B) \cup (A \cap \bar{B}) = A$

Q1) ii) $l \Rightarrow r = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix}$, $m \Rightarrow r = \begin{pmatrix}4\\0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$



a) $P_1 = (2,1,0)$, $P_2 = (3,2,1)$
$Q_1 = (4,0,2)$, $Q_2 = (5,0,3)$

b) Take the points P_1, P_2 and Q_2 First. we know the line equation through P_1P_2 is
$$l = r = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix}$$

Now we will Find the line equation through P_1Q_2 say l'.

Write the number of the question in this column

$$\overrightarrow{P_1Q_2} = \begin{pmatrix}5\\0\\3\end{pmatrix} - \begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$= \begin{pmatrix}3\\-1\\3\end{pmatrix}$$

$$l' \equiv r = \begin{pmatrix}2\\1\\0\end{pmatrix} + \eta\begin{pmatrix}3\\-1\\3\end{pmatrix}$$

Now we will find the plane equation through points P_1, P_2 and Q_2 say π_1.

$$\pi_1 = \pi = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \eta\begin{pmatrix}3\\-1\\3\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix}$$

$2+\lambda+3\eta = x$

$1+\lambda-\eta = y \quad \Rightarrow \quad 1+4\eta = x-y$

$0+\lambda+3\eta = z \quad \Rightarrow \quad -1+4\eta = z-y$

$$\Downarrow$$

$$x - z = 2 \quad \text{——} \quad (1)$$

Now take the points P_1, P_2 and Q_1. Now we will find the line equation through P_1Q_1 say l''.

$$l'' \equiv r = \begin{pmatrix}2\\1\\0\end{pmatrix} + \xi\begin{pmatrix}2\\-1\\2\end{pmatrix}$$

Now we will find the plane equation through points P_1, P_2 and Q_1 say π_2

$$\pi_2 = \pi = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} + \xi\begin{pmatrix}2\\-1\\2\end{pmatrix} \equiv \begin{pmatrix}x\\y\\z\end{pmatrix}$$

$2+\lambda+2\xi = x \qquad 0+\lambda+2\xi = z$

$1+\lambda+(-1)\xi = y$

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$1+3\xi = x-y$

$-1+3\xi = z-y$

$\Downarrow$

$x - z = 2$ —— ②

①, ② $\Rightarrow$ $\pi_1 \equiv \pi_2 \equiv 2 = x-z$

$\therefore$ We can say that the four points we found in Part (a) namely P_1, P_2, Q_1 and Q_2 lie on the plane $x-z=2$

(c) ①, ② give more sufficient evidence.

d) P_1, P_2, Q_1 and Q_2 lie on the plane $\pi = x-z=2$.

So, the lines $l \equiv r = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $m \equiv r = \begin{pmatrix}4\\0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$

should be intersected each other.
(We have sufficient evidence).

From this,

$$\begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}4\\0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$2+\lambda = 4+\mu$

$1+\lambda = 0+0 \Rightarrow \lambda = -1, \mu = -3$

$\therefore$ intersecting Point $\equiv (1,0,-1)$.

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3) a) $z^3 + n = 0$

$z^3 = -n$

$z^3 = n(-1 + 0i)$

$z^3 = n(\cos\pi + i\sin\pi)$

$z = n^{\frac{1}{3}}\left[\cos\left(\frac{2k\pi+\pi}{3}\right) + i\sin\left(\frac{2k\pi+\pi}{3}\right)\right]$

$z = n^{\frac{1}{3}}e^{i\left(\frac{\pi+2k\pi}{3}\right)}$ $\quad k = 0, 1, 2.$

when $k=0$ $\quad z = n^{\frac{1}{3}}e^{i\frac{\pi}{3}} = n^{\frac{1}{3}}(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$= n^{\frac{1}{3}}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$k=1$, $z = n^{\frac{1}{3}}e^{i\pi} = n^{\frac{1}{3}}(\cos\pi + i\sin\pi)$

$= -n^{\frac{1}{3}}$

$k=2$ $\quad z = n^{\frac{1}{3}}e^{i\left(\frac{5\pi}{3}\right)} = n^{\frac{1}{3}}(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$

$= n^{\frac{1}{3}}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

b) $u = 3 - 3i$

$u = 3\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

$= 3\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$u^4 = (3\sqrt{2})^4\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]^4$

$u^4 = 324\left(\cos\left(-\frac{\pi}{4}\times 4\right) + i\sin\left(-\frac{\pi}{4}\times 4\right)\right]$

$u^4 = 324\left[\cos(-\pi) + i\sin(-\pi)\right]$

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c) $w = -1+2i$

$w^3+7w^2+15w+25=0$

Since $-1+2i$ is a root of the equation $-1-2i$ is also a root of the equation.

$[w-(-1+2i)][w-(-1-2i)]$
$=(w+1-2i)(w+1+2i)$
$=[(w+1)^2-(2i)^2]$
$= w^2+2w+1-4i^2$
$= w^2+2w+5$

$w^3+7w^2+15w+25=(w^2+2w+5)(Aw+B)$
$[w^3] \Rightarrow A=1$
$[w^0] \Rightarrow 5B=25 \Rightarrow B=5$

~~other factor is~~ $(w+5)(w^2+2w+5)=0$
other two roots of the equation is $-5, -1-2i$

d) $|z-3-4i|=5$
$|z-(3+4i)|=5$
This is a circle with centre $=(3,4)$, radius $=5$