Faculty of Engineering
University of Jaffna, Sri Lanka
MC 1020 Mathematics - April 2023

Assignment Test - 01 **Duration: 90 Minutes**

Part I

Underline the correct answer

1. If $A = \{m, a, t, h\}$ then, How many non-empty subsets are available for A?

   (a) 16 (b) 4 (c) 15 (d) None of the above

2. The cardinality of the set $\{a, b, c, \{a, b, c\}\}$is

   (a) 2 (b) 4 (c) 6 (d) None of the above

3. The power set of $\mathbb{Z} = \{0, 1\}$ is

   (a) $\mathbb{P}(\mathbb{Z}) = \{\{0\}, \{1\}, \{\mathbb{Z}\}\}$
   (c) $\mathbb{P}(\mathbb{Z}) = \{\{\mathbb{Z}\}\}$
   (b) $\mathbb{P}(\mathbb{Z}) = \{\phi, \{\mathbb{Z}\}\}$
   (d) $\mathbb{P}(\mathbb{Z}) = \{\{0\}, \{1\}, \{\mathbb{Z}\}, \phi\})$

4. Let $A = \{x\epsilon\mathbb{R} | -3 < x < 2\}$. and $A = \left\{x\epsilon\mathbb{R} | x^2 + x - 6 < 0\right\}$. Which of the following is true?

   (a) $A = B$ (b) $A \subseteq B$ (c) $A \neq B$ (d) (a) and (b)

5. What is the another way of writing the set $B = \left\{x\epsilon\mathbb{R} : \left|x - 3\right| < 2\right\}$

   (a) $(2, 3]$ (b) $[2, 4]$ (c) $(2, 3)$ (d) $(1, 5)$

6. Find the angle between the vectors $u = \begin{bmatrix} -\cos t \\ \sin t \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix}$.

   (a) $\dfrac{\pi}{2}$ (b) 0 (c) π (d) $3t$

7. Calculate the area of the parallelogram whose edges are $P = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$.

   (a) 20.88 (b) 41.66 (c) 10.44 (d) 5.22

8. Given vector u, find its projection p in the direction of vector v.

(a) $p = \left[\frac{u.v}{v.v}\right] v$ (b) $p = \left[\frac{u.v}{u.u}\right] v$ (c) $p = \left[\frac{u.v}{v.v}\right] u$ (d) $p = \left[\frac{u.v}{u.u}\right] u$

9. Resolve the vector u, perpendicular $u_\perp$, to the vector v where $u = \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}$,

$v = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

(a) $u_\perp = \begin{bmatrix} 0 \\ -4 \\ 4 \end{bmatrix}$ (b) $u_\perp = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ (c) $u_\perp = \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$ (d) $u_\perp = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$

10. Find an equation that defines the line that passes through the point p in the direction of a vector u where $p = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, u = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(a) $x = 7 - 3t, y = 9 + 2t, z = 7 + 5t$ (c) $x = 3 + 2t, y = 4 + 4t, z = 1 - 3t$

(b) $x = 4 + 5t, y = 1 - 3t, z = 3 + 2t$ (d) $x = 1 - 3t, y = 3 + 2t, z = 4 + 5t$

11. If $z_1 = 3 + 4i$, $z_2 = 7 - 3i$, then $Im(z_1.z_2)$ is

(a) 33 (b) 19 (c) $33 + 19i$ (d) $19 + 33i$

12. Find the polar form of the complex number $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

(a) $2(\text{Cos}\frac{\pi}{3} + i\text{Sin}\frac{\pi}{3})$ (c) $2(\text{Cos}\frac{2\pi}{3} + i\text{Sin}\frac{2\pi}{3})$

(b) $2(\text{Cos}\frac{2\pi}{3} - i\text{Sin}\frac{2\pi}{3})$ (d) $2(\text{Cos}\frac{\pi}{3} - i\text{Sin}\frac{\pi}{3})$

13. It is known that the polynomial equation $z^4 - 4z^3 + 14z^2 - 36z + 45 = 0$ has $3i$ and $2 - i$ as two of its roots.What are other two roots.

(a) $3i, 2 + i$ (b) $2 - 3i, i$ (c) $-3i, 2 + i$ (d) $1 - 3i, 2 + i$

14. If $z = z^*$, then

(a) z is purely real (c) Re(z) = Im(z)

(b) z is purely imaginary (d) z is any complex number

15. The square roots of – 8i are

(a) $2 – 2i, –2 + 2i$

(b) $2 + 2i, –2 + 2i$

(c) $2 – 2i, –2 – 2i$

(d) $-2 – 2i, 2 + 2i$

Part II

Answer the following questions

1. (a) Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

   i. $A \cup B$

   ii. $B \setminus A$

   (b) A and B are sets, then Prove the following

   i. $A \setminus B = A \cap \bar{B}$.

   ii. $(A \cap B) \cup (A \cap \bar{B}) = A$.

2. The lines l, $r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, and m, $r = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, lie in the same plane π.

   (a) Find the co-ordinates of any two points on each of the lines.

   (b) Show that all the four points you found in part (i) lie on the plane $x - z = 2$.

   (c) Explain why you now have more than sufficient evidence to show that the plane π has equation $x - z = 2$.

   (d) Find the co ordinates of the point where the lines l and m intersect.

3. (a) Find all the solution of the equation $z^3 + n = 0$, where n is positive real number.

   (b) If $u = 3 - 3i$, find u^4 in the form rcisθ

   (c) Given that $w = -1 + 2i$ is a root of the equation $w^3 + 7w^2 + 15w + 25 = 0$, find other two roots of the equation.

   (d) Draw an argand diagram showing the set of points z for which $|z - 3 - 4i| = 5$