UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

MID SEMESTER EXAMINATION– MARCH 2022

MC1020: MATHEMATICS

Writing Time: ONE Hour. ***Registration Number:20. . . /E/. . .***

Instructions

1. This is a **closed book** exam.
2. This paper contains **20** questions:
3. Answer **all** questions in the space provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **30%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number**, **the module code** and the **title of the paper** in the answer book. Also write your registration number on each additional sheet attached.

1. The set of all possible subsets of a set A is called the power set of A, denoted by $\mathcal{P}(A)$. Set $A = \{1,\ 2,\ 3\}$. List the elements of $\mathcal{P}(A)$ .....
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2. What is the cardinality of the set $\{a, b, c, \{a, b, c\}\}$ is .....
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3. Which of the following statements is FALSE?

   (a) $\{2, 3, 4\} \subseteq A$ implies that $2 \in A$ and $\{3, 4\} \subseteq A$.

   (b) $\{2, 3, 4\} \in A$ and $\{2, 3\} \in B$ implies that $\{4\} \subseteq A - B$.

   (c) $A \cap B \supseteq \{2, 3, 4\}$ implies that $\{2, 3, 4\} \subseteq A$ and $\{2, 3, 4\} \subseteq B$.

   (d) $\{2, 3\} \subseteq A \cup B$ implies that if $\{2, 3\} \cap A = \emptyset$ then $\{2, 3\} \subseteq B$.

   (e) None of the above.

4. Let $A = \{x \in \mathbb{R} | -3 < x < 2\}$ and $B = \{x \in \mathbb{R} | x^2 + x - 6 < 0\}$. Which of the following is true?

   (a) $A = B$ (c) $A \subseteq B$ (e) None of the above.

   (b) $A \neq B$ (d) $B \subseteq A$

5. Find the angle between the vectors $\mathbf{u} = \begin{bmatrix} -\cos t \\ \sin t \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix}$ .....
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6. Calculate the area of the parallelogram whose edges are $\mathbf{P} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$
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7. Use the vector cross product formula to find $(\mathbf{i} \times \mathbf{j}) \times \mathbf{i}$. .....
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8. Given vector $\mathbf{u}$, which of the following is a projection p in the direction of vector $\mathbf{v}$.

(a) $\mathbf{p} = \left[\frac{\mathbf{u.v}}{\mathbf{v.v}}\right] \mathbf{v}$
(b) $\mathbf{p} = \left(\frac{\mathbf{u.v}}{\mathbf{u.u}}\right) \mathbf{v}$
(c) $\mathbf{p} = \left(\frac{\mathbf{u.v}}{\mathbf{v.v}}\right) \mathbf{u}$
(d) $\mathbf{p} = \left(\frac{\mathbf{u.v}}{\mathbf{u.u}}\right) \mathbf{u}$
(e) None of the above.

9. Which of the following are the solutions of the equation $z^3 + n = 0$, where n is a positive real number.

(a) $z_1 = n^{\frac{1}{3}} cis(\frac{\pi}{3})$, $z_2 = n^{\frac{1}{3}} cis(\pi)$, $z_3 = n^{\frac{1}{3}} cis(\frac{2\pi}{3})$
(b) $z_1 = n^{\frac{1}{3}} cis(\frac{\pi}{6})$, $z_2 = n^{\frac{1}{3}} cis(\pi)$, $z_3 = n^{\frac{1}{3}} cis(\frac{5\pi}{6})$
(c) $z_1 = ncis(\frac{\pi}{3})$, $z_2 = ncis(\pi)$, $z_3 = ncis(\frac{5\pi}{3})$
(d) $z_1 = n^{\frac{1}{3}} cis(\frac{\pi}{3})$, $z_2 = n^{\frac{1}{3}} cis(\pi)$, $z_3 = n^{\frac{1}{3}} cis(\frac{5\pi}{3})$
(e) None of the above.

10. Find the Cartesian equation of the locus described by $|z + 27i| = 2|z10 + 2i|$. Write your answer in the form $(x + A)^2 + (y + B)^2 = K$ .....
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11. Find $Im(z_1.z_2)$ if $z_1 = 3 + 4i$, and $z_2 = 7 - 3i$.....
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12. Find all possible values of k that make $u = \dfrac{k + 4i}{1 + ki}$ a purely real number. .....
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13. The function $f(x) = 2x + 3$ is one to one. Find its inverse function of f. .....
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14. Which one of the following is not a function.

(a) $\{(2,3),(3,0),(5,2),(4,3)\}$
(b) $\{(4,1),(5,2),(5,3),(1,9)\}$
(c) $\{(4,2),(3,4),(5,8),(6,2)\}$
(d) $\{(1,1),(2,3),(4,5),(5,4)\}$
(e) None of the above.

15. Find the domain of the function $h(x) = \dfrac{x}{x^2-9}$. .....
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16. Which one of the following is even function?

(a) $\sec x \tan x$
(b) $x^4 \sin x \cos^2 x$
(c) $\dfrac{x}{x^2-1}$
(d) $1 - x^4$
(e) None of the above.

17. Find the slope of the tangent line to the graph of $f(x) = -2x + 3$ at any point $(x, f(x))$. .....
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18. If $Q(K,L) = 2022K^{0.3}L^{0.7}$, find $K\dfrac{\partial Q}{\partial K} + L\dfrac{\partial Q}{\partial L}$ .....
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19. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at any point $(1,1,3)$.

(a) $z = 2x + 2y + 3$
(b) $z = 4x - 2y + 3$
(c) $z = 2x - 4y - 3$
(d) $z = 4x + 2y - 3$
(e) None of the above.

20. If $u = x^4y + y^2z^3$, where $x = rse^t, y = rs^2e^{-t}$, and $z = r^2s\sin t$, find the value of $\dfrac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$. .....
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End of Examination