Faculty of Engineering
University of Jaffna, Sri Lanka
MC 1020 Mathematics - June 2023

Assignment - 03 **Duration:90 Minutes**

Answer all the questions

1. Test the convergence of the following series using ratio test

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{(3n+2)\sqrt{n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n}{(n+1)^2}$

2. Check whether the series is converge or diverge,using appropriate test.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

3. Consider the series, $3 + 1 + \dfrac{3}{3^2} + \dfrac{1}{3^3} + \dfrac{3}{3^4}.......$

   (a) State the type of series.

   (b) State whether the series is convergent or divergent.

4. Consider the series, $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$

   (a) State the type of series.

   (b) Find the sum of the series corrected two decimal places

5. Write the number 3.5231231231.... as a ratio of integers

6. Find the radius of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{n(2x-1)^n}{5^n}$$

7. The relationship between the wave length L, the wave period T, and the water depth d, for a surface wave in water is given by:

$$L = \frac{gT^2}{2\pi} \sin\left(\frac{2\pi d}{L}\right)$$

In a particular case the wave period was $10s$ and the water depth was $6.1m$. Taking the acceleration due to gravity, g, as $9.81ms^{-2}$, determine the wave length. [Hint: Use the Maclaurin series expansion for $\sin x$ up to powers of x^1]

8. Find the power series representation of the function $f(x) = ln(x+3)$ and its radius of convergence

9. Find the Maclaurin series for $ln(1+x)$ and hence find for $ln\frac{(1+x)}{(1-x)}$.

10. A civil engineering company wants to analyze the relationship between the load capacity (in pounds) and the deflection (in inches) of a particular type of beam. They conducted tests on various beams and collected the following data:

Load Capacity (in pounds)	1000	2000	3000	4000	5000
Deflection (in inches)	0.5	1.2	1.8	2.6	3.5

Using linear regression, determine the equation of the regression line and predict the deflection for a load capacity of 12000 pounds.

Assignment-03

1) $$\sum_{n=1}^{\infty} \frac{x^{2n}}{(3n+2)\sqrt{n}}$$

$$a_n = \frac{x^{2n}}{(3n+2)\sqrt{n}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{x^{2n+2}}{(3n+5)\sqrt{n+1}} \times \frac{(3n+2)\sqrt{n}}{x^{2n}}$$

$$= \lim_{n\to\infty}\left|\frac{x^2.(3n+2)\sqrt{n}}{(3n+5)\sqrt{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{3+2/n}{3+5/n} \cdot \frac{1}{\sqrt{1+1/n}}\right| |x^2|$$

$$= |x^2| . \frac{3}{3}$$

$$= |x^2|$$

Now, to test the convergence. the series converges if $L<1$.

$$L = \lim_{n\to\infty} |x|^2$$

$|x|^2<1$ the series converges.
$|x|^2>1$. the series diverges.
So the convergence of the series depends on the value of $|x|^2$.

x^{2n}

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b) $\sum_{n=1}^{\infty} \frac{2^n n}{(n+1)^2}$

Let $U_n = \frac{2^n n}{(n+1)^2}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{2^{n+1}(n+1)}{(n+2)^2} \times \frac{(n+1)^2}{2^n \cdot n}$$

$$= \lim_{n\to\infty} \frac{2(n+1)^3}{n(n+2)^2}$$

$$= \lim_{n\to\infty} \frac{2\left(1+\frac{1}{n}\right)^3}{\left(1+\frac{2}{n}\right)^2}$$

$$= 2 > 1$$

By the ratio test $\sum U_n$ dgs.

ie) $\sum_{n=1}^{\infty} \frac{2^n n}{(n+1)^2}$ dgs.

Write the number of the question in this column

2)

a)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$$

Let $U_n = \frac{1}{\sqrt{n}+\sqrt{n+1}}$ and $V_n = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} \cdot \sqrt{n} = \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}.$$

The both series $\sum U_n$ and $\sum V_n$ have the same nature of convergence.
ie) both converge or both diverge.

Since $\sum_{n=1}^{\infty} V_n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ dgs by the p-series test with $p = \frac{1}{2}$.

by the lim comparison test $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ is also divergence.

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b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

$\lim_{n\to\infty} \left| \frac{(-1)^{n-1}}{n^2} \right|$

$\lim_{n\to\infty} \frac{1}{n^2}$

By the p-series test $\frac{1}{n^2}$ Converges.

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ converges

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3) a) $3 + 1 + \frac{3}{3^2} + \frac{1}{3^3} + \frac{3}{3^4} - \cdots$

a) The given series is an infinite geometric series.

b) In this series, the common ratio (r) can be observed as follows:

$3, 1, \frac{3}{3^2}, \frac{3}{3^3}, \frac{3}{3^4}, \cdots$

$\frac{1}{3}$

$\frac{3}{3^2}$

$r = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{3}{1} = \frac{1}{3}$

Since the r is between -1 and 1, $|r| < 1$

the series is convergent.

$|r| = |\frac{1}{3}| < 1$

$S_\infty = a$

I) Alternating Series.

$$\sum_{n=0}^{\alpha} \frac{(-1)^{n}}{(n+1)!} = -\frac{1}{1!} + \frac{-1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!}$$

$$b_4 = \frac{1}{5!} = 0.0083$$

$$\sum_{n=0}^{\alpha} \frac{(-1)^n}{n+1} \approx 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!}$$

$$= 0.625$$

$$S = 0.625$$

$$\approx 0.63$$

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5) 3.5231231231

$3.5231231 = 3.5 + \frac{231}{10^4} + \frac{231}{10^7} + \frac{231}{10^{10}} + \cdots$ 04

$a = \frac{231}{10^4} \quad r = \frac{1}{10^3}$

$3.5\overline{231} = 3.5 + \frac{\frac{231}{10^4}}{1 - \frac{1}{10^3}} = 3.5 + \frac{\frac{231}{10000}}{\frac{999}{1000}}$

$\frac{35}{10} + \frac{231}{9990} = \frac{35}{10} + \frac{77}{3330}$

$= \frac{5866}{1665}$

6) $\sum_{n=0}^{\alpha} \frac{n(2x-1)^n}{5^n}$

$= \sum_{n=0}^{n} \frac{n(2(x-\frac{1}{2}))^n}{5^n}$

$= \sum_{n=0}^{\alpha} \frac{n2^n(x-\frac{1}{2})^n}{5^n}$

Take $U_n = \frac{n2^n(x-\frac{1}{2})^n}{5^n}$ $\quad U_{n+1} = \frac{(n+1)2^{n+1}(x-\frac{1}{2})^{n+1}}{5^{n+1}}$

$\lim_{n\to\alpha}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to\alpha} \frac{2}{5}\left|x-\frac{1}{2}\right|$ $\quad \frac{n2^n(x-\frac{1}{2})^n}{5^n}$

$= \frac{2}{5}\frac{(n+1)(x-\frac{1}{2})}{n}$

$= \frac{2}{5}\left|x-\frac{1}{2}\right|$

6) The ratio test says that this power series converges if $\frac{2}{5}\left|x-\frac{1}{2}\right| < 1$.

$$\therefore \left|x-\frac{1}{2}\right| < \frac{5}{2}.$$

$\therefore$ Radius of converges is $\frac{5}{2}$.

7)

Using Maclaurin expansion of

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$-04-$

$$\sin x \simeq x$$

$-02-$

$$L = \frac{gT^2}{2\pi}\sin\left(\frac{2\pi d}{L}\right)$$

$$L = \frac{9.81\times 10^2}{2\pi}\sin\left(\frac{2\pi\times 6.1}{L}\right)$$

$$= \frac{9.81\times 10^2}{2\pi}\times\frac{12.2\pi}{L} \quad -02- \quad \left[\sin x \simeq x\right]$$

$$L^2 = (490.5)(12.2)$$

$$L = 77.36$$

$-03-$

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8)

$$f(x) = \ln(x+3)$$

$$\frac{df(x)}{dx} = \frac{1}{x+3}$$

$$= \frac{1}{3\left(1-\left(-\frac{x}{3}\right)\right)}$$

$$= \frac{1}{3}\sum_{n=0}^{\infty}\left(-\frac{x}{3}\right)^n \qquad \frac{1}{3}\left(1-\frac{x}{3}+\frac{x^2}{3^2}-\frac{x^3}{3^3}+\dots\right)$$

$$= \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{3^{n+1}}$$

$\ln\left(\frac{x}{3}+1\right)$ -

$$f(x) = \int\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{3^{n+1}}\,dx$$

-06

$$= \sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{3^{n+1}(n+1)} + c$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left(\frac{x}{3}\right)^{n+1} + c \qquad = \frac{x}{3}-\frac{x^2}{2\cdot 3^2}+\dots + \ln 3$$

$$f(0) = c = \ln 3$$

$$f(x) = \ln 3 + \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}\left(\frac{x}{3}\right)^{n+1}$$

-02

Interval of cgs is $(-3,3)$

-02

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9)

~~This~~

$f(x) = \ln(1+x)$ $\qquad$ $f(0) = \ln(1)$

$f^1(x) = \frac{1}{1+x}$ $\qquad$ $f^1(0) = 1$

$f^2(x) = \frac{-1}{(1+x)^2}$ $\qquad$ $f^2(0) = -1$

$f^3(x) = \frac{2}{(1+x)^3}$ $\qquad$ $f^3(0) = 2$

$$f(x) = f(0) + f^1(0)x + \frac{f''(0)x^2}{2!} + \frac{f^3(0).x^3}{3!} + \cdots$$

$$\ln(1+x) = 0 + 1x + \frac{-1x^2}{2!} + \frac{2.x^3}{3!} + \cdots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

- 05

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$$

- 01

$$\ln\frac{(1+x)}{(1-x)} = \ln(1+x) - \ln(1-x)$$

- 01

$$= \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right] - \left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots\right]$$

$$= 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$$

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$y_i = \alpha + \beta x_i$ ~~or~~ $y_i = a_0 + a_1 x_i + e.$

10) $y_i = \hat{\alpha} + \hat{\beta} x_i .$

$\hat{y} = -0.3 + 7.4 \times 10^{-4} x$ $a_1 =$

x - Load capacity

y - deflection.

When $x = 12\,000$

$y = -0.3 + 7.4 \times 10^{-4} \times 1.2 \times 10^{4}$

$= -0.3 + 8.88$

$= 8.58$

$y_i = a_0 + a_1 x_i + e .$

$a_1 = \dfrac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$

$n = 5$

$\sum x_i y_i = 362200$

$\sum x_i = 15000$

$\sum y_i = 9.6$

$\sum x_i^2 = 55 \times 10^6$

$a_0 = \bar{y} - a_1 \bar{x}$

-07

$y_i = -0.3 + 7.4 \times 10^{-4} x$

-01

$\alpha = -0.3$

$\beta = 7.4 \times 10^{-4}$

When $x = 12\,000$

$y = -0.3 + (7.4 \times 10^{-4} \times 1.2 \times 10^{4})$

$= -0.3 + 8.88$

$= 8.58 .$ -02

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