UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION– JULY 2023-SEMESTER I

MC 1020 – MATHEMATICS

Date:24/07/2023 *Duration: TWO Hours*

Instructions

1. This paper contains **SIX (6)** parts in 4 pages:
2. Read carefully each question and provide comprehensive answers that include all the required steps.
3. This examination accounts for **50%** of module assessment. Total maximum mark attainable is **100**.

Part 1[Set operations][10 marks]: You are advised to spend 15 minutes. This part have SIX questions and best of FIVE will be considered for your Final Mark. All questions carry equal marks.

1. List all non-empty subsets of set $A = \{m, a, t, h\}$
2. Determine the cardinality of the set $\{a, b, c, \{a, b, c\}\}$
3. Find the power set of $\mathbb{Z} = \{0, 1\}$
4. Write the interval of elements in set $B = \Big\{x\epsilon\mathbb{R} : \big|x - 3\big| < 2\Big\}$
5. Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.
   (a) $A \cup B$
   (b) $B \setminus A$
6. If A and B are sets, then demonstrate the following proof
   (a) $A \setminus B = A \cap \bar{B}$.
   (b) $(A \cap B) \cup (A \cap \bar{B}) = A$.

Part 2[Vectors][18 marks]: You are advised to spend 20 minutes. This part have FOUR questions and all will be considered for your Final Mark. All questions carry equal marks.

1. (a) Find the angle between the vectors $u = \begin{bmatrix} -\cos t \\ \sin t \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} \cos t \\ -\sin t \\ 0 \end{bmatrix}$.

   (b) Find the area of the parallelogram whose edges are $P = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$.

2. (a) For a given vector u, find its projection p in the direction of vector v.

   (b) Find $u_\perp$, to the vector v where $u = \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

   (c) Find an equation that defines the line that passes through the point $P(1, 3, 4)$ in the direction of a vector $u = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$.

3. The lines l, $r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, and m, $r = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, lie in the same plane π.

   (a) Find the co-ordinates of any two points on each of the lines.

   (b) Show that all the four points you found in part (i) lie on the plane $x - z = 2$.

   (c) Explain why you now have more than sufficient evidence to show that the plane π has equation $x - z = 2$.

   (d) Find the co ordinates of the point where the lines l and m intersect.

Part 3[Complex Numbers][18 marks]: You are advised to spend 20 minutes. This part have SIX questions and all will be considered for your Final Mark. All questions carry equal marks.

1. Show that $(1 + i)^3 = 2(i - 1)$

2. Find the locus of the points representing z if $\dfrac{z + 2i}{z - 2i}$ is purely imaginary.

3. Using De Moivre's Theorem simplify the expression $\dfrac{(\sqrt{3} - i)^6}{(-1 + i)^9}$. Give your answer in polar form and show all your working.

4. For the complex number $z = \cos\theta + i\sin\theta$ prove that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$.

5. The polynomial $p(z) = 2z^3 + az^2 + bz - 5$ where a and b are real has $2 - i$ as one of its roots. Find the value of the constants a and b.

6. Solve the equation $2z^3 = 9 + 3\sqrt{3}i$, giving all your solutions in polar forms.

Part 4[Functions][18 marks]: You are advised to spend 25 minutes. This part have EIGHT questions in total and best of SIX will be considered for your Final Mark. All questions carry equal marks.

1. Find the directional derivative of $z = xe^{2y}$ at $P(1, 0)$ in the direction from P to the point $Q(2, -1)$.

2. A rectangular container without a lid is to be made from $18m^2$ wood board. Find the maximum volume of such a container.

3. If $f(x, y) = x^4 + y^4 - 4xy + 1$, find the local maximum, minimum and saddle points.

4. Find $\dfrac{\mathrm{d}z}{\mathrm{d}t}$ if $z = f(x, y) = x^2 - y^2, x = \sin t, y = \cos t$.

5. Find the linear approximation of a function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the indicated point $P(3, 2, 6)$.

6. Use the total differential to estimate the change of the function $z = \sqrt{20 - 7x^2 - y^2}$ when (x, y) changes from $(1, 2)$ to $(0.98, 2.03)$.

7. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 3x + 2}}$.

8. Find the range of function $f(x) = x^2 - 8x + 11$ if the domain is $(-\infty, +\infty)$.

Part 5[Curve Fitting][12 marks]: You are advised to spend 15 minutes.
A process involving 2 variables C, and T, is known to obey the law

$$C^n T^m = K$$

with n, m, K constant. The following data was collected from this process:

$\log_{10} T$	5	8	14	17
$\log_{10} C$	513	960	2025	2623

(a) Write down the linear law equivalent to the law given.

(b) Verify the values in the table lie on a straight line using a suitable graph.

(c) Assuming $m = 0.33$ find n and K.

Part 6[Series][24 marks]: You are advised to spend 25 minutes. This part have EIGHT questions in total and best of SIX will be considered for your Final Mark. All questions carry equal marks.

1. Check whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ is converge using appropriate test.

2. Consider the series, $3 + 1 + \frac{3}{3^2} + \frac{1}{3^3} + \frac{3}{3^4}.......$
   (a) State the type of series.
   (b) State whether the series is convergent.

3. Calculate the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$ with precision up to two decimal places.

4. Write the number 3.5231231231.... as a ratio of integers

5. Find the radius of convergence of the following power series: $\sum_{n=0}^{\infty} \frac{n(2x-1)^n}{5^n}$

6. The relationship between the wave length L, the wave period T, and the water depth d, for a surface wave in water is given by:

$$L = \frac{gT^2}{2\pi} \sin\left(\frac{2\pi d}{L}\right)$$

   In a particular case the wave period was $10s$ and the water depth was $6.1m$. Taking the acceleration due to gravity, g, as $9.81ms^{-2}$, determine the wave length. [Hint: Use the Maclaurin series expansion for $\sin x$ up to powers of x^1]

7. Find the power series representation of the function $f(x) = ln(x+3)$ and its radius of convergence

8. Find the Maclaurin series for $ln(1+x)$ and hence find for $ln\frac{(1+x)}{(1-x)}$.

——— *End of Examination* ———