UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING
MID SEMESTER EXAMINATION- MAY 2023

Date:29/05/2023 **MC 1020 – MATHEMATICS** *Duration: ONE Hour*

Instructions

1. This paper contains **FIFTEEN (15)** questions:
2. Read carefully each question and provide comprehensive answers that include all the required steps.
3. This examination accounts for 30% of module assessment. Total maximum mark attainable is 60.

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1. Show on an Argand diagram the points representing the three cube roots of unity.

   [03 marks]

2. Find the exact roots of the equation $z^3 - 1 = \sqrt{3}i$, expressing them in the form $re^{i\theta}$, where $r > 0$ and $\pi < \theta < \pi$.

   [03 marks]

3. The points representing the cube roots of unity form a triangle D1 . The points representing the roots of the equation $z^3 - 1 = \sqrt{3}i$ form a triangle D2. State a sequence of two transformations that maps D1 onto D2.

   [02 marks]

4. Find the local minimum values and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

   [06 marks]

5. Find the linear approximation of a function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the indicated point $P(3, 2, 6)$.

   [06 marks]

6. A rectangular container without a lid is to be made from $18m^2$ woodboard. Find the maximum volume of such a container.

   [04 marks]

7. Calculate f_{xxyz} when $f(x, y, z) = \sin(3x + 2yz)$

   [04 marks]

8. Use the total differential to estimate the change of the function $z = \sqrt{20 - 7x^2 - y^2}$ when (x, y) changes from $(1, 2)$ to $(0.98, 2.03)$.

   [04 marks]

9. Resolve the vector, u, parallel, $u_{\parallel}$, and perpendicular, $u_{\perp}$, to the vector v, where

$$\mathbf{u} = \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}.$$

[04 marks]

10. Show that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ lies on both the planes $x + y + z = 3$ and $x - y + 2z = 2$. Then using a cross product, find the line of intersection of the planes $x + y + z = 3$ and $x - y + 2z = 2$.

[06 marks]

11. Calculate the area of the parallelogram whose edges are $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$.

[03 marks]

12. A lecturer points a laser pointer at a screen with equation $x = 0$ (i.e. the y-z plane). The lecture's hand is at the point $\mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ and the direction of the laser is u. She wants the point on the screen to appear at $y - z$ coordinates $y = 5$, $z = 5$. Compute a vector u that gives the direction in which she should point the laser.

[04 marks]

13. Show that $\{x : 2x^2 + 5x - 3 = 0\} \subseteq \{x : 2x^2 + 7x + 2 = 3/x\}$.

[03 marks]

14. Show by using Venn diagrams that $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$.

[04 marks]

15. Let X, Y be sets. We define the symmetric difference of X and Y to be

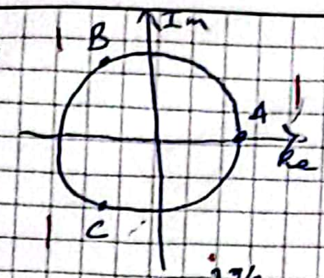
$$X \oplus Y = (X - Y) \cup (Y - X)$$

Draw diagrams to represent $A \oplus (B \oplus C)$ and $(A \oplus B) \oplus C$.

[04 marks]

—— End of Examination ——

Total = 60

1)


3

2) $z_1 = \sqrt[3]{2}\, e^{j\pi/9}$ 1

$z_2 = \sqrt[3]{2}\, e^{j7\pi/9}$ 1

$z_3 = \sqrt[3]{2}\, e^{-j5\pi/9}$ 1

3

3) Rotated through 20°, enlarged by $\sqrt[3]{2}$

2

4) critical points

$(0,0)$, $(1,1)$ & $(-1,-1)$

saddle point — Minimum — Minimum

6

8) $dz = \frac{\partial z(1,2)}{\partial x}\Delta x + \frac{\partial z(1,2)}{\partial y}\Delta y$

$= -\frac{1}{3}(0.98-1) - \frac{2}{3}(2.03-2)$

$= 0.0026667$

4

6)


$V = xyz$

$2xz + 2yz + xy = 18$

$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$

$\Rightarrow x^2 + 2xy - 18 = 0$ — (1)

$y^2 + 2xy - 18 = 0$ — (2)

$\Rightarrow x = \sqrt{6} = y$

$z = \sqrt{6}/2$

4

7) $f_x = 3\cos(3x+2yz)$

$f_{xx} = -9\sin(3x+2yz)$

$f_{xy} = -18z\cos(3x+2yz)$

$f_{xxyz} = -18\cos(3x+2yz) + 36yz\sin(3x+2yz)$

4

5) $L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$

9) $u_{11} = \left(\frac{u\cdot v}{v\cdot v}\right)v = \begin{bmatrix}2\\-2\\-2\end{bmatrix}$

$u_\perp = u - u_{11} = \begin{bmatrix}0\\-4\\4\end{bmatrix}$

9) $u_{11} = \left(\frac{u\cdot v}{v\cdot v}\right)v$

$= \frac{6}{3}\begin{bmatrix}1\\-1\\-1\end{bmatrix} = \begin{bmatrix}2\\-2\\-2\end{bmatrix}$

$u_\perp = u - u_{11}$

$= \begin{bmatrix}2\\-6\\2\end{bmatrix} - \begin{bmatrix}2\\-2\\-2\end{bmatrix}$

$= \begin{bmatrix}0\\-4\\4\end{bmatrix}$.

4

5) $f_x(3,2,6) = 3/7$ $\quad f_x = \frac{x}{\sqrt{x^2+y^2+z^2}}$

$f_y(3,2,6) = 2/7$ $\quad f_y = \frac{y}{\sqrt{x^2+y^2+z^2}}$

$f_z(3,2,6) = 6/7$ $\quad f_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$

6

$f(3,2,6) = \sqrt{3^2+2^2+6^2}$

$= \sqrt{49} = 7$

$L(3,2,6) = 7 + 3/7(x-3) + 2/7(y-2) + 6/7(z-6)$

File No. By Subject Date Sheet No.

1-2

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Write the number of the question in this column

10) If $\begin{bmatrix}1\\1\\1\end{bmatrix}$ lies on both planes then it will satisfy the equation.

-5-

$x+y+z=3$

apply $\begin{bmatrix}1\\1\\1\end{bmatrix}$ $\Rightarrow$ $L.H.S = 1+1+1 = 3 = R.H.S$ -1-

So that point lies on the plane $x+y+z=3$.

$x-y+2z=2$

apply $\begin{bmatrix}1\\1\\1\end{bmatrix}$ $\Rightarrow$ $L.H.S = 1-1+2(1)$

$= 2$

$= R.H.S$ -1-

So that point lies on the plane $x-y+2z=2$.

normal vector of $x+y+z=3$ is $\begin{pmatrix}1\\1\\1\end{pmatrix}$ -1-

normal vector of $x-y+2z=2$ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$

direction vector of the line of intersection

$= \begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}1\\-1\\2\end{pmatrix}$ -1-

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$

$= i(2+1) - j(2-1) + k(-1-1)$

$= 3i - j - 2k$

$\begin{pmatrix}3\\-1\\-2\end{pmatrix}$ -1-

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[illegible]
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So, line of intersection

$$\begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-2\end{pmatrix}$$ -1-

ii) $\boxed{-3-}$

$$p = \begin{bmatrix}1\\3\\6\end{bmatrix} \qquad q = \begin{bmatrix}-2\\0\\4\end{bmatrix}$$

$$p\times q = \begin{bmatrix}1\\3\\6\end{bmatrix} \times \begin{pmatrix}-2\\0\\4\end{pmatrix}$$ -1-

$$= \begin{vmatrix} i & j & k \\ 1 & 3 & 6 \\ -2 & 0 & 4 \end{vmatrix}$$

$$= i(12) - j(4+12) + k(6)$$
$$= 12i - 16j + 6k$$ -1-

$$|p\times q| = \sqrt{12^2 + 16^2 + 6^2}$$
$$= \sqrt{436}$$
$$= 20.88$$ -1-

Write the number of the question in this column

$u = 10$

4 b) Let's denote the desired point on the screen as $q = [0, 5, 5]$.

The vector u will be in the same direction as the vector from p to q.

$$v = \vec{pq} = [0, 5, 5] - [3, 3, 2]$$
$$= [-3, 2, 3]$$

Now, to ensure that u is in the same direction as v, we can normalize v to obtain a unit vector in the same direction:

$$u = \frac{v}{\|v\|}$$

$$\|v\| = \sqrt{(-3)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{9+4+9}$$
$$= \sqrt{22}$$

$$\therefore u = \frac{[-3, 2, 3]}{\sqrt{22}}$$

$$u = \begin{bmatrix} -3/\sqrt{22} \\ 2/\sqrt{22} \\ 3/\sqrt{22} \end{bmatrix} .$$

Write the number of the question in this column

8) (i) $z = \sqrt{20 - 7x^2 - y^2}$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,2)} = \left.\frac{-14x}{2\sqrt{20 - 7x^2 - y^2}}\right|_{(1,2)}$$

$$= \frac{-14 \times 1}{2\sqrt{20 - 7(1)^2 - 2^2}}$$

$$= -7/3$$

marks in previous paper.

$$\left.\frac{\partial z}{\partial y}\right|_{(1,2)} = \left.\frac{-2y}{2\sqrt{20 - 7x^2 - y^2}}\right|_{(1,2)}$$

$$= \frac{-2 \times 2}{2\sqrt{20 - 7(1)^2 - 2^2}}$$

$$= -2/3$$

The total differential is

$$dz = \left.\frac{\partial z}{\partial x}\right|_{(1,2)} (x-a) + \left.\frac{\partial z}{\partial y}\right|_{(1,2)} (y-b)$$

$$= -7/3\,(0.98 - 1) - 2/3\,(2.03 - 2)$$

$$= 0.0026667$$

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13) $\{x : 2x^2+5x-3=0\} \subseteq \{x : 2x^2+7x-2=\frac{3}{x}\}$

$A = \{x : 2x^2+5x-3=0\}$

$2x^2+5x-3=0$

$(x+3)(2x-1)=0$

$x=-3$ or $x=\frac{1}{2}$

$A = \{-3, \frac{1}{2}\}$ 1

$B = \{x : 2x^2+7x-2=\frac{3}{x}\}$

$\{x : 2x^3+7x^2-2x-3=0\}$

$2x^3+7x^2-2x-3=0$

$(x+3)(x+1)(2x-1)=0$

$x=-3$ or $x=-1$ or $x=\frac{1}{2}$

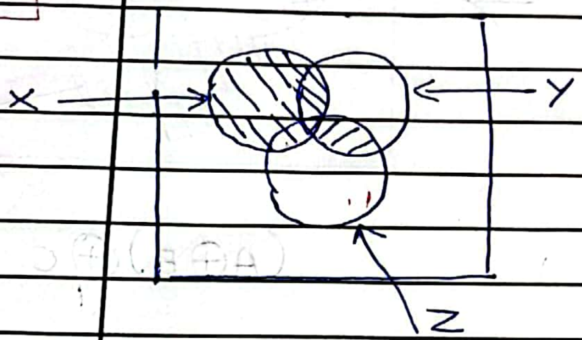
$B = \{-3, -1, \frac{1}{2}\}$ 1

$\{-3, \frac{1}{2}\} \subseteq \{-3, -1, \frac{1}{2}\}$

$A \subseteq B$ 1

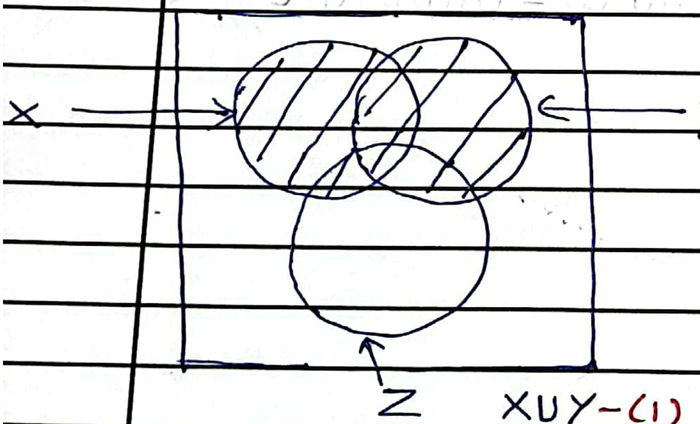
4

14) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

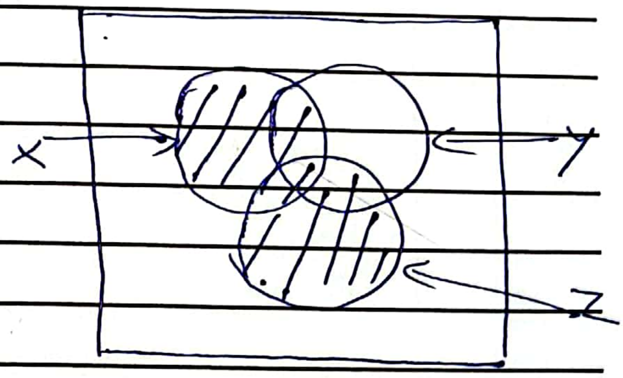


— I

$X \cup (Y \cap Z)$ — (1)

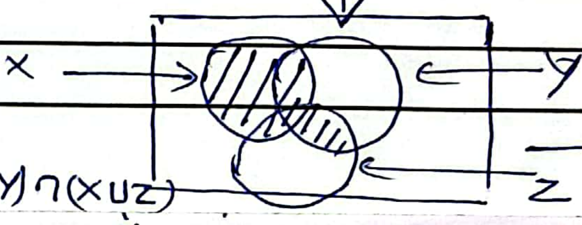


$X \cup Y$ — (1)



$X \cup Z$ — (1)

$\Downarrow$



$(X \cup Y) \cap (X \cup Z)$

— II

from I & II

$X \cup (Y \cap Z) = (X \cup Y$

(1)

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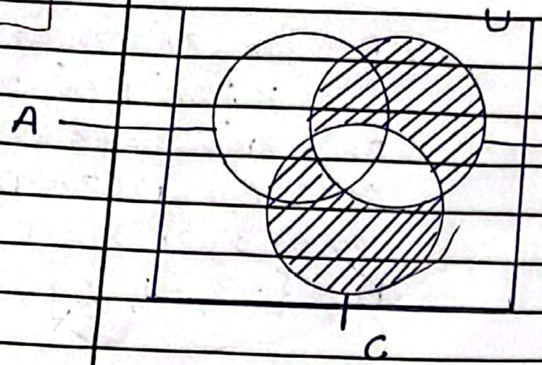
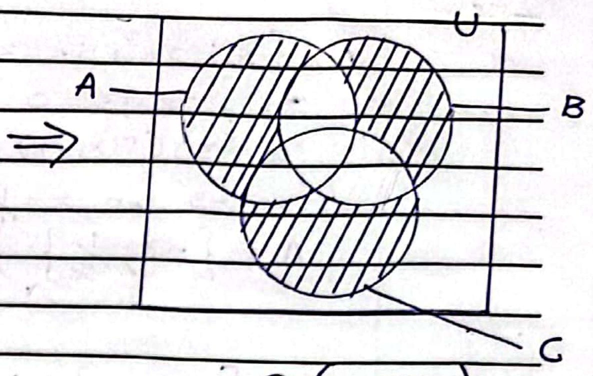
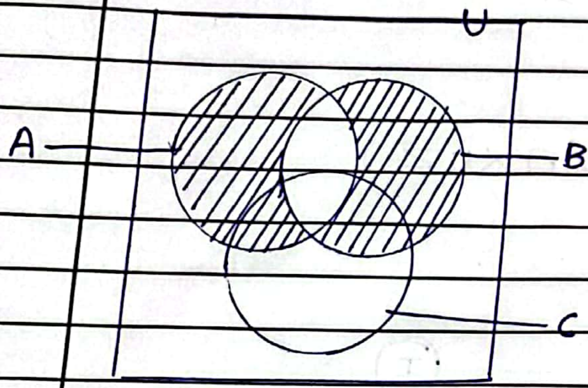
15 4

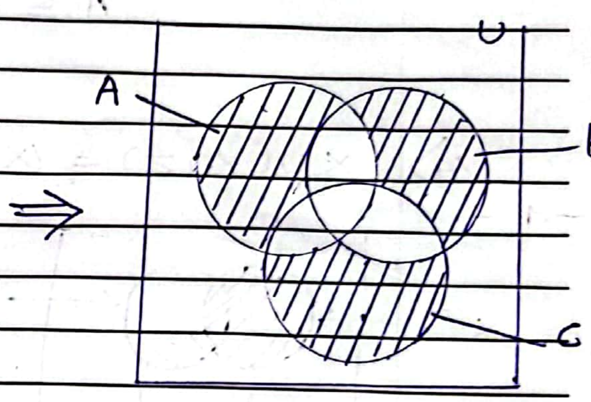
$X \oplus Y = (X - Y) \cup (Y - X)$





$B \oplus C$ — (1) $\qquad$ $A \oplus (B \oplus C)$ — (1)





$A \oplus B$ — (1) $\qquad$ $(A \oplus B) \oplus C$ — (1)

Therefore,

$A + (B \oplus C) = (A \oplus B) \oplus C$ //