Department of Inter Disciplinary Studies,
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MC 1020: Mathematics

Assignment Test - 02 **Duration: 90 Minutes**

Each questions are multiple choice questions with answer choices. Read each question and answer choice carefully and choose the ONE best answer.

1. Function $f(x) = log\left(x^3 + \sqrt{1+x^6}\right)$ is

   (a) even function
   (b) odd function
   (c) algebraic function
   (d) discontinuous function

2. If $f(x) = \dfrac{x-3}{x+1}$, then $f\left[f\left\{f\left(x\right)\right\}\right]$ is equal to,

   (a) x
   (b) $-x$
   (c) $\dfrac{x}{2}$
   (d) $-\dfrac{1}{x}$

3. If the domain of function $f(x) = x^2 - 8x + 11$ is $(-\infty, +\infty)$ then the range of function is

   (a) $[-5, 5]$
   (b) $[-5, \infty)$
   (c) $(-\infty, -5]$
   (d) $(-\infty, +\infty)$

4. The domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 3x + 2}}$ is

   (a) $(-\infty, 1)$
   (b) $(-\infty, 1) \bigcap (2, \infty)$
   (c) $(-\infty, 1) \bigcup (2, \infty)$
   (d) $(2, \infty)$

5. Graph of $y = |x|$ is

   (a) above x-axis
   (b) below x-axis
   (c) right side of y-axis
   (d) left side of y-axis

6. $\lim\limits_{x \to 0} \dfrac{5^x - 3^x}{\sqrt[3]{1+x} - 1}$ is equal to

   (a) $3\ log\dfrac{5}{3}$
   (b) $3\ log\dfrac{3}{5}$
   (c) $log\dfrac{3}{5}$
   (d) 1

7. If the function $f(x) = \begin{cases} \frac{1-cosx}{x^2} & for \quad x \neq 0 \\ k & for \quad x = 0 \end{cases}$

is continuous at $x = 0$, then value of k is

(a) 1
(b) 0
(c) $\frac{1}{2}$
(d) -1

8. The function $f(x) = |x| + \frac{|x|}{x}$ is

(a) Continuous at the origin
(b) Discontinuous at the origin because $|x|$ is discontinuous there
(c) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there
(d) Discontinuous at the origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there

9. If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then

(a) $f(x)$ is continuous but not differentiable at $x = 0$
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) None of these

10. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

(a) $\sqrt{ab}$
(b) $\frac{a}{b}$
(c) $2ab$
(d) ab

11. If $u = f\left(\frac{x}{y}\right)$ then

(a) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$
(b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
(c) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$
(d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

12. Use the total differential to estimate the change of the function

$$z = \sqrt{20 - 7x^2 - y^2}$$

when (x, y) changes from $(1, 2)$ to $(0.98, 2.03)$.
(Approximately in 4 decimal points)

(a) 0.0124
(b) 0.0041
(c) 0.0026
(d) 0.0365

13. Find du if $u = x + \sin\dfrac{y}{2} + e^{yz}$,

(a) $1 + \left(\dfrac{1}{2}\cos\dfrac{y}{2} + e^{yz}\right)dy + ye^{yz}$
(b) $dx + \left(\dfrac{1}{2}\cos\dfrac{y}{2} + ze^{yz}\right)dy + ye^{yz}dz$
(c) $dx + \left(\cos\dfrac{y}{2}\right)dy + ye^{yz}dz$
(d) $1 + \dfrac{1}{2}\cos\dfrac{y}{2} + ze^{yz} + ye^{yz}$

14. Calculate f_{xxyz} when $f(x, y, z) = \sin(3x + 2yz)$

(a) $-36\cos(3x + 2yz) + 36yz\sin(3x + 2yz)$
(b) $-18\cos(3x + 2yz) + 36yz\sin(3x + 2yz)$
(c) $-18\cos(3x + 2yz) + 18yz\sin(3x + 2yz)$
(d) $-36\cos(3x + 2yz) + 18yz\sin(3x + 2yz)$

15. Suppose $z = f(x, y) = x^2 - y^2, x = \sin t, y = \cos t$. Find $\dfrac{\mathrm{d}z}{\mathrm{d}t}$ using the chain rule.

(a) $2\sin 2t$
(b) $2\cos 2t$
(c) $\cos 2t$
(d) $\sin 2t$

16. Using the chain rule as an aid, find the partial derivatives of the function $z = e^x \sin y$, with respect to s where $x = st$ and $y = s + t$.

(a) $e^{st}(t\sin(s + t) + \cos(s + t))$
(b) $e^{st}(\sin(s + t) + t\cos(s + t))$
(c) $e^{st}(s\sin(s + t) + \cos(s + t))$
(d) $e^{st}(\sin(s + t) + s\cos(s + t))$

17. Find the directional derivative of $z = xe^{2y}$ at $P(1, 0)$ in the direction from P to the point $Q(2, -1)$.

(a) $\sqrt{2}$
(b) $-\dfrac{2}{\sqrt{2}}$
(c) $-\dfrac{\sqrt{2}}{2}$
(d) 2

18. A rectangular container without a lid is to be made from $18m^2$ woodboard. Find the maximum volume of such a container.

(a) $3\sqrt{6}$

(b) $\dfrac{3}{\sqrt{6}}$

(c) $\dfrac{\sqrt{6}}{3}$

(d) $\sqrt{6}$

19. Find the local minimum values and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

(a) $(0,0), (-1,-1), (1,-1)$

(b) $(-1,1), (-1,-1), (0,0)$

(c) $(1,1), (-1,-1), (0,0)$

(d) $(1,1), (0,0), (-1,-1)$

20. Find the linear approximation of a function $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at the indicated point $P(3,2,6)$.

(a) $\dfrac{3}{7}(x-3) + \dfrac{2}{7}(y-2) + \dfrac{6}{7}(z-6)$

(b) $\dfrac{2}{7} + \dfrac{3}{7}(x-3) + \dfrac{2}{7}(y-2) + \dfrac{6}{7}(z-6)$

(c) $7 + \dfrac{2}{7}(x-3) + \dfrac{6}{7}(y-2) + \dfrac{3}{7}(z-6)$

(d) $7 + \dfrac{3}{7}(x-3) + \dfrac{2}{7}(y-2) + \dfrac{6}{7}(z-6)$