Faculty of Engineering
University of Jaffna, Sri Lanka
MC 1020 Mathematics - June 2023

Assignment - 03 **Duration:90 Minutes**

Answer all the questions

1. Test the convergence of the following series using ratio test

   (a) $\sum_{n=1}^{\infty} \frac{x^{2n}}{(3n+2)\sqrt{n}}$

   (b) $\sum_{n=1}^{\infty} \frac{2^n n}{(n+1)^2}$

2. Check whether the series is converge or diverge,using appropriate test.

   (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

3. Consider the series, $3 + 1 + \frac{3}{3^2} + \frac{1}{3^3} + \frac{3}{3^4}$.....

   (a) State the type of series.

   (b) State whether the series is convergent or divergent.

4. Consider the series, $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$

   (a) State the type of series.

   (b) Find the sum of the series corrected two decimal places

5. Write the number 3.5231231231.... as a ratio of integers

6. Find the radius of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{n(2x-1)^n}{5^n}$$

7. The relationship between the wave length L, the wave period T, and the water depth d, for a surface wave in water is given by:

$$L = \frac{gT^2}{2\pi} \sin\left(\frac{2\pi d}{L}\right)$$

In a particular case the wave period was $10s$ and the water depth was $6.1m$. Taking the acceleration due to gravity, g, as $9.81ms^{-2}$, determine the wave length. [Hint: Use the Maclaurin series expansion for $\sin x$ up to powers of x^1]

8. Find the power series representation of the function $f(x) = ln(x + 3)$ and its radius of convergence

9. Find the Maclaurin series for $ln(1 + x)$ and hence find for $ln\dfrac{(1 + x)}{(1 - x)}$.

10. A civil engineering company wants to analyze the relationship between the load capacity (in pounds) and the deflection (in inches) of a particular type of beam. They conducted tests on various beams and collected the following data:

Load Capacity (in pounds)	1000	2000	3000	4000	5000
Deflection (in inches)	0.5	1.2	1.8	2.6	3.5

Using linear regression, determine the equation of the regression line and predict the deflection for a load capacity of 12000 pounds.