

UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

END SEMESTER EXAMINATION– SEPTEMBER 2022

MC 1020: MATHEMATICS

Writing Time: TWO Hours.

Answer all parts

Part 1[Set operations][15 marks]: You are advised to spend 20 minutes. This part have SIX questions and best of Five will be considered for your Final Mark. All questions carry equal marks.

1. Find all subsets of $Z = \{z \in \mathbb{C} | z^2 = -1\}$,
2. Let X, Y be sets. We define the symmetric difference of X and Y to be

$$X \oplus Y = (X - Y) \cup (Y - X)$$

Draw a diagram to represent $X \oplus Y$.

3. The set of all possible subsets of a set A is called the power set of A , denoted by $\mathcal{P}(A)$. Set $A = \{1, 2, 3\}$. List the elements of $\mathcal{P}(A)$.
4. Which of the following statements are True? (Justify your answers)
 - (a) $\{2, 3, 4\} \subseteq A$ implies that $2 \in A$ and $\{3, 4\} \subseteq A$.
 - (b) $\{2, 3, 4\} \in A$ and $\{2, 3\} \in B$ implies that $\{4\} \subseteq A - B$.
 - (c) $A \cap B \supseteq \{2, 3, 4\}$ implies that $\{2, 3, 4\} \subseteq A$ and $\{2, 3, 4\} \subseteq B$.
 - (d) $\{2, 3\} \subseteq A \cup B$ implies that if $\{2, 3\} \cap A = \emptyset$ then $\{2, 3\} \subseteq B$.
 - (e) None of the above.
5. True or false (Justify your answers):
 - (a) $\mathbb{Z} \subseteq \mathbb{Q}$
 - (b) $\mathbb{R} \subseteq \mathbb{C}$
 - (c) $\emptyset \subseteq \emptyset$
 - (d) $\emptyset \subseteq \mathbb{Z}$

6. In a group of 600 science students: 320 are taking Calculus I; 200 are taking Physics I; 150 are taking Chemistry I; 80 are taking both Calculus I and Physics I; 40 are taking both Chemistry I and Calculus I; 60 are taking both Chemistry I and Physics I; 12 are taking all the three subjects. How many are taking none of the three subjects.

Part 2[Vectors][20 marks]: You are advised to spend 20 minutes. This part have FIVE questions and best of FOUR will be considered for your Final Mark. All questions carry equal marks.

1. Find the projection of \mathbf{u} on \mathbf{v} , where $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.
2. Find the area of parallelogram with vertices A $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$, B $\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$, C $\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}$ and D $\begin{bmatrix} -1 \\ 8 \\ 9 \end{bmatrix}$.
3. Write down a vector equation of the line passing through the point $(-1, 1, 2)$ in the direction of the vector $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.
4. Decide whether the following pairs of lines are parallel, perpendicular or neither. Justify your answer. $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$.
5. Determine whether points A(2, -1, 3), B(4, 1, 1), C(3, 3, 2), and D(-3, 1, 5) lie in the same plane.

Part 3[Complex Numbers][16 marks]: You are advised to spend 20 minutes. This part have Five questions and best of FOUR will be considered for your Final Mark. All questions carry equal marks.

1. Evaluate $z = \frac{(1+i)^{10}}{4i}$.
2. Given that $1 - i\sqrt{6}$ is a root of the function $f(x) = x^3 + x^2 + x + 21$, find the remaining roots.
3. Find a complex number z such that $3z + 2z^* = 5 + 2i$ where z^* is the conjugate of z .
4. Find the locus of the points representing z if $\frac{z+2i}{z-2i}$ is purely imaginary.
5. Solve the equation $z^4 = 8 + 8\sqrt{3}i$.

Part 4[Functions][18 marks]: You are advised to spend 20 minutes. This part have EIGHT questions in total and best of SIX will be considered for your Final Mark. All questions carry equal marks.

1. A curve has the equation $f(x) = x^3 - 3x^2 + 1$.
 - (a) Determine $f'(x)$ and $f''(x)$
 - (b) Write down the coordinates of the point of inflection, maxima and minima of the curve.
 - (c) Write down the interval on which the function f is decreasing.
 - (d) Determine the equation of the tangent to the curve at the point (3,1) which is on the curve.
2.
 - (a) Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.
 - (b) Sketch the level curves of the function $9 - x^2 - y^2$ for $k = 0, 1, 2, 3$.
 - (c) If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$.
 - (d) If $z = \sin(x + \sin t)$, show that $\frac{\partial z}{\partial x} \frac{\partial^2 z}{\partial x \partial t} = \frac{\partial z}{\partial t} \frac{\partial^2 z}{\partial x^2}$

Part 5[Curve Fitting][16 marks]: You are advised to spend 20 minutes. This part have FIVE questions in total and best of FOUR will be considered for your Final Mark. All questions carry equal marks.

Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature (x) and sidewalk deflection (y). Summary quantities were ,

$$n = 20, \quad \sum y_i = 12.75, \quad \sum y_i^2 = 8.86, \quad \sum x_i^2 = 143,215.8, \quad \sum x_i = 1478, \quad \sum x_i y_i = 1083.67$$

1. Calculate the least squares estimates of the slope and intercept. Graph the regression line.
2. Use the equation of the fitted line to predict what sidewalk deflection would be observed when the surface temperature is 85°F.
3. What is the sidewalk deflection when the surface temperature is 90°F?
4. What change in sidewalk deflection would be expected for a 1°F change in surface temperature?
5. Calculate the coefficient of determination between surface temperature and sidewalk deflection.

Part 6[Series][15 marks]: You are advised to spend 20 minutes. This part have SIX questions in total and best of FIVE will be considered for your Final Mark. All questions carry equal marks.

1. Show that the sequence $a_n = \frac{n}{n^2 + 1}$ is decreasing.
2. Find the Maclaurin series of the function $f(x) = e^x$ and its radius of convergence.
3. Test the series $(-1)^n \frac{n^3}{3^n}$ for absolute convergence.
4. Determine whether series $\sum_{n=1}^{\infty} \frac{5}{2n^2 + 4n + 3}$ converges or diverges.
5. Write the number $1.2\overline{133} = 1.2133133133133\dots$ as a ratio of integers.
6. Find the power series representation of $\frac{x^2}{(3+x)}$.

————— *End of Examination* —————