UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 03 – January 2023

Linear Algebra

MC 2020

Reading Time: Five Minutes

Writing Time: 105 Minutes

1. What do you mean by the term "Vector Space"?

   (a) Let V be the set of all real numbers x such that $x > 0$. Define an operation of addition by $x \oplus y = xy, \forall x, y \in V$. Define another operation of scalar multiplication by $\alpha \odot x = x^{\alpha}, \forall \alpha \in \mathbb{R}$ and $x \in V$. Prove that under the operation $\oplus$ and $\odot$, the set of V is a vector space.

   (b) Let V be the set of all fifth degree polynomials with standard addition and multiplication. Is it a vector space? Justify your answer.

2. (a) Let $S = \{(a_1 \quad a_2) : a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in S$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 - b_2)$$

   and

$$c(a_1, a_2) = (ca_1, ca_2).$$

   Then S is not a vector space.

   (b) Given non-zero real numbers a and b, consider

$$X = \{(x \quad y) \in \mathbb{R}^2 \mid ax + by = 0\}.$$

   Show that X is a subspace of vector space $\mathbb{R}^2$

   (c) Given non-zero real numbers a, b and c, consider

$$X_c = \{(x \quad y) \in \mathbb{R}^2 \mid ax + by = c\}.$$

   Show that X_c is not a subspace of $\mathbb{R}^2$.

3. (a) Let the distance between two vectors x and y in a linear normed vector space be defined as:

$$d(x, y) = \|x - y\|_p$$

Show that this distance function satisfies the distance properties.

(b) Find the distance between $u = (7+3j, -2+5j)$ and $v = (6+3j, -2+6j)$ in $\mathbb{C}^2$

4. (a) Determine a value for q such that the following vectors are linearly independent (1, 1, 2, 1), (2, 1, 2, 3), (1, 4, 2, 1) (-1, 3, 5, q)

(b) Show that the vectors $u_1 = (0, 3, 1, -1), u_2 = (6, 0, 5, 1)$ and $u_3 = (4, -7, 1, 3)$ form a linearly dependent set in $\mathbb{R}^4$
Express each vector as a linear combination of the other two.

5. Determine whether the given set of vectors spans the given vector space

(a) In $\mathbb{R}^3 : (1, -1, 2), (1, 1, 2), (0, 0, 1)$

(b) In $P_2 : 1 - x, 3 - x^2, x$

(c) In $M_{22} : \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 3 & 1 \end{pmatrix}$

6. (a) Let V and W be the subspace of $\mathbb{R}^4$.
$V = \{(a, b, c, d) \,/ b - 2c + d = 0\}$,
$W = \{(a, b, c, d) \,/ a = d, b = 2c\}$
Find the basis.

(b) Find the basis for the subspace $\{ax^2 + bx + c \quad ; \quad a - 2b = c\}$ of $\mathbb{P}_2$.

Assignment – 03

1) A non-empty set V together with the operations namely vector addition denoted by '+' and scalar multiplication denoted by '.' such that.

(i) V is closed under vector addition.

(ii) Vector addition is commutative.

(iii) Vector addition is associative.

(iv) there is an element $0 \in V$ such that $u+0=u$ $\forall u \in V$.

(v) $\forall u \in V$, $\exists (-u) \in V$ such that $u+(-u)=0$.

(vi) V is closed under scalar multiplication.

(vii) $\alpha \cdot (u+v) = \alpha u + \alpha v$ $\quad \forall \alpha \in K$ and $u, v \in V$

(viii) $(\alpha \cdot \beta) u = \alpha (\beta u)$ $\quad \forall \alpha, \beta \in K$ and $u \in V$

(ix) $(\alpha+\beta) u = \alpha u + \beta u$ $\quad \forall \alpha, \beta \in K$ and $u \in V$ 305

(x) $1 \cdot u = u$ $\quad \forall u \in V$.

a) $V = \{x \mid x \in \mathbb{R}^+\}$

$x \oplus y = xy$

$\alpha \odot x = x^\alpha$

Let $x, y \in \mathbb{R}$

(i) $x \oplus y = xy \in \mathbb{R}^+$ So $x \oplus y \in V$

(ii) $x \oplus y = xy = yx = y \oplus x$

(iii) Let $x, y, z \in V$

$(x \oplus y) \oplus z = xy \oplus z = xyz = x \oplus yz$

$= x \oplus (y \oplus z)$

(iv) $\exists 1 \in V$ suchthat $x \oplus 1 = x \quad \forall x \in V$

(v) $x \in \mathbb{R}^+ \Rightarrow 1/x \in \mathbb{R}^+$

So $\exists (1/x) \in V$ suchthat $x \oplus 1/x = x \cdot 1/x = 1 \quad \forall x \in V$.

So addition inverse exist.

(vi) Let $x, y \in \mathbb{R}^+$

$r \odot x = x^r$ Since x is positive real number any real power of it is also a positive real number.

(vii) $\alpha \odot (x \oplus y) = \alpha \odot (xy)$

$= xy^{\alpha} = x^{\alpha} y^{\alpha} = x^{\alpha} \oplus y^{\alpha}$

$= (\alpha \odot x) \oplus (\alpha \odot y)$

$\alpha \odot (x \oplus y) = (\alpha \odot x) \oplus (\alpha \odot y) \quad \forall \alpha \in \mathbb{R}, \forall x, y \in V$

(viii) $(r+s) \odot x = x^{r+s} = x^r \cdot x^s$

$= x^r \oplus x^s$

$= r \odot x \oplus s \odot x$

i.e., $(r+s) \odot x = (r \odot x) \oplus (s \odot x) \quad \forall r, s \in \mathbb{R}, \forall x \in V$.

(ix) $(rs) \odot x = x^{rs}$

$r \odot (s \odot x) = r \odot (x^s) = (x^s)^r = x^{sr} = x^{rs} = (x^r)^s$

$= (rs) \odot x$

i.e., $r \odot (s \odot x) = (rs) \odot x. \quad \forall r, s \in \mathbb{R}, \forall x \in V$.

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(x) $\exists 1 \in \mathbb{R}$ such that $1 \odot x = x^1 = x \quad \forall x \in V$.

(b) $V = \{ a_0 x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5 \,;\, a_i \in \mathbb{R}$ and $a_0 \neq 0 \}$

take $x^5 + 2 \,,\, -x^5 + 3 \in V$

Consider $(x^5+2) + (-x^5+3)$

$= 5 \notin V$

so V is not closed under vector addition

therefore, V is not a vector space.

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08

02 a) $S = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in S$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1+b_1, a_2 - b_2)$$

and $c(a_1, a_2) = (ca_1, ca_2)$

Soln :- take $(a_1, a_2), (b_1, b_2) \in S$.

Consider $(a_1, a_2) + (b_1, b_2)$

$= (a_1+b_1, a_2-b_2)$ — ①

Now Consider, $(b_1, b_2) + (a_1, a_2)$

$= (b_1+a_1, b_2-a_2)$

$= (a_1+b_1, b_2-a_2)$ — ②

from ① and ② $\Rightarrow$ $a_2-b_2 \neq b_2-a_2$ if $a_2 \neq b_2$

Therefore, $(a_1, a_2)+(b_1, b_2) \neq (b_1, b_2)+(a_1, a_2)$

~~Hence is~~ S is not commutative

i.e., S is not a vector space.

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(b) $X = \{ (x,y) \in \mathbb{R}^2 \mid ax + by = 0 \}$.

Pick $l, m \in X$ and $\alpha, \beta \in \mathbb{R}$.

Say: $l = (x_1, y_1)$ and $m = (x_2, y_2)$ with $ax_1 + by_1 = 0$, and $ax_2 + by_2 = 0$.

Consider, $\alpha l + \beta m$

$$= \alpha (x_1, y_1) + \beta (x_2, y_2)$$

$$= (\alpha x_1 + \beta x_2), (\alpha y_1 + \beta y_2)$$

Now Consider,

$$a(\alpha x_1 + \beta x_2) + b(\alpha y_1 + \beta y_2)$$

$$= a\alpha x_1 + a\beta x_2 + b\alpha y_1 + b\beta y_2$$

$$= \alpha (ax_1 + by_1) + \beta (ax_2 + by_2)$$

$$= \alpha \cdot 0 + \beta \cdot 0$$

$$= 0$$

i,e) $(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2) \in X$

$\Rightarrow \alpha l + \beta m \in X$

Therefore X is a Subspace of vector space $\mathbb{R}^2$.

-05-

(c) $X_c = \{(x,y) \in \mathbb{R}^2 \mid ax+by = c\}$.

take $u, v \in X_c$

Since $u = (x_1, y_1)$ and $v = (x_2, y_2)$

with $ax_1 + by_1 = c$ and $ax_2 + by_2 = c$

Consider, $u+v$.

$= (x_1, y_1) + (x_2, y_2)$

$= (x_1 + x_2, y_1 + y_2)$

Now Consider, $a(x_1+x_2) + b(y_1+y_2)$

$= (ax_1 + by_1) + (ax_2 + by_2)$

$= c + c$

$= 2c$

$\neq c$

i.e., $(x_1+x_2, y_1+y_2) \notin X_c$

$\Rightarrow u+v \notin X_c$

Hence X_c is not a subspace of $\mathbb{R}^2$.

-05-

3) a) $d(x,y) = \|x-y\|_p$

(i) We know that $\|x-y\|_p = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p}$

Always $|x_i - y_i| \geq 0$

so, $|x_i - y_i|^p \geq 0$

$\Rightarrow \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p} \geq 0$

$\Rightarrow \|x-y\|_p \geq 0$

$\Rightarrow d(x,y) \geq 0$.

(ii) $d(x,y) = 0$.

$\Leftrightarrow \|x-y\|_p = 0$

$\Leftrightarrow \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p} = 0$

$\Leftrightarrow |x_i - y_i|^p = 0 \quad ; \; i = 1, 2 \ldots$

$\Leftrightarrow x_i = y_i \quad i = 1, 2 - -$

$\Leftrightarrow (x_i) = (y_i) \; ; \; i = 1, 2 - -$

$\Leftrightarrow x = y$

(iii) $d(x,y) = \|x-y\|_p = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p}$

$= \left(\sum_{i=1}^{\infty} |y_i - x_i|^p\right)^{1/p}$; as $|x_i - y_i| = |y_i - x_i|$

$= \|y-x\|_p$

$= d(y,x) \quad \forall x, y \in V$

V - normed vector space.

Additional question:

$x, y, z \in V$

$$d(x,y) = \|x-y\|_p = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p}$$

Consider, $|x_i - y_i| = |x_i - z_i + z_i - y_i|$

$\leq |x_i - z_i| + |y_i - z_i|$; $|z_i - y_i| = |y_i - z_i|$

$$|x_i - y_i|^p \leq |x_i - z_i|^p + |y_i - z_i|^p$$

$$\left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p} \leq \left(\sum_{i=1}^{\infty} |x_i - z_i|^p\right)^{1/p} + \left(\sum_{i=1}^{\infty} |y_i - z_i|^p\right)^{1/p}$$

$$= \|x-z\|_p + \|y-z\|_p$$

i.e., $\|x-y\|_p \leq \|x-z\|_p + \|y-z\|_p$. $\forall x, y, z \in V$.

Therefore, $d(x,y)$ is a distance function.

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b) 4

Ans. 6. (a) $\operatorname{dis}(u,v) = ||u-v|| = ||(7+3j, -2+5j) - (6+3j, -2+6j)||$

$$= ||(1,-j)|| = \sqrt{(1 \quad -j)\begin{pmatrix}1\\ j\end{pmatrix}} = \sqrt{1+1} = \sqrt{2}$$

— 05 —

(b) $||u|| = ||(j, 1+j, 1-j)|| = \sqrt{(j \quad 1+j \quad 1-j)\begin{pmatrix}-j\\ 1-j\\ 1+j\end{pmatrix}}$

$$= \sqrt{1+2+2} = \sqrt{5}$$

— ~~0.5~~ —

$$||v|| = ||(-j, j, 4)|| = \sqrt{(-j \quad j \quad 4)\begin{pmatrix}j\\ -j\\ 4\end{pmatrix}} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}.$$

~~Question – 2~~

4) a)

$$c_1\begin{pmatrix}1\\1\\2\\1\end{pmatrix} + c_2\begin{pmatrix}2\\1\\2\\3\end{pmatrix} + c_3\begin{pmatrix}1\\4\\2\\1\end{pmatrix} + c_4\begin{pmatrix}-1\\3\\5\\q\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \text{———} (*)$$

c_1, c_2, c_3, c_4 satisfy $(*)$ if & only if

$c_1 + 2c_2 + c_3 - c_4 = 0$

$c_1 + c_2 + 4c_3 + 3c_4 = 0$

$2c_1 + 2c_2 + 2c_3 + 5c_4 = 0$

$c_1 + 3c_2 + c_3 + qc_4 = 0$

So $(*)$ has the same solutions as

So $(*)$ has the same sol^{n}s as the systems represented by

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0\\ 1 & 1 & 4 & 3 & 0\\ 2 & 2 & 2 & 5 & 0\\ 1 & 3 & 1 & q & 0\end{array}\right) \xrightarrow[\substack{R_3 \to R_3 - 2R_1\\ R_4 \to R_4 - R_1}]{R_2 \to R_2 - R_1} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0\\ 0 & -1 & 3 & 4 & 0\\ 0 & -2 & 0 & 7 & 0\\ 0 & 1 & 0 & q+1 & 0\end{array}\right)$$

$$\xrightarrow[R_4 \to R_4 + R_2]{R_3 \to R_3 - 2R_2} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0\\ 0 & -1 & 3 & 4 & 0\\ 0 & 0 & -6 & -1 & 0\\ 0 & 0 & 3 & q+5 & 0\end{array}\right) \xrightarrow{R_4 \to 2R_4 + R_3} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0\\ 0 & -1 & 3 & 4 & 0\\ 0 & 0 & -6 & -1 & 0\\ 0 & 0 & 0 & 2q+9 & 0\end{array}\right)$$

The system has the unique solution $c_1 = c_2 = c_3 = c_4 = 0$ if and only if $2q+9 \neq 0$.

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$\det(A) = (-3)(-2a-9)$
$= (6a+27)$
$\neq 0$ for linearly independent
$a \neq -\frac{9}{2}$

(5)

Hence, the vectors are linearly independent iff $2q+9 \neq 0$. Thus $q = 1$, for example, yields a linearly independent set of vectors.

$u_1 = (0, 3, 1, -1)$
$u_2 = (6, 0, 5, 1)$
$u_3 = (4, -7, 1, 3)$

4 b)

Suppose that

$$c_1\begin{pmatrix}0\\3\\1\\-1\end{pmatrix} + c_2\begin{pmatrix}6\\0\\5\\1\end{pmatrix} + c_3\begin{pmatrix}4\\-7\\1\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$6c_2 + 4c_3 = 0$
$3c_1 - 7c_3 = 0$
$c_1 + 5c_2 + c_3 = 0$
$-c_1 + c_2 + 3c_3 = 0$

-04

$$\left(\begin{array}{ccc|c}0&6&4&0\\3&0&-7&0\\1&5&1&0\\-1&1&3&0\end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c}1&5&1&0\\3&0&-7&0\\0&6&4&0\\-1&1&3&0\end{array}\right) \xrightarrow[R_4 \to R_4 + R_1]{R_2 \to R_2 - 3R_1}$$

$$\left(\begin{array}{ccc|c}1&5&1&0\\0&3&2&0\\0&0&0&0\\0&0&0&0\end{array}\right) \xleftarrow{R_2 \to -\frac{1}{5}R_2} \left(\begin{array}{ccc|c}1&5&1&0\\0&-15&-10&0\\0&0&0&0\\0&0&0&0\end{array}\right) \xleftarrow[R_4 \to 5R_4 + 2R_2]{R_3 \to 5R_3 + 2R_2} \left(\begin{array}{ccc|c}1&5&1&0\\0&-15&-10&0\\0&6&4&0\\0&6&4&0\end{array}\right)$$

$c_1 + 5c_2 + c_3 = 0$
$3c_2 + 2c_3 = 0$

This has general sol^n $c_3 = \alpha$, $c_2 = -\frac{2}{3}\alpha$
$c_1 = -\alpha + \frac{10}{3}\alpha$
$= \frac{7}{3}\alpha$

-03

Taking $\alpha = 3$

$$7\begin{pmatrix}0\\3\\1\\-1\end{pmatrix} - 2\begin{pmatrix}6\\0\\5\\1\end{pmatrix} + 3\begin{pmatrix}4\\-7\\1\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

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$$\begin{pmatrix}0\\3\\1\\-1\end{pmatrix} = \frac{2}{7}\begin{pmatrix}6\\0\\5\\1\end{pmatrix} - \frac{3}{7}\begin{pmatrix}4\\-7\\1\\3\end{pmatrix}$$

$$\begin{pmatrix}6\\0\\5\\1\end{pmatrix} = \frac{7}{2}\begin{pmatrix}0\\3\\1\\-1\end{pmatrix} + \frac{3}{2}\begin{pmatrix}4\\-7\\1\\3\end{pmatrix}$$ -03

$$\begin{pmatrix}4\\-7\\1\\3\end{pmatrix} = -\frac{7}{3}\begin{pmatrix}0\\3\\1\\-1\end{pmatrix} + \frac{2}{3}\begin{pmatrix}6\\0\\5\\1\end{pmatrix}$$

7)

$$c_1\begin{pmatrix}1\\2\\-1\\2\end{pmatrix} + c_2\begin{pmatrix}-2\\3\\1\\-1\end{pmatrix} + c_3\begin{pmatrix}-1\\3\\2\\1\end{pmatrix} = \begin{pmatrix}3\\4\\-1\\6\end{pmatrix}$$

$c_1 - 2c_2 - c_3 = 3$ — (1)

$2c_1 + 3c_2 + 3c_3 = 4$ — (2)

$-c_1 + c_2 + 2c_3 = -1$ — (3)

$2c_1 - c_2 + c_3 = 6$ — (4)

Using Gaussian elimination,

$c_3 = 1$, $c_2 = -1$, $c_1 = 2$

Thus,

$$\begin{pmatrix}3\\4\\-1\\6\end{pmatrix} = 2\begin{pmatrix}1\\2\\-1\\2\end{pmatrix} - \begin{pmatrix}-2\\3\\1\\-1\end{pmatrix} + \begin{pmatrix}-1\\3\\2\\1\end{pmatrix}$$

$$\begin{pmatrix}3\\4\\-1\\6\end{pmatrix} \in \text{Span}\left\{\begin{pmatrix}1\\2\\-1\\2\end{pmatrix}, \begin{pmatrix}-2\\3\\1\\-1\end{pmatrix}, \begin{pmatrix}-1\\3\\2\\1\end{pmatrix}\right\}$$

5a) Let $(x,y,z) \in \mathbb{R}^3$,
we seek c_1, c_2, c_3 s.t

$$c_1(1,-1,2) + c_2(1,1,2) + c_3(0,0,1) = (x,y,z)$$

$$c_1 + c_2 = x$$
$$-c_1 + c_2 = y$$
$$2c_1 + 2c_2 + c_3 = z$$

Using Gaussian elimination

$$\left(\begin{array}{ccc|c}1 & 1 & 0 & x\\-1 & 1 & 0 & y\\2 & 2 & 1 & z\end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 + R_1} \left(\begin{array}{ccc|c}1 & 1 & 0 & x\\0 & 2 & 0 & x+y\\0 & 0 & 1 & z-2x\end{array}\right)$$

$$c_1 + c_2 = x \implies c_1 = x - \left(\frac{x+y}{2}\right) = \frac{x-y}{2}$$

$$2c_2 = x+y \implies c_2 = \frac{x+y}{2}$$

$$c_3 = z - 2x$$

Thus

$$(x,y,z) = \tfrac{1}{2}(x-y)(1,-1,2) + \tfrac{1}{2}(x+y)(1,1,2) + (z-2x)(0,0,1)$$

Hence the given vectors span $\mathbb{R}^3$

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b) Suppose $a_0 + a_1 x + a_2 x^2 \in P_2$. We seek c_1, c_2, c_3 such that

$$c_1(1-x) + c_2(3-x^2) + c_3 x = a_0 + a_1 x + a_2 x^2.$$

ie) $(c_1 + 3c_2) + (c_3 - c_1)x - c_2 x^2 = a_0 + a_1 x + a_2 x^2$.

$$c_1 + 3c_2 = a_0$$
$$c_3 - c_1 = a_1.$$
$$-c_2 = a_2$$

So $c_2 = -a_2$, $c_1 = a_0 + 3a_2$, $c_3 = a_1 + a_0 + 3a_2$.

Hence

$$a_0 + a_1 x + a_2 x^2 = (a_0 + 3a_2)(1-x) + (-a_2)(3-x^2) + (a_0 + a_1 + 3a_2)x$$

$\{1-x,\ 3-x^2,\ x\}$ span P_2. ~~05~~

c) Suppose $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{22}$

We seek c_1, c_2, c_3, c_4 such that

$$c_1\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + c_2\begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix} + c_3\begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix} + c_4\begin{pmatrix} 0 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\Rightarrow$ $2c_1 + 3c_3 = a$

$c_1 - c_3 = b$ $\qquad$ $c_3 = \frac{1}{5}(a - 2b)$

$2c_2 + 3c_4 = c$ $\qquad \Rightarrow$

$c_2 + c_4 = d$ $\qquad$ $c_1 = \frac{1}{5}(a + 3b)$

$c_2 = 3d - c$

$c_4 = c - 2d$

8.A

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{5}(a+3b)\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + (3d-c)\begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix} + \frac{1}{5}(a-2b)\begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix} + (c-2d)\begin{pmatrix} 0 & 0 \\ 3 & 1 \end{pmatrix}$$

Hence the given matrices span M_{22}.

-05

c) Suppose $v \in V$. Since u_1, u_2, u_3 span V, $\exists$ exist constant c_1, c_2, c_3 s.t $v = c_1u_1 + c_2u_2 + c_3u_3$

But $v_1 = u_1$, $v_2 = u_2 - u_1$, $v_3 = u_3$.

$$v = c_1v_1 + c_2(v_2 + v_1) + c_3v_3$$
$$= (c_1 + c_2)v_1 + c_2v_2 + c_3v_3$$

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6 a) Let $(a,b,c,d) \in V \iff b-2c+d=0$

$(a,b,c,d) = (a,b,c,2c-b)$
$= (a,0,0,0)+(0,b,0,-b)+(0,0,c,2c)$
$= a(1,0,0,0)+b(0,1,0,-1)+c(0,0,1,2)$.

So V is a linear combination three vectors.

$\alpha(1,0,0,0)+\beta(0,1,0,-1)+\gamma(0,0,1,2)=(0,0,0)$
$\alpha=0, \beta=0, \gamma=0$
So $(1,0,0,0), (0,1,0,-1), (0,0,1,2)$ are linearly independent
Thus, a basis of V is
$A=\{(1,0,0,0),(0,1,0,-1),(0,0,1,2)\}$

~~Hence, dim V=3~~

-05

Let $(a,b,c,d) \in W \iff a=d, b=2c$

$(a,b,c,d)=(a,2c,c,a)$
$=(a,0,0,a)+(0,2c,c,0)$
$= a(1,0,0,1)+c(0,2,1,0)$

$\alpha(1,0,0,1)+\beta(0,2,1,0)=(0,0,0,0)$
$\alpha=0 \quad \beta=0$
So $(1,0,0,1)(0,2,1,0)$ are linearly independent.

Thus basis of W is $\{(1,0,0,1),(0,2,1,0)\}$

~~Hence, dim W=2~~

-05

இந் நிரலில் வினாவின் இலக்கத்தை எழுதுக.
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Write the number of the question in this column

b) 2) Find the basis for the subspace $\{ax^2+bx+c \,;\, a-2b=c\}$ of $\mathbb{P}_2$.

$\mathbb{P}_2 = \{ax^2+bx+c \,/\, a-2b=c\}$
$= \{ax^2+bx+a-2b\}$
$= \{a(x^2+1)+b(x-2)\}$
$= \langle \{(x^2+1), (x-2)\} \rangle$

Consider $\alpha(x^2+1)+\beta(x-2)=0$

By using wronskin method

$\begin{vmatrix} x^2+1 & x-2 \\ 2x & 1 \end{vmatrix}$
$= x^2+1-2x(x-2)$
$= 1-x^2+4x$
$\neq 0$

05

So, (x^2+1) and $(x-2)$ are linearly independent.
So Basis of $\mathbb{P}_2 = \{(1+x^2), (x-2)\}$