UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION - APRIL 2021

MC2020: LINEAR ALGEBRA

(Duration: 2 hours)

Instructions

1. This is a **Closed-book** exam.
2. This paper contains **FIVE** questions:
3. Answer **all** questions in the space provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **60%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number** in the space provided. Also write your registration number on each additional sheet attached.

Registration Number	**201 . . . |E| . . . .**
Question Number	Marks
1	
2	
3	
4	
5	
TOTAL	

1. A system of equations is shown below:

$$\begin{aligned} 3x - ky - 6z &= k \\ kx + 3y + 3z &= 2 \\ -3x - y + 3z &= -2 \end{aligned}$$

For each of the following values of k, determine whether the system of equations is consistent or inconsistent. If the system is consistent, determine whether there is a unique solution or an infinity of solutions. In each case, identify the geometric configuration of the planes corresponding to each value of k.

(a) $k = 0$ ....

(b) $k = 1$ ....

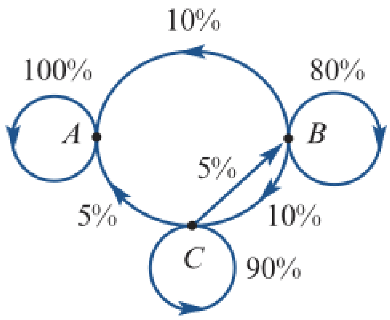
(c) $k = -6$ ....

[20 marks]

2. On Windy Island, sea birds are observed nesting at three sites: A, B and C. The following transition matrix and accompanying transition diagram can be used to predict the movement of sea birds between these sites from year to year. Initially,

$$T = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} & \textit{This year} & \\ A & B & C \end{array}$$

$$T = \begin{bmatrix} 1.0 & 0.10 & 0.05 \\ 0 & 0.80 & 0.05 \\ 0 & 0.10 & 0.90 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{Next year} \\ C \end{array}$$



10000 sea birds were observed nesting at each site, so $\mathbf{S_0} = \begin{bmatrix} 10000 \\ 10000 \\ 10000 \end{bmatrix}$

(a) Use the recurrence rule $S_{n+1} = TS_n$ to

i. determine S_1, the state matrix after 1 year,

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ii. predict the number of sea birds nesting at site B after 2 years.

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(b) Without calculation, write down the number of sea birds predicted to nest at each of the three sites in the long term. Explain why this can be done without calculation. To help solve the problem of having all the birds eventually nesting at site A, the ranger suggests that 2000 sea birds could be removed from site A each year and relocated in equal numbers to sites B and C.

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The state matrix, S_2, is now given by $S_2 = TS_1 + N$

where $\mathbf{S_1} = \begin{bmatrix} 10000 \\ 10000 \\ 10000 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 1.0 & 0.1 & 0.05 \\ 0 & 0.80 & 0.05 \\ 0 & 0.10 & 0.90 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} -2000 \\ 1000 \\ 1000 \end{bmatrix}$.

(c) Evaluate:

i. $\mathbf{S_2}$

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ii. $\mathbf{S_3}$ (assuming that $\mathbf{S_3} = \mathbf{TS_2} + \mathbf{N}$)

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iii. $\mathbf{S_4}$ (assuming that $\mathbf{S_4} = \mathbf{TS_3} + \mathbf{N}$)

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[20 marks]

3. The matrix $\mathbf{A} = \begin{bmatrix} 7 & 5 & 5 \\ 5 & -2 & 4 \\ 5 & 4 & -2 \end{bmatrix}$

(a) Verify that $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of $\mathbf{A}$ and find the corresponding eigenvalue.

(b) Show that (-6) is another eigenvalue of $\mathbf{A}$ and find the corresponding eigenvector.

(c) Given that the third eigenvector of $\mathbf{A}$ is $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$, find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$.

(d) Diagonalisation makes calculating the powers of matrices much simpler and faster. Evaluate $\mathbf{A^5}$ using the result you obtained from 3(c) above.

[20 marks]

4. (a) Show that vector $V = [1\ 8\ 11]^T$ is a linear combination of $V_1 = [1\ 2\ 1]^T$ and $V_2 = [-1\ 1\ 4]^T$.

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(b) Are the following three vectors linearly independent?

$\mathbf{V_1}$ is $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{V_2}$ is $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{V_3}$ is $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

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(c) Is $S = \{(1, 2, 3), (0, 1, 2), (-1, 0, 1)\}$ a basis for $\mathbb{R}^3$?

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(d) Find the null space of the 2×3 matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & -3 & -5 \end{bmatrix}$.

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(e) Compute L_1, L_2 and L_∞ norms of the following two vectors in $\mathbb{R}^3$.
$x = [1\ \ -2\ \ 3\ \ -4]^T$, $y = [3\ \ 0\ \ 5\ \ -2]^T$.

i. $\|x\|_1$

. . . .

ii. $\|x\|_2$

. . . .

iii. $\|x\|_\infty$

. . . .

iv. $\|y\|_1$

. . . .

v. $\|y\|_2$

. . . .

vi. $\|y\|_\infty$

. . . .

[20 marks]

5. Consider the following system of equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 5 \\ 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

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(a) Perform LU factorisation on matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

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(b) Using your LU factorisation from part (a), evaluate the determinant of matrix A. What does the determinants value tell us about the nature of the solution?

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(c) What is the rank of matrix $\mathbf{A}$?

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(d) Briefly, explain how the factorisation could be used to determine solution in above system of equations, given $\mathbf{b}$.
Note: Do not attempt to solve the system of equations.

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[20 marks]

——— *End of Examination* ———