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MC 2020 : Linear Algebra

Answer all the questions

1. (a) A triangle T has vertices at (0,0),(7,7) and (3,-2).
    i. Write down the matrix M, which represents a rotation through 45° anticlockwise about (0,0).
    ii. Find the exact coordinates of the image of T when T is transformed using M.
    iii. Show that det M = 1.
    iv. Hence find the area of the original triangle T

   (b)
$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$
    i. The point $A = (3, -1, 4)$ is transformed using this matrix. Find the coordinates of the image of A.
    ii. The point $B = (a, -a, 2a - 1)$ is transformed to the point with coordinates $(a, a - 5, -a)$ using matrix M.Find the value of a.

2. (a) Use Gaussian elimination method to solve the system of linear equation .
$$-5x_1 - 2x_2 + 2x_3 = 14$$
$$3x_1 + x_2 - x_3 = -8$$
$$2x_1 + 2x_2 - x_3 = -3$$
   (b) Use Cramer's rule to solve for x' and y' in terms of x and y
$$x = x'cos\theta - y'sin\theta$$
$$y = x'sin\theta + y'cos\theta$$

3. (a) Find the steady-state vector of the regular transition matrix A
$$A = \begin{pmatrix} 1/3 & 3/4 \\ 2/3 & 1/4 \end{pmatrix}$$
   (b) A country is divided into three demographic regions. It is found that each year 5% of the residents of region 1 move to region 2, and 5% move to region 3. Of the residents of region 2, 15% move to region 1 and 10% move to region 3, and of the residents region 3, 10% move to region 1 and 5% move to region 2. What percentage of the population resides in each of the three regions after a long period of time.

4. (a) Find the eigen values λ_1 and λ_2 of the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$ also find the eigenvalues of A^T and A^{-1}.

   (b) Verify that

      i. $\lambda_1 + \lambda_2 = trace(A)$.

      ii. $\lambda_1.\lambda_2 = det(A)$.

      iii. Eigenvalues of A^T are also λ_1 and λ_2.

      iv. Eigenvalues of A^{-1} are $\dfrac{1}{\lambda_1}$ and $\dfrac{1}{\lambda_2}$.

   (c) Find the eigenvectors of A.

5. $\mathbf{M} = \begin{pmatrix} 0 & -1 & 1 \\ 6 & -2 & 6 \\ 4 & 1 & 3 \end{pmatrix}$.

   (a) Show that -2 and -1 are eigenvalues of $\mathbf{M}$. and find the other eigenvalue.

   (b) Show that $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ is an eigenvector corresponding to the eigenvalue -2, and find an eigenvector corresponding to the eigenvalue -1.