UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 01 – November 2022

Linear Algebra

MC 2020

Reading Time: Five Minutes

Writing Time: 105 Minutes

Permitted Materials: Calculators; Notes from the class

Question 1[10 marks]

1. The following matrices are needed for Question (a) to (j)

- $\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$
- $\mathbf{B} = \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$
- $\mathbf{C} = \begin{bmatrix} -3 & 3 \end{bmatrix}$
- $\mathbf{D} = \begin{bmatrix} -8 \\ 8 \end{bmatrix}$
- $\mathbf{E} = \begin{bmatrix} -2 & 3 & 5 \\ 2 & 7 & -6 \end{bmatrix}$
- $\mathbf{F} = \begin{bmatrix} 3 & -6 \\ -8 & 9 \end{bmatrix}$

(a) The square matrices are :

i. A and F ii. B and D iii. A and E iv. A and C

(b) The order of matrix B is :

i. 1×3 ii. 3×3 iii. 3×1 iv. 1×2

(c) The following matrices can be added :

i. D and E ii. A and B iii. C and E iv. A and F

(d) The following matrix product is not defined :

i. BC ii. AD iii. AF iv. EA

(e) The order of matrix product DC is :

i. 1×2 ii. 2×1 iii. 1×1 iv. 2×2

★ Write the answers for the following questions.

(f) $-2E$

(g) B^T

(h) $EB - D$

(i) F^{-1}

(j) det A

Question 2[10 marks]

1. $\mathbf{U} = \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}$ and $\mathbf{V} = \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}$:

   (a) Find the matrix product $\mathbf{UV}$ in terms of x.

   (b) If $\mathbf{UV} = \begin{pmatrix} 10x & -9 \\ -4 & 3 \end{pmatrix}$, find the possible values of x.

   (c) Find the possible matrix products $\mathbf{VU}$.

Question 3[10 marks]

1. $P = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

   (a) Describe fully the single geometrical transformation U represented by the matrix P.

   (b) Given that U maps the point with coordinates (a, b) onto the point with coordinates $(3 + 2a, b + 1)$, find the values of a and b.

Question 4[10 marks]

1. Four local football teams took part in a competition in which they each played each other twice, once at home and once away. Figure 1.1 shows the results matrix after half of the games had been played.

	Win	Draw	Lose	Goals for	Goals against
City	2	1	0	6	3
Rangers	0	0	3	2	8
Town	2	0	1	4	3
United	1	1	1	5	3

Figure 1.1

   (a) The results of the next three matches are as follows :

   - City 2 · Rangers 0

- Town 3 United 3
- City 2 Town 4

Find the results matrix for these three matches and hence find the complete results matrix for all the matches so far.

(b) Here is the complete results matrix for the whole competition.

$$\begin{pmatrix} 4 & 1 & 1 & 12 & 8 \\ 1 & 1 & 4 & 5 & 12 \\ 3 & 1 & 2 & 12 & 10 \\ 1 & 3 & 2 & 10 & 9 \end{pmatrix}$$

Find the results matrix for the last three matches (City vs United, Rangers Vs Town and Rangers vs United) and deduce the result of each of these three matches.

Question 5[30 marks]

1. Solve the following system by using

   (a) Gaussian Elimination

   (b) LU Factorization

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 2 \\ x_1 + 2x_2 - x_3 + x_4 &= 1 \\ x_1 - x_2 + 2x_3 + 2x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 3x_4 &= 1 \end{aligned}$$

Question 6[30 marks]

1. Solve the following simultaneous equations

   (a) Using the Matrix Method [by finding Adjoint Matrix]

   (b) Using by Determinant.

   (c) Using Cramer's Rule.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 178 \\ x_1 + 2x_2 + 4x_3 &= 180 + N \\ x_1 + x_2 &= 176 \end{aligned}$$

Where N is your Registration Number.

***** **END** *****

Assignment- 01

1 a) A, F (i)

b) 3×1 (iii)

c) A and F (iv)

d) EA (iv)

e) 2×2 (iv)

f) $-2E = -2\begin{pmatrix} -2 & 3 & 5 \\ 2 & 7 & -6 \end{pmatrix}$

$= \begin{pmatrix} 4 & -6 & -10 \\ -4 & -14 & 12 \end{pmatrix}$

g) $B^T = \begin{pmatrix} -1 & 3 & -8 \end{pmatrix}$

h) $EB - D = \begin{pmatrix} -2 & 3 & 5 \\ 2 & 7 & -6 \end{pmatrix}_{2\times3} \begin{pmatrix} -1 \\ 3 \\ -8 \end{pmatrix}_{3\times1} - \begin{pmatrix} -8 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} -29 \\ 67 \end{pmatrix}_{2\times1} - \begin{pmatrix} -8 \\ 8 \end{pmatrix} = \begin{pmatrix} -21 \\ 59 \end{pmatrix}_{2\times1}$

27 − 48 = −21

i) $F^{-1} = \frac{1}{\det F}\begin{bmatrix} 9 & 6 \\ 8 & 3 \end{bmatrix} = \frac{1}{-21}\begin{pmatrix} 9 & 6 \\ 8 & 3 \end{pmatrix}$

j) $\det A = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$

$= 12 - 2$

$= 10$

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2) $U = \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}$, $V = \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}$

a) $UV = \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}_{2\times2} \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}_{2\times2} = \begin{pmatrix} 2x^2+12 & -9 \\ -4 & 3 \end{pmatrix}_{2\times2}$

b) $\begin{pmatrix} 2x^2+12 & -9 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 10x & -9 \\ -4 & 3 \end{pmatrix}$

$2x^2 + 12 = 10x$

$2x^2 - 10x + 12 = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or $x = 2$.

c) $VU = \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}_{2\times2} \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}_{2\times2} = \begin{pmatrix} 2x^2 & 6x \\ 4x & 15 \end{pmatrix}$

IF $x = 2$

$VU = \begin{pmatrix} 8 & 12 \\ 8 & 15 \end{pmatrix}$

$x = 3$

$VU = \begin{pmatrix} 18 & 18 \\ 12 & 15 \end{pmatrix}$

3) a) $P = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ maps to $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ maps to $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$,

hence the transformation U represented by matrix P is a reflection in the line $y = -x$.

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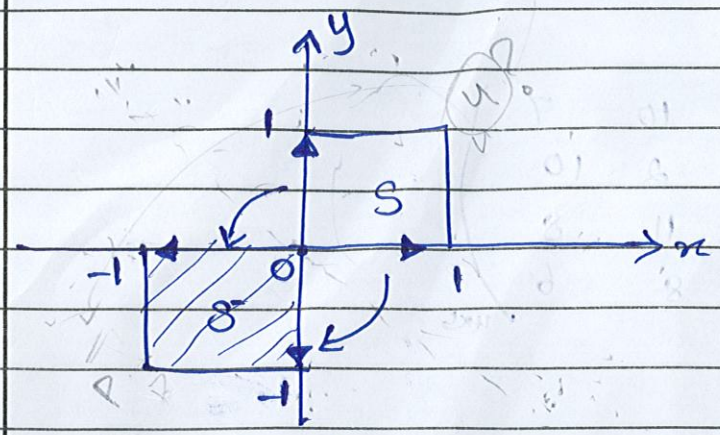

b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3+2a \\ b+1 \end{pmatrix}$

$\begin{pmatrix} -b \\ -a \end{pmatrix} = \begin{pmatrix} 3+2a \\ b+1 \end{pmatrix}$

So $-b = 3+2a$ —— (1)

and $-a = b+1$ —— (2).

Solving simultaneously gives $a=-2$ and $b=1$.

4) a) i) Result matrix for three matches

$$= \begin{pmatrix} 1 & 0 & 1 & 4 & 4 \\ 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 7 & 5 \\ 0 & 1 & 0 & 3 & 3 \end{pmatrix}$$

ii) Result matrix for all the matches so far,

$$\begin{pmatrix} 2 & 1 & 0 & 6 & 3 \\ 0 & 0 & 3 & 2 & 8 \\ 2 & 0 & 1 & 4 & 3 \\ 1 & 1 & 1 & 5 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & 4 & 4 \\ 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 7 & 5 \\ 0 & 1 & 0 & 3 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & 1 & 10 & 7 \\ 0 & 0 & 4 & 2 & 10 \\ 3 & 1 & 1 & 11 & 8 \\ 1 & 2 & 1 & 8 & 6 \end{pmatrix}$$ 4 marks

b) Result matrix for last three matches

$$\begin{pmatrix} 4 & 1 & 1 & 12 & 8 \\ 1 & 1 & 4 & 5 & 12 \\ 3 & 1 & 2 & 12 & 10 \\ 1 & 3 & 2 & 10 & 9 \end{pmatrix} - \begin{pmatrix} 3 & 1 & 1 & 10 & 7 \\ 0 & 0 & 4 & 2 & 10 \\ 3 & 1 & 1 & 11 & 8 \\ 1 & 2 & 1 & 8 & 6 \end{pmatrix}$$ 4 marks

$$= \begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \end{pmatrix}$$ 4 marks

United	1	Rangers	1
Rangers	2	Town	1
City	2	United	1

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$$x_1 + x_2 + x_3 + x_4 = 2$$
$$x_1 + 2x_2 - x_3 + x_4 = 1$$
$$x_1 - x_2 + 2x_3 + 2x_4 = 0$$
$$x_1 + 2x_2 + x_3 + 3x_4 = 1$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

a)

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 1 & 1 \\ 1 & -1 & 2 & 2 & 0 \\ 1 & 2 & 1 & 3 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & -2 & 1 & 1 & -2 \\ 0 & 1 & 0 & 2 & -1 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & -3 & 1 & -4 \\ 0 & 0 & 2 & 2 & 0 \end{array}\right)$$

$$\longleftarrow$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & -3 & 1 & -4 \\ 0 & 0 & 0 & \frac{8}{3} & -\frac{8}{3} \end{array}\right)$$

$$x_4 = -1$$
$$-3x_3 + x_4 = -4 \Rightarrow x_3 = 1$$
$$x_2 - 2x_3 = -1 \Rightarrow x_2 = 1$$
$$x_1 + x_2 + x_3 + x_4 = 2 \Rightarrow x_1 = 1$$

b).

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 2 & 1 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & \frac{8}{3} \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -\frac{2}{3} & 1 \end{pmatrix}$$

$m_{21} = 1 \quad m_{22} = -2 \quad m_{43} = -\frac{2}{3}$

$m_{31} = 1 \quad m_{42} = 1$

$m_{41} = 1$

$$U = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & \frac{8}{3} \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -\frac{2}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & \frac{8}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 2 & 1 & 3 \end{pmatrix}$$

$= A$ $\quad \therefore LU = A.$

$Lc = b$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -\frac{2}{3} & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$c_1 = 2$

$c_1 + c_2 = 1 \Rightarrow c_2 = -1$

$c_1 - 2c_2 + c_3 = 0 \Rightarrow c_3 = -4$

$c_1 + c_2 - \frac{2}{3}c_3 + c_4 = 1 \Rightarrow c_4 = -\frac{8}{3}$

$Ux = c$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & \frac{8}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -4 \\ -\frac{8}{3} \end{pmatrix}$$

$x_4 = -1$

$-3x_3 + x_4 = -4 \Rightarrow x_3 = 1$

$x_2 - 2x_3 = -1 \Rightarrow x_2 = 1$

$x_1 + x_2 + x_3 + x_4 = 1 \Rightarrow x_1 = 1$

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6)

$x_1 + x_2 + 2x_3 = 178$ ... 1

$x_1 + 2x_2 + 4x_3 = 180 + N$

$x_1 + x_2 = 176$

a) Matrix form.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 178 \\ 180+N \\ 176 \end{bmatrix}$$

Inverse matrix.

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$C_{ij} = (-1)^{i+j} M_{ij} \quad (i = \text{row}, j = \text{column})$$

$$C = \begin{pmatrix} (-1)^{1+1}(-4) & (-1)^{1+2}(-4) & (-1)^{1+3}(-1) \\ (-1)^{2+1}(-2) & (-1)^{2+2}(-2) & (-1)^{2+3}(0) \\ (-1)^{3+1}(0) & (-1)^{3+2}(-2) & (-1)^{3+3}(1) \end{pmatrix}$$

$$C = \begin{pmatrix} -4 & 4 & -1 \\ 2 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

3

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= 1(0-4) - 1(0-4) + 2(1-2)$$

$$= -4 + 4 - 2$$

$$= -2$$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} -4 & 2 & 0 \\ 4 & -2 & -2 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ -2 & +1 & +1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

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$$Ax = b$$
$$A^{-1}Ax = A^{-1}b$$
$$(A^{-1}A)x = A^{-1}b$$
$$Ix = A^{-1}b$$
$$x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ -2 & 1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \begin{bmatrix} 178 \\ 180+N \\ 176 \end{bmatrix}$$

$$= \begin{pmatrix} 2\cdot 178 + (-1)(180+N) + 0 \\ -2\cdot 178 + 1(180+N) + 176 \\ \frac{1}{2}\times 178 + 0 - \frac{1}{2}\cdot 176 \end{pmatrix}$$

$$= \begin{pmatrix} 176 - N \\ N \\ 1 \end{pmatrix}$$

b) Using by determinant

$$x_1 + x_2 + 2x_3 - 178 = 0$$
$$x_1 + 2x_2 + 4x_3 - 180 - N = 0$$
$$x_1 + x_2 - 176 = 0$$

Soln is given by

$$\frac{x_1}{D_{x_1}} = \frac{-x_2}{D_{x_2}} = \frac{x_3}{D_{x_3}} = \frac{-1}{D} \quad \text{— (1)}$$

$$D_{x_1} = \begin{vmatrix} 1 & 2 & -178 \\ 2 & 4 & -180-N \\ 1 & 0 & -176 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & -180-N \\ 0 & -176 \end{vmatrix} - 2\begin{vmatrix} 2 & -180-N \\ 1 & -176 \end{vmatrix} + (-178)\begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 352 - 2N$$

$$D_{x_2} = \begin{vmatrix} 1 & 2 & -178 \\ 1 & 4 & -180-N \\ 1 & 0 & -176 \end{vmatrix}$$

$$= -2N$$

$$D_{x_3} = \begin{vmatrix} 1 & 1 & -178 \\ 1 & 2 & -180-N \\ 1 & 1 & -176 \end{vmatrix}$$

$$= 2$$

$$D = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -2$$

Thus

$$\frac{x_1}{Dx_1} = \frac{-x_2}{Dy/Dx_2} = \frac{x_3}{Dz/Dx_3} = \frac{-1}{D}$$

$$\frac{x_1}{352-2N} = \frac{-x_2}{-2N} = \frac{x_3}{2} = \frac{-1}{-2}$$

$2x_1 = 352-2N \Rightarrow x_1 = 176-N$

$2x_2 = 2N \Rightarrow x_2 = N$

$2x_3 = 2 \Rightarrow x_3 = 1$

c) Using Cramer's Rule.

$$D = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 0 \end{vmatrix} = -2$$

D_{x_1} by replacing the x_1-values in the first col^{n} with the values after the equal sign leaving the x_2 and x_3 col^{n} unchanged.

$$D_{x_1} = \begin{vmatrix} 178 & 1 & 2 \\ 180+N & 2 & 4 \\ 176 & 1 & 0 \end{vmatrix} = 2N-352$$

$$D_{x_2} = \begin{vmatrix} 1 & 178 & 2 \\ 1 & 180+N & 4 \\ 1 & 176 & 0 \end{vmatrix} = -2N$$

$$D_{x_3} = \begin{vmatrix} 1 & 1 & 178 \\ 1 & 2 & 180+N \\ 1 & 1 & 176 \end{vmatrix} = -2$$

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Use cramer's rule to find the values of x_1, x_2, and x_3.

$$x_1 = \frac{D_x}{D} = \frac{2N - 352}{-2} = 176 - N$$

$$x_2 = \frac{D_y}{D} = \frac{-2N}{-2} = N$$

$$x_3 = \frac{D_z}{D} = \frac{-2}{-2} = 1.$$