19. Find a transition matrix for above phenomenon.

(a) $\begin{bmatrix}0.15 & 0.8\\ 0.85 & 0.2\end{bmatrix}\begin{bmatrix}0.3\\ 0.15\end{bmatrix}$ (c) $\begin{bmatrix}0.2 & 0.85\\ 0.8 & 0.15\end{bmatrix}\begin{bmatrix}0.15\\ 0.8\end{bmatrix}$

(b) $\begin{bmatrix}0.15 & 0.8\\ 0.85 & 0.2\end{bmatrix}\begin{bmatrix}0.15\\ 0.85\end{bmatrix}$ (d) $\begin{bmatrix}0.15 & 0.2\\ 0.8 & 0.75\end{bmatrix}\begin{bmatrix}0.2\\ 0.8\end{bmatrix}$

20. If the mouse chooses type I today, what is the probability that it will choose type I two days from now?

(a) $\begin{bmatrix}0.15 & 0.8\\ 0.85 & 0.2\end{bmatrix}$ (b) $\begin{bmatrix}0.15 & 0.2\\ 0.85 & 0.8\end{bmatrix}$ (c) $\begin{bmatrix}0.2 & 0.85\\ 0.8 & 0.15\end{bmatrix}$ (d) $\begin{bmatrix}0.15 & 0.2\\ 0.8 & 0.75\end{bmatrix}$

21. Over the long term, what is the probability that it will choose type I ?

(a) $\frac{85}{165}$ (b) $\frac{80}{160}$ (c) $\frac{80}{165}$ (d) None of the above

22. Select the regular transition matrix

(a) $\begin{bmatrix}-0.1 & 0.8\\ 0.9 & 0.2\end{bmatrix}$ (b) $\begin{bmatrix}0.3 & 0.2\\ 0 & 0.8\end{bmatrix}$ (c) $\begin{bmatrix}-0.2 & 0.85\\ 0.8 & -0.15\end{bmatrix}$ (d) $\begin{bmatrix}0.4 & 0.25\\ 0.6 & 0.75\end{bmatrix}$

The following matrix is needed for Question 23to 24

$$\begin{bmatrix}3 & 1 & 1\\ 2 & 3 & 1\\ 1 & 3 & 4\end{bmatrix}$$

23. Find the minor M_{32}

(a) 2 (b) 3 (c) -1 (d) 1

24. Find the co-factor C_{32}

(a) -1 (b) -2 (c) 3 (d) None of the above

25. Which of the following statement is not true about matrices

(a) Matrix multiplication is commutative
(b) $A(BC) = (AB)C$
(c) $A(B+C) = AB+AC$
(d) None of the above

End of Examination

Answers

UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

MID SEMESTER EXAMINATION –DECEMBER 2022

MC2020 : LINEAR ALGEBRA

Time Allowed: One (01) Hour

Answer all the questions.

Underline the correct answer.

Use the matrices U, V, W, X for Questions 1 to 4.

$U = \begin{bmatrix}2 & 0\\ -1 & 1\end{bmatrix}, V = \begin{bmatrix}2 & 1\end{bmatrix}, W = \begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}, X = \begin{bmatrix}3\\ 2\end{bmatrix}$

1. Find $det\,(W)$

(a) 5 (b) -5 (c) 4 (d) 3

2. Find U^{-1}

(a) $U = \begin{bmatrix}1/2 & 0\\ -1/2 & 2/3\end{bmatrix}$ (c) $U = \begin{bmatrix}1/2 & 0\\ 1/2 & 2/3\end{bmatrix}$

(b) $U = \begin{bmatrix}-2/3 & -1\\ 0 & -1/3\end{bmatrix}$ (d) $U = \begin{bmatrix}1 & 0\\ 1 & 3\end{bmatrix}$

3. The order of matrix product UX

(a) (1×2) (b) (2×3) (c) (2×1) (d) None of the above.

4. The following matrix product is not defined

(a) UW (b) VX (c) XW (d) WX

5. Describe the transformation of Matrix A, $A = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

(a) Rotation of 90° anti-clock wise
(b) Rotation of 90°clock wise
(c) y-direction shear by -1
(d) x-direction shear by -1

6. Find the image of matrix $\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}$, after rotation of 180° anticlockwise about y-axis

(a) $\begin{bmatrix}4 & 2\\ -1 & 3\end{bmatrix}$ (b) $\begin{bmatrix}-4 & 2\\ -1 & -3\end{bmatrix}$ (c) $\begin{bmatrix}-4 & -2\\ -1 & -3\end{bmatrix}$ (d) $\begin{bmatrix}4 & -2\\ -1 & 3\end{bmatrix}$

7. The point with position vector $\begin{bmatrix}1\\ 3\\ a\end{bmatrix}$ is transformed by the linear transformation represented by the matrix $\begin{bmatrix}4 & -1 & 0\\ -2 & 2 & 3\\ 5 & -2 & 1\end{bmatrix}$ to the point with position vector $\begin{bmatrix}b\\ -5\\ c\end{bmatrix}$. Find the values of the constants a, b and c.

(a) $a=-3, b=1, c=2$ (c) $a=3, b=1, c=-2$

(b) $a=-3, b=1, c=-2$ (d) $a=3, b=-1, c=2$

8. Which of the following system of linear equations have a unique solution

$I.$ $x-2y=6, 2x+y=3$

$II.$ $2x+2y=6, 4x+4y=3$

$III.$ $4x-3y=6, 8x-12y=3$

(a) I, II, III (b) I (c) III (d) I, III

9. Which pair of simultaneous equations will be generated by this matrix equation

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(a) $2x-y=2$, $3x+2y=3$ (b) $2x+y=2$, $3x+2y=3$ (c) $2x-y=3$, $3x+2y=2$ (d) $2x+3y=2$, $-x+2y=3$

10. Which of the following statement is true

(a) The determinant of 2 ×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $Ad+bc$.

(b) Two square matrices that have the same determinants must have the same size.

(c) The minor M_{ij} is same as the co factor C_{ij} if $i+j$ is even.

(d) If A is the 3×3 symmetric matrix, then $C_{ij} \neq C_{ji}$ for all i and j.

11. Which of the following statement is not true

(a) If A is a square matrix whose minors are all zero, then $\det(A)=0$

(b) The determinant of the lower triangular matrix is the sum of the entries along the main diagonal.

(c) For every square matrix A and every scalar c,it is true that $\det(cA) \neq c\det(A)$

(d) For all square matrix A and B,it is true that,$\det(AB)=\det(A)*\det(B)$

12. Consider the matrices,

$$U=\begin{bmatrix} 0.1 & 0.2 \\ 0.85 & 0.8 \end{bmatrix}, V=\begin{bmatrix} 0.43 \\ 0.45 \end{bmatrix}, W=\begin{bmatrix} 0.3 & 0.4 \\ 0.6 & 0.55 \end{bmatrix}, X=\begin{bmatrix} 0.35 & 0.45 \\ 0.65 & 0.55 \end{bmatrix}$$

The matrix that could be a transition matrix for a Markov chain is,

(a) U (b) U,X (c) X (d) W,V

13. Which is not the property of row echelon matrix

(a) The leading entry of each non-zero row equals 1

(b) All zero rows, if there are any, are at the bottom of the matrix

(c) If a column of the matrix contain the leading entry of some non-zero row, then all other entries in that column are 0.

(d) None of the above

The following system of equations are needed for Question 14 to 17

$$2y+4z=-2$$
$$2x+y+z=1$$
$$x+2y-2z=7$$

14. Identify the corresponding matrix (A),

(a) $\begin{bmatrix} 0 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 5 & 1 & 2 \end{bmatrix}$

15. Using LU factorization method, Find U matrix

(a) $\begin{bmatrix} 0 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 1 & -4 \end{bmatrix}$

16. Using LU factorization method, Find L matrix

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 3/4 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1/2 & 3/4 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1/2 & 1/4 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & -4 \end{bmatrix}$

17. Find $det(A)$

(a) -20 (b) -20 (c) 4 (d) -4

18. Find the equation of the circle that passes through the points (2,6), (2,0) and (5,3)

(a) $det\begin{bmatrix} x^2 & y^2 & 1 \\ 2 & 6 & 40 \\ 2 & 0 & 4 \end{bmatrix}$ (c) $det\begin{bmatrix} x & y & 1 \\ 4 & 36 & 40 \\ 4 & 0 & 4 \end{bmatrix}$

(b) $det\begin{bmatrix} x & y & 1 \\ 2 & 6 & 8 \\ 2 & 0 & 2 \end{bmatrix}$ (d) $det\begin{bmatrix} x^2 & y^2 & 1 \\ 4 & 36 & 40 \\ 4 & 0 & 4 \end{bmatrix}$

Answer the questions 19 to 21.

In a laboratory experiment, a mouse can choose one of two food types each day, type I or type II. Records show that if the mouse chooses type I on a given day, then there is a 15% chance that it will choose type I the next day, and if it chooses type II on one day, then there is a 20% chance that it will choose type II the next day.