Part 1

1 Which of the following transformations are linear transformations?

a $P: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2x \\ y+1 \end{pmatrix}$ **b** $Q: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y \end{pmatrix}$ **c** $R: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2x+y \\ x+xy \end{pmatrix}$

d $S: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ -x \end{pmatrix}$ **e** $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y+3 \\ x+3 \end{pmatrix}$ **f** $U: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2x \\ 3y-2x \end{pmatrix}$

2 Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a $S: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2x-y \\ 3x \end{pmatrix}$ **b** $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2y+1 \\ x-1 \end{pmatrix}$ **c** $U: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} xy \\ 0 \end{pmatrix}$

d $V: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2y \\ -x \end{pmatrix}$ **e** $W: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ x \end{pmatrix}$

3 Identify which of these are linear transformations and give their matrix representations. Give reasons to explain why the other transformations are not linear.

a $S: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$ **b** $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} -y \\ x \end{pmatrix}$ **c** $U: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x-y \\ x-y \end{pmatrix}$

d $V: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **e** $W: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \end{pmatrix}$

4 Find matrix representations for these linear transformations:

a $P: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y+2x \\ -y \end{pmatrix}$ **b** $Q: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} -y \\ x+2y \end{pmatrix}$

5 The triangle T has vertices at $(-1, 1)$, $(2, 3)$ and $(5, 1)$.
Find the vertices of the image of T under the transformations represented by these matrices:

a $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$ **c** $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

6 The square S has vertices at $(-1, 0)$, $(0, 1)$, $(1, 0)$ and $(0, -1)$.
Find the vertices of the image of S under the transformations represented by these matrices:

a $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ **c** $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

7 The rectangle R has vertices at $(2, 1)$, $(4, 1)$, $(4, 2)$ and $(2, 2)$.

a Find the vertices of the image of R under the transformation represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

b Sketch R and its image, R', on a coordinate grid.

c Describe fully the transformation that maps R onto R'.

8 A quadrilateral Q has coordinates (1, 0), (4, 2), (3, 4) and (0, 2).

a Find the vertices of the image of Q under the transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

b Sketch Q and its image, Q', on a coordinate grid.

c Describe fully the transformation that maps Q onto Q'.

9 A square S has coordinates (−1, 0), (−3, 0), (−3, 2) and (−1, 2).

a Find the vertices of the image of S under the transformation represented by the matrix $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

b Sketch S and the image of S on a coordinate grid.

c Describe fully the two transformations that map S onto S'.

10 A triangle T has vertices (4, 1), (4, 3) and (1, 3).

a Find the vertices of the image of T under the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

b Describe the effect of the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.