UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

END SEMESTER EXAMINATION - JUNE 2022

MC2020: LINEAR ALGEBRA

(Duration: 2 hours)

Instructions

1. This is a **Closed-book** exam.
2. This paper contains **FIVE** questions:
3. Answer **all** questions in the space provided.
4. Read all the problems first before beginning to answer any of them. Start with the one you feel most comfortable with, and only move on to the next problem when you are certain you have completed it perfectly.
5. If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state in the script.
6. This examination accounts for **60%** of module assessment. Total maximum mark attainable is **100**.
7. Write your **registration number** in the space provided. Also write your registration number on each additional sheet attached.

Registration Number	**20** . . . . . . |**E**| . . . .
Question Number	Marks
1	
2	
3	
4	
5	
TOTAL	

1. Suppose a automobile plant manufactures three different types of cars,Camry, Corolla and Yaris.

   - The Camry takes 4 hours to assemble, 3 hours to paint and 2 hours to check.
   - The Corolla takes 5 hours to assemble, 2 hours to paint and 3 hours to check.
   - The Yaris takes 3 hours to assemble, 3 hours to paint and 2 hours to check.

   The plant is limited to 47 hours for assembley, 33 hours for painting and 27 hours for checking.

   (a) Prepare a table to summarise the above information. ....
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   (b) How many of each type should we produced to use up all the resources.

      i. System of equation in matrix form: ....
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      ii. The LU factorisation: ....
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      iii. Solve using forward and backward substitution: ....
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(c) What happens if we increase the number of hours for painting up to 38.

   i. Prepare a new table:

   ii. Solve using forward and backward substitution:

(d) Explain which solution method (LU factorisation, Gaussian elimination) is the best approach for the above problem.

   Justify your answer:

[14 marks]

2. The matrix $\mathbf{A} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 1 \\ 2 & -1 & 0 \end{bmatrix}$. Find $\mathbf{A^{-1}}$ according to the following steps.

(a) Find the determinant of $\mathbf{A}$ ....

(b) Form the matrix $\mathbf{M}$ of the minors of $\mathbf{A}$ ....

(c) Form the matrix $\mathbf{C}$ of cofactors. ....

(d) Write down the transpose, $\mathbf{C}^T$, of the matrix of cofactors. ....

(e) Find the inverse of the matrix $\mathbf{A}$ using (c) and (d). ....

3. The transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ is represented by the matrix $\mathbf{T} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

The points with position vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ are respectively transformed by $\mathbf{T}$ to the points with position vectors $\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(a) Find the values of a,b,c,d,e,f,g,h and i. ....

(b) Find the image of the point with position vector $\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$. ....

4. Suppose that a car rental firm has two branches, one in city and the other in Airport. Cars are usually rented for a week and returned to the same place. However, the probability that a car rented in city will be returned to airport is known to be 0.1, and the probability that a car rented in airport will be returned to city is 0.2. Initially the company places 85 cars in city and 40 in Airport.
   Hints: For a Markov chain where:

   - $\mathbf{S_0}$ is an $m \times 1$ column matrix that describes the states at step 0
   - $\mathbf{T}$ is a corresponding $m \times m$ transition matrix
   - $\mathbf{S_n}$ is an $m \times 1$ column matrix giving information about the states at step n of the Markov chain then

   $$\mathbf{S_n} = \mathbf{T} \times \mathbf{S_{n-1}} = \mathbf{T^n} \times \mathbf{S_0}$$

   Determine

   (a) the transition matrix, $\mathbf{T}$, which can be used to represent this information. ..

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   (b) the estimated number of cars in each location at the end of week 4. ....

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5. The matrix $\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Reduce $\mathbf{A}$ to a diagonal matrix $\mathbf{D}$ using the following steps.

(a) Find all eigen values of $\mathbf{A}$. ....

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(b) Find all eigen vectors corresponding to each eigen values. ....

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(c) Find normalised eigen vectors of $\mathbf{A}$ ....

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(d) Form a matrix $\mathbf{P}$ with columns consisting of the normalised eigenvectors of $\mathbf{A}$.

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(e) Write down $\mathbf{P}^T$, the transpose of the matrix $\mathbf{P}$.

(f) A diagonal matrix $\mathbf{D}$ is given by $\mathbf{P}^T\mathbf{A}\mathbf{P} = \mathbf{D}$

6. (a) Let V be the set of all fifth degree polynomials with standard addition and multiplication. Is it a vector space? Justify your answer.

(b) Given non-zero real numbers a, b and c, consider

$$X_c = \{(x \;\; y) \in \mathbb{R}^2 \mid ax + by = c\}.$$

Show that X_c is not a subspace of $\mathbb{R}^2$.

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(c) i. For which real values of λ do the following vectors form a linearly dependent set in R^3?
$v_1 = (\lambda, -\frac{1}{2}, -\frac{1}{2})$, $v_2 = (-\frac{1}{2}, \lambda, -\frac{1}{2})$ and $v_3 = (-\frac{1}{2}, -\frac{1}{2}, \lambda)$ . . . .
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ii. Use the Wronskian to show that the following functions $f_1(x) = e^x$, $f_2(x) = xe^x$, and $f_3(x) = x^2e^x$ are linear independent vectors in $C^\infty(-\infty, \infty)$ . . .
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(d) Find the null space of the 2×3 matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -3 & -5 \end{bmatrix}$. . . . .
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(e) Consider the set of polynomials $S = \{1\ , t\ , t^2\}$ defined over the intervals of $-1 \le t \le 1$. Using the Gram -Schmidt orthogonalization process, obtain an orthonormal set. . . . .
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——— *End of Examination* ———