Faculty of Engineering
University of Jaffna, Sri Lanka
MC 2020 Linear Algebra - 2023

Model Paper-03 Duration 2 Hours

1. Let, $\mathbf{M} = \begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

   (a) Find the Characteristic polynomial of M

   (b) Find all eigenvalues of M

   (c) for each eigenvalues find a basis for the corresponding eigen space.

2. $\mathbf{A} = \begin{pmatrix} 4 & x & 0 \\ 2 & -3 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -5 \\ 4 & x \\ x & 7 \end{pmatrix}$:

   (a) Find the product $\mathbf{AB}$ in terms of x.
   A symmetric matrix is one in which the entries are symmetrical about the leading diagonal, for example $\begin{pmatrix} 2 & 5 \\ 5 & 0 \end{pmatrix}$ and $\begin{pmatrix} 3 & 4 & -6 \\ 4 & 2 & 5 \\ -6 & 5 & 1 \end{pmatrix}$.

   (b) Given that the matrix $\mathbf{AB}$ is symmetric, find the possible values of x.

   (c) Write down the possible matrices of $\mathbf{AB}$.

4. (a) For Each of the following Matrices decide whether or not the matrix is diagonalizable over R. The eigen values for each matrix are given. you should justify your answer.

   i. $\begin{pmatrix} 7 & -14 & -10 \\ -2 & 7 & 4 \\ 7 & -19 & -12 \end{pmatrix}$ has eigenvalues: -1,1,2

   ii. $\begin{pmatrix} 3 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$ has eigenvalues: 1,1,2

   iii. $\begin{pmatrix} 0 & 0 & 1 \\ 6 & 4 & -6 \\ 1 & 1 & 0 \end{pmatrix}$ has eigenvalues: 1,1,2

   (b) i. Write down a symmetric 2×2 matrix A Such that
   $\begin{pmatrix} \mathrm{x} & \mathrm{y} \end{pmatrix} A \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \end{pmatrix} = \begin{pmatrix} 8x^2 + 6xy \end{pmatrix}$

   ii. Find an orthogonal matrix Q and a diagonal matrix D such that $D = Q^T AQ$

5. (a) Consider the vectors $u = (1, 0)$ and $v = (0, 1)$, with the standard Euclidean inner product, they have a norm of 1, and the distance is $\sqrt{2}$. Use the weighted Euclidean inner product $< u, v >= 7u_1v_1 + 3u_2v_2$ to find the norm of u, v and the distance between them.

   (b) Find the distance between $u = (8, -2, 1)$ and $v = (3, 5, 0)$ in $\mathbb{R}^3$ using the weighted Euclidean inner product $< u, v >= 2u_1v_1 + 3u_2v_2 + 4u_3v_3$.

6. Consider the Matrices,

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 2 & 0 & -1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & -1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

Evaluate,If possible.

(a) BA

(b) $CD + A^T$

(c) CB^2A

(d) $B^TB - 5C$

7. Determine whether the following are vector space over $\mathbb{R}$.

(a) Under the usual addition and scalar multiplication in $\mathbb{R}^3$
$S_1 = \{(x, y, z) \,|x^2 + y^2 + z^2 = 1\}$

(b) Under the usual addition and scalar multiplication on matrices
$S_2 = \left\{ \begin{pmatrix} a & 1 \\ b & c \end{pmatrix} |a, b, c \;\varepsilon\; \mathbb{R} \right\}$

(c) Under the usual addition and scalar multiplication on matrices
$S_3 = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} |a, b, c \;\varepsilon\; \mathbb{R} \right\}$