1. For an $n \times n$ upper triangular matrix $\mathbf{A}$,

(a) $a_{ij} = 0,\ i > j$
(b) $a_{ij} = 0,\ j > i$
(c) $a_{ij} \neq 0,\ i > j$
(d) $a_{ij} \neq 0,\ j > i$

2. The order of the matrix $\begin{bmatrix} 4 & -6 & -7 & 2 \\ 3 & 2 & -5 & 6 \end{bmatrix}$ is

(a) 4×2
(b) 2×4
(c) 8×1
(d) not defined

3. For a square $n \times n$ matrix $\mathbf{A}$ to be an identity matrix,

(a) $a_{ij} \neq 0,\ i = j; a_{ij} = 0,\ i = j$
(b) $a_{ij} = 0,\ i \neq j; a_{ij} = 1,\ i = j$
(c) $a_{ij} = 0,\ i \neq j; a_{ij} = i,\ i = j$
(d) $a_{ij} = 0,\ i \neq j; a_{ij} > 0,\ i = j$

4. If $\mathbf{A} = \begin{bmatrix} 5 & 6 \\ 7 & -3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ then $\mathbf{AB} =$

(a) $\begin{bmatrix} -8 \\ 23 \end{bmatrix}$
(b) $\begin{bmatrix} 10 & 12 \\ 14 & 9 \end{bmatrix}$
(c) $\begin{bmatrix} -2 & 5 \end{bmatrix}$
(d) not possible

5. For the product $\mathbf{AB}$ to be possible

(a) the number of rows of $\mathbf{A}$ needs to be the same as the number of columns of $\mathbf{B}$
(b) the number of columns of $\mathbf{A}$ needs to be the same as the number of rows of $\mathbf{B}$
(c) the number of rows of $\mathbf{A}$ and $\mathbf{B}$ needs to be the same
(d) the number of columns of $\mathbf{A}$ and $\mathbf{B}$ needs to be the same

6. $\mathbf{A}$ and $\mathbf{B}$ are square matrices of $n \times n$ order. Then $(\mathbf{A} - \mathbf{B})(\mathbf{A} - \mathbf{B})$ is equal to

(a) $\mathbf{A}^2 + \mathbf{B}^2 - 2\mathbf{AB}$
(b) $\mathbf{A}^2 + \mathbf{B}^2$
(c) $\mathbf{A}^2 - \mathbf{B}^2$
(d) $\mathbf{A}^2 + \mathbf{B}^2 - \mathbf{AB} - \mathbf{BA}$

7. You sell Jupiter and Fickers Candy bars. The sales in January are 25 and 30 of Jupiter and Fickers, respectively. In February, the sales are 75 and 35 of Jupiter and Fickers, respectively. If a Jupiter bar costs \$2 and a Fickers bar costs \$7, then if $\mathbf{A} = \begin{bmatrix} 25 & 30 \\ 75 & 35 \end{bmatrix}$, and $\mathbf{B} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, the total sales amount in each month is given by

(a) **BA**
(b) **AB**
(c) 2**A**
(d) 7**A**

8. If $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$, the eigenvalue corresponding to the eigen vector is

   (a) 1
   (b) 4
   (c) -4.5
   (d) 6

9. The eigenvalues of the following matrix

$$\begin{bmatrix} 3 & 2 & 9 \\ 7 & 5 & 13 \\ 6 & 17 & 19 \end{bmatrix}$$

   are given by solving the cubic equation

   (a) $\lambda^3 - 27\lambda^2 + 167\lambda - 285 = 0$
   (b) $\lambda^3 - 27\lambda^2 - 122\lambda - 313 = 0$
   (c) $\lambda^3 + 27\lambda^2 + 167\lambda + 285 = 0$
   (d) $\lambda^3 + 23.23\lambda^2 - 158.3\lambda + 313 = 0$

10. The eigenvalues of a 4×4 matrix **A** are given as 2,3,13, and 7. The $det(\mathbf{A})$ then is

   (a) 546
   (b) 19
   (c) 25
   (d) cannot be determined

11. If one of the eigenvalues of $\mathbf{A}_{n\times n}$ is zero, it implies

   (a) The solution to $\mathbf{AX} = \mathbf{C}$ system of equations is unique.
   (b) The determinant of **A** is zero.
   (c) The solution to $\mathbf{AX} = \mathbf{0}$ system of equations is trivial.
   (d) The determinant of **A** is nonzero.

12. Given that matrix $\mathbf{A} = \begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -3 \end{bmatrix}$ has an eigenvalue of 4 with the corresponding eigenvectors of $\mathbf{X} = \begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix}$, then $\mathbf{A}^5\mathbf{X}$ is

   (a) $\begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix}$
   (b) $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$
   (c) $\begin{bmatrix} -4608 \\ -4096 \\ 1024 \end{bmatrix}$
   (d) $\begin{bmatrix} -0.004395 \\ -0.003906 \\ 0.0009766 \end{bmatrix}$

13. The goal of forward elimination steps in the Gauss elimination method is to reduce the coefficient matrix to a(an) . . . matrix.

(a) diagonal

(b) identity

(c) lower triangular

(d) upper triangular

14. Division by zero during forward elimination steps in Nave Gaussian elimination of the set of equations $\mathbf{AX} = \mathbf{C}$ implies the coefficient matrix $\mathbf{A}$

(a) is invertible

(b) is nonsingular

(c) may be singular or nonsingular

(d) is singular

15. Using a computer with four significant digits with chopping, the Gaussian elimination with partial pivoting solution to

$$\begin{aligned} 0.0030x_1 + 55.23x_2 &= 58.12 \\ 6.239x_1 - 7.123x_3 &= 47.23 \end{aligned}$$

is

(a) $x_1 = 26.66;\ x_2 = 1.051$

(b) $x_1 = 8.769;\ x_2 = 1.051$

(c) $x_1 = 8.800;\ x_2 = 1.000$

(d) $x_1 = 8.771;\ x_2 = 1.052$

16. At the end of the forward elimination steps of the Naive Gauss elimination method on the following equations

$$\begin{bmatrix} 4.2857 \times 10^7 & -9.2307 \times 10^5 & 0 & 0 \\ 4.2857 \times 10^7 & -5.4619 \times 10^5 & -4.2857 \times 10^7 & 5.4619 \times 10^5 \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857 \times 10^7 & -3.6057 \times 10^5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887 \times 10^3 \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

the resulting equations in matrix form are given by

$$\begin{bmatrix} 4.2857 \times 10^7 & -9.2307 \times 10^5 & 0 & 0 \\ 4.2857 \times 10^7 & -5.4619 \times 10^5 & -4.2857 \times 10^7 & 5.4619 \times 10^5 \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857 \times 10^7 & -3.6057 \times 10^5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887 \times 10^3 \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

the resulting equations in matrix form are given by

$$\begin{bmatrix} 4.2857 \times 10^7 & -9.2307 \times 10^5 & 0 & 0 \\ 0 & 3.7688 \times 10^5 & -4.2857 \times 10^7 & 5.4619 \times 10^5 \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500 \times 10^5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887 \times 10^3 \\ 7.887 \times 10^3 \\ 1.19530 \times 10^{-2} \\ 1.90336 \times 10^4 \end{bmatrix}$$

The determinant of the original coefficient matrix is

(a) 0.00 (b) 4.2857×10^7 (c) 5.486×10^{19} (d) -2.445×10^{20}

17. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is

(a) 0 (b) 1 (c) 2 (d) 3

18. The following data is given for the velocity of the rocket as a function of time.

t	(s)	0	14	15	20	30	35
v(t)	(m/s)	0	227.04	362.78	517	30	35

To find the velocity at $t = 21$ s, you are asked to use a quadratic polynomial,$v(t) = at^2 + bt + c$ to approximate the velocity profile. The correct set of equations that will find a, b and c are

(a) $\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$

(b) $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$

(d) $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$

19. The [L][U] decomposition method is computationally more efficient than Naive Gauss elimination for solving

(a) a single set of simultaneous linear equations.

(b) multiple sets of simultaneous linear equations with different coefficient matrices and the same right hand side vectors.

(c) multiple sets of simultaneous linear equations with the same coefficient matrix and different right hand side vectors.

(d) less than ten simultaneous linear equations.

20. If $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ and $det(\mathbf{A}) = 0$ then rank of a matrix $\mathbf{A}$ is

(a) Greater than or equal to 3

(b) Strictly less than 3

(c) Less than or equal to 3

(d) Strictly greater than 3

21. In given system of linear equations $\mathbf{AX} = \mathbf{b}$, if $det(\mathbf{A}) \neq 0$ then system has

(a) Unique solution
(b) No solution
(c) infinite solutions
(d) None of the above

22. In set of vectors , if at least one vector of the set can be expressed as a linear combination of the remaining vectors then these vectors are called

(a) Linearly independent
(b) linearly dependent
(c) Orthogonal vectors
(d) none of these

23. Rank of a matrix is nothing but

(a) number of zero rows in that matrix
(b) number of zero rows in its echelon form of matrix
(c) number of non-zero rows in that matrix
(d) number of non-zero rows in its echelon form of matrix.

24. Consider the matrices:
$\mathbf{U} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, $\mathbf{V} = \begin{bmatrix} 0.43 \\ 0.45 \end{bmatrix}$, $\mathbf{W} = \begin{bmatrix} 0.9 & 0.5 \\ 0.1 & 0.5 \end{bmatrix}$, $\mathbf{Y} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$
The matrix that could be a transition matrix for a Markov chain is

(a) $\mathbf{U}$
(b) $\mathbf{V}$
(c) $\mathbf{W}$
(d) $\mathbf{Y}$

The following information is needed for Questions 25 and 26: A factory has a large number of machines that can be in one of two states, operating or broken. The probability that an operating machine breaks down by the end of the day is 0.05 and the probability that a broken machine is repaired by the end of the day is 0.80.

25. A transition matrix $\mathbf{T}$ that can be used to represent this information is:

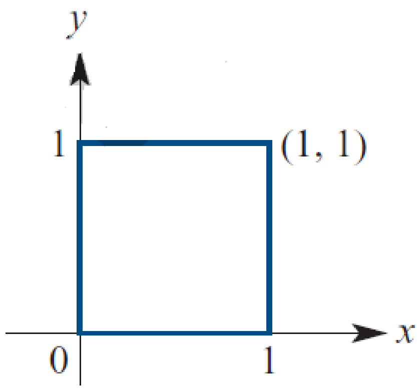
(a) $\begin{bmatrix} 0.95 & 0.20 \\ 0.05 & 0.80 \end{bmatrix}$
(b) $\begin{bmatrix} 0.05 & 0.20 \\ 0.95 & 0.80 \end{bmatrix}$
(c) $\begin{bmatrix} 0.05 \\ 0.80 \end{bmatrix}$
(d) $\begin{bmatrix} 0.95 & 0.80 \\ 0.05 & 0.20 \end{bmatrix}$

26. If a machine is operating at the end of day 1, then the probability that it is broken by the end of day 6 can be found by evaluating:

(a) $\mathbf{T}^6 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
(b) $\mathbf{T}^5 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
(c) $\mathbf{T}^6 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
(d) $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \times \mathbf{T}^6$

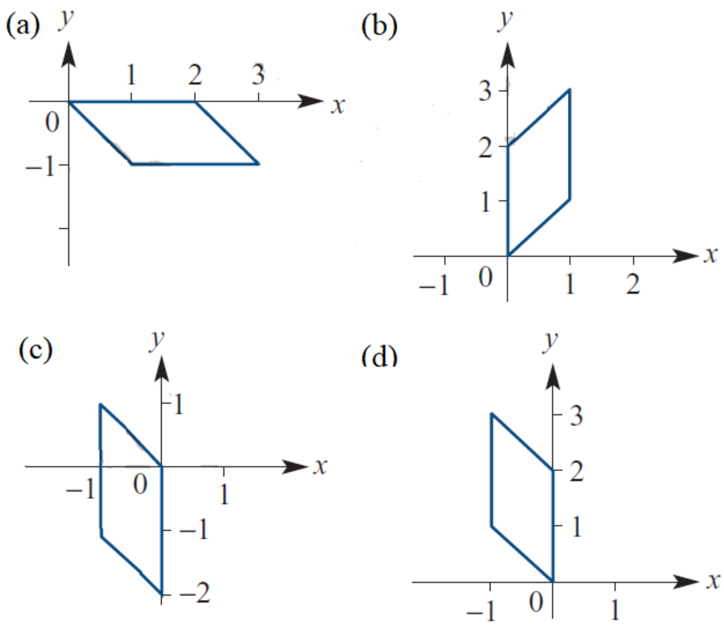
27. The square shown is subject to successive transformations.

- The first transformation has matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ and



- the second transformation has matrix $\begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix}$

Which one of the following shows the image of the square after these two transformations?



28. The matrix which determines the transformation, stretch in the x-direction of factor 2 followed by a stretch of factor 3 in the y-direction, is

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}$

29. The matrix which determines the transformation, rotation of -30^o about the origin, is

(a) $\begin{bmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -0.5 \\ 0.5 & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{bmatrix}$

(d) $\begin{bmatrix} -0.5 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0.5 \end{bmatrix}$

30. The inverse of matrix $\mathbf{A}$, when $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$

(a) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$

31. If $\mathbf{P}$ is an $m \times n$ matrix, and $\mathbf{Q}$ is an $n \times p$ matrix, the dimension of matrix $\mathbf{QP}$ is:

(a) $n \times n$,

(b) $m \times p$,

(c) $n \times p$,

(d) cannot be determined.

32. Situation in which not a single row of a matrix is linear combination of remaining rows is classified as

(a) identity dependence

(b) identity independence

(c) linearly dependant

(d) linearly independent

33. According to determinant properties, when two rows are interchanged then signs of determinant

(a) must changes

(b) remains same

(c) multiplied

(d) divided

34. Let $\mathbf{H}$ be a subspace of a vector space $\mathbf{V}$, and suppose that $\mathbf{V}$ has dimension d. Which of the following statements are true?

A- $\dim(\mathbf{H}) \leq \dim(\mathbf{V})$

B- a linearly independent set of vectors in $\mathbf{H}$ is also linearly independent in $\mathbf{V}$;

C- d vectors which span $\mathbf{V}$ will be linearly independent;

D- d vectors which span $\mathbf{H}$ will also span $\mathbf{V}$

(a) A, B, C and D

(b) A, B and C only

(c) B, C and D only

(d) A and D only

35. Let $\mathbf{V}$ be a five dimensional vector space, and let $\mathbf{S}$ be a subset of $\mathbf{V}$. Then $\mathbf{S}$

(a) Must consist of at least five elements

(b) Must be linearly dependent

(c) Must have at most five elements

(d) Must have exactly five elements

36. If $\mathbf{T} : \mathbf{U} \longrightarrow \mathbf{V}$ is any linear transformation from $\mathbf{U}$ to $\mathbf{V}$ and $\mathbf{B} = \{u_1, u_2, \ldots, u_n\}$ is a basis for U, then set $T(B) = \{T(u_1), T(u_2), \ldots, T(u_n)\}$

(a) spans $\mathbf{V}$

(b) spans $\mathbf{U}$

(c) is a basis for $\mathbf{V}$

(d) is linearly independent

37. Let $\mathbf{V}$ be the vector space of all functions $f(x)$ where $f : \mathbf{R} \to \mathbf{R}$. Which of the following, is not subspace of $\mathbf{V}$?

(a) the constant functions;

(b) functions with $\lim_{x\to\infty} f(x) = 3$

(c) functions with $f(0) = 0$

(d) none of the above.

38. Let $\mathbf{V}$ be an vector space, and $\mathbf{W}$ be a subset of $\mathbf{V}$. What does it mean when we say that $\mathbf{W}$ is closed under addition?

(a) whenever x and y are in $\mathbf{V}$, then $x + y$ is in $\mathbf{V}$.

(b) whenever x and y are in $\mathbf{W}$, then $x + y$ is in $\mathbf{W}$.

(c) $\mathbf{W}(x + y) = \mathbf{W}x + \mathbf{W}y$ for every two vectors x and y.

(d) whenever x and y are in $\mathbf{V}$, then $x + y$ is in $\mathbf{W}$.

39. Consider the following sets of vectors:

$\mathbf{S_1} = \{(1, 0, 1)^T, (2, 1, 1)^T, (1, 1, 0)^T\}$

$\mathbf{S_2} = \{(2, 1, 0)^T, (3, -2, 0)^T, (0, 1, 0)^T\}$

$\mathbf{S_3} = \{(1, -1, 0)^T, (0, 1, -1)^T, (2, 0, 2)^T\}$

Of these sets, those which are basis of $\mathbb{R}^3$:

(a) All of them (b) $\mathbf{S_1}, \mathbf{S_3}$ (c) $\mathbf{S_2}$ (d) $\mathbf{S_3}$

40. The trace of a matrix $\begin{bmatrix} 5 & 6 & -7 \\ 9 & -11 & 13 \\ -17 & 19 & 23 \end{bmatrix}$ is

(a) 17 (b) 39 (c) 40 (d) 110