

## MC2020- Linear Algebra

### Tutorial-01

1)

- a** Write down the transpose of  $\begin{bmatrix} 7 & 4 \\ 8 & 1 \end{bmatrix}$ .
- b** Write down the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 5 \end{bmatrix}^T$ .
- c** If  $A = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}$ , write down the matrix  $A^T$ .

2)

Consider the following square matrices.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 4 & 7 \\ 5 & 7 & 2 \end{bmatrix}$$

Write down:

- a** the upper triangular matrices      **b** the identity matrix  
**c** the diagonal matrices      **d** the symmetric matrices.

3)

For the matrices  $A$  and  $B$ , opposite, write down the values of:

- a**  $a_{12}$       **b**  $a_{21}$       **c**  $a_{33}$       **d**  $b_{31}$ .

$$A = \begin{bmatrix} 1 & 5 & 3 \\ -1 & 0 & 4 \\ 2 & -2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

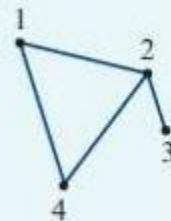
The table opposite shows the three types of membership of a local gym and the number of males and females enrolled in each. Construct a matrix to display the numerical information in the table.

Gender	Gym membership		
	Weights	Aerobics	Fitness
Males	16	104	86
Females	75	34	94

4)

Represent the network diagram shown opposite as a  $4 \times 4$  matrix  $A$ , where the:

- matrix element = 1 if the two points are joined by a line
- matrix element = 0 if the two points are not connected.



5)

The diagram opposite shows the roads connecting four towns: town 1, town 2, town 3, and town 4. This diagram has been represented by a  $4 \times 4$  matrix,  $A$ . The elements show the number of roads between each pair of towns.



- a In the matrix  $A$ ,  $a_{24} = 1$ . What does this tell us?
- b In the matrix  $A$ ,  $a_{34} = 3$ . What does this tell us?
- c In the matrix  $A$ ,  $a_{41} = 0$ . What does this tell us?
- d What is the sum of the elements in row 3 of matrix  $A$  and what does this tell us?
- e What is the sum of all the elements of matrix  $A$  and what does this tell us?

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix} \end{matrix}$$

6)

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix} \quad \text{Find } A + B.$$

7)

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix} \quad \text{Find } A - B.$$

8)

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix} \quad \text{Find } 3A \text{ and } 0.5C.$$

9)

$$G = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} \quad H = \begin{bmatrix} 9 & 0 \\ -6 & 3 \end{bmatrix} \quad \text{Show that } 3G - 2H = O.$$

10)

The sales data for two used car dealers, Honest Joe's and Super Deals, are displayed below.

<i>Car sales</i>	<i>2014</i>			<i>2015</i>		
	<i>Small</i>	<i>Medium</i>	<i>Large</i>	<i>Small</i>	<i>Medium</i>	<i>Large</i>
Honest Joe's	24	32	11	26	38	16
Super Deals	32	34	9	35	41	12

- Construct two matrices,  $A$  and  $B$ , to represent the sales data for 2014 and 2015 separately.
- Construct a new matrix  $C = A + B$ . What does this matrix represent?

- c** Construct a new matrix,  
 $D = B - A$ . What does this matrix  
 represent?

11)

$$A = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The following matrix products are defined. What is their order?

- a**  $BA$                       **b**  $BC$                       **c**  $AC$

12)

#### Order of a matrix

- 1** How many elements are there in a  $3 \times 5$  matrix?
- 2** A matrix has 12 elements. What are its possible orders? (There are six.)

#### The transpose of a matrix

**3** **a**  $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^T =$       **b**  $\begin{bmatrix} 3 \\ 5 \end{bmatrix}^T =$       **c**  $\begin{bmatrix} 9 & 1 & 0 & 7 \\ 8 & 9 & 1 & 5 \end{bmatrix}^T =$

13)

- 1** The table opposite gives the  
 number of residents, TVs and  
 computers in three households.

<i>Household</i>	<i>Residents</i>	<i>TVs</i>	<i>Computers</i>
A	4	2	1
B	6	2	3
C	2	1	0

Use the table to:

- a** construct a matrix to display the numbers in the table. What is its order?
- b** construct a row matrix to display the numbers in the table relating to household B.  
 What is its order?
- c** construct a column matrix to display the numbers in the table relating to computers.  
 What is its order? What does the sum of its elements tell you?

14)

### Square matrices and their elements

4 Complete the sentences below that relate to the following matrices.

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 6 \\ -1 & 0 \\ 1 & 3 \\ 4 & -4 \end{bmatrix} \quad E = \begin{bmatrix} 4 & 3 & 1 \\ 0 & -1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

- a The square matrices are  and .
- b Matrix  $B$  has rows .
- c The row matrix is .
- d The column matrix is .
- e Matrix  $D$  has  rows and  columns.
- f The order of matrix  $E$  is   $\times$  .
- g The order of matrix  $A$  is   $\times$  .
- h The order of matrix  $B$  is   $\times$  .
- i The order of matrix  $D$  is   $\times$  .
- j There are  elements in matrix  $E$ .
- k There are  elements in matrix  $A$ .
- l  $a_{14} =$
- m  $b_{31} =$
- n  $c_{11} =$
- o  $d_{41} =$
- p  $e_{22} =$
- q  $d_{32} =$
- r  $b_{11} =$
- s  $c_{12} =$

15)

The table opposite gives the yearly car sales for two car dealers.

Car sales	Small	Medium	Large
Honest Joe's	24	32	11
Super Deals	32	34	9

Use the table to:

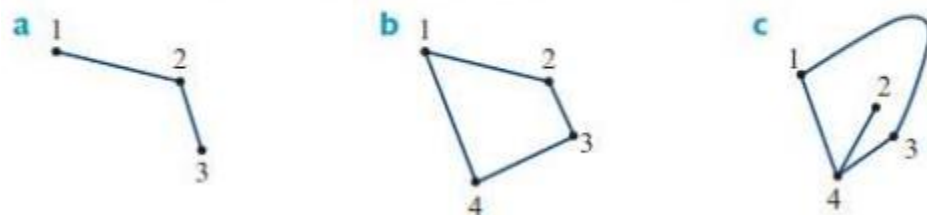
- a construct a matrix to display the numbers in the table. What is its order?
- b construct a row matrix to display the numbers in the table relating to Honest Joe's. What is its order?
- c construct a column matrix to display the numerical information in the table relating to small cars. What is its order? What does the sum of its elements tell you?



16)

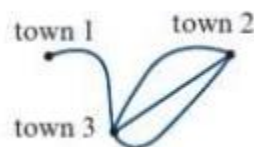
Represent each of the following network diagrams by a matrix  $A$  using the rules:

- matrix element = 1 if points are joined by a line
- matrix element = 0 if points are not joined by a line.



17)

The diagram opposite shows the roads interconnecting three towns: town 1, town 2 and town 3. Represent this diagram with a  $3 \times 3$  matrix where the elements represent the number of roads between each pair of towns.



18)

The network diagram opposite shows a friendship network between five girls: girl 1 to girl 5.

This network has been represented by a  $5 \times 5$  matrix,  $F$ , using the rule:

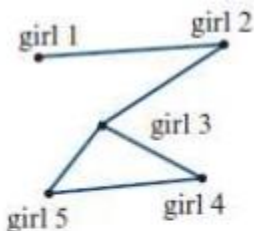
- element = 1 if the pair of girls are friends
- element = 0 if the pair of girls are not friends.

**a** In the matrix  $F$ ,  $f_{34} = 1$ . What does this tell us?

**b** In the matrix  $F$ ,  $f_{25} = 0$ . What does this tell us?

**c** What is the sum of the elements in row 3 of the matrix and what does this tell us?

**d** Which girl has the least friends? The most friends?



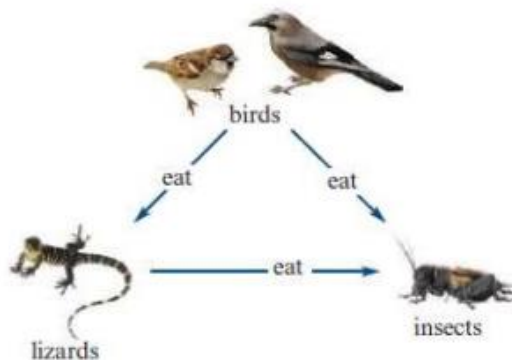
$$F = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

19)

- a** The diagram below shows the feeding patterns for insects (*I*), birds (*B*) and lizards (*L*).

The matrix *E* below has been set up to represent the information in the diagram.

$$E = \begin{array}{ccc|c} I & B & L & \\ \hline 0 & 1 & 0 & I \\ 0 & 0 & 0 & B \\ 0 & 1 & 0 & L \end{array}$$



Referring to insects, birds or lizards:

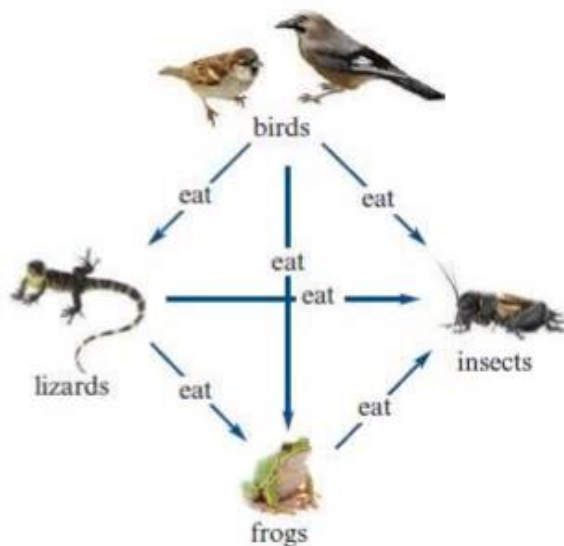
- i What does the '1' in column *B*, row *L*, of matrix *E* represent?
- ii What does the row of zeroes in matrix *E* represent?

The diagram below shows the feeding patterns for insects (*I*), birds (*B*), lizards (*L*) and frogs (*F*).

The matrix *Z* below the diagram has been set up to represent the information in the diagram.

Matrix *Z* is incomplete.

$$Z = \begin{array}{cccc|c} I & B & L & F & \\ \hline 0 & 1 & \square & \square & I \\ 0 & 0 & \square & \square & B \\ 0 & 1 & \square & \square & L \\ \square & \square & \square & \square & F \end{array}$$



- b** Complete matrix *Z*.

20)

- 1** The questions below relate to the following six matrices. Computations will be quicker if done by hand.

$$A = \begin{bmatrix} 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

- a** Which matrices are equal?  
**b** Which matrices have the same order?  
**c** Which matrices can be added or subtracted?  
**d** Compute each of the following, where possible.

<b>i</b> $A + B$	<b>ii</b> $D + E$	<b>iii</b> $C - F$	<b>iv</b> $A - B$	<b>v</b> $E - D$
<b>vi</b> $3B$	<b>vii</b> $4F$	<b>viii</b> $3C + F$	<b>ix</b> $4A - 2B$	<b>x</b> $E + F$