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MC 2020 : Linear Algebra

Tutorial-06

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1. The following matrices are needed for Question (a) to (j)

• $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

• $D = \begin{bmatrix} -8 \\ 8 \end{bmatrix}$

• $B = \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$

• $E = \begin{bmatrix} -2 & 3 & 5 \\ 2 & 7 & -6 \end{bmatrix}$

• $C = \begin{bmatrix} -3 & 3 \end{bmatrix}$

• $F = \begin{bmatrix} 3 & -6 \\ -8 & 9 \end{bmatrix}$

(a) The square matrices are :

- i. A and F ii. B and D iii. A and E iv. A and C

(b) The order of matrix B is :

- i. 1×3 ii. 3×3 iii. 3×1 iv. 1×2

(c) The following matrices can be added :

- i. D and E ii. A and B iii. C and E iv. A and F

(d) The following matrix product is not defined :

- i. BC ii. AD iii. AF iv. EA

(e) The order of matrix product DC is :

- i. 1×2 ii. 2×1 iii. 1×1 iv. 2×2

★ Write the answers for the following questions.

(f) $-2E$

(h) $EB - D$

(j) $\det A$

(g) B^T

(i) F^{-1}

2. $P = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Given that U maps the point with coordinates (a, b) onto the point with coordinates $(3 + 2a, b + 1)$, find the values of a and b .

3. $U = \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}$ and $V = \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}$:

(a) Find the matrix product \mathbf{UV} in terms of x .

(b) If $\mathbf{UV} = \begin{pmatrix} 10x & -9 \\ -4 & 3 \end{pmatrix}$, find the possible values of x .

(c) Find the possible matrix products \mathbf{VU} .

4. $\mathbf{A} = \begin{pmatrix} 4 & x & 0 \\ 2 & -3 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -5 \\ 4 & x \\ x & 7 \end{pmatrix}$:

(a) Find the product \mathbf{AB} in terms of x .

A symmetric matrix is one in which the entries are symmetrical about the leading diagonal, for example $\begin{pmatrix} 2 & 5 \\ 5 & 0 \end{pmatrix}$ and $\begin{pmatrix} 3 & 4 & -6 \\ 4 & 2 & 5 \\ -6 & 5 & 1 \end{pmatrix}$.

(b) Given that the matrix \mathbf{AB} is symmetric, find the possible values of x .

(c) Write down the possible matrices of \mathbf{AB} .

5. Four local football teams took part in a competition in which they each played each other twice, once at home and once away. Figure 1.1 shows the results matrix after half of the games had been played.

		Win	Draw	Lose	Goals for	Goals against
City		2	1	0	6	3
Rangers		0	0	3	2	8
Town		2	0	1	4	3
United		1	1	1	5	3

Figure 1.1

(a) The results of the next three matches are as follows :

- City 2 Rangers 0
- Town 3 United 3
- City 2 Town 4

Find the results matrix for these three matches and hence find the complete results matrix for all the matches so far.

(b) Here is the complete results matrix for the whole competition.

$$\begin{pmatrix} 4 & 1 & 1 & 12 & 8 \\ 1 & 1 & 4 & 5 & 12 \\ 3 & 1 & 2 & 12 & 10 \\ 1 & 3 & 2 & 10 & 9 \end{pmatrix}$$

Find the results matrix for the last three matches (City vs United, Rangers Vs Town and Rangers vs United) and deduce the result of each of these three matches.

6. Solve the following system by using

- (a) Gaussian Elimination
- (b) LU Factorization

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 2 \\x_1 + 2x_2 - x_3 + x_4 &= 1 \\x_1 - x_2 + 2x_3 + 2x_4 &= 0 \\x_1 + 2x_2 + x_3 + 3x_4 &= 1\end{aligned}$$

7. $M = \begin{pmatrix} -2\sqrt{2} & -2\sqrt{2} \\ 2\sqrt{2} & -2\sqrt{2} \end{pmatrix}$ The matrix M represents an enlargement with scale factor k followed by a rotation through angle θ anticlockwise about the origin.

- (a) Find the value of k .
- (b) Find the value of θ .

8. $M = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

- (a) Find the image of T' of a triangle T with vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$ under the transformation represented by M .
- (b) Sketch T and T' on the same set of coordinate axes.
- (c) Describe geometrically the transformation represented by M .

9. The transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix T where $T = \begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 2 \\ -3 & 2 & 8 \end{pmatrix}$.

- (a) Find T^{-1} .

The vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is transformed by T to the vector $\begin{pmatrix} -5 \\ 5 \\ 16 \end{pmatrix}$.

- (b) Find the values of constants a , b , c .

***** **END** *****