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MC 2020 : Linear Algebra

Tutorial-04 **October 2023**

1. Given that $T : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x - y \\ y + z \\ 2x - 3z \end{pmatrix}$ and $U : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} 2x - 3y - z \\ 2y + 3z \\ 5z \end{pmatrix}$, find matrices representing

   (a) T (b) U (c) TU

2. The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix T.
   The vector $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ is transformed by T to the vector $\begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$.
   The vector $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ is transformed by T to the vector $\begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$.
   The vector $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is transformed by T to the vector $\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$.
   Find T.

3. The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix T where $T = \begin{pmatrix} 0 & -1 & 2 \\ 2 & 5 & -4 \\ 3 & 2 & 1 \end{pmatrix}$.
   The line l_1 is transformed by the line l_2. The line l_1 has the vector equation $r = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$, where t is a real parameter. Find a vector equation of l_2.

4. The point A and B have position vector $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$ respectively. The points A and B are transformed by the linear transformation T to the point A' and B' respectively. The transformation T is represented by the matrix T, where $T = \begin{pmatrix} 1 & -3 & 4 \\ 2 & 3 & -2 \\ 0 & 2 & 5 \end{pmatrix}$.

   (a) Find the position vectors of A' and B'.
   (b) Hence find a vector equation of the line $A'B'$.

5. The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix T where $T = \begin{pmatrix} 3 & -2 & -2 \\ -2 & -8 & 4 \\ -2 & 4 & 0 \end{pmatrix}$.
   The plane π_1 is transformed by T to the plane π_2. The plane π_1 has Cartesian equation $x - 2y + z = 0$. Find a Cartesian equation of π_2.