Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 2020 : Linear Algebra

Tutorial-03 **October 2023**

1. Determine whether the vectors (1, 2, 3), (1, 1, 2), (1, 4, 2) in $\mathbb{R}^3$ are linearly independent.

2. Determine a value for q such that the following vectors are linearly independent
(1, 1, 2, 1), (2, 1, 2, 3), (1, 4, 2, 1) (1, 3, 5, q)

3. Let V be a vector space and let $u, v, w \in V$. Show that the vectors $u - v, v - w$ and $w - u$ are linearly dependent.

4. Let V be a vector space and suppose that u_1, u_2, u_3 are linearly independent vectors in V. Prove that $u_1 + u_2, u_2 + u_3, u_3$are also linearly independent.

5. Let V be a vector space and let $v \in V$. If v is a linear combination of the vectors $\{u_1, \ ....u_n\}$ and if each u_i is a linear combination of the vectors $\{w_1, \ ....w_m\}$, prove that v is a linear combination of$\{w_1, \ ....w_m\}$.

6. Show that the vectors $u_1 = (0, 3, 1, 1), u_2 = (6, 0, 5, 1)$ and $u_3 = (4, 7, 1, 3)$ form a linearly dependent set in $\mathbb{R}^4$
Express each vector as a linear combination of the other two.

7. Determine if (3, 4, 1, 6) lies in span (1, 2, -1, 2), (-2, 3, 1, -1), (-1, 3, 2, 1) in $\mathbb{R}^4$

8. Determine whether the given set of vectors spans the given vector space

   (a) In $\mathbb{R}^3 : (1, -1, 2), (1, 1, 2), (0, 0, 1)$

   (b) In $P_2 : 1 - x, 3 - x^2, x$

   (c) In $M_{22} : \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 3 & 1 \end{pmatrix}$

9. Let V be a vector space and suppose $u_1 u_2 u_3$ span V. Let $v_1 = u_1, v_2 = u_2 - u_1, v_3 = u_3$. Prove that v_1, v_2, v_3 also span V.

10. Let V be the set of all 2×2 matrices with real entries. Consider the set, $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$,
Find Span S.
Lets S be a subset of vector space of V, Span(S) consists of all of the linear combination of vector in S.

11. Let $v_1 = t + 2$,$v_2 = t^2 + 1$ and $S = \{v_1,\ v_2\}$. Show that S does not span of P_2 (2 degree of polynomials).

12. Let V be a vector space and let $S = u_1, ...u_n \subseteq V$. If $S1$ is a nonempty subset of S, prove or give counterexamples to the following statements.

    (a) If S spans V, then S_1 spans V.

    (b) If S_1 is linearly independent, then S is linearly independent.

13. (a) Let V and W be the subspace of $\mathbb{R}^4$.
    $V = \{(a, b, c, d)\,/b - 2c + d = 0\}$,
    $W = \{(a, b, c, d)\,/a = d, b = 2c\}$
    Find the basis.

    (b) Find the basis for the subspace $\{ax^2 + bx + c \quad ; \quad a - 2b = c\}$ of $\mathbb{P}_2$.