UNIVERSITY OF JAFFNA

FACULTY OF ENGINEERING

MID SEMESTER EXAMINATION- MAY 2023

MC 3010 – DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

Date:22/05/2023 *Duration: ONE Hour*

Instructions

1. This paper contains **TWENTY (20)** questions:
2. Each question in this paper is a multiple choice with four answer choices. Read each question and answer carefully and choose the ONE best answer.
3. This examination accounts for **30%** of module assessment. Total maximum mark attainable is **100**.

---

1. If $f(x)$is a real continuous function in [a,b], and $f(a)f(b) < 0$, then for $f(x) = 0$, there is (are) ... in the domain [a,b].

   (a) one root
   (b) an undeterminable number of roots
   (c) no root
   (d) at least one root

2. Assuming an initial bracket of [1,5], the second (at the end of 2 iterations) iterative value of the root of $te^{-t} - 0.3 = 0$ using the bisection method is

   (a) 0 (b) 1.5 (c) 2 (d) 3

3. The following data of the velocity of a body is given as a function of time.

Time(s)	10	15	18	22	24
Velocity(m/s)	22	24	37	25	123

   A quadratic Lagrange interpolant is found using three data points, $t = 15$, 18 and 22. From this information, at what of the times given in seconds is the velocity of the body $26m/s$ during the time interval of $t = 15$ to $t = 22$ seconds.

   (a) 20.173 (b) 21.858 (c) 21.667 (d) 22.020

4. Given $3\frac{dy}{dx} + 5y = 2x$, $y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.6)$ using the Runge-Kutta $4th$ order method is most nearly

   (a) 3.1067
   (b) 3.2067
   (c) 4.2587
   (d) none of the above

5. The equation in Question 4 with the same step size, the value of $y(0.9)$ using Eulers method is most nearly

(a) 1.3 (b) 1.2 (c) 1.5 (d) 1.4

6. Truncation error is caused by approximating

(a) irrational numbers
(c) rational numbers
(b) fractions
(d) exact mathematical procedures

7. $\int_0^1 \frac{\sin t}{t} dt$ is exactly

(a) $\int_{-1}^{1} \frac{\sin(\frac{x+1}{2})}{x+1} dx$
(c) $\int_{-1}^{1} \frac{\sin(x+1)}{x+1} dx$
(b) $\int_{0}^{1} \frac{\sin(\frac{x+1}{2})}{x+1} dx$
(d) $\int_{0}^{1} \frac{\sin(x+1)}{x+1} dx$

8. Consider the second-order initial value problem

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 0, x(0) = 3, \frac{dx(0)}{dt} = -5$$

write down the equivalent system of two first-order equations

(a) $\frac{dx}{dt} = y, \frac{dy}{dt} = -x - 4y, x(0) = 3, y(0) = -3$
(b) $\frac{dx}{dt} = y, \frac{dy}{dt} = -5x - 4y, x(0) = 3, y(0) = -5$
(c) $\frac{dx}{dt} = y, \frac{dy}{dt} = -x - 4y, x(0) = 2, y(0) = -5$
(d) $\frac{dx}{dt} = y, \frac{dy}{dt} = -x - 4y, x(0) = 3, y(0) = -5$

9. In a circuit with an inductor of inductance L , a resistor with resistance R , and a variable voltage source $E(t) = L(di/dt) + Ri$. The current i , is measured at several values of time as

t (secs)	1.00	1.01	1.03	1.1
i (Amperes)	3.10	3.12	3.18	3.24

If $L = 0.98$ H and $R = 0.142\ \Omega$, the most accurate expression for $E(1.00)$ is

(a) $0.98\left(\frac{3.24-3.10}{0.1}\right) + (0.142)(3.10)$
(c) $0.98\left(\frac{3.12-3.10}{0.01}\right) + (0.142)(3.10)$
(b) 0.142×3.10
(d) $0.98\left(\frac{3.12-3.10}{0.01}\right)$

10. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at x = 2 is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

(a) 6.697 (b) 7.389 (c) 7.438 (d) 8.180

11. Find the number $2m$(No.of.sub-intervals) so that the error $E_s(f,h)$ for the Simpson rule is less than 4×10^{-9} for the approximation $\int_2^5 \frac{dx}{x^2}$.The maximum value of $|f^{(4)}(\hat{t})|$ taken over [2,5] occurs at the end point $x = 2$. [Hint:The Error formula for Simpson rule is given by $E_s = -\frac{(b-a)^5}{180(2m)^4} f^{(4)}(\hat{t}) = -\frac{b-a}{180} h^4 f^{(4)}(\hat{t})$]

(a) 448 (b) 449 (c) 224 (d) 225

12. Using Newton-Raphson method evaluate to decimal figures, the root of the equation $x \log_{10} x - 1.2 = 0$, (Assume $x_0 = 3$ as first approximation point).

(a) 2.76 (b) 2.75 (c) 2.74 (d) 2.73

13. Find the value of $\int_0^1 \sin \sqrt{x} dx$ by using trapezoidal rule with $h = 0.5$ is most nearly

(a) 2.140745 (b) 1.070037 (c) 0.535186 (d) 0.537818.

14. Find the absolute error, use question (13) and assume true value as 0.602337

(a) 0.067151 (b) 0.064519 (c) 0.467700 (d) 0.004677

15. Find $\frac{\partial^2 f}{\partial y \partial x}$ of $f(x,y) = e^{x+5y}$

(a) $3e^{x+5y}$ (b) $6e^{x+5y}$ (c) $6e^{2x+5y}$ (d) $3e^{2x+y}$.

16. Consider the function $w(x,y) = 4x^2 + 3y^2$ and find the value of $\frac{dw}{dr}$ in polar coordinates (r, θ) where $x = r\cos\theta$, $y = r\sin\theta$.

(a) $5r$ (b) $50r$ (c) $25r^2$ (d) 50

17. If $Z = f(x,y)$ and $Z = 4 - x^3 + y^2$, write the total differential, dz

(a) $-2x^2dx + 4ydy$
(b) $-2x^3dx + 2ydy$
(c) $-3x^2dx + 2ydy$
(d) none of the above.

18. Find the work done by the force $\mathbb{F}(x,y,z) = (y^2, xy, 0)$ in moving an object along the shortest path between point $(1,1,0)$ and point $(2,2,0)$ at constant speed in unit time.

(a) 7/3 (b) 29/3 (c) 15/2 (d) 14/3

19. The temperature T at the point (x, y) is $T(x, y)$ and it is measured using the Celsius scale. A fly crawls so that its position after t seconds is given by $x = t^2$ and $y = 2 + \frac{1}{3}t$ where x and y are measured in centimeters. The temperature function satisfies $\frac{\partial T}{\partial x}(2, 3) = 4$ and $\frac{\partial T}{\partial y}(2, 3) = 3$. How fast is the temperature increasing on the flys path after 3s

(a) 2 (b) 8 (c) 9 (d) 2.5

20. The motion of a point is described by $r(t) = (cost, sint, t^2)$ Find the speed of the motion.

(a) $\sqrt{t^2 + 1}$ (b) $\sqrt{4t^4 + 1}$ (c) $\sqrt{4t^2 + \cos^2 t}$ (d) $\cos^{2t}$

Formula sheet

1. Newton's iteration formula: $x_{n+1} = x_n - \frac{f(x_n}{f'(x_n)}$

2. Trapezoidal rule: $\int_a^b f(x)dx \approx h\left[\frac{1}{2}f_0 + f_1 + \ldots + f_{n-1} + \frac{1}{2}f_n\right]$, where $h = \frac{(b-a)}{n}$. The x_j's and a and b are called notes.

3. Eulers method: $y_{k+1} = y_k + hf(t_k, y_k)$, $t_{k+1} = t_k + h$ for $k = 0, 1, \ldots, M-1$

4. Runge-Kutta method of order $N = 4$: $y_{k+1} = y_k + \frac{h(f_1 + 2f_2 + 2f_3 + f_4)}{6}$, where $f_1 = f(t_k, y_k)$, $f_2 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2}f_1\right)$, $f_3 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2}f_2\right)$, $f_4 = f(t_k + h, y_k + hf_3)$.

5. Quadratic Lagrange Polynomial $P_2(x) = l_0f_0 + l_1f_1 + l_2f_2$ where, $l_0 = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$, $l_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$, $l_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

6. 2-point forward difference-$f'(x_i) = \frac{f(x_{i+1})-f(x_i)}{h}$, 2-point backward-$f'(x_i) = \frac{f(x_i)-f(x_{i-1})}{h}$

——— *End of Examination* ———