UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 01– May 2023

MC3010 - Differential Equations and Numerical Methods

Duration: 90 minutes

Answer all questions

1. Complete the following computation and state what type of error is present in this situation.

$$\int_0^{0.5} e^{x^2} dx \simeq \int_0^{0.5} (1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!}) dx$$

   If true value $p = 0.544987104$, find the absolute error.

2. Given the Taylor polynomial Expansions

$$tan^{-1}(h) = h - \frac{h^3}{3} + \frac{h^5}{5} + O(h^7)$$

   and

$$ln(h+1) = h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + O(h^5)$$

   determine the order of approximation for their sum and product.

3. Let $f(x) = x + \frac{2}{x}$. Use quadratic Lagrange interpolation based on the nodes $x_0 = 1, x_1 = 2, x_2 = 2.5$ to approximate $f(1.5)$

4. Apply Newton-Raphson method to find an approximate solution of the equation $e^x - 3x = 0$ correct up-to three decimal figures (assume $x_0 = 0.4$)

5. Find the root of the equation $2x - log_{10}x = 7$ which lies between 3 and 4, correct to four places of decimal, using bisection method.

6. In a vibration experiment, the position (x) of a block of mass is given as a function of time(t). The recorded data for the first 2 seconds are given in the table below.

t(s)	0	0.5	1	1.5	2
x(mm)	200	123	27	-56	-100

   The velocity of the block is the derivative of the position w.r.t time. Use Forward divided difference, backward divided difference or central divided difference approximation method to find the velocity at time $t = 1.5s$

7. Calculate the integral value, to using trapezoidal rule . The step size $h = 0.25$

$$\int_0^1 (1 + e^{-x} cos(4x)) dx$$

8. Find the number m(no.of.sub interval) and the step size h so that the error $E_s(f,h)$ for the Simpson rule is less than 5×10^{-9} for the approximation $\int_2^7 \frac{dx}{x}$.
The $f^{(4)}(x) = \frac{24}{x^5}$ and the maximum value of taken over [2,7] occurs at the end point $x = 2$.

9. Show that two integrals are equivalent and calculate $G_2(f)$ (Two-point Gauss-Legendre)

$$\frac{1}{\pi}\int_0^{\pi} \cos(0.6\sin(t))dt = 0.5\int_{-1}^{1} \cos(0.6\sin((x+1)\frac{\pi}{2}))dx$$

10. Compare the truncation error term for the two-point Gauss-Legendre rule $(\frac{f^{(4)}(c)}{135})$ and Simpson's rule $(\frac{-h^5 f^{(4)}(c)}{90})$ on the closed interval[-1,1] and state which method do you think best? Why?