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MC 3010 - Differential Equations and Numerical Methods

Tutorial-1

1. Find the error E_x and relative error R_x. Also determine the number of significant digits in the approximation.

   (a) $x = 2.71828182, \widehat{x} = 2.7182$

   (b) $y = 98,350, \widehat{y} = 98,000$

   (c) $z = 0.000068, \widehat{z} = 0.00006$

   (d) $a = 3.62728182, \widehat{a} = 3.6272$

   (e) $b = 1.52006323, \widehat{b} = 1.32006323$

   (f) $c = 48.460544, \widehat{c} = 47.060543$

   (g) $c = 78.3501, \widehat{c} = 78.0001$

2. Polynomial evaluation. Let $P(x) = x^3 – 3x^2 + 3x - 1, Q(x) = ((x - 3)x + 3)x – 1$, and $R(x) = (x - 1)^3$.

   (a) Use four-digit rounding arithmetic and compute $P(2.72), Q(2.72)$, and $R(2.72)$.In the computation of $P(x)$, assume that $(2.72)^3 = 20.12$ and $(2.72)^2 = 7.398$.

   (b) Use four-digit rounding arithmetic and compute $P(0.975), Q(0.975)$, and $R(0.975)$. In the computation of $P(x)$, assume that $(0.975)^3 = 0.9268$ and $(0.975)^2 = 0.9506$.

3. Given the Taylor polynomial expansions

$$\frac{1}{1-h} = 1 + h + h^2 + h^3 + O(h^4)$$

   and

$$\cos h = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^6)$$

   determine the order of approximation for their sum and product.

4. Given the Taylor polynomial expansions

$$e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)$$

   and

$$\sin h = h - \frac{h^3}{3!} + O(h^5)$$

   determine the order of approximation for their sum and product.

5. Given the Taylor polynomial expansions

$$\cos h = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^6)$$

and

$$\sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)$$

determine the order of approximation for their sum and product.

6. Given the Taylor polynomial Expansions

$$tan^{-1}(h) = h - \frac{h^3}{3} + \frac{h^5}{5} + O(h^7)$$

and

$$ln(h+1) = h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + O(h^5)$$

determine the order of approximation for their sum and product.

7. Complete the following computation and state what type of error is present in this situation.

$$\int_0^{0.5} e^{x^2} dx \simeq \int_0^{0.5} (1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!}) dx$$

If true value $p = 0.544987104$, find the absolute error.