Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 3010 - Differential Equations and Numerical Methods

Tutorial-2

1. Using the bisection method to find a root of equation; Provide the approximations to four decimal places.
   (a) $f(x) = x^3 + x^2 - 1 = 0$ compute the first four approximations.
   (b) $f(x) = x^3 + 2x^2 + x - 1 = 0$ compute the first five approximations.
   (c) $2x - log_{10}x = 7$ which lies between 3 and 4. Find the root to within an absolute error tolerance 0.01.

2. Use Newton's method to, find a root of equation; Provide the approximations to four decimal places
   (a) $f(x) = x^3 - 2x - 5 = 0$ starting with $x_0 = 2$,
   (b) Find an interval of length in which the root of $f(x) = 3x^3 - 4x^2 - 4x - 7 = 0$ lies. Take the middle point of this interval as the starting approximation.
   (c) Find the root between 0 and 1 of $x^3 = 6x - 4$
   (d) Find an approximate solution of the equation $e^x - 3x = 0$ (assume $x_0 = 0.4$)

3. Apply the fixed-point method to determine the initial approximate root, up to four decimal places, for the equation $2x^3 - 2x - 5 = 0$ and indicate whether the iterations will converge or not near $x = 1.5$

4. Using Linear Lagrange polynomial find the $P_1(x)$. Given $f(-1) = 0, f(1) = 1$.

5. Using Quadratic Lagrange polynomial find the $P_2(x)$ and find value of $P_2(x)$ at $x = 2$. Given $f(0) = 15, f(1) = 48, f(5) = 85$.

6. Using Lagrange formula,find $P(10)$ from the given data.
   Given $f(5) = 12, f(6) = 13, f(9) = 14, f(11) = 16$

7. The velocity (v) of a rocket is given as a function of time (t) as

t(s)	0	0.5	1.2	1.5	1.8
v(m/s)	0	213	223	275	300

   Allowed to use the forward divided difference, backward divided difference or central divided difference approximation of the first derivative, find the best estimate for the acceleration $a = \dfrac{dv}{dt}$ in m/s^2 of the rocket at $t = 1.5$ seconds.

8. Let $f(x) = x + \dfrac{2}{x}$. Use quadratic Lagrange interpolation based on the nodes $x_0 = 1, x_1 = 2, x_2 = 2.5$ to approximate $f(1.5)$