UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 04– July 2023

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

MC 3010

1. Find a parametric representation of the straight line through the points $A(3,-1,5)$ and $B(5,-5,5)$.

   (a) $r(t) = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

   (b) $r(t) = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$

   (c) $r(t) = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$

   (d) $r(t) = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$

2. What curves are represented by the parametric representations $\cos(t)j + (2 + 2\sin(t))k$.

   (a) $x = 0, y^2 + \left(\frac{z-2}{2}\right)^2 = 1$

   (b) $x = 0, \left(\frac{y-2}{2}\right)^2 + z^2 = 2$

   (c) $x = 0,\ y^2 + z^2 = 2$

   (d) $x = 0,\ (y-2) + (z-2) = 1$

3. Find the work done by the force $\mathbb{F}(x,y,z) = (y^3, xy^2, 0)$ in moving an object along the shortest path between point $(1,1,0)$ and point $(2,2,0)$ at constant speed in unit time.

   (a) 15 (b) 15/2 (c) 45 (d) 45/2

4. Find the directional derivative of $f(x,y,z) = x^2yz$ at the point $P(1,2,1)$ in the direction of the vector $b = i + k$.

   (a) $2\sqrt{2}$ (b) $6\sqrt{2}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

5. Evaluate the following integral by first converting to polar coordinates.

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dxdy$$

   (a) $\left(\sqrt{3}-1\right)\frac{\pi}{4}$ (b) $\left(\sqrt{2}-1\right)\frac{\pi}{6}$ (c) $\left(\sqrt{3}-1\right)\frac{\pi}{3}$ (d) $\left(\sqrt{5}-1\right)\frac{\pi}{4}$

6. Find the linear approximation of the function $z = e^{(7x^2+4y^2)}$ at the point $(0,0,1)$.

   (a) $z = 2$ (b) $z = 1$ (c) $z = 7$ (d) $z = -2$

7. $f(x,y) = \dfrac{y}{x}$ at $(2,0)$ Give its components, its direction, and its magnitude at the point specified.

(a) $\langle 1, \frac{1}{2}\rangle$, $\frac{1}{2}$, $\frac{\pi}{4}$ (b) $\langle 0, \frac{1}{2}\rangle$, $\frac{1}{2}$, $\frac{\pi}{2}$ (c) $\langle 0, \frac{1}{2}\rangle$, $\frac{1}{2}$, $\frac{\pi}{6}$ (d) $\langle 0, 2\rangle$, $\frac{1}{2}$, $\frac{\pi}{2}$

8. For the differential equation $\left(\dfrac{dy}{dx}\right)^2 + \dfrac{d^2y}{dx^2} = y^2$ state its order, whether it is linear(L) or not(NL), and whether it is homogeneous(H) or not(NH).]

(a) 2^{nd} , L, NH. (b) 2^{nd} , NL, NH (c) 2^{nd} , L, H (d) 1^{st} , L, NH

9. The integrating factor of $\dfrac{dy}{dx} + 3(cosec\ x)y = \cot x$

(a) $\tan\left(\dfrac{x}{2}\right)$ (b) $\tan^2\left(\dfrac{x}{2}\right)$ (c) $\tan^3\left(\dfrac{x}{2}\right)$ (d) $\tan^4\left(\dfrac{x}{2}\right)$

10. Find the general and particular solution; $\dfrac{dy}{dx} = y^2,\ y(0) = 4$

(a) $y(x) = -\dfrac{1}{x+c},\ y(x) = \dfrac{4}{1-4x}$

(b) $y(x) = -\dfrac{1}{x+c},\ y(x) = \dfrac{1}{x-\frac{1}{4}}$

(c) $y(x) = -\dfrac{1}{x^2+c},\ y(x) = -\dfrac{1}{x^2-\frac{1}{4}}$

(d) $y(x) = -\dfrac{1}{x+c},\ y(x) = -\dfrac{1}{x-\frac{1}{8}}$

11. The solution of $(D-2)(D+1)^2 y = e^{2x} + e^x$ is

(a) $y = C_1e^x + (C_2 + C_3x)\,e^{2x} + \dfrac{x}{9}e^{2x} - \dfrac{e^x}{4}$

(b) $y = C_1e^x + (C_2 + C_3x)\,e^{2x} + \dfrac{x}{9}e^{x} - \dfrac{e^{2x}}{4}$

(c) $y = C_1e^{2x} + (C_2 + C_3x)\,e^{-x} + \dfrac{x}{9}e^{2x} - \dfrac{e^x}{4}$

(d) none of the above.

12. A curve is passing through the origin and the slope of the tangent at a point $R(x,y)$ where $-1 < x < 1$ is given as $\dfrac{(x^4 + 2xy + 1)}{(1-x^2)}$. What will be the equation of the curve?

(a) $y = \dfrac{x^5}{5} + \dfrac{x}{(1-x^2)} + C$

(b) $y = \dfrac{x^5}{5\,(1-x^2)} + \dfrac{x}{(1-x^2)} + C$

(c) $y = \left(1-x^2\right)\left(\dfrac{x^5}{5} + x\right) + C$

(d) none of the above.

13. Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\dfrac{1-y^2}{1-x^2}}$ and that $y = \dfrac{\sqrt{3}}{2}$ when $x = \dfrac{1}{2}$, show that

$2y = x\sqrt{k} + \sqrt{1-x^2}$, where k is a constant to be found.

(a) $k = \sqrt{3}$ (b) $k = 2$ (c) $k = 3$ (d) $k = \frac{1}{2}$

14. Using the substitution $z = y^2$ or otherwise, solve the equation $2y\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y^2}{x} = x^2$ given that when $x = 4,\ y = -5$.Give your answer in the form y=f(x).

(a) $y = -\sqrt{\frac{x^4}{4} + \frac{36}{x}}$

(b) $y = \sqrt{\frac{x^4}{4} + \frac{6}{x}}$

(c) $y = -\sqrt{\frac{x^3}{4} + \frac{36}{x}}$

(d) $y = \sqrt{\frac{x^3}{4} + \frac{36}{x}}$

15. Find the general solution of the non-homogeneous differential equations,

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = e^x$$

(a) $y = C_1e^x + C_2\cos 2x + C_3\sin 2x + \frac{xe^x}{5}$

(b) $y = C_1e^{2x} + C_2\cos x + C_3\sin x + \frac{xe^x}{5}$

(c) $y = C_1e^x + C_2\cos 2x + \frac{xe^x}{5}$

(d) $y = C_1e^x + C_2\cos 2x + C_3\sin 2x + \frac{xe^{2x}}{5}$

16. Assuming an initial bracket of [0.5,1.5], the fourth (at the end of 4 iterations) iterative value of the root of $e^{-x} = 3log(x)$ using the bisection method is

(a) 1.197 (b) 1.187 (c) 1.167 (d) 1.176

17. Using Newton-Raphson method evaluate to second approximation correct upto 2 decimal figures, the root of the equation $x^3 - x^2 + 4x - 4 = 0$ (Assume $x_0 = 2$ as first approximation point).

(a) 0.67 (b) 1.33 (c) 1.00 (d) 1.50

18. The following data of the velocity of a body is given as a function of time.

Time(s)	10	15	18	22	24
Velocity(m/s)	22	24	37	25	123

A Linear Lagrange interpolation is found using two data points, $t = 15,\ 18$. The velocity in m/s at $16s$ using linear lagrange interpolation most nearly

(a) 27.867

(b) 28.333

(c) 30.429

(d) 43.000

19. Using Euler's method an approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2y, y(0) = 1,\ h = 0.1$ is

(a) 0.900

(b) 1.00o

(c) 0.994

(d) none of the above.

20. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at $x = 2$ is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

(a) 6.697
(b) 7.389
(c) 7.438
(d) None of the above

21. Find the value of $\int_{-2}^{2} x^3 e^x dx$ by using Trapezoidal rule with $h = 1$ is most nearly

(a) 31.3654
(b) 30.5346
(c) 31.5643
(d) none of the above.

22. To find the value of Question (21) using Simpson's rule with $h = 1$ is most nearly

(a) 22.498
(b) 22.477
(c) 22.485
(d) none of the above.

23. Find the inverse transforms of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$

(a) $e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t$
(b) $2e^{2t}\cos 2t - 7e^{2t}\sin 2t$
(c) $e^t - e^{-t}\cos 2t + \frac{3}{2}e^{-t}\sin 2t$
(d) none of the above.

24. Using the Laplace transforms, find the solution for the initial value problem

$$y_1^{''} = y_1 + 3y_2 \quad \text{and} \quad y_2^{''} = 4y_1 - 4e^t$$
$$y_1(0) = 2,\ y_1^{'}(0) = 3, y_2(0) = 1,\ y_2^{'}(0) = 2$$

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(a) $y_1 = e^t + \cos t,\ y_2 = e^t + e^{2t}$
(b) $y_1 = e^t,\ y_2 = e^t + e^{2t}$
(c) $y_1 = e^t + e^{2t},\ y_2 = e^{2t}$
(d) $y_1 = e^{2t} + \sin t,\ y_2 = e^t + e^{2t}$

25. Find the Laplace Transform of $t^2 \cos \alpha t$

(a) $\dfrac{2as}{(s^2+a^2)^3}$
(b) $\dfrac{2s(s^2-a^2)}{(s^2+a^2)^3}$
(c) $\dfrac{2s(s^2-3a^2)}{(s^2+a^2)^3}$
(d) $\dfrac{s(s^2-a^2)}{(s^2+a^2)^2}$