UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 04– July 2023

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

MC 3010

1. Find a parametric representation of the straight line through the points $A(3, -1, 5)$ and $B(5, -5, 5)$.

   (a) $r(t) = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

   (b) $r(t) = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$

   (c) $r(t) = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$

   (d) $r(t) = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$

2. What curves are represented by the parametric representations $\cos(t)j + (2 + 2\sin(t))k$.

   (a) $x = 0, y^2 + \left(\frac{z-2}{2}\right)^2 = 1$

   (b) $x = 0, \left(\frac{y-2}{2}\right)^2 + z^2 = 2$

   (c) $x = 0,\ y^2 + z^2 = 2$

   (d) $x = 0,\ (y - 2) + (z - 2) = 1$

3. Find the work done by the force $\mathbb{F}(x, y, z) = (y^3, xy^2, 0)$ in moving an object along the shortest path between point $(1, 1, 0)$ and point $(2, 2, 0)$ at constant speed in unit time.

   (a) 15 (b) 15/2 (c) 45 (d) 45/2

4. Find the directional derivative of $f(x, y, z) = x^2yz$ at the point $P(1, 2, 1)$ in the direction of the vector $b = i + k$.

   (a) $2\sqrt{2}$ (b) $6\sqrt{2}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

5. Evaluate the following integral by first converting to polar coordinates.

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1 + x^2 + y^2}}\, dxdy$$

   (a) $\left(\sqrt{3} - 1\right)\frac{\pi}{4}$ (b) $\left(\sqrt{2} - 1\right)\frac{\pi}{6}$ (c) $\left(\sqrt{3} - 1\right)\frac{\pi}{3}$ (d) $\left(\sqrt{5} - 1\right)\frac{\pi}{4}$

6. Find the linear approximation of the function $z = e^{(7x^2+4y^2)}$ at the point $(0, 0, 1)$.

   (a) $z = 2$ (b) $z = 1$ (c) $z = 7$ (d) $z = -2$

7. $f(x, y) = \dfrac{y}{x}$ at $(2, 0)$ Give its components, its direction, and its magnitude at the point specified.

(a) $\left\langle 1, \frac{1}{2} \right\rangle, \frac{1}{2}, \frac{\pi}{4}$ (b) $\left\langle 0, \frac{1}{2} \right\rangle, \frac{1}{2}, \frac{\pi}{2}$ (c) $\left\langle 0, \frac{1}{2} \right\rangle, \frac{1}{2}, \frac{\pi}{6}$ (d) $\langle 0, 2 \rangle, \frac{1}{2}, \frac{\pi}{2}$

8. For the differential equation $\left(\dfrac{dy}{dx}\right)^2 + \dfrac{d^2y}{dx^2} = y^2$ state its order, whether it is linear(L) or not(NL), and whether it is homogeneous(H) or not(NH).]

(a) 2^{nd} , L, NH. (b) 2^{nd} , NL, NH (c) 2^{nd} , L, H (d) 1^{st} , L, NH

9. The integrating factor of $\dfrac{dy}{dx} + 3(cosec\ x)y = \cot x$

(a) $\tan\left(\dfrac{x}{2}\right)$ (b) $\tan^2\left(\dfrac{x}{2}\right)$ (c) $\tan^3\left(\dfrac{x}{2}\right)$ (d) $\tan^4\left(\dfrac{x}{2}\right)$

10. Find the general and particular solution; $\dfrac{dy}{dx} = y^2,\ y(0) = 4$

(a) $y(x) = -\dfrac{1}{x+c},\ y(x) = \dfrac{4}{1-4x}$

(b) $y(x) = -\dfrac{1}{x+c},\ y(x) = \dfrac{1}{x-\frac{1}{4}}$

(c) $y(x) = -\dfrac{1}{x^2+c},\ y(x) = -\dfrac{1}{x^2-\frac{1}{4}}$

(d) $y(x) = -\dfrac{1}{x+c},\ y(x) = -\dfrac{1}{x-\frac{1}{8}}$

11. The solution of $(D-2)(D+1)^2 y = e^{2x} + e^x$ is

(a) $y = C_1e^x + (C_2 + C_3x)\, e^{2x} + \dfrac{x}{9}e^{2x} - \dfrac{e^x}{4}$

(b) $y = C_1e^x + (C_2 + C_3x)\, e^{2x} + \dfrac{x}{9}e^{x} - \dfrac{e^{2x}}{4}$

(c) $y = C_1e^{2x} + (C_2 + C_3x)\, e^{-x} + \dfrac{x}{9}e^{2x} - \dfrac{e^x}{4}$

(d) none of the above.

12. A curve is passing through the origin and the slope of the tangent at a point $R(x, y)$ where $-1 < x < 1$ is given as $\dfrac{(x^4 + 2xy + 1)}{(1-x^2)}$. What will be the equation of the curve?

(a) $y = \dfrac{x^5}{5} + \dfrac{x}{(1-x^2)} + C$

(b) $y = \dfrac{x^5}{5\,(1-x^2)} + \dfrac{x}{(1-x^2)} + C$

(c) $y = \left(1-x^2\right)\left(\dfrac{x^5}{5} + x\right) + C$

(d) none of the above.

13. Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\dfrac{1-y^2}{1-x^2}}$ and that $y = \dfrac{\sqrt{3}}{2}$ when $x = \dfrac{1}{2}$, show that

$2y = x\sqrt{k} + \sqrt{1-x^2}$, where k is a constant to be found.

(a) $k = \sqrt{3}$ (b) $k = 2$ (c) $k = 3$ (d) $k = \frac{1}{2}$

14. Using the substitution $z = y^2$ or otherwise, solve the equation $2y\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y^2}{x} = x^2$ given that when $x = 4,\ y = -5$.Give your answer in the form y=f(x).

(a) $y = -\sqrt{\frac{x^4}{4} + \frac{36}{x}}$

(b) $y = \sqrt{\frac{x^4}{4} + \frac{6}{x}}$

(c) $y = -\sqrt{\frac{x^3}{4} + \frac{36}{x}}$

(d) $y = \sqrt{\frac{x^3}{4} + \frac{36}{x}}$

15. Find the general solution of the non-homogeneous differential equations,

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = e^x$$

(a) $y = C_1e^x + C_2\cos 2x + C_3\sin 2x + \frac{xe^x}{5}$

(b) $y = C_1e^{2x} + C_2\cos x + C_3\sin x + \frac{xe^x}{5}$

(c) $y = C_1e^x + C_2\cos 2x + \frac{xe^x}{5}$

(d) $y = C_1e^x + C_2\cos 2x + C_3\sin 2x + \frac{xe^{2x}}{5}$

16. Assuming an initial bracket of [0.5,1.5], the fourth (at the end of 4 iterations) iterative value of the root of $e^{-x} = 3log(x)$ using the bisection method is

(a) 1.197 (b) 1.187 (c) 1.167 (d) 1.176

17. Using Newton-Raphson method evaluate to second approximation correct upto 2 decimal figures, the root of the equation $x^3 - x^2 + 4x - 4 = 0$ (Assume $x_0 = 2$ as first approximation point).

(a) 0.67 (b) 1.33 (c) 1.00 (d) 1.50

18. The following data of the velocity of a body is given as a function of time.

Time(s)	10	15	18	22	24
Velocity(m/s)	22	24	37	25	123

A Linear Lagrange interpolation is found using two data points, $t = 15,\ 18$. The velocity in m/s at $16s$ using linear lagrange interpolation most nearly

(a) 27.867

(b) 28.333

(c) 30.429

(d) 43.000

19. Using Euler's method an approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2y, y(0) = 1,\ h = 0.1$ is

(a) 0.900

(b) 1.00o

(c) 0.994

(d) none of the above.

20. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at $x = 2$ is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

(a) 6.697
(b) 7.389
(c) 7.438
(d) None of the above

21. Find the value of $\int_{-2}^{2} x^3 e^x dx$ by using Trapezoidal rule with $h = 1$ is most nearly

(a) 31.3654
(b) 30.5346
(c) 31.5643
(d) none of the above.

22. To find the value of Question (21) using Simpson's rule with $h = 1$ is most nearly

(a) 22.498
(b) 22.477
(c) 22.485
(d) none of the above.

23. Find the inverse transforms of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$

(a) $e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t$
(b) $2e^{2t}\cos 2t - 7e^{2t}\sin 2t$
(c) $e^{t} - e^{-t}\cos 2t + \frac{3}{2}e^{-t}\sin 2t$
(d) none of the above.

24. Using the Laplace transforms, find the solution for the initial value problem

$$y_1^{''} = y_1 + 3y_2 \quad \text{and} \quad y_2^{''} = 4y_1 - 4e^t$$
$$y_1(0) = 2,\ y_1^{'}(0) = 3, y_2(0) = 1,\ y_2^{'}(0) = 2$$

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(a) $y_1 = e^t + \cos t,\ y_2 = e^t + e^{2t}$
(b) $y_1 = e^t,\ y_2 = e^t + e^{2t}$
(c) $y_1 = e^t + e^{2t},\ y_2 = e^{2t}$
(d) $y_1 = e^{2t} + \sin t,\ y_2 = e^t + e^{2t}$

25. Find the Laplace Transform of $t^2 \cos \alpha t$

(a) $\dfrac{2as}{(s^2+a^2)^3}$
(b) $\dfrac{2s(s^2-a^2)}{(s^2+a^2)^3}$
(c) $\dfrac{2s(s^2-3a^2)}{(s^2+a^2)^3}$
(d) $\dfrac{s(s^2-a^2)}{(s^2+a^2)^2}$

Answers

1. (b)	6. (b)	11. (c)	16. (b)	21. (a)
2. (a)	7. (b)	12. (d)	17. (b)	22. (b)
3. (b)	8. open	13. (c)	18. (b)	23. (c)
4. (c)	9. (c)	14. (c) or (d)	19. (b)	24. (c)
5. (d)	10. (a)	15. (a)	20. (d)	25. (c)