Department of Inter Disciplinary Studies,
Faculty of Engineering,
University of Jaffna, Sri Lanka
MC 3010 : Multi-Variable Calculus

Tutorial-01 **April 2024**

1. Find a parametric representation of the straight line represented by

   (a) $y = x,\ z = 0$

   (b) $x = y,\ y = z$

   (c) $x + y + z = 1,\ y - z = 1$

2. What curves are represented by the following parametric representations?

   (a) $\begin{bmatrix} t & \frac{1}{t} & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & t^2 & t \end{bmatrix}$

   (c) $\begin{bmatrix} \cos(3t) & \sin(3t) & -t \end{bmatrix}$

   (d) $\cos(t)\mathbf{j} + (2 + 2\sin(t))\mathbf{k}$

   (e) $\cosh(t)\mathbf{i} + 4\sinh(t)\mathbf{j}$

3. Represent the following curves in parametric form.

   (a) $x^2 + y^2 = 1,\ z = 0$

   (b) $(x + 2)^2 + (y - 2)^2 = 4,\ z = 6$

   (c) $\frac{1}{4}x^2 + \frac{1}{16}y^2 = 1,\ z = 1$

   (d) $y^2 + 25z^2 = 25,\ x = -2$

   (e) $4x^2 - 9y^2 = 36,\ z = 0$

4. For the following curves, $r(t)$, find

   (a) Tangent vector $r'(t)$, and the corresponding unit tangent vector $u(t)$.

   (b) r' and u at the given point P.

   i. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$ $\qquad P = (1, 1, 0)$

   ii. $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$ $\qquad P = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$

   iii. $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{k}$ $\qquad P = (2, 8, 0)$

   iv. $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + \sin(t)\mathbf{j}$ $\qquad P = \left(\sqrt{2}, \frac{1}{\sqrt{2}}, 0\right)$

   v. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ $\qquad P = (1, 1, 1)$

5. Let $r(t)$ be the position vector of a moving particle where $t(\geq 0)$ is time. Describe the geometric shape of the path and find the velocity vector, the speed, and the acceleration vector.

   (a) $\mathbf{r}(t) = 3t\mathbf{i}$

   (b) $\mathbf{r}(t) = 4t\mathbf{i} - 4t\mathbf{j} + 2t\mathbf{k}$

   (c) $\mathbf{r}(t) = 4t^2\mathbf{k}$

   (d) $\mathbf{r}(t) = (1 + t^3)\mathbf{i} + 2t^3\mathbf{j} + (2 - t^3)\mathbf{k}$

   (e) $\mathbf{r}(t) = 3\cos(2t)\mathbf{i} + 3\sin(2t)\mathbf{j}$

6. Let a motion given by $r(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$. Find the tangential acceleration.

7. What curve is represented by the parametric equation :

$$\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} - \frac{1}{2}\mathbf{k}?$$

8. Represent

$$4(x + 2)^2 + (y - 4)^2 = 4, z = 0$$

   in parametric form.

9. Find the work done by a force field :

$$\mathbf{F}(x, y, z) = (y^2, x^2, 0)$$

   on a particle that moves from $(1, 0, 1)$ to $(-1, 0, 1)$ along the curve

$$r(t) = (1 - 2t, t(1 - t), 1), \quad 0 \leq t \leq 1.$$

10. Find the work done by the force field :

$$\mathbf{F}(x, y) = (x\sin y, y)$$

    on a particle that moves along the parabola

$$y = x^2$$

    from $(-1, 1)$ to $(2, 4)$.

11. Obtain $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the point $(2, 1)$ for the following functions :

    (a) $3x + 7y - 2$

    (b) $-2x + 3y + 4$

    (c) $2x^2 - 3y^2 - 2xy - x - y + 1$

    (d) $\dfrac{1}{8}x^3 + y^3 - 2y - 1$

    (e) $x^4y^2 - 1$

    (f) $(x - 1)(y - 2)$