UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 02– June 2023

Multi-Variable Calculus

MC 3010

Reading Time: 05 Minutes

Writing Time: 90 Minutes

Permitted Materials: Calculators; Mathematical tables

1. Write a parametric representation of the curve given by:

$$(y-5)(z+5)=1, \ x=1$$

2. Write the velocity of a particle moving with position: $\mathbf{r}(t)=(\cosh t, \sinh t, t)$

3. Write the length of the semicircle defined by

$$\mathbf{r}(t)=(2\cos t, 2\sin t, t), \ t\in[0,\pi]$$

4. Write a tangent vector to the curve: $\mathbf{r}(t)=(\cos t, \sin t), \ t\in[0,\pi]$ at the point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

5. Write both components of the acceleration of the curve: $\mathbf{r}(t)=(2\cos t, 2\sin t, t)$.

   (a) The tangential acceleration (b) The normal acceleration

6. Write the work done by the force field: $\mathbf{F}(x,y)=(x\sin y, y)$ on the particle that moves along the parabola $y=x^2$ from $(-1,1)$ to $(2,4)$.

7. Obtain ∇f, where $f(x,y)$ is given by:

$$f(x,y)=\frac{1}{x+y} \text{ at } (1,-2)$$

   Give its components, its direction, and its magnitude at the point specified.

8. Consider the level surface given by:

$$f(x,y,z)=z-\frac{x}{y}$$

   At the point $P=(2,1,2)$ on the surface find the normal vector and the equation for the tangent plane.

9. use the total differential to approximate the change in the function $z=f(x,y)=\sqrt{4-x^2-y^2}$ as (x,y) changes from $(1,1)$ to $(1.01, 0.97)$.

10. Find all 2-nd partials of $f(x, y) = \cos(x^2 + y^2)$

(a) $\dfrac{\partial^2 f}{\partial x^2}$ (b) $\dfrac{\partial^2 f}{\partial y^2}$ (c) $\dfrac{\partial^2 f}{\partial x \partial y}$ (d) $\dfrac{\partial^2 f}{\partial y \partial x}$

11. A rectangular container is required to have total surface area S, and a volume as large as possible. Find its dimensions:

(a) if it has a lid,

(b) if it does not have a lid.

12. Find all stationary points of $f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$ and classify them.

Points	Classification of stationary points				
	$\frac{\partial^2 f}{\partial x^2}$	$\frac{\partial^2 f}{\partial y^2}$	$\frac{\partial^2 f}{\partial x \partial y}$	Δ	Classification
(5,0)					
(-5,0)					
(3,4)					
(-3,-4)					

13. Sketch the region of integration for the integral $\displaystyle\int_0^{\infty} \int_{\frac{1}{4}y^{-2}}^{y^{-2}} x^2 y e^{-x^2 y^2}\, dx dy$ and write an equivalent integral with the order of integration reversed.

14. Evaluate the following integral by first converting to polar coordinates.

$$\int_0^{\sqrt{2}} \int_1^{\sqrt{4-y^2}} \frac{1}{\sqrt{1 + x^2 + y^2}}\, dx dy$$

15. Evaluate the following integral by first converting to polar coordinates.

$$\int_0^{2} \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\, dy dx$$

16. A washer has inner radius r_1 and outer radius r_2. The thickness of the washer is given by $f(x, y) = ae^{-b(x^2+y^2)}$. What is the average thickness of the washer?

17. If $Z = f(x, y)$ and $Z = \sqrt{9 - x^2 - y^2}$, write the total differential, dz

18. Calculate the divergence of the velocity vector field v defined by

$$v = \cos(x)\cosh(y)\ i + \sin(x)\sinh(y)\ j$$

19. Calculate the curl of the velocity vector field v defined by

$$v = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}} (xi + yj + zk)$$

——— *End of Examination* ———