UNIVERSITY OF JAFFNA
FACULTY OF ENGINEERING

Assignment Test 03 – June 2023

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

MC 3010

Reading Time: Five Minutes

Writing Time: 90 minutes

Permitted Materials: Calculators; Notes from the class

1. A population of bacteria grows at a rate that is proportional to the size of the population multiplied by an antibiotic effectiveness factor. The effectiveness factor of the antibiotic is inversely proportional to the square of time:

   (a) Write down an Ordinary Differential Equation for the number of bacteria, denoted by n.

   (b) Solve the resulting Ordinary Differential Equation, assuming that at time $t = 1$ there is one bacterium.

   (c) Hence determine if the number of bacteria reaches a steady state.

2. A large cylindrical tank has radius 2 m. Water flows out of a tap at the bottom of the tank at a rate proportional to the square root of the depth of the water within it. Initially the tank is full to a depth of 9 m. After 15 minutes the depth of water is 4 m. How long will it take for the tank to empty?

3. Solve the differential equation $x\dfrac{dy}{dx} - y = x$, subject to the initial condition $y(1) = 2$, according to the following steps:

   (a) Put the differential equation in the standard form $\dfrac{dy}{dx} + p(x)y = r(x)$ and so determine $p(x)$ and $r(x)$.

   (b) Determine $h(x)$ using the formula: $h(x) = \displaystyle\int p(x)dx$

   (c) Find y by using the linear differential equation formula: $y = e^{-h(x)} \displaystyle\int e^{h(x)} r(x)dx$

   (d) Apply the initial condition to find the constant of integration, and hence write down the solution for y.

4. Solve the differential equation $\left(D^2 - 3D + 2\right) y = e^x$ according to the following steps:

   (a) Find the complementary function

   (b) Find a particular integral

   (c) Find the solution.

5. (a) Find the Laplace transform of $f\left(x\right) = 5 \sin 3x - 17 \exp\left(-2x\right)$

   (b) Find $\mathcal{L}^{-1}\left(\dfrac{1}{s^2 - 2s + 9}\right)$

   (c) The vertical motion of a particle is modelled by:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 9; \; y(0) = 0, \; \frac{dy}{dx}\left(0\right) = 0$$

   Use Laplace transforms to solve according to the following steps:

   i. Taking the Laplace transform of both sides of the differential equation by applying the formulae for the Laplace transforms.

   ii. Put in the given initial conditions $y(0)$ and $\dfrac{dy}{dx}\left(0\right)$

   iii. Re-arrange the equation to make $\mathcal{L}\left(y\right)$ the subject.

   iv. Determine y by using, where necessary, partial fractions, and taking the inverse of each term.