



Tutorial-3

1. Consider the function $f(x) = x^3$. Calculate its first derivative at point $x = 3$ numerically with the forward, backward, and central finite difference formulas and using the h values of 1, 0.25. Also find the error percentages.

2. Use the three-point Gauss-Legendre rule to approximate $\int_1^5 \frac{dt}{t} = \ln(5) - \ln(1) \approx 1.609438$ and compare the result with Simpson and Trapezoidal rule for $h = 1$.

3. Calculate the integral value, to using Trapezoidal rule and Simpson rule .

(a) $\int_1^2 x \ln x \, dx$ with $h = 0.5$

(b) $\int_0^1 (1 + e^{-x} \cos(4x)) \, dx$ with $h = 0.25$

(c) $\int_{-2}^2 x^3 e^x \, dx$ with $h = 1$

4. Find the number m (no.of.sub interval) and the step size h so that the error $E_s(f, h)$ for the Simpson rule is less than 5×10^{-9} for the approximation $\int_2^7 \frac{dx}{x}$. The $f^{(4)}(x) = \frac{24}{x^5}$ and the maximum value of taken over $[2,7]$ occurs at the end point $x = 2$.

5. Show that two integrals are equivalent and calculate $G_2(f)$ (Two-point Gauss-Legendre)

$$\frac{1}{\pi} \int_0^\pi \cos(0.6 \sin(t)) dt = 0.5 \int_{-1}^1 \cos(0.6 \sin((x+1)\frac{\pi}{2})) dx$$