Question 1:

a)

q1-a

$$x(t) = 2\cos(2\pi t) + \cos 6\pi t$$

$$= 2 \times \frac{1}{2}\left[e^{j2\pi t} + e^{-j2\pi t}\right] + \frac{1}{2}\left[e^{j6\pi t} + e^{-j6\pi t}\right]$$

$$\Rightarrow e^{j2\pi t} + e^{-j2\pi t} + \frac{1}{2}e^{j6\pi t} + \frac{1}{2}e^{-j6\pi t} \rightarrow (1)$$

eq (1) is of form,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \rightarrow (1)$$

from comparing eq (1) with (2) we get

for $k = 1$, $a_k = 1 = a_1$

$k = -1$, $a_k = a_{-1} = 1$

for $k = 3$, $a_3 = 1/2 = 0.5$

$k = -3$, $a_{-3} = 1/2 = 0.5$

$a_1 = a_{-1} = 1$

$a_3 = a_{-3} = 0.5$

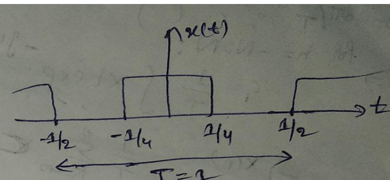
Verified the matlab code result with the above computed answer both are same.

b)

q1-b

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$T = 1$, and signal exists from $-\frac{1}{4}$ to $\frac{1}{4}$,



$$a_k = \frac{1}{1}\int_{-1/4}^{1/4} 1 \cdot e^{-jk\frac{2\pi}{1}t}\, dt$$

$$a_0 = 1/2 \quad (\text{for } k=0) \longrightarrow [t \cdot 1]_{-1/4}^{1/4} \Rightarrow \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$a_{k\neq 0} = \frac{1}{-j2\pi k} e^{-jk\pi 2t}\Big|_{-1/4}^{1/4}$$

$$= \frac{1}{-j2\pi k}\left[e^{-jk\pi 2\times\frac{1}{4}} - e^{-jk2\pi(-1/4)}\right]$$

$$\Rightarrow \frac{1}{\pi k}\left[\frac{e^{jk\pi/2} - e^{-jk\pi/2}}{2j}\right]$$

$$a_{k\neq 0} = \frac{\sin(k\pi/2)}{\pi k}$$

for even values of k, $a_k = 0 \Rightarrow a_2 = a_{-2} = a_4 = a_{-4} = 0$

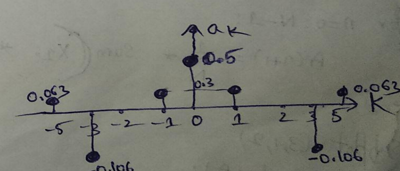
for odd values we get alternate sign, $a_k = a_{-k}$

$$a_{-1} = a_1 = \frac{1}{\pi} = 0.3185$$

$$a_{-3} = a_3 = \frac{-1}{3\pi} = -0.106$$

$$a_{-5} = a_5 = \frac{1}{5\pi} = 0.063$$

$$a_{-7} = a_7 = \frac{-1}{7\pi} = -0.04$$



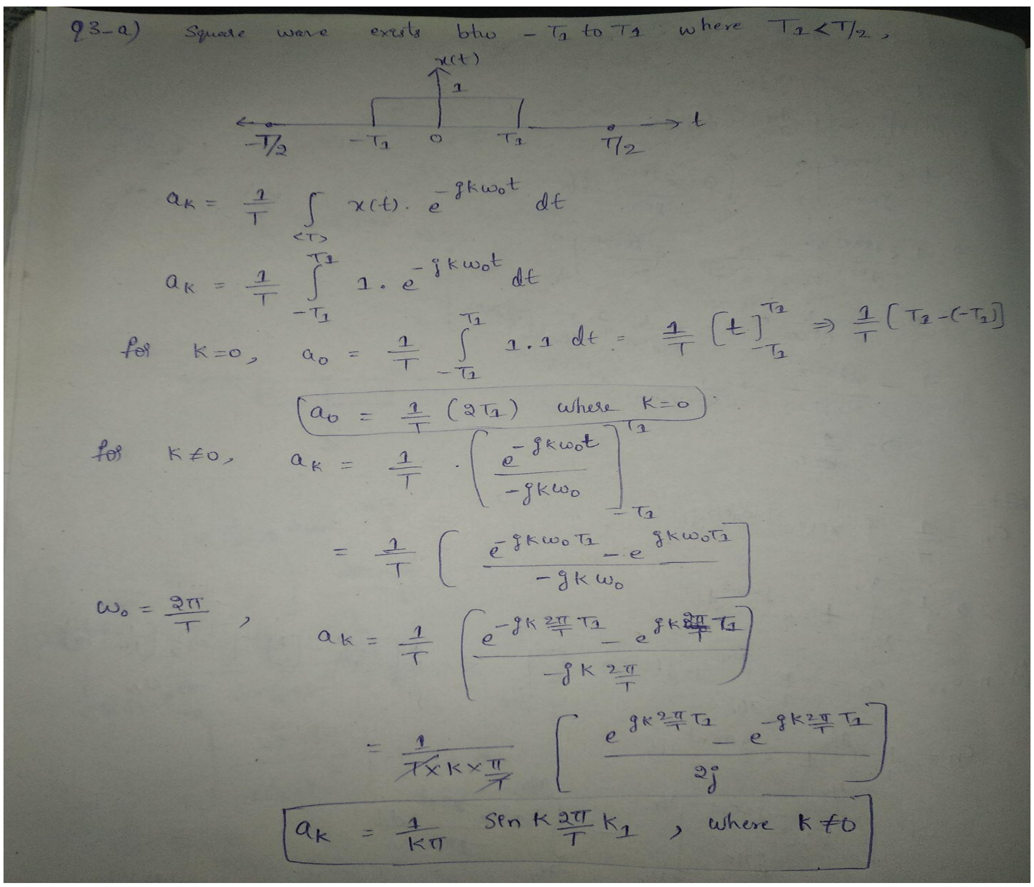
Verified the matlab result with the above computed answer both are same.

Question 2:

c) Between original and reconstructed signal the obtained maximum absolute error is "1/4503599627370496" and
root mean squared error is (5*65^(1/2)*202^(1/2))/7277816997830721536". Both are very small in number.

Question 3:

a)



Q3-a) Square wave exists b/w $-T_1$ to T_1 where $T_1 < T/2$,

$$a_K = \frac{1}{T}\int_{\langle T\rangle} x(t)\cdot e^{-jk\omega_0 t}\,dt$$

$$a_K = \frac{1}{T}\int_{-T_1}^{T_1} 1\cdot e^{-jk\omega_0 t}\,dt$$

for $K=0$, $$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} 1\cdot 1\,dt = \frac{1}{T}\left[t\right]_{-T_1}^{T_1} \Rightarrow \frac{1}{T}\left[T_1-(-T_1)\right]$$

$$\boxed{a_0 = \frac{1}{T}(2T_1) \quad \text{where } K=0}$$

for $K\neq 0$, $$a_K = \frac{1}{T}\cdot\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T_1}^{T_1}$$

$$= \frac{1}{T}\left[\frac{e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}}{-jk\omega_0}\right]$$

$\omega_0 = \frac{2\pi}{T}$, $$a_K = \frac{1}{T}\left(\frac{e^{-jk\frac{2\pi}{T}T_1} - e^{jk\frac{2\pi}{T}T_1}}{-jk\frac{2\pi}{T}}\right)$$

$$= \frac{1}{\cancel{T}\times K\times\frac{\pi}{\cancel{T}}}\left[\frac{e^{jk\frac{2\pi}{T}T_1} - e^{-jk\frac{2\pi}{T}T_1}}{2j}\right]$$

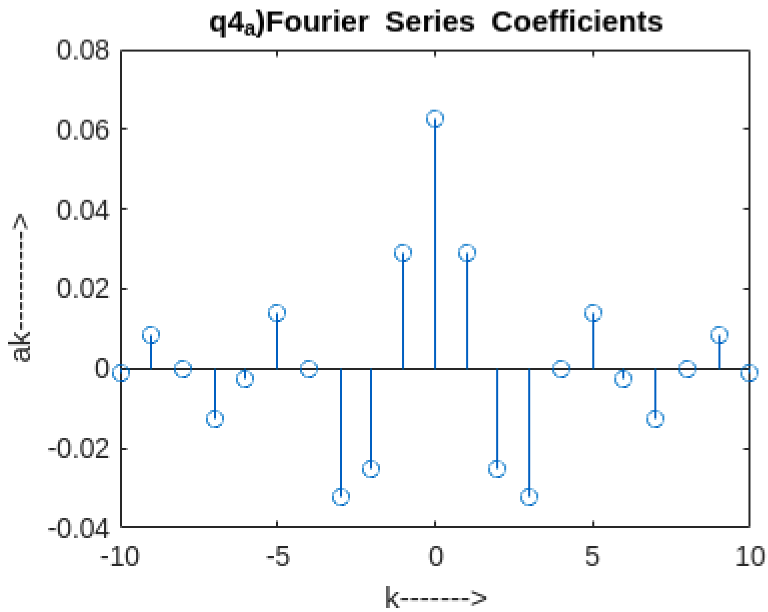
$$\boxed{a_K = \frac{1}{K\pi}\sin K\frac{2\pi}{T}K_1, \quad \text{where } K\neq 0}$$

b)The spacing between coefficients decreased as T -> infinity. The amplitude of them remain as we are multiplying with T otherwise it would decrease.

c) The ripples at the edges got decreased with increasing N.

Question 4:

c) x1(t) is real and even, the obtained coefficients $C_n = C_{-n}$ they are also real and even.



x2(t) is real and odd, the obtained coefficients are odd and purely imaginary, the conjugate symmetry exists.For plot took absolute coefficient values.