EE22BTECH11032 - Meenakshi

Question 9.3.26 Question: A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch? **Solution:**

parameter	**value**	**description**
n	8	Number of watches selected
p	$\frac{1}{10}$	Chosen watch is defective
q	$\frac{9}{10}$	Chosen watch is non-defective
$\mu = np$	$\frac{8}{10}$	Mean of the distribution
$\sigma^2 = npq$	$\frac{72}{100}$	Variance of the distribution

TABLE 1: Gaussian Info Table

(i) Gaussian Distribution
Let Y is the Gaussian obtained by approximating binomial with parameters n,p then By Central limit theroem,

$$Y \sim \mathcal{N}(np, npq) \tag{1}$$

CDF of Y is:

$$\begin{aligned} F_Y(x) &= \Pr(Y \le x) && (2)\\ &= \Pr(Y - \mu \le x - \mu) && (3)\\ &= \Pr\left(\frac{Y-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) && (4) \end{aligned}$$

Since,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{5}$$

Q function is defined

$$Q(x) = \Pr(Y > x)\ \forall x \in Y \sim \mathcal{N}(0,1) \tag{6}$$

$$\begin{aligned} F_Y(x) &= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) && (7)\\ &= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right), & x > \mu \\ Q\left(\frac{\mu-x}{\sigma}\right), & x < \mu \end{cases} && (8) \end{aligned}$$

(a) For atleast one watch to be defective, we need to find

$$1 - \Pr(Y = 0) \tag{9}$$

$$\begin{aligned} \Pr(Y = 0) &= \Pr(Y \le 1) && (10)\\ &= F_Y(1) && (11) \end{aligned}$$

$$F_Y(1) = 1 - Q\left(\frac{1-0.8}{0.848}\right) \tag{12}$$
$$= 1 - Q(0.235) \tag{13}$$
$$= 0.58 \tag{14}$$
$$\Pr(Y = 0) = F_Y(1) \tag{15}$$
$$= 0.58 \tag{16}$$

(ii) Binomial Distribution
Lets define a random variable X which represents the number of defective bulbs.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{17}$$

The pmf is given by

$$P_X(r) = {}^nC_r p^r (1-p)^{n-r} \tag{18}$$

If we consider atleast one watch to be defective, we need,

$$1 - P_X(0) \tag{19}$$
$$P_X(0) = 0.430 \tag{20}$$
$$1 - P_X(0) = 0.569 \tag{21}$$

(iii) Binomial vs Gaussian Graph