EE22BTECH11032 - Meenakshi

Question 12.13.3.66

If

$$\Pr(B) = \frac{3}{5}, \Pr(A|B) = \frac{1}{2} \text{ and } \Pr(A + B) = \frac{4}{5}, \text{ then } \Pr(A + B)' + \Pr(A' + B) \tag{1}$$

Solution:

$$\Pr(AB) = \Pr(A|B)\Pr(B) \tag{2}$$

$$= \frac{1}{2} \times \frac{3}{5} \tag{3}$$

$$= \frac{3}{10} \tag{4}$$

$$= \Pr(A)\Pr(B) \tag{5}$$

Hence, A and B are independent of each other.
We know that

$$\Pr(A'B) = \Pr(B) - \Pr(AB) \tag{6}$$

$$= \frac{3}{5} - \frac{3}{10} \tag{7}$$

$$= \frac{3}{10} \tag{8}$$

Calculating the probability of A

$$\Pr(A) = \Pr(A + B) - \Pr(B) + \Pr(AB) \tag{9}$$

$$= \frac{4}{5} - \frac{3}{5} + \frac{3}{10} \tag{10}$$

$$= \frac{5}{10} \tag{11}$$

$$= \frac{1}{2} \tag{12}$$

Complement of A is given by

$$\Pr(A') = 1 - \Pr(A) \tag{13}$$

$$= 1 - \frac{1}{2} \tag{14}$$

$$= \frac{1}{2} \tag{15}$$

$$\Pr(A + B)' = 1 - \Pr(A + B) \tag{16}$$

$$= 1 - \frac{4}{5} \tag{17}$$

$$= \frac{1}{5} \tag{18}$$

$$\Pr(A' + B) = \Pr(A') + \Pr(B) - \Pr(A'B) \tag{19}$$

$$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10} \tag{20}$$

$$= \frac{8}{10} \tag{21}$$

$$= \frac{4}{5} \tag{22}$$

Therefore, the required probability is

$$\Pr(A + B)' + \Pr(A' + B) = \frac{1}{5} + \frac{4}{5} \tag{23}$$

$$= \frac{5}{5} \tag{24}$$

$$= 1 \tag{25}$$