

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (1)$$

parameters	values	description	
$\mathbf{m}_1$	$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	$AB$	
$\mathbf{m}_2$	$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$	$BC$	
$\mathbf{m}_3$	$\begin{pmatrix} -1 \\ 8 \end{pmatrix}$	$CA$	
$\ A - B\ $	5.38	length of $AB$	
$\ B - C\ $	4.24	length of $BC$	
$\ C - A\ $	8.06	length of $CA$	
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non-collinear	
$\mathbf{n}_1$	$\begin{pmatrix} 5 \\ -2 \end{pmatrix}$	2[1]	$AB$
$\mathbf{c}_1$	-5	2[1]	$AB$
$\mathbf{n}_2$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	2[1]	$BC$
$\mathbf{c}_2$	-3	2[1]	$BC$
$\mathbf{n}_3$	$\begin{pmatrix} -8 \\ -1 \end{pmatrix}$	2[1]	$CA$
$\mathbf{c}_3$	-13	2[1]	$CA$
Area	10.5	Area of Triangle	
$\angle A$	$28.93^\circ$	3[1]	Angles
$\angle B$	$113.20^\circ$		
$\angle C$	$37.88^\circ$		

TABLE I  
TRIANGLE ABC

parameters	value	description	
<b>D</b>	$\begin{pmatrix} 0.5 \\ -1.5 \end{pmatrix}$	midpoint of $BC$	
<b>E</b>	$\begin{pmatrix} 1.5 \\ 1.0 \end{pmatrix}$	midpoint of $CA$	
<b>F</b>	$\begin{pmatrix} 0 \\ 2.5 \end{pmatrix}$	midpoint of $AB$	
<b>n<sub>1</sub></b>	$\begin{pmatrix} -6.5 \\ 0.5 \end{pmatrix}$	2[1]	$AD$
<b>c<sub>1</sub></b>	-4.0	2[1]	$AD$
<b>n<sub>2</sub></b>	$\begin{pmatrix} 1 \\ -2.5 \end{pmatrix}$	2[1]	$BE$
<b>c<sub>2</sub></b>	-1	2[1]	$BE$
<b>n<sub>3</sub></b>	$\begin{pmatrix} 5.5 \\ 2 \end{pmatrix}$	2[1]	$CF$
<b>c<sub>3</sub></b>	5.0	2[1]	$CF$
<b>G</b>	$\begin{pmatrix} 0.67 \\ 0.67 \end{pmatrix}$	Centroid	
$\frac{AG}{GD}$	3[1]                  2	3[1]	Division ratio by <b>G</b>
$\frac{BG}{GE}$			
$\frac{CG}{GF}$			
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	3[1]                  2	3[1]	collinear
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$			
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$			

TABLE II  
MEDIANS

parameters	value	description	
<b>n<sub>1</sub></b>	$\begin{pmatrix} -3 \\ 3 \end{pmatrix}$	2[1]	Altitude $AP$
<b>c<sub>1</sub></b>	12	2[1]	Altitude $AP$
<b>n<sub>2</sub></b>	$\begin{pmatrix} 1 \\ -8 \end{pmatrix}$	2[1]	Altitude $BQ$
<b>c<sub>2</sub></b>	-1	2[1]	Altitude $BQ$
<b>n<sub>3</sub></b>	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$	2[1]	Altitude $CR$
<b>c<sub>3</sub></b>	-11	2[1]	Altitude $CR$
<b>H</b>	$\begin{pmatrix} -4.43 \\ -0.43 \end{pmatrix}$	Orthocentre	
	$(O - ((B+C)/2) \cdot (B-C) = 0$	Verified	

TABLE III  
ALTITUDES

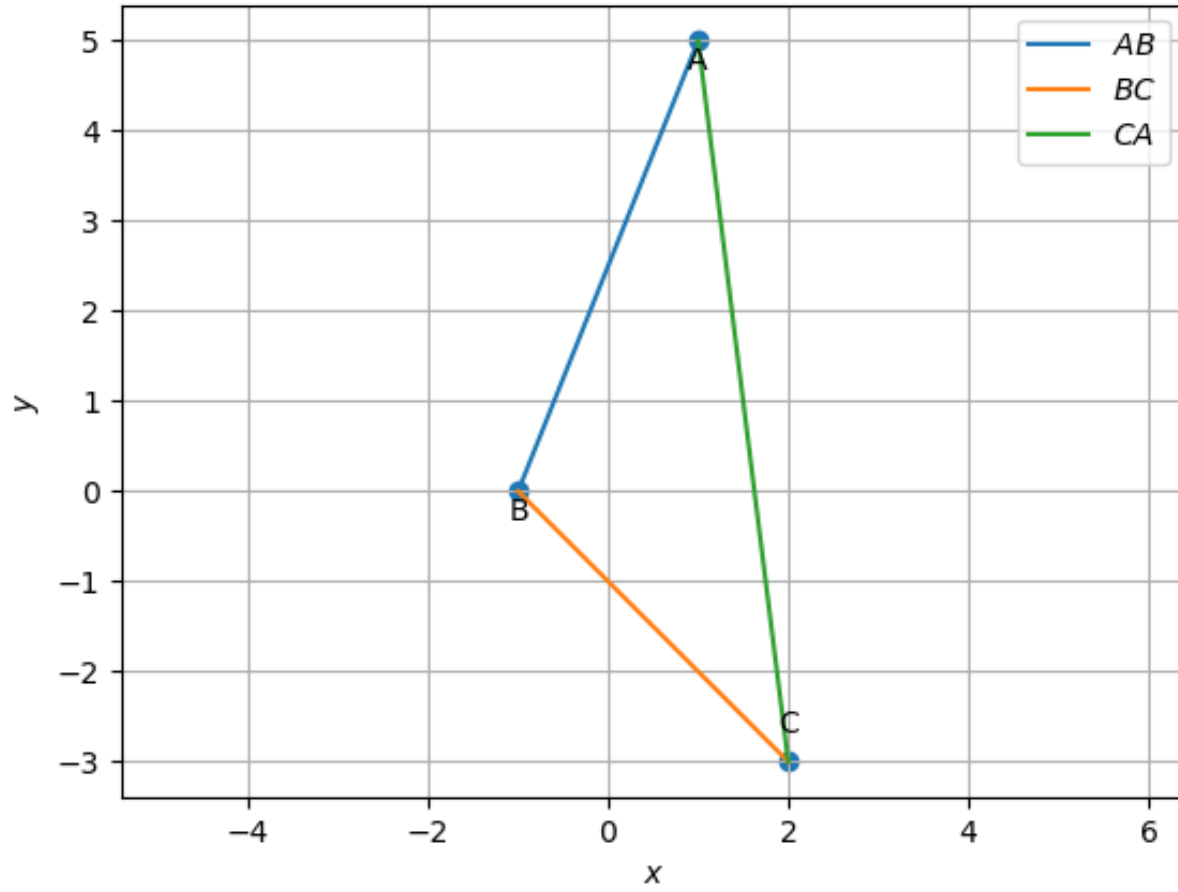


Fig. 1. Triangle ABC

parameters	value	description	
$\mathbf{n}_1$	$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	2[1]	Perpendicular bisector of $AB$
$\mathbf{c}_1$	-12.5	2[1]	Perpendicular bisector of $AB$
$\mathbf{n}_2$	$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$	2[1]	Perpendicular bisector of $BC$
$\mathbf{c}_2$	6	2[1]	Perpendicular bisector of $BC$
$\mathbf{n}_3$	$\begin{pmatrix} -1 \\ 8 \end{pmatrix}$	2[1]	Perpendicular bisector of $CA$
$\mathbf{c}_3$	6.5	2[1]	Perpendicular bisector of $CA$
$\mathbf{O}$	$\begin{pmatrix} 3.21 \\ 1.21 \end{pmatrix}$	Circumcentre	
$\ \mathbf{O} - \mathbf{A}\ $	4[2]      6.24	4[2]	$OA = OB = OC = R$
$\ \mathbf{O} - \mathbf{B}\ $			
$\ \mathbf{O} - \mathbf{C}\ $			
$\angle BOC$	57.85°	2[1]	$\angle BOC = 2\angle BAC$
$\angle BAC$	28.92°		

TABLE IV  
PERPENDICULAR BISECTORS

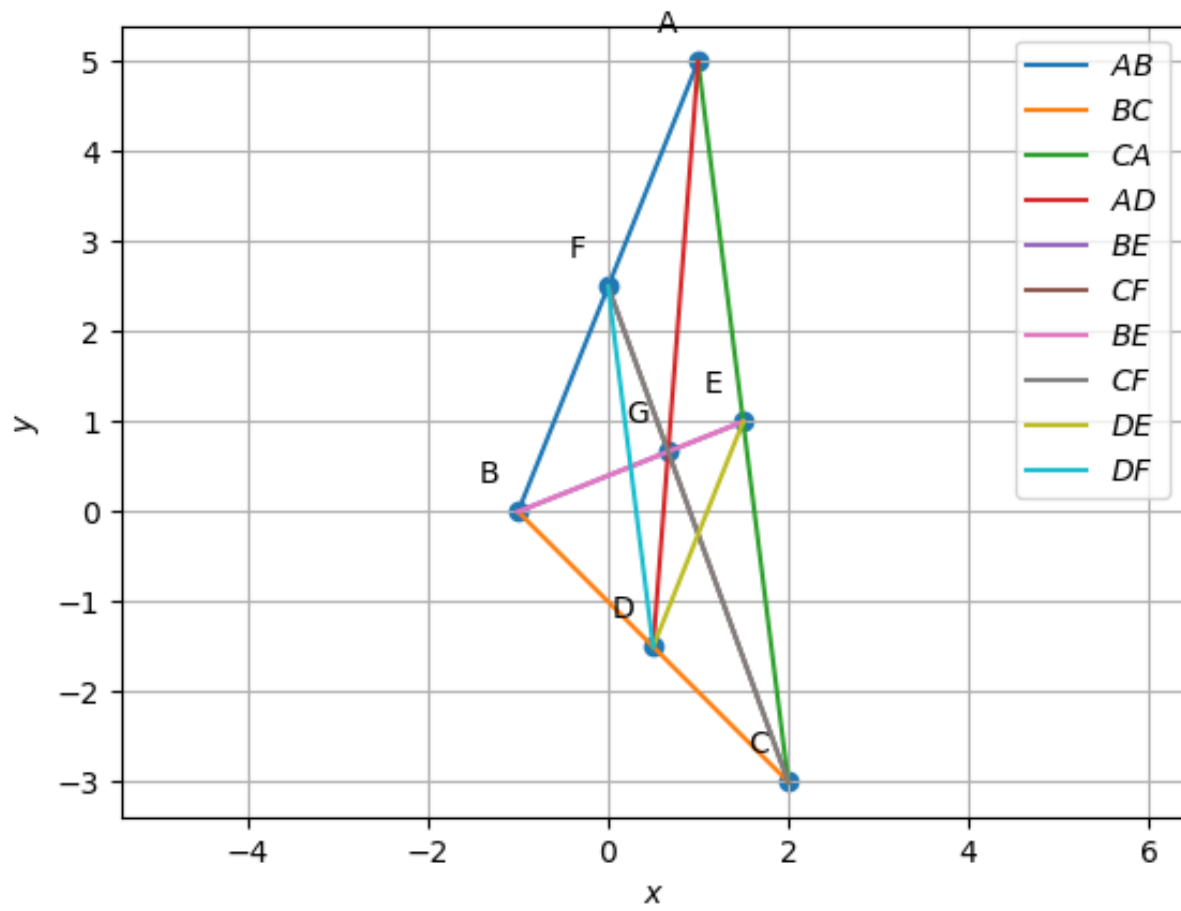


Fig. 2. Medians

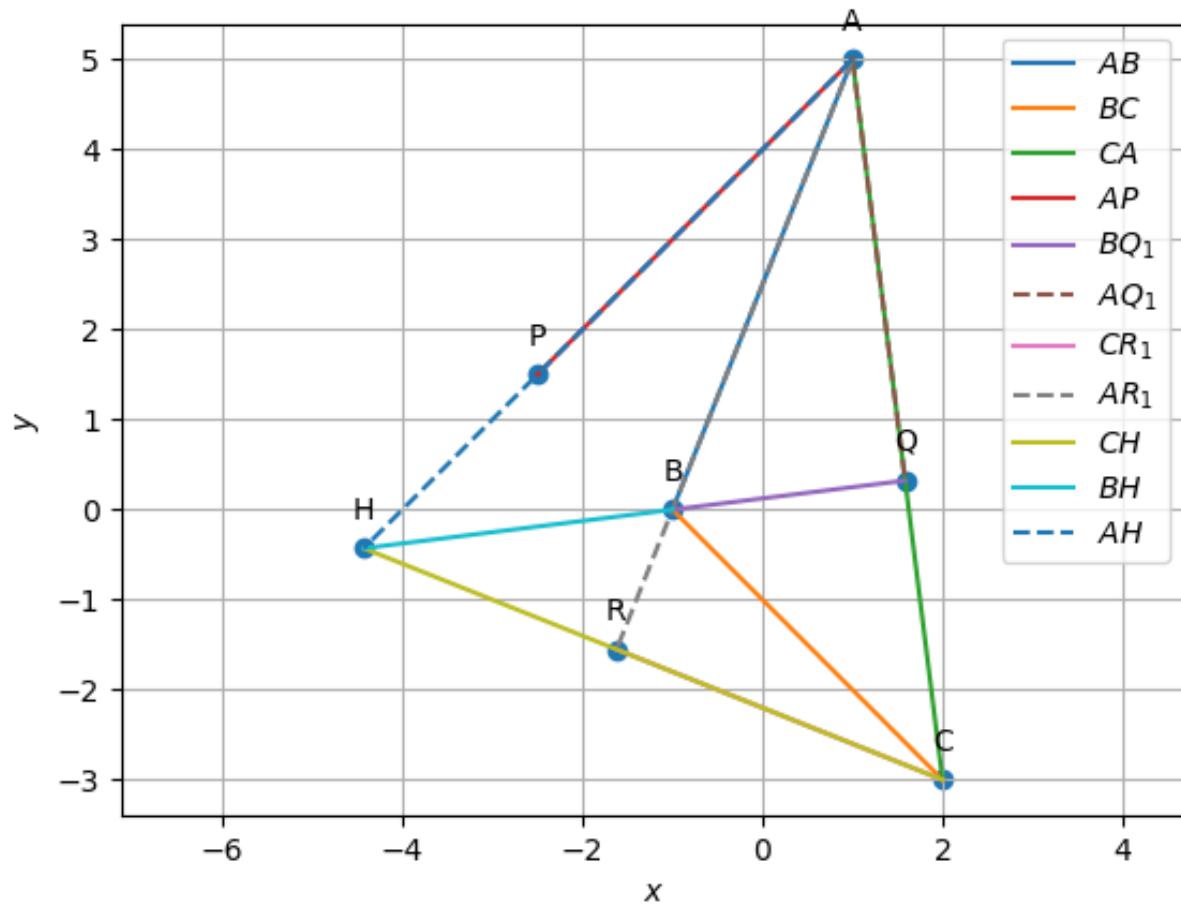


Fig. 3. Altitudes

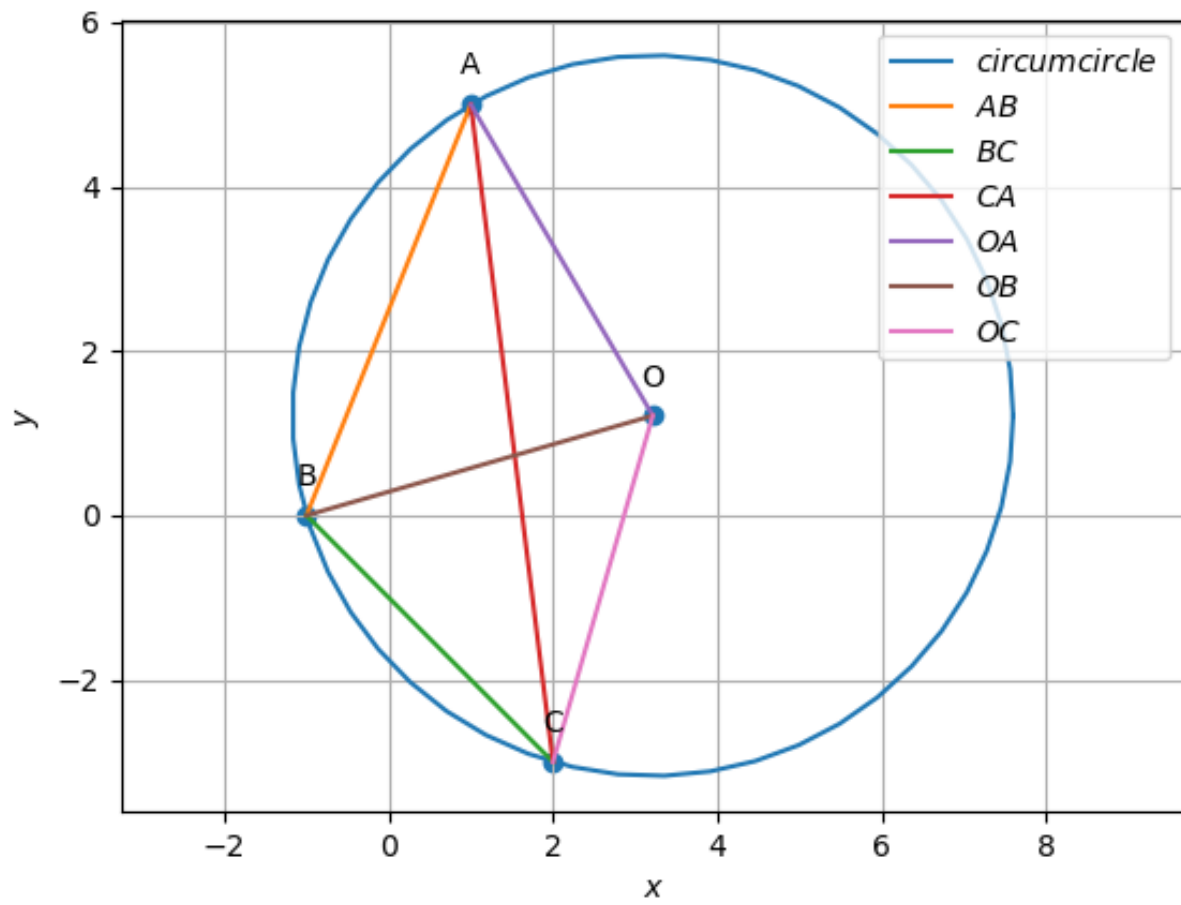


Fig. 4. Perpendicular bisectors

parameters	value		description
$\mathbf{n}_1$	$\begin{pmatrix} -1.92 \\ 0.25 \end{pmatrix}$	2[1]	Angle bisector of $A$
$\mathbf{c}_1$	-0.68	2[1]	Angle bisector of $A$
$\mathbf{n}_2$	$\begin{pmatrix} 0.22 \\ -1.08 \end{pmatrix}$	2[1]	Angle bisector of $B$
$\mathbf{c}_2$	-0.22	2[1]	Angle bisector of $B$
$\mathbf{n}_3$	$\begin{pmatrix} 1.7 \\ 0.83 \end{pmatrix}$	2[1]	Angle bisector of $C$
$\mathbf{c}_3$	0.90	2[1]	Angle bisector of $C$
$\mathbf{I}$	$\begin{pmatrix} 0.39 \\ 0.29 \end{pmatrix}$		Incentre
$\mathbf{D}_3$	$\begin{pmatrix} -0.45 \\ -0.55 \end{pmatrix}$		Point of contact with $BC$
$\mathbf{E}_3$	$\begin{pmatrix} 1.57 \\ 0.43 \end{pmatrix}$		Point of contact with $AC$
$\mathbf{F}_3$	$\begin{pmatrix} -0.71 \\ 0.73 \end{pmatrix}$		Point of contact with $AB$
$\ \mathbf{I} - \mathbf{D}_3\ $	4[2] 1.187	4[2]	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $			
$\ \mathbf{I} - \mathbf{F}_3\ $			
$r$			
$\angle BAI$	2[1] 14.46°	2[1]	$\angle BAI = \angle CAI$
$\angle CAI$			
$\mathbf{AE}_3$	4.60		$\mathbf{AE}_3 = \mathbf{AF}_3 = m$
$\mathbf{AF}_3$	4.60		
$\mathbf{BD}_3$	0.78		$\mathbf{BD}_3 = \mathbf{BF}_3 = n$
$\mathbf{BF}_3$	0.78		
$\mathbf{CD}_3$	3.45		$\mathbf{CD}_3 = \mathbf{CE}_3 = p$
$\mathbf{CE}_3$			

TABLE V  
ANGLE BISECTORS

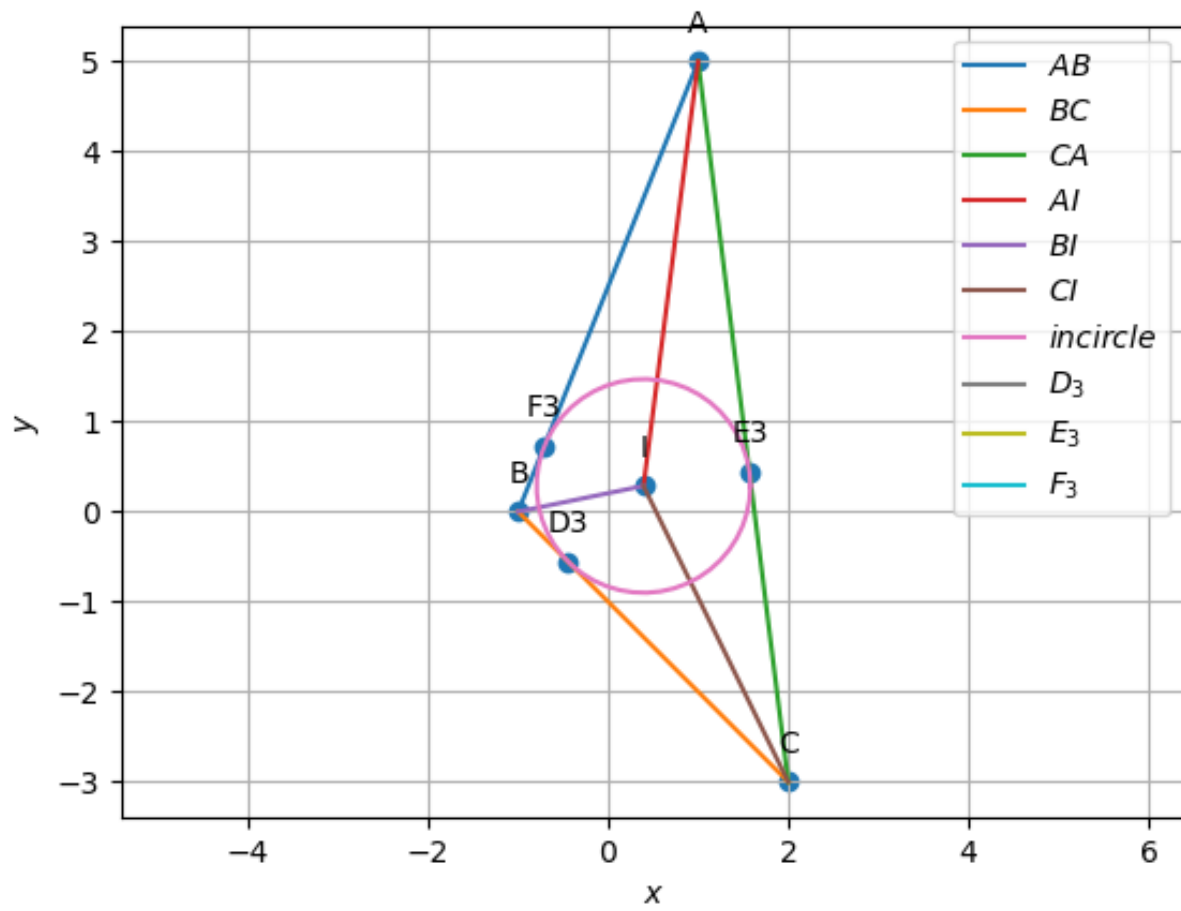


Fig. 5. Angle bisectors