Question 1.3.3 D_1 is a point on BC such that $AD_1 \perp BC$and AD_1 is defined to be the altitude. Find the equations of the altitude BE_1 and CF_1 to the sides AC and AB respectively.

Solutions: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{2}$$

Similarly,

Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{3}$$

To find the directional vector perpendicular to line AB and AC

Therefore

$$\mathbf{m}_{AC_\perp} = \mathbf{m}_{BE} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{4}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{5}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{n}_{BE}^\top = \begin{pmatrix} -4 & -4 \end{pmatrix} \tag{8}$$

$$\mathbf{m}_{AB_\perp} = \mathbf{m}_{CF} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{9}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{10}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{12}$$

$$\implies \mathbf{n}_{CF}^{\top} = \begin{pmatrix} 5 & -7 \end{pmatrix} \tag{13}$$

The equation of a line can be represented by:

$$\mathbf{m}^T(\mathbf{x} - \mathbf{p}) = 0 \tag{14}$$

Therefore, the equation of line BE

$$\mathbf{n}_{BE}^{\top}(\mathbf{x} - \mathbf{B}) = 0 \tag{15}$$

Therefore, the equation of line BE

$$\mathbf{n}_{CF}^{\top}(\mathbf{x} - \mathbf{C}) = 0 \tag{16}$$