Question 1.3.3 $D_1$ is a point on BC such that $AD_1 \perp BC$ and $AD_1$ is defined to be the altitude. Find the equations of the altitude $BE_1$ and $CF_1$ to the sides AC and AB respectively.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{2}$$

Similarly, Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{3}$$

Directional vectors perpendicular to AB and AC

$$\mathbf{n}_{AC} = \mathbf{m}_{BE} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{4}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{5}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{7}$$

$$\mathbf{n}_{AB} = \mathbf{m}_{CF} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{8}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{9}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{11}$$

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The equation of a line can be represented by:

$$\mathbf{n}^T(\mathbf{x} - \mathbf{p}) = 0 \tag{12}$$

Therefore, the equation of line BE

$$\begin{pmatrix} -4 & -4 \end{pmatrix} (\mathbf{x} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}) = 0 \tag{13}$$

Therefore, the equation of line CF

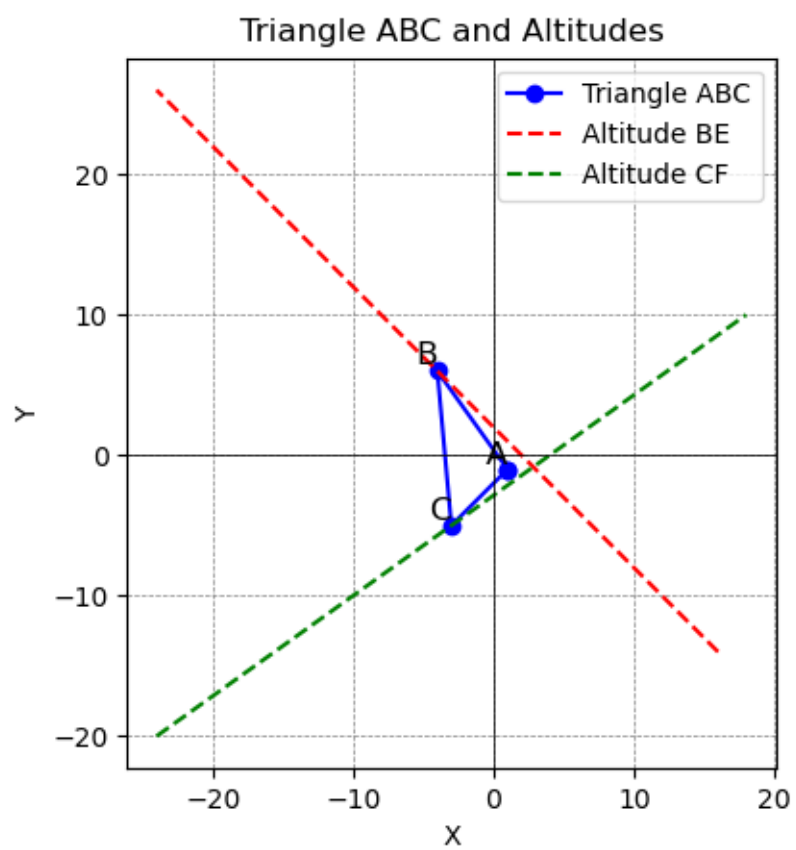$$\begin{pmatrix} 5 & -7 \end{pmatrix} (\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix}) = 0 \tag{14}$$

Figure 1: Triangle ABC with altitudes BE and CF