Question 1.3.3 D_1 is a point on BC such that $AD_1 \perp BC$and AD_1 is defined to be the altitude. Find the equations of the altitude BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{2}$$

Similarly, Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{3}$$

1. Calculating the equation of altitude BE
   Directional vector of BE = Directional vector perpendicular to AC
   Since,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{4}$$

$$\tag{5}$$

$$\mathbf{n}_{AC} = \mathbf{m}_{BE} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{6}$$

$$\mathbf{n}_{BE} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m}_{BE} \tag{7}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{9}$$

   The equation of a line can be represented by:

$$\mathbf{n}^T(\mathbf{x} - \mathbf{p}) = 0 \tag{10}$$

Therefore, the equation of line BE is

$$\begin{pmatrix}-4 & -4\end{pmatrix}(\mathbf{x}-\begin{pmatrix}-4\\6\end{pmatrix})=0 \tag{11}$$

$$\begin{pmatrix}-4 & -4\end{pmatrix}\mathbf{x}=\begin{pmatrix}-4 & -4\end{pmatrix}\begin{pmatrix}-4\\6\end{pmatrix} \tag{12}$$

$$\tag{13}$$

2. Calculating the equation of altitude CF
Directional vector of CF = Directional vector perpendicular to AB
Since,

$$\mathbf{n}=\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\mathbf{m} \tag{14}$$

$$\tag{15}$$

$$\mathbf{n}_{AB}=\mathbf{m}_{CF}=\begin{pmatrix}7\\5\end{pmatrix} \tag{16}$$

$$\mathbf{n}_{CF}=\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\mathbf{m}_{CF} \tag{17}$$

$$=\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}7\\5\end{pmatrix} \tag{18}$$

$$=\begin{pmatrix}5\\-7\end{pmatrix} \tag{19}$$

The equation of a line can be represented by:

$$\mathbf{n}^T(\mathbf{x}-\mathbf{p})=0 \tag{20}$$

Therefore, the equation of line CF is

$$\begin{pmatrix}5 & -7\end{pmatrix}(\mathbf{x}-\begin{pmatrix}-3\\-5\end{pmatrix})=0 \tag{21}$$

$$\begin{pmatrix}5 & -7\end{pmatrix}\mathbf{x}=\begin{pmatrix}5 & -7\end{pmatrix}\begin{pmatrix}-3\\-5\end{pmatrix} \tag{22}$$

$$\tag{23}$$

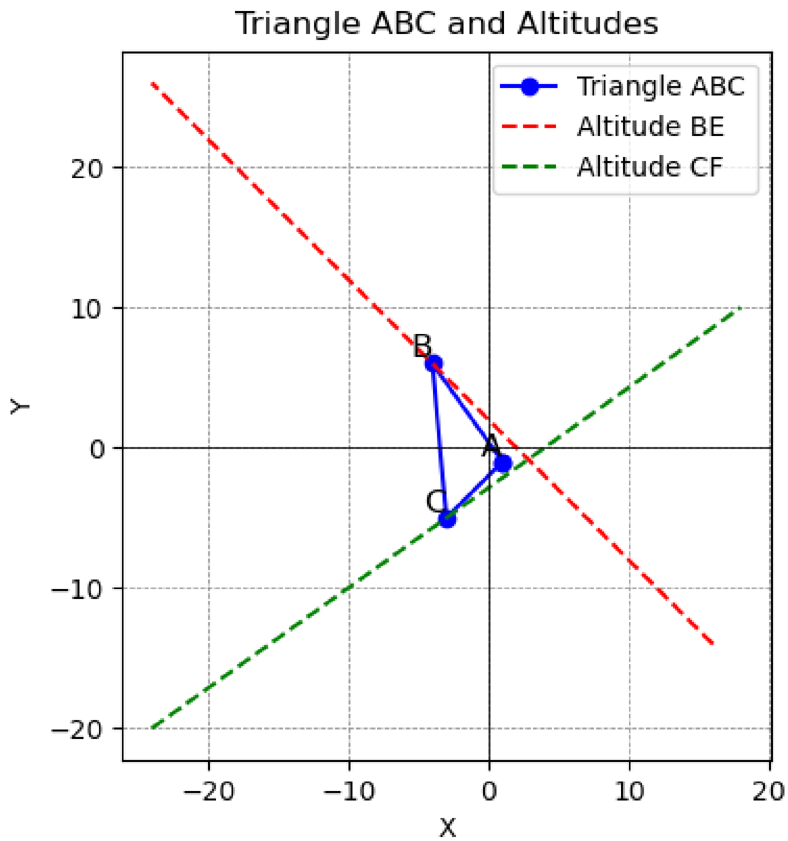

Figure 1: Triangle ABC with altitudes BE and CF