

Question 1.3.3 D_1 is a point on BC such that $AD_1 \perp BC$ and AD_1 is defined to be the altitude. Find the equations of the altitude BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1)$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (2)$$

Similarly, Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (3)$$

1. Calculating the equation of altitude BE

Directional vector of $BE =$ Directional vector perpendicular to AC

$$\mathbf{n}_{BE} = \mathbf{m}_{AC} \quad (4)$$

$$(5)$$

Therefore, the equation of line BE is

$$(-4 \quad -4) \left(\mathbf{x} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \right) = 0 \quad (6)$$

$$(-4 \quad -4) \mathbf{x} = (-4 \quad -4) \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (7)$$

$$(8)$$

2. Calculating the equation of altitude CF

Directional vector of $CF =$ Directional vector perpendicular to AB

$$\mathbf{n}_{CF} = \mathbf{m}_{AB} \quad (9)$$

$$(10)$$

Therefore, the equation of line CF is

$$(5 \quad -7) \left(\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \right) = 0 \quad (11)$$

$$(5 \quad -7) \mathbf{x} = (5 \quad -7) \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (12)$$

$$(13)$$

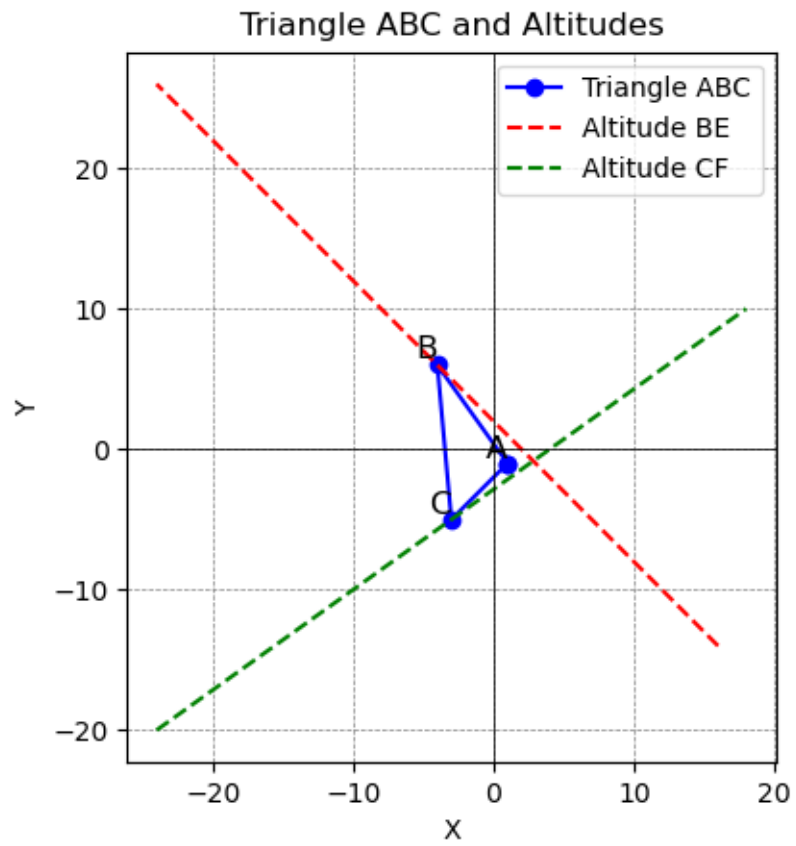


Figure 1: Triangle ABC with altitudes BE and CF