Question 1.3.3 D_1 is a point on BC such that $AD_1 \perp BC$and AD_1 is defined to be the altitude. Find the equations of the altitude BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{2}$$

Similarly, Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{3}$$

1. Calculating the equation of altitude BE
   Directional vector perpendicular to BE = Directional vector of AC

$$\mathbf{n}_{BE} = \mathbf{m}_{AC} \tag{4}$$

$$\tag{5}$$

   Therefore, the equation of line BE is

$$\begin{pmatrix} -4 & -4 \end{pmatrix} (\mathbf{x} - \begin{pmatrix} -4 \\ 6 \end{pmatrix}) = 0 \tag{6}$$

$$\begin{pmatrix} -4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -4 & -4 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{7}$$

$$\tag{8}$$

2. Calculating the equation of altitude CF
   Directional vector perpendicular to CF = Directional vector of AB

$$\mathbf{n}_{CF} = \mathbf{m}_{AB} \tag{9}$$

$$\tag{10}$$

   Therefore, the equation of line CF is

$$\begin{pmatrix} 5 & -7 \end{pmatrix} (\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix}) = 0 \tag{11}$$

$$\begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 & -7 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{12}$$

$$\tag{13}$$

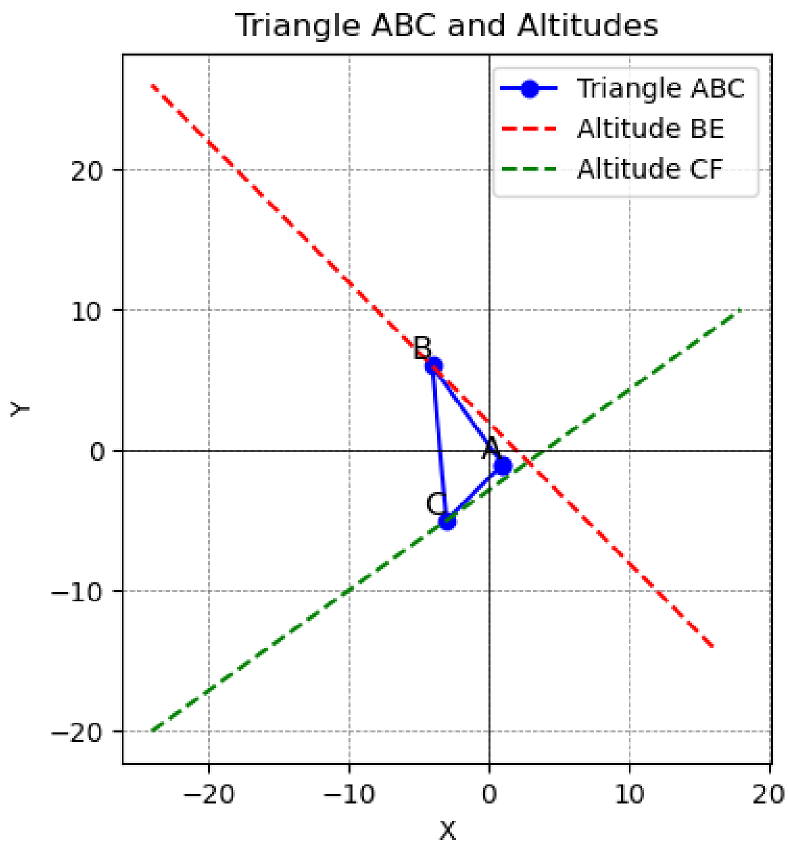

Figure 1: Triangle ABC with altitudes BE and CF