

Question 1.3.3 D_1 is a point on BC such that $AD_1 \perp BC$ and AD_1 is defined to be the altitude. Find the equations of the altitude BE_1 and CF_1 to the sides AC and AB respectively.

Solutions: Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1)$$

Directional vector of $AB = \mathbf{B} - \mathbf{A}$

$$\mathbf{m}_{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (2)$$

Similarly,

Directional vector of $AC = \mathbf{C} - \mathbf{A}$

$$\mathbf{m}_{AC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (3)$$

To find the directional vector perpendicular to line AB and AC
Therefore

$$\mathbf{m}_{AC_{\perp}} = \mathbf{m}_{BE} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad (4)$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (5)$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (7)$$

$$\implies \mathbf{n}_{BE}^{\top} = (-4 \quad -4) \quad (8)$$

$$\mathbf{m}_{AB_{\perp}} = \mathbf{m}_{CF} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad (9)$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (10)$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (12)$$

$$\implies \mathbf{n}_{CF}^\top = (5 \quad -7) \quad (13)$$

The equation of a line can be represented by:

$$\mathbf{m}^T(\mathbf{x} - \mathbf{p}) = 0 \quad (14)$$

Therefore, the equation of line BE

$$\mathbf{n}_{BE}^\top(\mathbf{x} - \mathbf{B}) = 0 \quad (15)$$

Therefore, the equation of line CF

$$\mathbf{n}_{CF}^\top(\mathbf{x} - \mathbf{C}) = 0 \quad (16)$$