Q:The difference between any two cosecutive interior angles of a polygon is 5°.If the smallest angle is 120°,find the number of sides of polygon.
Solution: The interior angles of a polygon are in AP with

$$a = 120$$
$$d = 5$$

The sum of n terms of an AP is given by

$$S = \frac{n}{2}(2a + (n-1)d) \tag{1}$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \tag{2}$$

$$\frac{n}{2}(2a + (n-1)d) = (n-2)180$$
$$\frac{n}{2}(240 + (n-1)5) = (n-2)180$$
$$n(235 + 5n) = 360n - 720$$
$$5n^2 + 235n = 360n - 720$$
$$5n^2 - 125 + 720 = 0$$
$$n^2 - 25n + 144 = 0$$

solving equation we get

$$n = 16, 9$$

$$x(n) = \begin{cases} 115 + 5n & \text{if } 0 \le n \le 15 \\ 0 & \text{if } n > 15, n < 0 \end{cases}$$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$X(n) = \sum_{n=0}^{15}(120 + 5n) \cdot z^{-n} X(n) = 120 \sum_{n=0}^{15} z^{-n} + 5 \sum_{n=0}^{15} n \cdot z^{-n}$$

Let $p = 120 \cdot \sum_{n=0}^{15} z^{-n}$, $q = 5 \sum_{n=0}^{15} n \cdot z^{-n}$
sum of n terms of G.P with a= first term and r= common ratio is given by

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$\therefore \ p = 120 \times \frac{(z^{-16} - 1)}{(z^{-1} - 1)} = 120 \times \frac{(1 - z^{-16})}{(1 - z^{-1})}$$

$$p = \frac{120 - 120z^{-16}}{(1 - z^{-1})}$$

$$q = 5 \sum_{n=0}^{15} n \cdot z^{-n}$$

The terms of q are in AGP with

$$a = 1$$
$$d = 1$$
$$r = z^{-1}$$
$$n = 16$$

$$S = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{(a+(n-1)d)r^n}{1-r}$$

$$\begin{aligned}
\sum_{n=0}^{15} n \cdot z^{-n} &= \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{(1+(16-1)1)\times z^{-16}}{1-z^{-1}} \\
&= \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{16\times z^{-16}}{1-z^{-1}} \\
&= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16\cdot z^{-16}(1-z^{-1})}{(1-z^{-1})^2} \\
&= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16z^{-16} - z^{-17})}{(1-z^{-1})^2} \\
&= \frac{1-z^{-1}+z^{-1}-z^{-16}-16\cdot z^{-16}+16\cdot z^{-17}}{(1-z^{-1})^2} \\
&= \frac{1-17\cdot z^{-16}+16\cdot z^{-17}}{(1-z^{-1})^2}
\end{aligned}$$

$$\begin{aligned}
q &= 5\times\sum_{n=1}^{16} n\cdot z^{-(n-1)} \\
&= 5\times\frac{1-17\cdot z^{-16}+16\cdot z^{-17}}{(1-z^{-1})^2}
\end{aligned}$$

$$\therefore\ q = \frac{5-85\cdot z^{-16}+80\cdot z^{-17}}{(1-z^{-1})^2}$$

$$\begin{aligned}
X(n) &= p + q \\
&= \frac{120-120z^{-16}}{1-z^{-1}} + \frac{5-85\cdot z^{-16}+80\cdot z^{-17}}{(1-z^{-1})^2} \\
&= \frac{(120-120z^{-16})(1-z^{-1}) + (5-85\cdot z^{-16}+80\cdot z^{-17})}{(1-z^{-1})^2} \\
&= \frac{(120-120z^{-16}-120z^{-1}+120\cdot z^{-17}) + (5-85\cdot z^{-16}+80\cdot z^{-17})}{(1-z^{-1})^2} \\
&= \frac{125-120z^{-1}-205z^{-16}+200z^{-17}}{(1-z^{-1})^2}
\end{aligned}$$

$$\therefore\ X(n) = \frac{125-120z^{-1}-205z^{-16}+200z^{-17}}{(1-z^{-1})^2}$$