Assignment

EE23BTECH11008 - Meenakshi

Q:The difference between any two cosecutive interior angles of a polygon is 5°.If the smallest angle is 120°,find the number of sides of polygon.
Solution: The interior angles of a polygon are in AP with $x(0) = 120$, $d = 5$ For the AP x(n), sum of

Variable	Description	Value
$x(0)$	first term of AP	120
d	common difference of AP	5
$x(n)$	general term of AP	none

n+1 terms of can be expressed as

$$y(n) = \sum_{k=1}^{n} x(k) \tag{1}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)u(n-k) \tag{2}$$

$$= x(n) * u(n) \tag{3}$$

$$\tag{4}$$

Applying Z-transform on both sides

$$Y(Z) = X(Z)U(Z) \tag{5}$$

$$= \left(\frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2}\right) \cdot \frac{1}{1-z^{-1}} \quad |z| > 1 \tag{6}$$

$$= \frac{x(0)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{7}$$

Seperating terms ,

$$Y(Z) = x(0)\left[\frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}\right] + \frac{d}{2}\left[-\frac{d}{dz}\left(\frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}\right)\right] \quad |z| > 1 \tag{8}$$

Taking the inverse Z-transform and applying the derivative property ,

$$y(n) = \left(x(0)(n+1) + \frac{d}{2}n(n+1)\right)u(n) \tag{9}$$

$$= \frac{n+1}{2}\left(2x(0) + nd\right)u(n) \tag{10}$$

Therefore, the sum of n terms of an AP is given by

$$y(n) = \frac{n}{2}\left(2x(0) + (n-1)d\right)u(n) \tag{11}$$

$$= \frac{n}{2}\left(2x(0) + (n-1)d\right) \forall n \geq 0 \tag{12}$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \tag{13}$$

$$\frac{n}{2}(2 \cdot x(0) + (n-1)d) = (n-2)180 \tag{14}$$

$$\frac{n}{2}(240 + (n-1)5) = (n-2)180 \tag{15}$$

$$n(235 + 5n) = 360n - 720 \tag{16}$$

$$5n^2 + 235n = 360n - 720 \tag{17}$$

$$5n^2 - 125 + 720 = 0 \tag{18}$$

$$n^2 - 25n + 144 = 0 \tag{19}$$

solving the above equation we get

$$n = 16, 9 \tag{20}$$

$$\boxed{x(n) = (120 - 5n) \times u(n)} \tag{21}$$

Now,

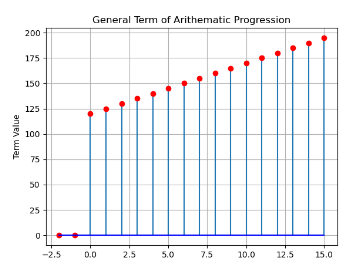

Fig. 0: Plot of the general term taken from Python

$$X(z) = x(0)U(z) + d\left(-z\frac{d(U(z))}{dz}\right) \tag{22}$$

$$\text{ROC: } |z| > 1 \tag{23}$$

$$\boxed{X(z) = 120U(z) - 5z\frac{d(U(z))}{dz}} \tag{24}$$