Assignment

EE23BTECH11001 - Aashna Sahu

Q:The difference between any two cosecutive interior angles of a polygon is 5°.If the smallest angle is 120°,find the number of sides of polygon.
Solution: The interior angles of a polygon are in AP with $x(0) = 120$, $d = 5$

The sum of n terms of an AP is given by

$$S = \frac{n}{2}(2 \cdot x(0) + (n-1)d) \qquad (1)$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \qquad (2)$$

$$\frac{n}{2}(2 \cdot x(0) + (n-1)d) = (n-2)180 \qquad (3)$$

$$\frac{n}{2}(240 + (n-1)5) = (n-2)180 \qquad (4)$$

$$n(235 + 5n) = 360n - 720 \qquad (5)$$

$$5n^2 + 235n = 360n - 720 \qquad (6)$$

$$5n^2 - 125 + 720 = 0 \qquad (7)$$

$$n^2 - 25n + 144 = 0 \qquad (8)$$

solving the above equation we get

$$n = 16, 9 \qquad (9)$$

$$x(n) = \begin{cases} 120 + 5n & \text{if } 0 \le n \le 15 \\ 0 & \text{if } n > 15, n < 0 \end{cases}$$
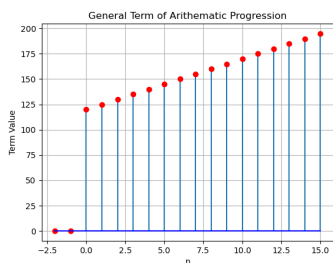

Fig. 0: Plot of the general term taken from Python

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \cdot u(n) \qquad (10)$$

The expression for $X(z)$ is given by:

$$X(z) = \sum_{n=0}^{15}(120 + 5n) \cdot z^{-n} \cdot u[n] \qquad (11)$$

The above sequence converges for $|z| > 15$
The expression for u(n) is

$$u(n) = \begin{cases} 1 & \text{if } n \ge 0, \\ 0 & \text{if } n < 0. \end{cases}$$

Evaluating the sum term by term, we get:

$$X(z) = 120 \cdot z^0 + (120 + 5 \cdot 1) \cdot z^{-1} + (120 + 5 \cdot 2) \cdot z^{-2} + \dots \qquad (12)$$

$$= 120 + 125 \cdot z^{-1} + 130 \cdot z^{-2} + \dots + 195 \cdot z^{-15} \qquad (13)$$

$$X(n) = \sum_{n=0}^{15}(120 + 5n) \cdot z^{-n} \qquad (14)$$

$$X(n) = 120 \sum_{n=0}^{15} z^{-n} + 5 \sum_{n=0}^{15} n \cdot z^{-n} \qquad (15)$$

Let $p = 120 \cdot \sum_{n=0}^{15} z^{-n}$, $q = 5\sum_{n=0}^{15} n \cdot z^{-n}$
By using formula of sum of n terms of GP we get

$$\therefore \; p = 120 \times \frac{(z^{-16} - 1)}{(z^{-1} - 1)} = 120 \times \frac{(1 - z^{-16})}{(1 - z^{-1})} \qquad (16)$$

$$q = 5\sum_{n=0}^{15} n \cdot z^{-n} \qquad (17)$$

The terms of q are in AGP. Hence the sum is given by

$$\sum_{n=0}^{15} n \cdot z^{-n} = \frac{1}{1 - z^{-1}} + \frac{z^{-1}(1 - z^{-15})}{(1 - z^{-1})^2} - \frac{(1 + (16 - 1)1) \times z^{-16}}{1 - z^{-1}} \qquad (18)$$

$$= \frac{1 - 17 \cdot z^{-16} + 16 \cdot z^{-17}}{(1 - z^{-1})^2} \qquad (19)$$

$$q = 5 \times \sum_{n=1}^{16} n \cdot z^{-(n-1)} \qquad (20)$$

$$\therefore \; q = \frac{5 - 85 \cdot z^{-16} + 80 \cdot z^{-17}}{(1 - z^{-1})^2} \qquad (21)$$

$X(n) = p + q$

$$\therefore X(z) = \frac{125 - 120z^{-1} - 205z^{-16} + 200z^{-17}}{(1 - z^{-1})^2} u(z) \quad \forall |z| > 15.$$
(22)

The expression for $u(z)$ is given by:

$$u(z) = \begin{cases} 0, & \text{for } z < 0, z \in \mathbb{Z}, \\ 1, & \text{for } z > 0, z \in \mathbb{Z}. \end{cases}$$

now the expression simplifies to

$$X(z) = \frac{125 - 120z^{-1} - 205z^{-16} + 200z^{-17}}{(1 - z^{-1})^2} u(z) \quad \forall z > 5$$
(23)

Variable	Description	Value
$x(0)$	first term of AP	120
d	common difference of AP	5
$x(n)$	general term of AP	none
n	Describing the order of term	none
$u(n)$	unit step function	mentioned above
$u(z)$	z-transform of u(n)	mentioned above
$X(z)$	z-transform of u(n)	none