Q:The difference between any two cosecutive interior angles of a polygon is 5°.If the smallest angle is 120°,find the number of sides of polygon.

Solution: The interior angles of a polygon are in AP with

$$x(0) = 120$$
$$d = 5$$

The sum of n terms of an AP is given by

$$S = \frac{n}{2}(2 \cdot x(0) + (n-1)d) \tag{1}$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \tag{2}$$
$$\frac{n}{2}(2 \cdot x(0) + (n-1)d) = (n-2)180$$
$$\frac{n}{2}(240 + (n-1)5) = (n-2)180$$
$$n(235 + 5n) = 360n - 720$$
$$5n^2 + 235n = 360n - 720$$
$$5n^2 - 125 + 720 = 0$$
$$n^2 - 25n + 144 = 0$$

solving the above equation we get

$$n = 16, 9$$

$$x(n) = \begin{cases} 120 + 5n & \text{if } 0 \le n \le 15 \\ 0 & \text{if } n > 15, n < 0 \end{cases}$$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \cdot u(n)$$

The expression for $X(z)$ is given by:

$$X(z) = \sum_{n=0}^{15}(120 + 5n) \cdot z^{-n} \cdot u[n]$$

The above sequence converges for $|z| > 15$

The expression for u(n) is

$$u(n) = \begin{cases} 1 & \text{if } n \ge 0, \\ 0 & \text{if } n < 0. \end{cases}$$

We can evaluate the summation term by term. The unit step function u[n] ensures that only the terms with $n \ge 0$ are considered. Evaluating the sum term by term, we get:

$$X(z) = 120 \cdot z^0 + (120 + 5 \cdot 1) \cdot z^{-1} + (120 + 5 \cdot 2) \cdot z^{-2} + \ldots + (120 + 5 \cdot 15) \cdot z^{-15}$$
$$= 120 + 125 \cdot z^{-1} + 130 \cdot z^{-2} + \ldots + 195 \cdot z^{-15}$$

$$X(n) = \sum_{n=0}^{15}(120 + 5n) \cdot z^{-n}$$

$$X(n) = 120 \sum_{n=0}^{15} z^{-n} + 5 \sum_{n=0}^{15} n \cdot z^{-n}$$

Let $p = 120 \cdot \sum_{n=0}^{15} z^{-n}$, $q = 5 \sum_{n=0}^{15} n \cdot z^{-n}$
By using formula of sum of n terms of GP we get

$$\therefore\ \ p = 120 \times \frac{(z^{-16}-1)}{(z^{-1}-1)} = 120 \times \frac{(1-z^{-16})}{(1-z^{-1})}$$

$$p = \frac{120-120z^{-16}}{(1-z^{-1})}$$

$$q = 5\sum_{n=0}^{15} n \cdot z^{-n}$$

The terms of q are in AGP. Hence the sum is given by

$$\begin{aligned}
\sum_{n=0}^{15} n \cdot z^{-n} &= \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{(1+(16-1)1)\times z^{-16}}{1-z^{-1}} \\
&= \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{16\times z^{-16}}{1-z^{-1}} \\
&= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16\cdot z^{-16}(1-z^{-1})}{(1-z^{-1})^2} \\
&= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16z^{-16} - z^{-17})}{(1-z^{-1})^2} \\
&= \frac{1-z^{-1}+z^{-1}-z^{-16}-16\cdot z^{-16}+16\cdot z^{-17}}{(1-z^{-1})^2} \\
&= \frac{1-17\cdot z^{-16}+16\cdot z^{-17}}{(1-z^{-1})^2}
\end{aligned}$$

$$q = 5\times\sum_{n=1}^{16} n\cdot z^{-(n-1)}$$

$$\therefore\ \ q = \frac{5-85\cdot z^{-16}+80\cdot z^{-17}}{(1-z^{-1})^2}$$

$$\begin{aligned}
X(n) &= p+q \\
&= \frac{120-120z^{-16}}{1-z^{-1}} + \frac{5-85\cdot z^{-16}+80\cdot z^{-17}}{(1-z^{-1})^2} \\
&= \frac{(120-120z^{-16})(1-z^{-1}) + (5-85\cdot z^{-16}+80\cdot z^{-17})}{(1-z^{-1})^2} \\
&= \frac{(120-120z^{-16}-120z^{-1}+120\cdot z^{-17}) + (5-85\cdot z^{-16}+80\cdot z^{-17})}{(1-z^{-1})^2}
\end{aligned}$$

$$\therefore X(z) = \frac{125-120z^{-1}-205z^{-16}+200z^{-17}}{(1-z^{-1})^2}u(z) \quad \forall |z| > 15. \tag{3}$$

The expression for $u(z)$ is given by:

$$u(z) = \begin{cases} 0, & \text{for } z < 0, z \in \mathbb{Z}, \\ 1, & \text{for } z > 0, z \in \mathbb{Z}. \end{cases}$$

now the expression simplifies to

$$X(z) = \frac{125-120z^{-1}-205z^{-16}+200z^{-17}}{(1-z^{-1})^2}u(z) \quad \forall z > 5 \tag{4}$$