Assignment

EE23BTECH11001 - Aashna Sahu

Q:The difference between any two cosecutive interior angles of a polygon is 5°.If the smallest angle is 120°,find the number of sides of polygon.
Solution: The interior angles of a polygon are in

Variable	Description	Value
$x(0)$	first term of AP	120
d	common difference of AP	5
$x(n)$	general term of AP	none
n	Describing the order of term	none
$u(n)$	unit step function	mentioned above
$u(z)$	z-transform of u(n)	mentioned above
$X(z)$	z-transform of u(n)	none

AP with $x(0) = 120$, $d = 5$

The sum of n terms of an AP is given by

$$S = \frac{n}{2}(2 \cdot x(0) + (n-1)d) \tag{1}$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \tag{2}$$

$$\frac{n}{2}(2 \cdot x(0) + (n-1)d) = (n-2)180 \tag{3}$$

$$\frac{n}{2}(240 + (n-1)5) = (n-2)180 \tag{4}$$

$$n(235 + 5n) = 360n - 720 \tag{5}$$

$$5n^2 + 235n = 360n - 720 \tag{6}$$

$$5n^2 - 125 + 720 = 0 \tag{7}$$

$$n^2 - 25n + 144 = 0 \tag{8}$$

solving the above equation we get

$$n = 16, 9 \tag{9}$$

$$x(n) = \begin{cases} 120 + 5n & \text{if } 0 \leq n \leq 15 \\ 0 & \text{if } n > 15, n < 0 \end{cases}$$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \cdot u(n) \tag{10}$$

The expression for $X(z)$ is given by:

$$X(z) = \sum_{n=0}^{15} (120 + 5n) \cdot z^{-n} \cdot u[n] \tag{11}$$

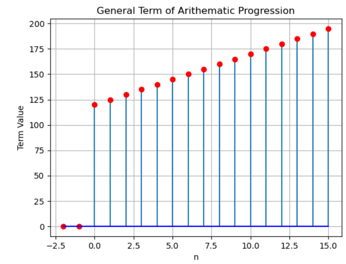

Fig. 0: Plot of the general term taken from Python

The above sequence converges for $|z| > 15$
The expression for u(n) is

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0, \\ 0 & \text{if } n < 0. \end{cases}$$

Evaluating the sum term by term, we get:

$$X(z) = 120 \cdot z^0 + (120 + 5 \cdot 1) \cdot z^{-1} + (120 + 5 \cdot 2) \cdot z^{-2} + \ldots \tag{12}$$

$$= 120 + 125 \cdot z^{-1} + 130 \cdot z^{-2} + \ldots + 195 \cdot z^{-15} \tag{13}$$

$$X(n) = \sum_{n=0}^{15} (120 + 5n) \cdot z^{-n} \tag{14}$$

$$X(n) = 120 \sum_{n=0}^{15} z^{-n} + 5 \sum_{n=0}^{15} n \cdot z^{-n} \tag{15}$$

Let $p = 120 \cdot \sum_{n=0}^{15} z^{-n}$,$q = 5 \sum_{n=0}^{15} n \cdot z^{-n}$
By using formula of sum of n terms of GP we get

$$\therefore \;\; p = 120 \times \frac{(z^{-16} - 1)}{(z^{-1} - 1)} = 120 \times \frac{(1 - z^{-16})}{(1 - z^{-1})} \tag{16}$$

$$q = 5 \sum_{n=0}^{15} n \cdot z^{-n} \tag{17}$$

The terms of q are in AGP. Hence the sum is given by

$$\sum_{n=0}^{15} n \cdot z^{-n} = \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{(1+(16-1)1) \times z^{-16}}{1-z^{-1}} \tag{18}$$

$$= \frac{1 - 17 \cdot z^{-16} + 16 \cdot z^{-17}}{(1-z^{-1})^2} \tag{19}$$

$$q = 5 \times \sum_{n=1}^{16} n \cdot z^{-(n-1)} \tag{20}$$

$$\therefore \; q = \frac{5 - 85 \cdot z^{-16} + 80 \cdot z^{-17}}{(1-z^{-1})^2} \tag{21}$$

$X(n) = p + q$

$$\therefore X(z) = \frac{125 - 120z^{-1} - 205z^{-16} + 200z^{-17}}{(1-z^{-1})^2} u(z) \quad \forall |z| > 15. \tag{22}$$

The expression for $u(z)$ is given by:

$$u(z) = \begin{cases} 0, & \text{for } z < 0, z \in \mathbb{Z}, \\ 1, & \text{for } z > 0, z \in \mathbb{Z}. \end{cases}$$

now the expression simplifies to

$$X(z) = \frac{125 - 120z^{-1} - 205z^{-16} + 200z^{-17}}{(1-z^{-1})^2} u(z) \quad \forall z > 5 \tag{23}$$