Gate2022.IN.39

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q: A car is moving collinearly with a laser beam emitted by a transceiver. A laser pulse emitted at $t = 0$ s is received back by the transceiver 100 ns (nanoseconds) later after reflection from the car.A second pulse emitted at $t = 0.1$ s is received back 90 ns later. Given the speed of light is 3×10^8 m/s, the average speed of the car in this interval is __.

(A) 54 kmph, moving towards the transceiver
(B) 108 kmph,moving towards the transceiver
(C) 54 kmph, moving away from the transceiver
(D) 108 kmph,moving away from the transceiver

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Solution:

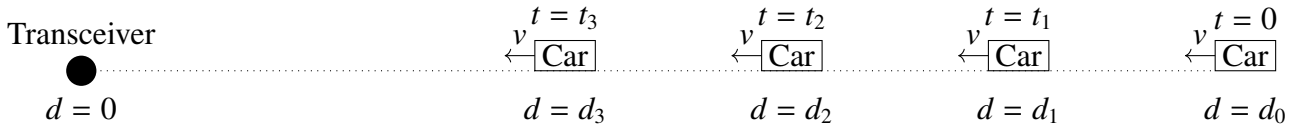

Fig. 0: block diagram of the system

Variable	Description	Value
v_c	velocity of laser	3×10^8 m/s
v	average speed of car	none
t_1	time at which first pulse hits car	none
t_2	time at which second pulse is emmited	0.1s
t_3	time at which second pulse hits car	none
d_0	Distance between tranceiver and car at $t = 0$	none
d_i for $i = 1, 2, 3$	Distance between tranceiver and car at $t = t_i$	none

TABLE 0: Input parameters

From Fig. 0

$$t_1 = \frac{d_1}{v_c} = \frac{d_0 - d_1}{v} \tag{1}$$

$$\implies d_1 = \frac{d_0}{\left(1 + \frac{v}{v_c}\right)} \tag{2}$$

Distance travelled by first pulse is given by

$$2d_1 = v_c \times 100\,\text{ns} \tag{3}$$

$$2\frac{d_0}{\left(1 + \frac{v}{v_c}\right)} = v_c \times 100\,\text{ns} \tag{4}$$

similarly time taken by car to move from d_2 to d_3 is given by

$$t_3 - t_2 = \frac{d_3}{v_c} = \frac{d_2 - d_3}{v} \tag{5}$$

$$\implies d_3 = \frac{d_2}{\left(1 + \frac{v}{v_c}\right)} \tag{6}$$

from Fig. 0

$$d_2 = d_0 - 0.1v \tag{7}$$

$$\therefore d_3 = \frac{d_0 - 0.1v}{\left(1 + \frac{v}{v_c}\right)} \tag{8}$$

Distance travelled by second pulse is given by

$$2d_3 = v_c \times 90\,\text{ns} \tag{9}$$

$$2\frac{d_0 - 0.1v}{\left(1 + \frac{v}{v_c}\right)} = v_c \times 90\,\text{ns} \tag{10}$$

solving (4) and (10) we get

$$v = 15\,\text{m/s} \tag{11}$$

$$v = 54\,\text{kmph} \tag{12}$$

since v is same but time taken by pulses to reach tranceiver is decreasing, v is towards tranceiver.
$\therefore$ Average speed of car is 54 kmph, moving towards the transceiver (option A)