12.10.4

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Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$$

let $u(t)$ be the unit step function.The input $x(t)$ that results in a steady-state output $y(t) = sin(\pi t)$ is __.

Solution:

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0
INPUT PARAMETERS

$$H(s) = \frac{s-\pi}{s+\pi} \tag{1}$$

Converting transfer function to frequency response, we get

$$H(j\omega) = \frac{j\omega-\pi}{j\omega+\pi} \tag{2}$$

$$|H(j\omega)| = \frac{\sqrt{(j\omega)^2+(\pi)^2}}{\sqrt{(j\omega)^2+(\pi)^2}} = 1 \tag{3}$$

$$\angle H(j\omega) = -2\tan^{-1}\frac{\omega}{\pi} \tag{4}$$

Here , $\omega = \pi$

$$\angle H(j\omega)\Big|_{\omega=\pi} = -90^\circ \tag{5}$$

Using the magnitude and phase information, the input signal $x(t)$is given by:

$$x(t) = |H(j\omega)|\sin(\omega t + \angle H(j\omega)) \tag{6}$$

Substituting $|H(j\pi)| = 1$ and $\angle H(j\pi) = -\frac{\pi}{2}$:

$$x(t) = 1\sin\left(\pi t - \frac{\pi}{2}\right) \tag{7}$$

$$\therefore x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \tag{8}$$