

12.10.4

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let $u(t)$ be the unit step function. The input $x(t)$ that results in a steady-state output $y(t) = \sin(\pi t)$ is ____.

Solution:

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0

INPUT PARAMETERS

$$H(s) = \frac{s - \pi}{s + \pi} \quad (1)$$

Converting transfer function to frequency response, we get

$$H(j\omega) = \frac{j\omega - \pi}{j\omega + \pi} \quad (2)$$

$$|H(j\omega)| = \frac{\sqrt{j^2 + (\pi)^2}}{\sqrt{j^2 + (\pi)^2}} = 1 \quad (3)$$

$$\angle H(j\omega) = -2 \tan^{-1} \frac{\omega}{\pi} \quad (4)$$

Here , $\omega = \pi$

$$\angle H(j\omega) \Big|_{\omega=\pi} = -90^\circ \quad (5)$$

Using the magnitude and phase information, the input signal $x(t)$ is given by:

$$x(t) = |H(j\omega)| \sin(\omega t + \angle H(j\omega)) \quad (6)$$

Substituting $|H(j\pi)| = 1$ and $\angle H(j\pi) = -\frac{\pi}{2}$:

$$x(t) = 1 \sin\left(\pi t - \frac{\pi}{2}\right) \quad (7)$$

$$\therefore x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \quad (8)$$