12.10.4

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$$

let $u(t)$ be the unit step function.The input $x(t)$ that results in a steady-state output $y(t) = sin(\pi t)$ is __.

Solution:

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0
INPUT PARAMETERS

$$H(s) = \frac{s-\pi}{s+\pi} \tag{1}$$

Converting transfer function to frequency response, we get

$$H(j\omega) = \frac{j\omega-\pi}{j\omega+\pi} \tag{2}$$

Here , $\omega = \pi$

$$H(j\pi) = \frac{j-1}{j+1} = j \tag{3}$$

$$|H(j\pi)| = 1 \tag{4}$$

$$\angle H(j\pi) = 90^\circ \tag{5}$$

$$y(t) = \sin(\pi t) = \cos\left(\pi t + \frac{3\pi}{2}\right) \tag{6}$$

$$|X|\cos(\omega t + \phi) \overset{H(j\omega)}{\rightarrow} |X||H(j\omega)|\cos(\omega t + \phi + \angle H(j\omega)) \tag{7}$$

$$\cos(\pi t + \pi) \overset{H(j\omega)}{\rightarrow} |H(j\omega)|\sin(\pi t + \pi + \angle H(j\omega)) \tag{8}$$

Therefore by (6) and (7) , we get

$$x(t) = cos(\pi t + \pi) \tag{9}$$

$$= \sin\left(\pi t - \frac{\pi}{2}\right) \tag{10}$$