12.10.4

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Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let u(t) be the unit step function. The input x(t) that results in a steady-state output $y(t) = sin(\pi t)$ is ___. **Solution:**

Variable	Description	Value
x(t)	input function	none
y(t)	output function	$\sin(\pi t)$
H(s)	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0: input parameters

$$H(s) = \frac{s - \pi}{s + \pi} \tag{1}$$

Converting transfer function to frequency response, we get

$$H(j\omega) = \frac{j\omega - \pi}{j\omega + \pi} \tag{2}$$

Here, $\omega = \pi$

$$H(j\pi) = \frac{j-1}{j+1} = j$$
 (3)

$$|H(j\pi)| = 1\tag{4}$$

$$\angle H(j\pi) = 90^{\circ} \tag{5}$$

$$y(t) = \sin(\pi t) \tag{6}$$

$$|X|\sin(\omega t + \phi) \stackrel{H(j\omega)}{\to} |X||H(j\omega)|\sin(\omega t + \phi + \angle H(j\omega))$$

-(7)

$$\sin\left(\pi t - \frac{\pi}{2}\right) \stackrel{H(j\omega)}{\to} |H(j\omega)| \sin\left(\pi t - \frac{\pi}{2} + \angle H(j\omega)\right) \tag{8}$$

Therefore by (6) and (7), we get

$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \tag{9}$$

$$\frac{\sin(\pi t - \frac{\pi}{2})}{H(s)} \xrightarrow{\sin(\pi t)}$$

Fig. 0: Block diagram of the System