

# Gate 2023-IN-21

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Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let  $u(t)$  be the unit step function. The input  $x(t)$  that results in a steady-state output  $y(t) = \sin(\pi t)$  is \_\_\_\_.

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**Solution:**

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0: input parameters

$$H(s) = \frac{s - \pi}{s + \pi} \quad (1)$$

let

$$H_i(s) = \frac{s + \pi}{s - \pi} \quad (2)$$

This is the Transfer function of inverse system having  $y(t)$  as input and  $x(t)$  as output.

Converting transfer function to frequency response, we get

$$H_i(j\omega) = \frac{j\omega + \pi}{j\omega - \pi} \quad (3)$$

Here ,  $\omega = \pi$

$$H_i(j\pi) = \frac{j + 1}{j - 1} = -j \quad (4)$$

$$|H(j\pi)| = 1 \quad (5)$$

$$\angle H(j\pi) = -90^\circ \quad (6)$$

$$y(t) = \sin(\pi t) \quad (7)$$

$$|X| \sin(\omega t + \phi) \xrightarrow{H(j\omega)} |X||H(j\omega)| \sin(\omega t + \phi + \angle H(j\omega)) \quad (8)$$

$$\sin(\pi t) \xrightarrow{H(j\omega)} |H(j\omega)| \sin(\pi t + \angle H(j\omega)) \quad (9)$$

Therefore by (7) and (9) , we get

$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \quad (10)$$

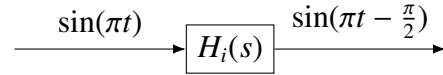


Fig. 0: Block diagram of the System

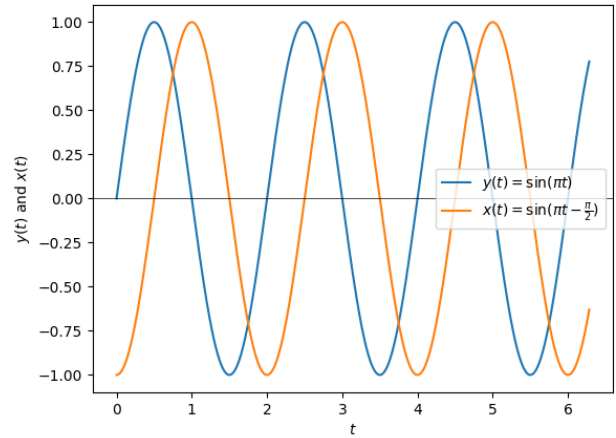


Fig. 0: Plot of  $x(t)$  and  $y(t)$  taken from Python