Gate 2023-IN-21

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q:A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$$

let $u(t)$ be the unit step function.The input $x(t)$ that results in a steady-state output $y(t) = sin(\pi t)$ is __.

Solution:

Variable	**Description**	**Value**
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0: input parameters

$$H(s) = \frac{s-\pi}{s+\pi} \tag{1}$$

let

$$H_i(s) = \frac{s+\pi}{s-\pi} \tag{2}$$

This is the Transfer function of inverse system having $y(t)$ as input and $x(t)$ as output.
Converting transfer function to frequency response, we get

$$H_i(j\omega) = \frac{j\omega+\pi}{j\omega-\pi} \tag{3}$$

Here , $\omega = \pi$

$$H_i(j\pi) = \frac{j+1}{j-1} = -j \tag{4}$$

$$|H(j\pi)| = 1 \tag{5}$$

$$\angle H(j\pi) = -90^\circ \tag{6}$$

$$y(t) = \sin(\pi t) \tag{7}$$

$$|X|\sin(\omega t+\phi) \xrightarrow{H(j\omega)} |X||H(j\omega)|\sin(\omega t+\phi+\angle H(j\omega)) \tag{8}$$

$$\sin(\pi t) \xrightarrow{H(j\omega)} |H(j\omega)|\sin(\pi t+\angle H(j\omega)) \tag{9}$$

Therefore by (7) and (9) , we get

$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \tag{10}$$

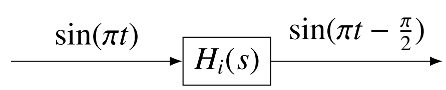

Fig. 0: Block diagram of the System

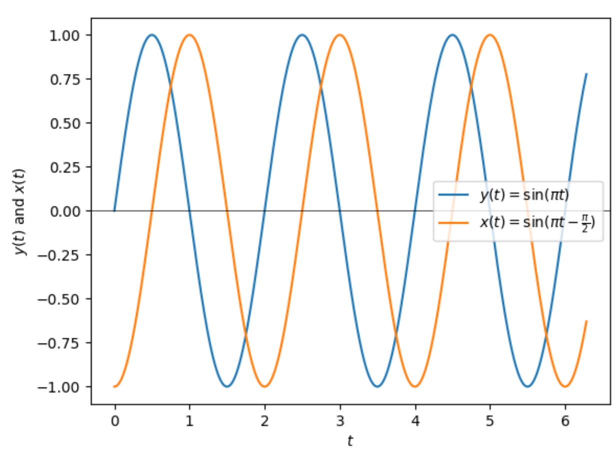

Fig. 0: Plot of $x(t)$ and $y(t)$ taken from Python