Gate.in.21

EE22BTECH11008 - Annapureddy Siva Meenakshi*

Q: A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$$

let $u(t)$ be the unit step function. The input $x(t)$ that results in a steady-state output $y(t) = \sin(\pi t)$ is __.

Solution:

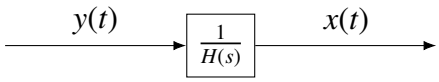


Fig. 0: Block diagram of the inverse system

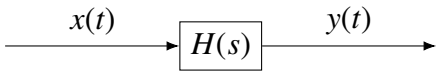


Fig. 0: Block diagram of the system

Variable	Description	Value
$x(t)$	input function	none
$y(t)$	output function	$\sin(\pi t)$
$H(s)$	Transfer-function	$\frac{s-\pi}{s+\pi}$

TABLE 0: Input parameters

$$H(s) = \frac{s-\pi}{s+\pi} \tag{1}$$

from Fig. 0

$$\frac{1}{H(s)} = \frac{s+\pi}{s-\pi} \tag{2}$$

Converting transfer function to frequency response, we get

$$\frac{1}{H(j\omega)} = \frac{j\omega+\pi}{j\omega-\pi} \tag{3}$$

from Table 0 $\omega = \pi$

$$\frac{1}{H(j\pi)} = \frac{j+1}{j-1} = -j = e^{-j\frac{\pi}{2}} \tag{4}$$

from (4)

$$\left|\frac{1}{H(j\pi)}\right| = 1 \quad \text{and} \quad \arg\left(\frac{1}{H(j\pi)}\right) = -90^\circ \tag{5}$$

$$y(t) = \sin(\pi t) \tag{6}$$

$$\sin(\pi t) \overset{\frac{1}{H(j\omega)}}{\rightarrow} \left|\frac{1}{H(j\omega)}\right| \sin\left(\pi t + \arg\left(\frac{1}{H(j\omega)}\right)\right) \tag{7}$$

Therefore, by (6) and (7), we get

$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) \tag{8}$$

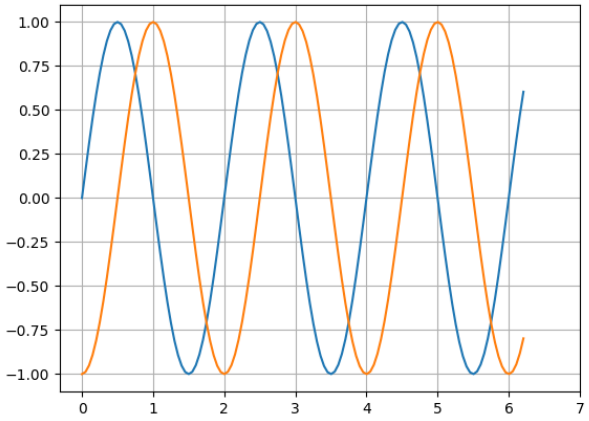
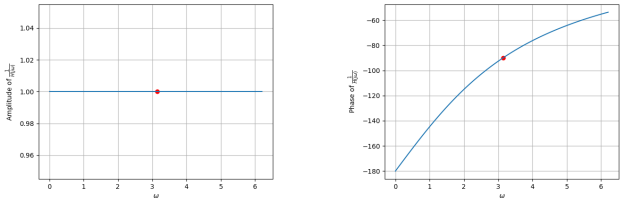


Fig. 0: Plot of $x(t)$ and $y(t)$ taken from Python



(a) Amplitude of $\frac{1}{H(j\omega)}$ (b) Phase of $\frac{1}{H(j\omega)}$

Fig. 0: Amplitude and Phase of $\frac{1}{H(j\omega)}$