

Q:The difference between any two cosecutive interior angles of a polygon is 5° .If the smallest angle is 120° ,find the number of sides of polygon.

Solution: The interior angles of a polygon are in AP with

$$a = 120$$

$$d = 5$$

The sum of n terms of an AP is given by

$$S = \frac{n}{2}(2a + (n-1)d) \quad (1)$$

Sum of interior angles of AP is given by

$$S = (n-2)180 \quad (2)$$

$$\frac{n}{2}(2a + (n-1)d) = (n-2)180$$

$$\frac{n}{2}(240 + (n-1)5) = (n-2)180$$

$$n(235 + 5n) = 360n - 720$$

$$5n^2 + 235n = 360n - 720$$

$$5n^2 - 125 + 720 = 0$$

$$n^2 - 25n + 144 = 0$$

solving equation we get

$$n = 16, 9$$

$$x(n) = \begin{cases} 115 + 5n & \text{if } 1 \leq n \leq 16 \\ 0 & \text{if } n > 16 \end{cases}$$

$$X(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$X(n) = \sum_{n=1}^{16} (115 + 5n) \cdot z^{-n} \quad X(n) = 115 \sum_{n=1}^{16} z^{-n} + 5 \sum_{n=1}^{16} n \cdot z^{-n}$$

Let $p = 115 \cdot \sum_{n=1}^{16} z^{-n}$, $q = 5 \sum_{n=1}^{16} n \cdot z^{-n}$

sum of n terms of G.P with a= first term and r= common ratio is given by

$$S_n = a \left(\frac{r^n - 1}{r - 1} \right)$$

$$\therefore p = 115 \times \frac{z^{-1} \cdot (z^{-16} - 1)}{(z^{-1} - 1)} = 115 \times \frac{z^{-1} \cdot (1 - z^{-16})}{(1 - z^{-1})}$$

$$p = \frac{115 \cdot z^{-1} - 115 \cdot z^{-17}}{1 - z^{-1}}$$

$$q = 5 \sum_{n=1}^{16} n \cdot z^{-n} = 5 \times z^{-1} \sum_{n=1}^{16} n \cdot z^{-(n-1)}$$

The terms of q are in AGP with

$$a = 1$$

$$d = 1$$

$$r = z^{-1}$$

$$n = 16$$

$$S = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{(a+(n-1)d)r^n}{1-r}$$

$$\sum_{n=1}^{16} n \cdot z^{-(n-1)} = \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{(1+(16-1)1) \times z^{-16}}{1-z^{-1}}$$

$$= \frac{1}{1-z^{-1}} + \frac{z^{-1}(1-z^{-15})}{(1-z^{-1})^2} - \frac{16 \times z^{-16}}{1-z^{-1}}$$

$$= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16 \cdot z^{-16}(1-z^{-1})}{(1-z^{-1})^2}$$

$$= \frac{(1-z^{-1}) + (z^{-1}-z^{-16}) - 16z^{-16} - z^{-17}}{(1-z^{-1})^2}$$

$$= \frac{1-z^{-1} + z^{-1} - z^{-16} - 16 \cdot z^{-16} + 16 \cdot z^{-17}}{(1-z^{-1})^2}$$

$$= \frac{1 - 17 \cdot z^{-16} + 16 \cdot z^{-17}}{(1-z^{-1})^2}$$

$$q = 5 \times z^{-1} \sum_{n=1}^{16} n \cdot z^{-(n-1)}$$

$$= 5 \times z^{-1} \frac{1 - 17 \cdot z^{-16} + 16 \cdot z^{-17}}{(1-z^{-1})^2}$$

$$\therefore q = \frac{5 \cdot z^{-1} - 85 \cdot z^{-17} + 80 \cdot z^{-18}}{(1-z^{-1})^2}$$

$$X(n) = p + q$$

$$= \frac{115 \cdot z^{-1} - 115 \cdot z^{-17}}{1-z^{-1}} + \frac{5 \cdot z^{-1} - 85 \cdot z^{-17} + 80 \cdot z^{-18}}{(1-z^{-1})^2}$$

$$= \frac{(115 \cdot z^{-1} - 115 \cdot z^{-17})(1-z^{-1}) + (5 \cdot z^{-1} - 85 \cdot z^{-17} + 80 \cdot z^{-18})}{(1-z^{-1})^2}$$

$$= \frac{(115 \cdot z^{-1} - 115 \cdot z^{-17} - 115 \cdot z^{-2} + 115 \cdot z^{-18}) + (5 \cdot z^{-1} - 85 \cdot z^{-17} + 80 \cdot z^{-18})}{(1-z^{-1})^2}$$

$$= \frac{120z^{-1} - 115z^{-2} - 200z^{-17} + 195z^{-18}}{(1-z^{-1})^2}$$

$$\therefore X(n) = \frac{120z^{-1} - 115z^{-2} - 200z^{-17} + 195z^{-18}}{(1-z^{-1})^2}$$