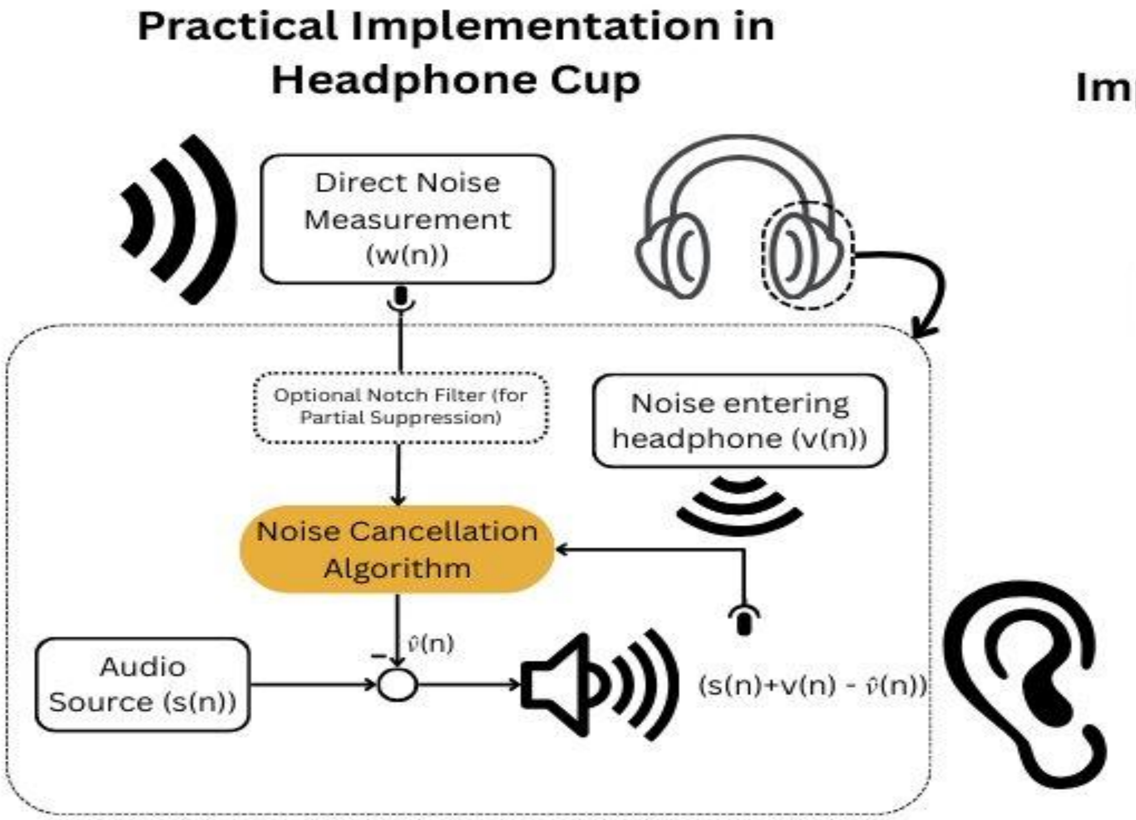
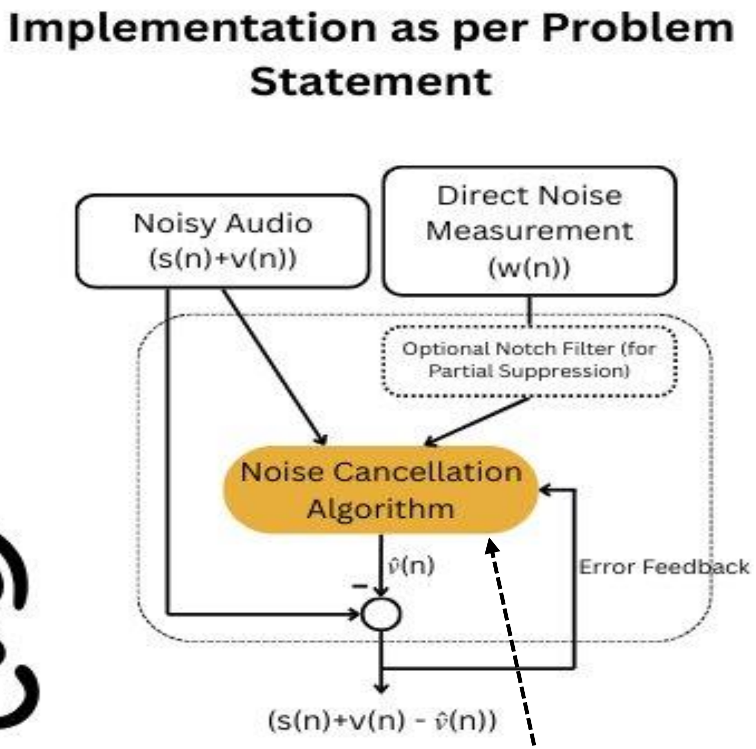
Design Overview





s(n) = Clean Signal (to be estimated)
w(n) = External Noise (outside headphones)
v(n) = Noise entering headphone cup
$\hat{v}$(n) = Estimate of v(n) using w(n)
s(n) + v(n) - $\hat{v}$(n) = Estimate of clean signal (required output)

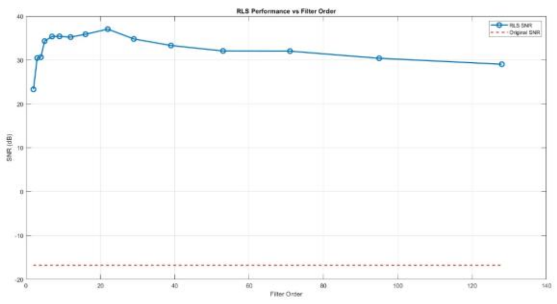
This block is to be designed. We use the Recursive Least Squares (RLS) to estimate the clean signal.

We do not have access to s(n), only s(n) + v(n), hence pre-subtraction is not possible.

Design Choices

Plots:

1. SNR vs. Filter Order
2. Computation Time vs. Filter Order
3. SNR vs. Forgetting Factor
4. Partial Performance Metrics vs. Notch Pole Radius

1


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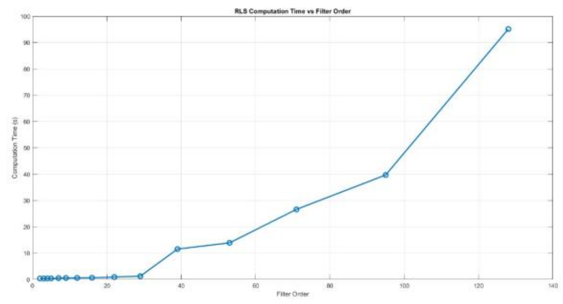

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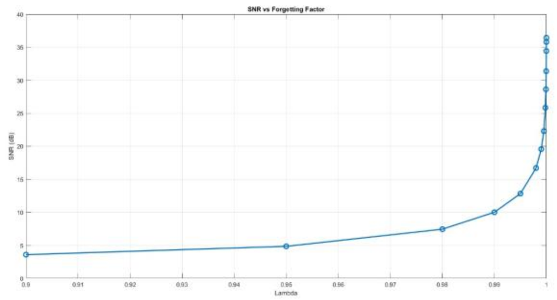

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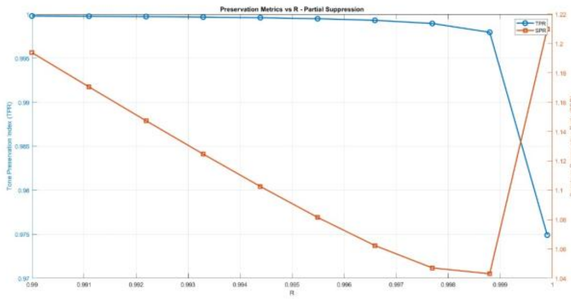


RLS Parameters Chosen:

1. Filter Order = 8 (Tradeoff SNR vs. Complexity)
2. Forgetting Factor (λ) = 0.99999 (for provided audio)
3. Notch Filter Pole Radius (R) = 0.999

Results and Comparison with LMS/NLMS [2]:

1. LMS SNR = 8-9 dB (gain 23-25 dB)
2. NLMS SNR = 10-11 dB (gain 25-27 dB)
3. RLS SNR = 35.80 dB (gain 52.57 dB)

Partial Suppression Results:

1. TPR = 0.997522
2. SPR = 1.045753

Partial Suppression Measurement Metrics:

1. Tonal Preservation Ratio = $\frac{|FFT_{filtered}(f_0)|}{|FFT_{noisy}(f_0)|}$
2. Spectral Preservation Ratio = $\frac{\sum_{f \notin B} |FFT_{filtered}(f)|^2}{\sum_{f \notin B} |FFT_{clean}(f)|^2}$
(in regions excluding tonal freq.)

$(B = (f_0 - 20, f_0 + 20)$ band)

Ideal values of both are 1.

Algorithm Details

Adaptive Filter Used: Recursive Least Squares (RLS)

- Minimize Cost Function: $\boldsymbol{J(n) = \sum_{k=0}^{n} \lambda^{n-k} e^2(k)}$, where e(k) = noisy signal – estimated noise. Its minimum value would be the clean speech. λ < 1 is the forgetting factor, that weights the error according to recentness. [1]
- Minimizing J(n) leads to the following equations (x(n) = external noise):

i. Gain vector: $g(n) = \frac{P(n-1)x(n)}{\lambda + x^T(n)P(n-1)x(n)}$ (how aggressively to update weights)

ii. Noise Estimate: $\hat{v}(n) = w^T(n-1)x(n)$

iii. True Signal Estimate: $e(n) = d(n) - \hat{v}(n)$

iv. Weight Update: $w(n) = w(n-1) + g(n)e(n)$

v. Inverse-Correlation Matrix Update: P(n) = $\frac{1}{\lambda}(P(n-1) - g(n)x^T(n)P(n-1))$

where P is the estimate of $\left(\sum_{i=0}^{n} \lambda^{n-i} x(i)x^T(i)\right)^{-1}$, i.e. the inverse of auto correlation matrix of reference noise, weighted in increasing order of recentness.

Partial Suppression Approach

- We require output as clean signal + tonal. Therefore, give reference noise as the non-tonal component.
- As tone frequency is known, a simple approach is applying a notch at that frequency to remove it from the reference. This way, only non-tonal component will be estimated.
- Construction: Create 2nd Order Notch Filter Coefficients using frequency and pole location. For every index n, apply the recursive IIR equation on the external noise for all tones successively.

Pros	Cons
Modifiable to filter stationary/non-stationary noise.	High Computational Complexity + Memory Requirements [2]
Adjustable Gain Vector according to noise characteristics.	No dynamic adjustment of forgetting factor and filter order.
Partial and Full Suppression handled efficiently.	Partial Suppression: Notch Filter would affect nearby frequencies and clean signal content.

References

[1] Paulo S. R. Diniz, ”Adaptive Filtering: Algorithms and Practical Implementation” (Chapter 5)

[2] Ali, F., Rawat, P., & Malvia, S. (2017). Comparative analysis and survey of LMS and RLS adaptive algorithms. International Journal of Computer Applications, 161(3), 26–29. https://doi.org/10.5120/ijca2017913136

Slides following this are extra, not to be considered.

Notch Filter Design

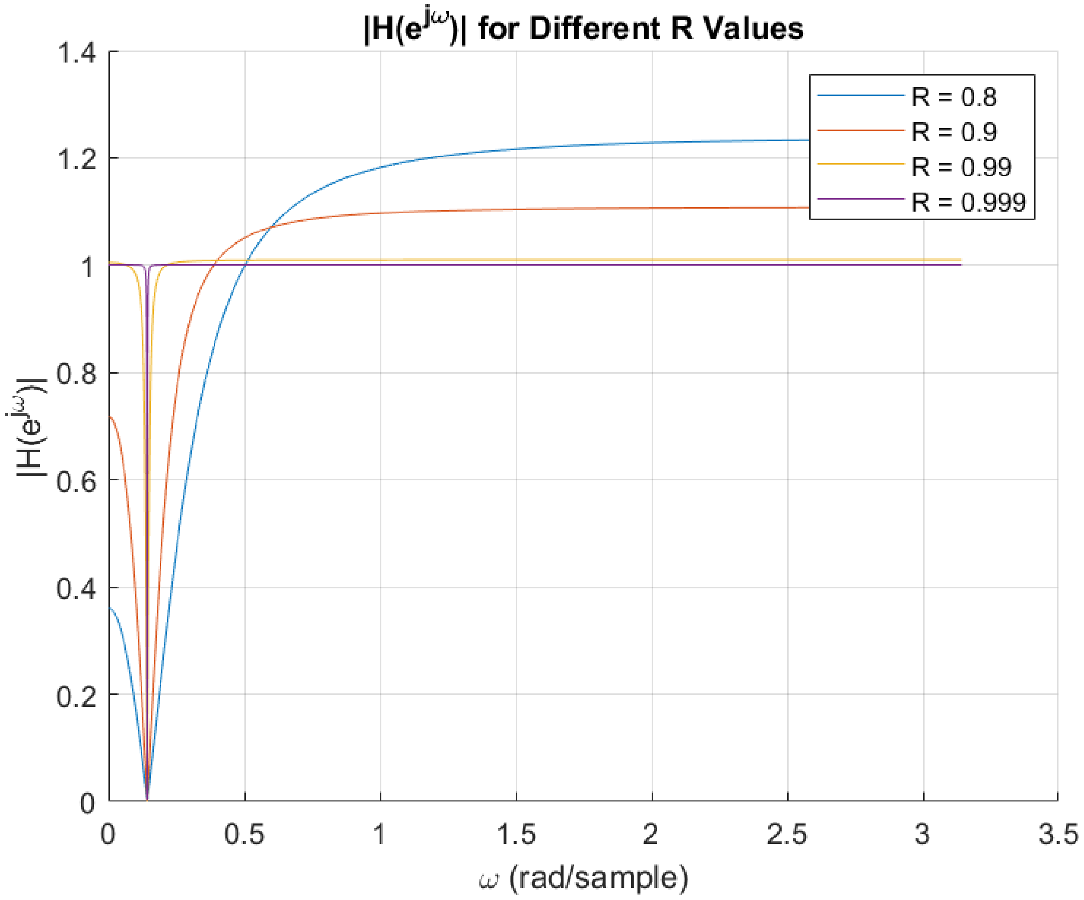
The following is the transfer function of the designed Notch Filter:

$$H(z) = \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2R\cos(\omega_0)z^{-1} + R^2 z^{-2}}$$

where R is very close to 1.

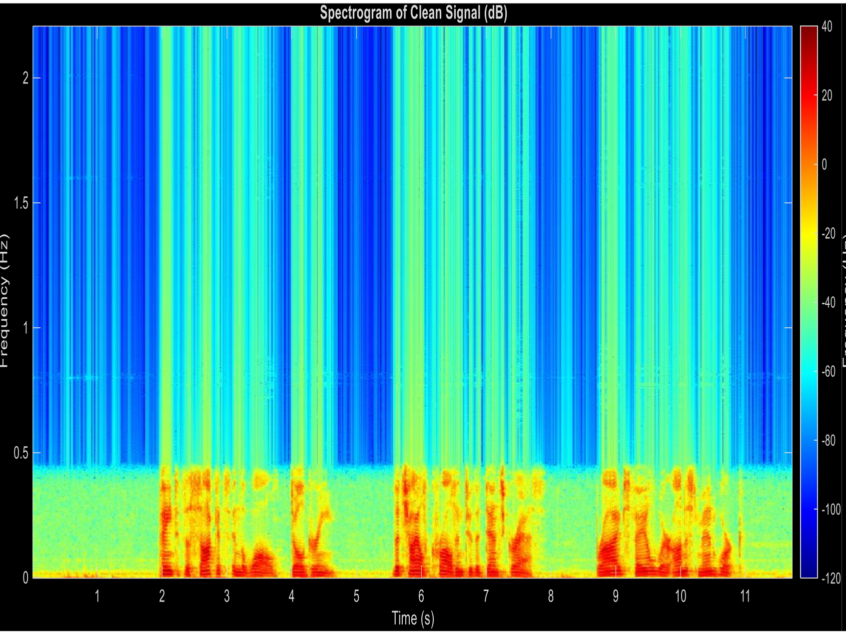
The principle is that we place a zero at $\pm\omega_0$, the frequency to be notched out, so H(z) is 0. But to ensure that the magnitude of H(z) quickly returns to 1, we also place two poles close to (inside unit circle) the zeros at the same frequency.

As ω moves away from ω_0, the numerator and denominator magnitudes become close (due to closeness of pole-zero placement) and overall magnitude of H(z) tends to 1.

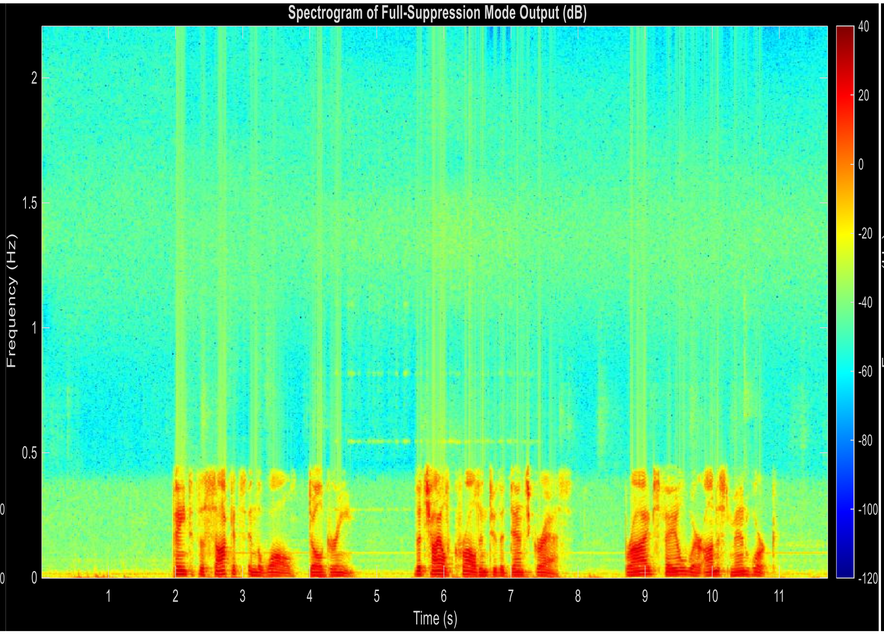


Output Spectrograms

Clean Signal Spectrogram



Full Suppression Spectrogram



Partial Suppr. Spectrogram