Problem 1

(1) $\{x = 2k + 1 \wedge x = x + 1 \wedge y = 10\} \Rightarrow \{x = 18 \wedge y = 20\} \Rightarrow \{x \text{ is divisible by } 6 \wedge y = x + 2\} \Rightarrow \{x\%2 = 0 \wedge \text{y is even}\}$

$\{x = 2k + 1 \wedge x = x + 1 \wedge y = 10\} \iff \{\text{ false }\}$ because $x = x + 1$ can't be satisfied by any value ($x = x \neq x + 1$). So, this must be the strongest condition since anything implying false is false.

$\{x = 18 \wedge y = 20\} \Rightarrow \{x \text{ is divisible by } 6 \wedge y = x+2\}$ is true because 18 is divisible by 6 and $20 = 18+2$. $\{x \text{ is divisible by } 6 \wedge y = x + 2\} \Rightarrow \{x = 18 \wedge y = 20\}$ is false because if $x = 12$, then x is divisible by 6 and $y = 14$, but $x = 18 \wedge y = 20$ isn't satisfied.

$\{x \text{ is divisible by } 6 \wedge y = x + 2\} \Rightarrow \{x\%2 = 0 \wedge \text{y is even}\}$ is true because if x is divisible by 6, then $x = 6k$ ($k \in \mathbb{Z}$) and $6k\%2 = 0$, so $x\%2 = 0$. Since x is divisible by 6 implies that x is even and $y = x+2$, that implies that y is even since adding 2 to an even number gets you an even number.
$\{x\%2 = 0 \wedge \text{y is even}\} \Rightarrow \{x \text{ is divisible by } 6 \wedge y = x + 2\}$ is false because if $x = 2$ and $y = 2$, then x isn't divisible by 6 and $y \neq x + 2$.

(2) $\{10 \leq k \leq -10\} \Rightarrow \{-10 < k \leq 1\} \Rightarrow \{k \geq -10\}$

$\{10 \leq k \leq -10\} \iff \{\text{ false }\}$ because $k \geq 10$ and $k \leq -10$ are two ranges that never overlap and so it's impossible to ever satisfy this condition. So, this must be the strongest condition since anything implying false is false.
$\{5 \leq k < 5\} \iff \{\text{ false }\}$ because $k \geq 5$ and $k < 5$ are two ranges with no overlap, and so there are no values to satisfy this condition.

$\{-10 < k \leq 1\} \Rightarrow \{k \geq -10\}$ is true because all the values that satisfy $k > -10$ also satisfy $k \geq -10$.
$\{k \geq -10\} \Rightarrow \{-10 < k \leq 1\}$ is false because if $k = 2$, then that satisfies $k \geq -10$ but not $k \leq 1$.

(3) $\{x = 3 \wedge y > 10\} \Rightarrow \{x \geq 0 \wedge y > x\}$

$\{x = 3 \wedge y > 10\} \Rightarrow \{x \geq 0 \wedge y > x\}$ is true because $x = 3 \geq 0$ and since $y > 10$, $y > 3 = x$.
$\{x \geq 0 \wedge y > x\} \Rightarrow \{x = 3 \wedge y > 10\}$ is false because $x = 1$ and $y = 2$ satisfies $x \geq 0 \wedge y > x$ but not $x = 3 \wedge y > 10$.

(4) $\{z \in \mathbb{N}\} \Rightarrow \{z \in \mathbb{Z}\} \Rightarrow \{z \in \mathbb{Q}\} \Rightarrow \{z \in \mathbb{R}\}$

$\{z \in \mathbb{N}\} \Rightarrow \{z \in \mathbb{Z}\}$ is true because $\mathbb{N} \subseteq \mathbb{Z}$.
$\{z \in \mathbb{Z}\} \Rightarrow \{z \in \mathbb{N}\}$ is false because if $z = 0$, then $z \in \mathbb{Z}$ but not $\mathbb{N}$.

$\{z \in \mathbb{Z}\} \Rightarrow \{z \in \mathbb{Q}\}$ is true because $\mathbb{Z} \subseteq \mathbb{Q}$.
$\{z \in \mathbb{Q}\} \Rightarrow \{z \in \mathbb{Z}\}$ is false because if $z = \frac{2}{3}$, then $z \in \mathbb{Q}$ but not $\mathbb{Z}$.

$\{z \in \mathbb{Q}\} \Rightarrow \{z \in \mathbb{R}\}$ is true because $\mathbb{Q} \subseteq \mathbb{R}$.
$\{z \in \mathbb{R}\} \Rightarrow \{z \in \mathbb{Q}\}$ is false because if $z = \pi$, then $z \in \mathbb{R}$ but not $\mathbb{Q}$.

(5) $\{x = 1 \wedge y \geq 1\} \Rightarrow \{-1 \leq x \leq 1\} \Rightarrow \{-5 < x \leq 10\}$

$\{x = 1 \wedge y \geq 1\} \Rightarrow \{-1 \leq x \leq 1\}$ is true because $x = 1$ satisfies $-1 \leq x \leq 1$.
$\{-1 \leq x \leq 1\} \Rightarrow \{x = 1 \wedge y \geq 1\}$ is false because $x = 0$ satisfies $-1 \leq x \leq 1$ but not $x = 1$.

$\{-1 \leq x \leq 1\} \Rightarrow \{-5 < x \leq 10\}$ is true because $x \geq 1 \Rightarrow x > -5$ and $x \leq 1 \Rightarrow x \leq 10$.
$\{-5 < x \leq 10\} \Rightarrow \{-1 \leq x \leq 1\}$ is false because $x = -2$ satisfies $x > -5$ but not $-1 \leq x$.

(6) $\{|\text{result} - \sin(x)| \leq -0.01\} \Rightarrow \{|\text{result} - \sin(x)| \leq 10^{-10}\} \Rightarrow \{|\text{result} - \sin(x)| \leq 0.01\} \Rightarrow \{|\text{result} - \sin(x)| \leq 1\}$

$\{|\text{result} - \sin(x)| \leq -0.01\} \Rightarrow \{|\text{result} - \sin(x)| \leq 10^{-10}\} \Rightarrow \{|\text{result} - \sin(x)| \leq 0.01\} \Rightarrow \{|\text{result} - \sin(x)| \leq 1\}$ is true because if $|\text{result} - \sin(x)| \leq -0.01$, then $|\text{result} - \sin(x)| \leq 10^{-10} \leq 0.01 \leq 1$.

$\{|\text{result} - \sin(x)| \leq 10^{-10}\} \Rightarrow \{|\text{result} - \sin(x)| \leq -0.01\}$ is false because if $|\text{result} - sin(x)| = 10^{-10}$, then $|\text{result} - sin(x)| \leq -0.01$ isn't satisfied. The same goes for $\{|\text{result} - \sin(x)| \leq 0.01\} \Rightarrow \{|\text{result} - \sin(x)| \leq 10^{-10}\}$ and $\{|\text{result} - \sin(x)| \leq 1\} \Rightarrow \{|\text{result} - \sin(x)| \leq 0.01\}$.

Problem 2

(1) Valid

$\{x = 5\}$
$x = x * 2; \Rightarrow x = 5 * 2 = 10$
$\{x = 10 \vee x \neq 0\} \Rightarrow$ both conditions are satisfied, making the postcondition true

(2) Invalid

$\{\sqrt{x-1} > k\}$
$x = x + 1;$
$\{k \geq 0\} \Rightarrow k$ never changes in the code, so it's possible for k to originally be a negative number. That would satisfy the precondition because $\sqrt{\cdot} \geq 0$. This doesn't satisfy the postcondition though.

A postcondition that would be valid is $\{x \geq 2\}$ because $\sqrt{n}$'s domain is $n \geq 0$. Then if you replace n with $x - 1$, you would get $x \geq 1$. Then in the code, you add 1 to x, so after execution, $x \geq 2$.

(3) Valid

$\{i + j \neq 0 \wedge i \cdot j = 0\} \Rightarrow (i = 0 \wedge j \neq 0) \vee (i \neq 0 \wedge j = 0)$
$i = j - 1; \Rightarrow (i = j - 1 \wedge j \neq 0) \vee (i = -1 \wedge j = 0) \Rightarrow i = j - 1$
$j = i + 1; \Rightarrow i = j - 1 \wedge j = j - 1 + 1 \Rightarrow i = j - 1$
$\{(i = 0 \vee i \neq -j) \wedge k \in \mathbb{Q}\} \Rightarrow i = j - 1 \wedge k \in \mathbb{Z} \subseteq \mathbb{Q}$ satisfies the postcondition ($i, j \in \mathbb{Z}$ and the only way to make $i \neq j$ false is for them to be fractions)

(4) Invalid
In both branches, one of x and y are undefined, so it's impossible to determine whether or not the postcondition holds.

$\{n < 0 \wedge n = \sqrt{m}\} \Rightarrow m > 0$ because if $m = 0$, $n < 0$ is false and if $m < 0$, $\sqrt{m}$ can't be an integer.
if $(n > m) \Rightarrow$ guaranteed to be true since $n < 0 \wedge m > 0$
  $x = n;$
else $\Rightarrow$ this part of the code is never reached
  $y = m;$
$\{x \neq y\}$

Since $n > m$ is always true, a valid postcondition would be $\{x = n\}$.

Problem 3

(1) Invalid

Let B = $x \geq 0$, code = $x = x + 1$, and E = $x > 0$.
Let C = $(x = 0) \vee (x > 0)$ and D = $x > 10$.

B $\Leftrightarrow$ C because if $x \geq 0$, then $x = 0$ or $x > 0$, which is what C is.
D $\Rightarrow$ E because $x > 10 > 0$.

{B} code {E} is true since after execution, $x \geq 1 > 0$. But, {C} code {D} is false because if $x = 0$, then $x = 1$ after execution, violating D.

(2) Invalid

Let B = $x \geq 0$, code = $x = x - 1$, and E = $x \geq -1$.
Let C = $(x = 0) \vee (x > 0)$.

B $\Leftrightarrow$ C because if $x \geq 0$, then $x = 0$ or $x > 0$, which is what C is.

{B} code {E} is true since $x \geq 0$, so after execution, $x \geq -1$. But, {B} code {C} is false because if $x = 0$, then $x = -1$ after execution, violating C.

(3) Invalid

Let B = $x \geq 0$, code = $x = x + 1$, and E = $x > 0$.
Let A = $x > 0$ and D = $x > 10$.

A $\Rightarrow$ B because if $x > 0$, then $x \geq 0$ must be true.
D $\Rightarrow$ E because $x > 10 > 0$.

{B} code {E} is true because after execution, $x \geq 1 > 0$. But, {A} code {D} is false because if $x = 1$, then $x = 2$ after execution, violating D.

(4) Valid

Since A $\Rightarrow$ B, any values satisfying A will also satisfy B. Since E $\Rightarrow$ F, any values that satisfy E must satisfy F. So, {A} code {F} must be true.

Problem 4

(1) $\{z \neq 0\}$
$y = 0;$
$\{y = 0 \wedge z \neq 0\}$
$x = y + 2;$
$\{x = 2 \wedge y = 0 \wedge z \neq 0\}$
$z = x + y;$
$\{z = 2 \wedge x = 2 \wedge y = 0\}$

(2) $\{|x| > 5\}$
$x = x\%10;$
$\{0 \leq |x| \leq 9\}$
$x = x * x;$
$\{0 \leq x \leq 81\}$
$x = -x;$
$\{0 \geq x \geq -81\}$

(3) $\{z < 5\}$
if $(z > 0)$ {
$\{0 < z < 5\}$
$z = -z;$
$\{0 > z > -5\}$
}
$\{(z \leq 0) \vee (0 > z > -5)\}$

Problem 5

(1) $\{\text{wp}(\text{"}x = -1;\text{"},\ y > -3x \wedge y < 10 - 3x)\} = \{y > 3 \wedge y < 13\} = \{3 < y < 13\}$

$x = -1;$

$\{\text{wp}(\text{"}z = 3 * x + y;\text{"},\ 0 < z < 10)\} = \{0 < 3x + y < 10\} = \{y > -3x \wedge y < 10 - 3x\}$

$z = 3 * x + y;$

$\{0 < z < 10\}$

(2) $\{(x > y > 0) \vee \text{false}\} = \{x > y > 0\}$

if $y > 0$ {

$\{\text{wp}(\text{"}x = x/y;\text{"},\ x > 1)\} = \{x/y > 1\} = \{x > y\}$

$x = x/y;$

$\{\text{wp}(\text{"}y = 0;\text{"},\ x > 1 \wedge y = 0)\} = \{x > 1 \wedge 0 = 0\} = \{x > 1\}$

$y = 0;$

$\{x > 1 \wedge y = 0\}$

} else {

$\{\text{wp}(\text{"}y = 4 * x;\text{"},\ x > 1 \wedge y = 0)\} = \{x > 1 \wedge 4 * x = 0\} = \{\text{ false }\}$

$y = 4 * x;$

$\{x > 1 \wedge y = 0\}$

}

$\{x > 1 \wedge y = 0\}$

(3) $\{(x < 0 \wedge (z \neq 0 \wedge y \geq 0 \vee x \geq 0)) \vee (x \geq 0 \wedge (((z > x \wedge z \neq 0 \vee y \geq -z) \vee (x \geq z \wedge x \neq 0 \vee y \geq -x)) \wedge y \geq 0))\} =$
$\{((z > x \wedge z \neq 0 \vee y \geq -z) \vee (x \geq z \wedge x \neq 0 \vee y \geq -x)) \wedge y \geq 0 \wedge x \geq 0)\} =$
$\{(z > x \wedge x \geq 0) \vee (x \geq z \wedge x > 0) \wedge y \geq 0\}$

if $(x \geq 0)$ {

$\{\text{wp}(\text{"}z = \text{Math.min}(z, x);\text{"},\ z \neq 0 \wedge y \geq 0 \wedge y \geq -z)\} = \{\text{Math.min}(z, x) \neq 0 \wedge y \geq 0 \wedge y \geq \text{Math.min}(z, x)\} =$
$\{((z > x \wedge z \neq 0 \vee y \geq -z) \vee (x \geq z \wedge x \neq 0 \vee y \geq -x)) \wedge y \geq 0\}$

$z = \text{Math.min}(z, x);$

$\{\text{wp}(\text{"}x = z + y;\text{"},\ z \neq 0 \wedge y \geq 0 \vee x \geq 0)\} = \{z \neq 0 \wedge y \geq 0 \vee z + y \geq 0\} = \{z \neq 0 \wedge y \geq 0 \wedge y \geq -z\}$

$x = z + y;$

$\{z \neq 0 \wedge y \geq 0 \vee x \geq 0\}$

}

$\{z \neq 0 \wedge y \geq 0 \vee x \geq 0\}$

(4) $\{(|x| \leq 5 \wedge -1 \leq x \leq 5) \vee (|x| > 5 \wedge (-9 \leq x \leq -5) \vee (-1 \leq x \leq 1))\} = \{(-1 \leq x \leq 5) \vee (-9 \leq x < -5)\}$

if (Math.abs(x) ≤ 5) {
$\{\text{wp}(\text{"}z = x - 2;\text{"}, -3 \leq z \leq 3)\} = \{-3 \leq x - 2 \leq 3\} = \{-1 \leq x \leq 5\}$
$z = x - 2;$
$\{-3 \leq z \leq 3\}$
} else {
$\{(x \leq -5 \wedge -9 \leq x \leq -3) \vee (x > -5 \wedge -1 \leq x \leq 1)\} = \{(-9 \leq x \leq -5) \vee (-1 \leq x \leq 1)\}$
if ($x \leq -5$) {
$\{\text{wp}(\text{"}z = x + 6;\text{"}, -3 \leq z \leq 3)\} = \{-3 \leq x + 6 \leq 3\} = \{-9 \leq x \leq -3\}$
$z = x + 6;$
$\{-3 \leq z \leq 3\}$
} else {
$\{\text{wp}(\text{"}z = 3 * x;\text{"}, -3 \leq z \leq 3)\} = \{-3 \leq 3 * x \leq 3\} = \{-1 \leq x \leq 1\}$
$z = 3 * x;$
$\{-3 \leq z \leq 3\}$
}
$\{-3 \leq z \leq 3\}$
}

$\{-3 \leq z \leq 3\}$

(5) $\{\text{wp}(\text{"}x = y/2;\text{"}, x > -1)\} = \{y/2 > -1\} = \{y > -2\}$

$x = y/2;$
$\{\text{wp}(\text{"}z = x + 1;\text{"}, x \neq 0.5 \wedge z > 0)\} = \{x \neq 0.5 \wedge x + 1 > 0\} = \{x > -1\}$
$z = x + 1;$

$\{x \neq 0.5 \wedge z > 0\}$

Problem 6

(1) $\{x < 2\}$

$\{\text{wp}(\text{"}z = x - z;\text{"}, x > 3)\} = \{x > 3\}$
$z = x - z;$
$\{\text{wp}(\text{"}w = x - 1;\text{"}, w > 2)\} = \{x - 1 > 2\} = \{x > 3\}$
$w = x - 1;$
$\{\text{wp}(\text{"}z = w - 1;\text{"}, z > 1)\} = \{w - 1 > 1\} = \{w > 2\}$
$z = w - 1;$

$\{z > 1\}$

The precondition is insufficient because as seen above, x must be > 3 for the postcondition to hold, but $x < 2$ violates that condition.

(2) $\{x = y \wedge y > 0 \vee y \neq x \wedge x \leq 0\}$

$\{(x > y \wedge 0 \leq x < y + 1) \vee (x \leq y \wedge y \geq 0)\} = \{y \geq x \wedge y \geq 0\}$
if $(x > y)$ {
$\{\text{wp}(\text{"}x - -;\text{"}, x < y \wedge x \geq 0)\} = \{x - 1 < y \wedge x - 1 \geq 0\} = \{1 \leq x < y + 1\}$
$x - -;$
$\{x < y \wedge x \geq 0\}$
} else {
$\{\text{wp}(\text{"}x = y/2;\text{"}, x < y \wedge x \geq 0)\} = \{y/2 < y \wedge y/2 \geq 0\} = \{y \geq 0\}$
$x = y/2;$
$\{x < y \wedge x \geq 0\}$
}

$\{x < y \wedge x \geq 0\}$

The precondition is insufficient because while $x = y \wedge y > 0$ implies $y \geq x \wedge y \geq 0$, $y \neq x \wedge x \leq 0$ doesn't imply $y \geq x \wedge y \geq 0$. This means that the precondition is either weaker or unrelated, but it's not possible to be weaker since $y \geq x \wedge y \geq 0$ is the weakest precondition. So, the precondition is insufficient.