

\* Homework 2 Meer Modi: 01/29/2024

$$A = \{1, 2, 3\} \quad B = \{2, 3\} \quad C = \{A, B\} \quad D = \{a'\} \quad E = \{0, 4\}$$

(Q1)

(a)  $(A, B) \in C \rightarrow \text{False}$

(f)  $a \in P(B) \rightarrow \text{False}$

(b)  $(1, 3) \in A \times B \rightarrow \text{true}$

Power of a set returns a set with sets inside of it

$$\{1, 2, 3\} \times \{2, 3\}$$

Hence  $a$  has no chances of being in  $P(B)$

$$= \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

(c)  $(1, 3) \in \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} \rightarrow \text{True}$

(g)  $\{2\} \subseteq P(B) \rightarrow \text{False}$

$$\{2\} \subseteq \{\{2, 3\}, \{2\}, \{3\}, \emptyset\}$$

True

(c)  $(1, 3) \subseteq A \times B \rightarrow \text{syntax error}$

(h)  $a \subseteq P(B) \rightarrow \text{syntax error}$

$(1, 3) \rightarrow \text{tuple}$

(d)  $(1, 3) \in C \rightarrow \text{False}$

$a$  is not a set

(e)  $\{2\} \in P(B) \rightarrow \text{True}$

$$\{2\} \in \{\{2, 3\}, \{2\}, \{3\}, \emptyset\}$$

(i)  $\{2, 3\} \subseteq P(A) \rightarrow \text{true}$

according to them

$$\{2, 3\} \subseteq \{\{1, 2, 3\}, \{1, 3\}, \{2, 3\}, \{3\}, \emptyset, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$$

$$|P(B)| = 2^{|B|}$$

$$|P(B)| = 2^2 = 4$$

the order within a

set does not matter

$$\{2\} \in \{\{2, 3\}, \{2\}, \{3\}, \emptyset\}$$

$$\{3, 2\} = \{2, 3\}$$

hence true

$$(j) \{ \{3, 2\} \} \subseteq \mathcal{P}(C) \rightarrow \text{False}$$

$$\mathcal{P}(C) = \{ \{A\}, \{B\}, \{A, B\}, \emptyset \}$$

$$\{ \{3, 2\} \} \notin \{ \{A\}, \{B\}, \{A, B\}, \emptyset \}$$

$$(k) \{ \{2, 3\} \} \in \mathcal{P}(C) \rightarrow \text{False}$$

$$\{ \{2, 3\} \} \notin \{ \{A\}, \{B\}, \{A, B\}, \emptyset \}$$

(Q2)

$$(a) B \times A$$

$$\{2, 3\} \times \{1, 2, 3\}$$

$$= \{ (2, 1), (2, 2), (2, 3), \\ (3, 1), (3, 2), (3, 3) \}$$

$$(e) E \times B \times D$$

$$\{ \{0, 4\} \} \times \{2, 3\} \times \{ 'a' \}$$

$$\{ (\{0, 4\}, 2), (\{0, 4\}, 3) \} \times \{ 'a' \}$$

$$\{ ((\{0, 4\}, 2), 'a'), ((\{0, 4\}, 3), 'a') \}$$

$$(b) A \times D$$

$$\{1, 2, 3\} \times \{ 'a' \}$$

$$= \{ (1, 'a'), (2, 'a'), \\ (3, 'a') \}$$

$$(f) \mathcal{P}(D) \text{ Power set of } D$$

$$D = \{ 'a' \}$$

$$\mathcal{P}(D) = \{ \emptyset, \{ 'a' \} \}$$

$$(g) \mathcal{P}(E) \sim E = \{ \{0, 4\} \}$$

$$\mathcal{P}(E) = \{ \emptyset, \{ \{0, 4\} \} \}$$

$$(c) \emptyset \times B$$

$$\emptyset \times \{2, 3\} = \emptyset$$

$$(h) \mathcal{P}(B) \quad B = \{2, 3\}$$

$$\mathcal{P}(B) = \{ \{2\}, \{3\}, \emptyset, \{2, 3\} \}$$

$$(d) C \times C$$

$$\{A, B\} \times \{A, B\}$$

$$= \{ (A, A), (A, B), (B, A), \\ (B, B) \}$$

(Q3)

$$(a) \{ (x, y) \mid x \in \mathbb{N} \text{ \& } x < 5 \text{ \& } y = 0 \}$$

$$= \{ (1, 0), (2, 0), (3, 0), (4, 0) \}$$

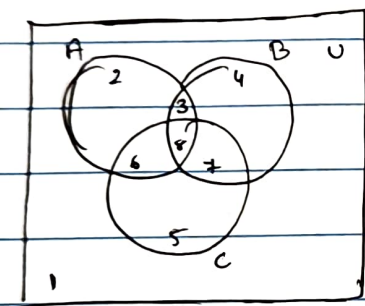
$$(b) \{ 4x \mid x \in \mathbb{Z} \text{ \& } -1 \leq x \leq 1 \}$$

$$= \{ -4, 0, 4 \}$$

$$(c) \{ a+b \mid a \in \mathbb{N} \text{ \& } a < 3 \text{ \& } b \in \{5, 6\} \}$$

$$= \{ 6, 7, 8 \}$$

(Q4)



$$(a) (A \cup B) \cup C = A \cup (B \cup C)$$

$$\{2, 3, 4, 5, 6, 7, 8\} = \{2, 3, 4, 5, 6, 7, 8\}$$

$$(b) A \cup B = B \cup A$$

$$\{2, 6, 3, 8, 4, 7\} = \{2, 6, 3, 7, 4, 7\}$$

$$(c) (A \cap B) \cap C = A \cap (B \cap C)$$

$$\{3, 8\} \cap \{6, 7, 7, 5\} = \{2, 3, 8, 6\} \cap \{8, 7\}$$

$$\{8\} = \{8\}$$

$$(d) A \cap B = B \cap A$$

$$\{3, 8\} = \{3, 8\}$$

$$(e) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\{2, 3, 8, 6\} \cap \{4, 3, 8, 7, 6, 5\} = \{3, 8\} \cup \{6, 8\}$$

$$\{6, 8, 3\} = \{3, 8, 6\}$$

$$(h) A \cup \bar{A} = U$$

$$\{2, 6, 8, 3\} \cup \{1, 5, 7, 4\}$$

$$= U$$

$$A \cap \bar{A} = \emptyset$$

$$\{2, 6, 8, 3\} \cap \{1, 5, 7, 4\}$$

$$= \emptyset$$

$$(f) A \cup \emptyset = A$$

$$\{2, 3, 8, 6\} \cup \{3\} = A$$

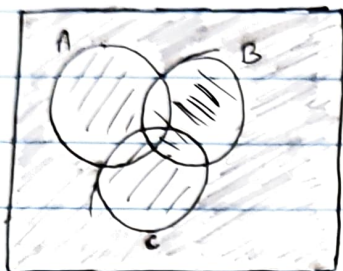
$$(g) A \cap U = A$$

$$\{2, 3, 8, 6\} \cap U = A$$

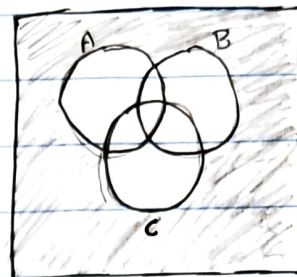
→ at the end of POP



(os)  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$



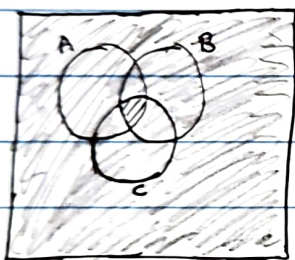
$\equiv \rightarrow A \cup B \cup C$   
 $\Rightarrow \overline{A \cup B \cup C}$



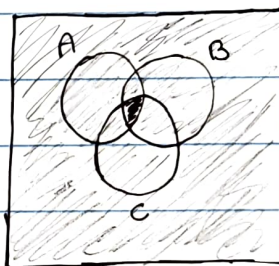
$\Rightarrow \bar{A} \cap \bar{B} \cap \bar{C}$

Hence  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

(b)  $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$



$\equiv \rightarrow A \cap B \cap C$   
 $\Rightarrow \overline{A \cap B \cap C}$



$\Rightarrow \bar{A} \cup \bar{B} \cup \bar{C}$

- Primes, divisors, modulo

### (01) Prime Factorization

(a) 40  
 $\begin{array}{c} \swarrow \quad \searrow \\ 10 \quad 4 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 5 \quad 2 \quad 2 \quad 2 \end{array}$

$2 \times 2 \times 2 \times 5$   
 $2^3 \times 5$

(b) 792  
 $\begin{array}{c} \wedge \\ 99 \quad 8 \\ \wedge \quad \wedge \\ 11 \quad 9 \quad 4 \quad 2 \\ \wedge \quad \wedge \\ 3 \quad 3 \quad 2 \quad 2 \end{array}$

$2 \times 2 \times 2 \times 3 \times 3 \times 11$   
 $2^3 \times 3^2 \times 11$

(c) 93  
 $\begin{array}{c} \wedge \\ 31 \quad 3 \end{array}$

$3 \times 31$

(d) 85  
 $\begin{array}{c} \wedge \\ 17 \quad 5 \end{array}$

$5 \times 17$

→ 98, 85 are relatively prime.

(Q2) Definition (Quotient Remainder Thm): let  $n, d$  be  $\mathbb{Z}$

$d > 0$  there are integers  $q, r$  such that

$$n = d \cdot q + r \text{ and } 0 \leq r < d$$

(a)  $4 \nmid 7$

$$n = 7 \quad d = 4$$

$$\frac{7}{4} = 1.75 \notin \mathbb{Z}$$

False

(e)  $0 \nmid 12$

$$n = 12 \quad d = 0$$

$$\frac{12}{0} \rightarrow \text{undefined}$$

(f)  $12 \nmid 0$

$$n = 0 \quad d = 12$$

$$\frac{0}{12} = 0 \in \mathbb{Z}$$

True

(b)  $4 \mid 12$

$$n = 12 \quad d = 4$$

$$\frac{12}{4} = 3 \in \mathbb{Z}$$

True

(g)  $\frac{4}{12} = 0.\bar{3}$

(c)  $4 \mid -12$

$$n = -12 \quad d = 4$$

$$\frac{-12}{4} = -3 \in \mathbb{Z}$$

True

(h)  $\frac{12}{4} = 3$

(i)  $\frac{0}{12} = 0$

(d)  $12 \nmid 4 \rightarrow \text{True}$

$$n = 4 \quad d = 12$$

$$\frac{4}{12} = 0.\bar{3} \notin \mathbb{Z}$$

False

(j)  $\frac{12}{0} \rightarrow \text{undefined}$

# 8 Homework 2 continued

(13)  $n = d \cdot q + r$  Find integers  $q$  and  $r$   $0 \leq r < d$

(a)  $n = 28$   $d = 7$

$$28 = 7 \cdot q + r$$

$$28 = 7 \cdot 4 + 0$$

$$q = 4 \quad r = 0$$

$$4R0$$

(e)  $n = 0$   $d = 6$

$$0 = 6 \times q + r$$

$$0 = 6 \times 0 + 0$$

$$q = 0 \quad r = 0$$

$$0R0$$

(b)  $n = -28$   $d = 7$

$$-28 = 7 \cdot q + r$$

$$-28 = 7 \cdot (-4) + 0$$

$$q = -4 \quad r = 0$$

$$-4R0$$

(f)  $n = 5$   $d = 6$

$$5 = 6 \times q + r$$

$$5 = 6 \times 0 + 5$$

$$q = 0 \quad r = 5$$

$$0R5$$

(c)  $n = 31$   $d = 7$

$$31 = 7 \cdot q + r$$

$$31 = 7 \cdot 4 + 3$$

$$q = 4 \quad r = 3$$

$$4R3$$

(g)  $n = -5$   $d = 6$

$$-5 = 6 \times q + r$$

$$-5 = 6 \times (-1) + 1$$

$$q = (-1) \quad r = 1$$

$$-1R1$$

(d)  $n = -31$   $d = 7$

$$-31 = 7 \cdot q + r$$

$$-31 = 7 \cdot (-5) + 4$$

$$q = -5 \quad r = 4$$

$$-5R4$$

(h)  $n = -10$   $d = 6$

$$-10 = 6 \times q + r$$

$$-10 = 6 \times (-2) + 2$$

$$q = -2 \quad r = 2$$

$$-2R2$$

(e4)

(a)  $7 \bmod 5$

$$n = 7 \quad d = 5$$

$$7 = 5 \times q + r$$

$$7 = 5 \times 1 + 2$$

$$R = 2$$

$$7 \div 5 = 2$$

(e)  $0 \bmod 5$

$$n = 0 \quad d = 5$$

$$0 = 5 \times q + r$$

$$0 = 5 \times 0 + 0$$

$$R = 0$$

$$0 \div 5 = 0$$

(b)  $-7 \bmod 5$

$$n = -7 \quad d = 5$$

$$-7 = 5 \times (-2) + 3$$

$$R = 3$$

$$-7 \div 5 = 3$$

(f)  $0 \bmod 3$

$$n = 0 \quad d = 3$$

$$0 = 3 \times 0 + 0$$

$$R = 0$$

$$0 \div 3 = 0$$

(c)  $3 \bmod 5$

$$n = 3 \quad d = 5$$

$$3 = 5 \times (0) + 3$$

$$R = 3$$

$$3 \div 5 = 3$$

(g)  $48 \bmod 12$

$$n = 48 \quad d = 12$$

$$48 = 12 \times 4 + 0$$

$$R = 0$$

$$48 \div 12 = 0$$

(d)  $-3 \bmod 5$

$$n = -3 \quad d = 5$$

$$-3 = 5 \times (-1) + 2$$

$$R = 2$$

$$-3 \div 5 = 2$$

(h)  $48 \bmod 11$

$$48 = 11 \times 4 + 4$$

$$R = 4$$

$$48 \div 11 = 4$$

$$48 \div 11 = 4$$



i  $-48 \bmod 11$

$$-48 = 11 \times (-5) + 7$$

$$-48 \div 11 = 7$$

(95)

(a)  $6 \mid 42$

$$\frac{42}{6} = 7 \in \mathbb{Z}$$

True  $q = 7$

(d)  $41 \bmod 10 = 3$

$$41 = 10 \times q + 1$$

$$41 = 10 \times 4 + 1$$

$$R = 1$$

$$1 \neq 3 \text{ false}$$

(b)  $6 \mid 40$

$$\frac{40}{6} = \frac{20}{3} \notin \mathbb{Z}$$

False  $q = \frac{20}{3}$

(c)  $39 \bmod 8 = 7$

$$39 = 8 \cdot q + r$$

$$39 = 8 \times 4 + 6$$

$$q = 4 \quad R = 7$$

$$7 = 7 \text{ True}$$



→ section 1.1 Q2, 25, 56, 77, 96, 103

$$(Q22) \quad U = \{0, 1, 2, 3, \dots\}$$

$$X = \{1, 2, 3, 4, 5\}$$

$$Y = \{2n \mid n \in \mathbb{Z}^+\} = \{0, 2, 4, 6, 8, 10, \dots\}$$

$$\bar{X} \cap Y$$

$\bar{X}$  = all elements not in  $X$  but in the universe

$$\bar{X} = \{0, 6, 7, 8, 9, 10, \dots\}$$

$$Y = \{0, 2, 4, 6, 8, 10, \dots\}$$

$$\bar{X} \cap Y = \{t \mid t \in U \text{ \& } t \in \bar{X} \text{ \& } t \in Y\}$$

$$(Q25) \quad X \cup \bar{Y}$$

$\bar{Y}$  = all elements not in  $Y$  but in universe

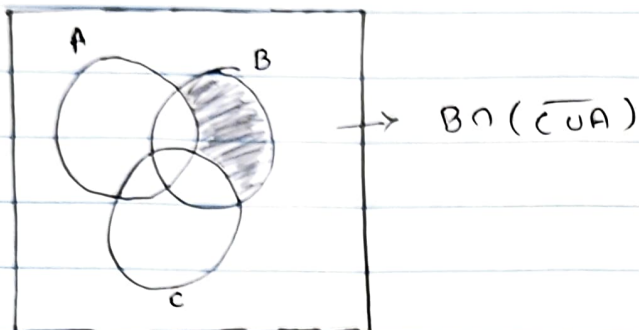
$X$  = all element in  $X$

$$X = \{1, 2, 3, 4, 5\} \rightarrow \text{Finite}$$

$$\bar{Y} = \{1, 3, 5, 7, \dots\} \rightarrow \text{Infinite}$$

$$X \cup \bar{Y} = \{t \mid t \in U \text{ \& } (t \in X \text{ or } t \in \bar{Y})\}$$

(Q76)  $B \cap (\overline{C \cup A})$



(Q77)  $\mathbb{Z} \times \mathbb{R}$

$\mathbb{Z}$  Represents all integers  $\{-1, 0, 1, \dots\}$

$\mathbb{R}$  Represents all Real numbers  $\{0, \dots\}$

$\mathbb{Z} \times \mathbb{R}$  will return a set with tuples inside of it which will have coordinate points  $(x, n)$  in them

$$\mathbb{Z} \times \mathbb{R} = \{(x, n) \mid x \in \mathbb{Z} \text{ and } n \in \mathbb{R}\}$$

(Q96) if  $x$  has  $n$  members how many proper subsets does  $x$  have

let  $w =$  a subset of  $x$

Proper subset  $\rightarrow w \subset x$  if  $w \subseteq x$  and  $w \neq x$

$$\underline{2^n - 1}$$

(Q103) Given universe  $\mathcal{U}$  describe  $A \Delta A$

$A \Delta B =$  symmetric difference

$$A \Delta B = (A \cup B) - (A \cap B)$$

$$(a) A \Delta A = (A \cup A) - (A \cap A)$$

$$= A - A$$

$$A - A = \{t \mid t \in \mathcal{U} \text{ and } t \in A \text{ and } t \notin A\}$$
$$= \{\} \quad \emptyset$$

$$(b) A \Delta \bar{A} = (A \cup \bar{A}) - (A \cap \bar{A})$$

$$(A \cup \bar{A}) - \emptyset$$

$$= \mathcal{U} - \emptyset$$

$$\mathcal{U} - \emptyset = \{t \mid t \in \mathcal{U} \text{ and } t \notin \emptyset\}$$

$$(c) \mathcal{U} \Delta A = (\mathcal{U} \cup A) - (\mathcal{U} \cap A)$$

$$\mathcal{U} - A$$

$$= \{t \mid t \in \mathcal{U} \text{ and } t \notin A\}$$

$$(d) \emptyset \Delta A = (\emptyset \cup A) - (\emptyset \cap A)$$

$$A - \emptyset$$

$$A - \emptyset = \{t \mid t \in \mathcal{U} \text{ and } t \in A\}$$

$$(i) A \cup A = A$$

$$\{2, 3, 8, 6\} \cup \{2, 3, 8, 6\} = A$$

$$(v) A \cap A = A$$

$$\{2, 3, 8, 6\} \cap \{2, 3, 8, 6\} = A$$

$$(k) A \cup \emptyset = A$$

$$\{2, 3, 8, 6\} \cup \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$= U$$

$$(l) A \cap \emptyset = \emptyset$$

$$\{2, 3, 8, 6\} \cap \{3\} = \emptyset$$

$$(m) A \cup (A \cap B) = A$$

$$\{2, 3, 8, 6\} \cup \{3, 8\}$$

$$\{2, 3, 8, 6, 3, 8\} = A$$

$$(n) A \cap (A \cup B) = A$$

$$\{2, 3, 8, 6\} \cap \{2, 3, 8, 6, 4, 7\}$$

$$= A$$

$$(o) \overline{\overline{A}} = A$$

$$\overline{\{1, 4, 5, 7\}} = \overline{A}$$

$$\overline{\overline{A}} = \{2, 3, 8, 6\} = A$$

$$(p) \overline{\emptyset} = U$$

$$\overline{\{3\}} = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$(q) \overline{\overline{U}} = \emptyset$$

$$\overline{\{1, 2, 3, 4, 5, 6, 7, 8\}} = \{3\}$$

$$(r) \overline{(A \cup B)} = \overline{A} \cap \overline{B}$$

$$\overline{\{1, 5\}} = \overline{\{4, 7, 5, 3\} \cap \{2, 6, 5, 1\}}$$

$$\{1, 5\} = \{1, 5\}$$

$$(s) \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

$$\overline{\{1, 2, 6, 4, 7, 5\}} = \overline{\{1, 2, 6, 4, 7, 5\}}$$