- Homework 1

$A = \{1, 2, 3\}$ $B = \{2, 3\}$ $C = \{A, B\}$ $D = \{'a'\}$

$E = \{\{0, 4\}\}$

(Q1) (a) $A \subseteq B$ = False

(b) $B \subseteq A$ = True

(c) $A \subseteq A$ = True

(d) $A \subset A$ True False

$\underbrace{A \subset A}_{\text{False}}$ if $\underbrace{\overset{\text{True}}{A \subseteq A} \;\&\; \overset{\text{False}}{A \neq A}}_{\text{False}}$

$A \not\subset A$ = $A \subset A$ = False

(e) $B \subset A$ =

$\underbrace{B \subset A}_{\text{True}}$ if $\underbrace{\overset{\text{true}}{B \subseteq A} \;\&\; \overset{\text{true}}{B \neq A}}_{\text{True}}$

$B \subset A$ = True

(f) = False = $B \in A$

(g) $B \in C$ = True

(h) = False = $0 \in E$

(Q2)

(a) $\{x \mid x - 2 = 0\}$ =

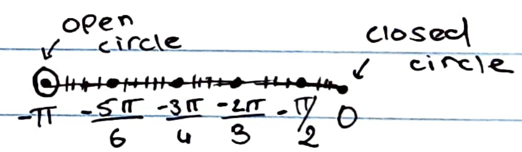
= $\{2\}$

(b) $\{x \mid x^2 - 2 = 0\}$ =

= $\{\sqrt{2}, -\sqrt{2}\}$

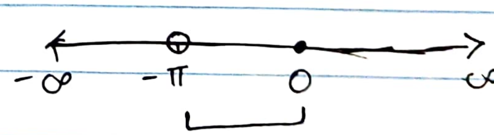
(Q3)

Subset $\mathbb{R}\ \{t \mid t \in \mathbb{R}\ \&\ -\pi < t \leq 0\}$



all numbers in between

$-\pi < t \leq 0$



not including $-\pi$

including 0

(Q4) (a) $|A| = 3$

(b) $|C| = 2$

(c) $|B| = |C|$ → $2 = 2$ → true

(d) $|B| = |D|$ → $2 \neq 1$ → False

(e) $|B = D|$ → undefined

(Q5) $x \subset y$ if $x \subseteq y$ and $x \neq y$

(a) $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

natural numbers = $\{1, 2, 3 \ldots\}$

intigers = $\{-\ldots -1, 0, 1 \ldots\}$

From the defination we see that $\mathbb{N} \subseteq \mathbb{Z}$ because all elements in $\mathbb{N}$ are in $\mathbb{Z}$

$\mathbb{Q}$ - $\{a/b \mid a \in \mathbb{Z}$ and $b \in \mathbb{Z}$ & $b \neq 0\}$

From this defination we see that Rationals are intiger division both a & b should be intigers hence will have all intigrs so $\mathbb{Z} \subseteq \mathbb{Q}$ ✓

$\mathbb{R}$ → rations contain every number in a number line so Hence $\mathbb{Z} \subseteq \mathbb{R}$

in conclusion the statement is true

(b) $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$A \subset B$ if $A \subseteq B$ & $A \neq B$

From A we proved that the all are $\subseteq$ $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$ and we know $\mathbb{N} \neq \mathbb{Z} \neq \mathbb{Q} \neq \mathbb{R}$ Hence they are proper subset

(Q6)

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

$C = \{7, 8\}$

(a) $A - B = \{t \mid t \in U$ and $t \in A$ and $t \notin B\}$

$A - B = \{1, 2\}$

(b) $B - A = \{t \mid t \in U$ and $t \in B$ and $t \notin A\}$

$B - A = \{4, 5\}$

(c) $C - A = \{7, 8\}$

(d) $U - A = \{4, 5, 6, 7, 8, 9, 10\}$

(e) $U - (A \cup B) = U - \{1,2,3,4,5\}$

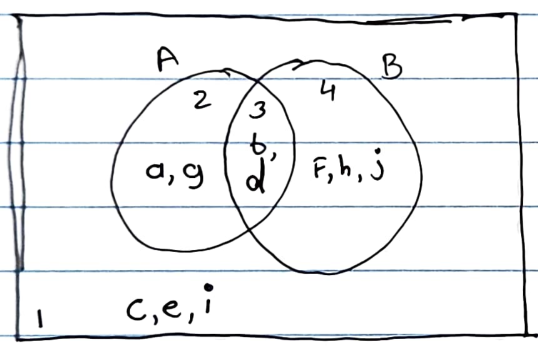
$= \{6,7,8,9,10\}$

(f) $U - \bar{A} = U - \{4,5,6,7,8,9,10\}$

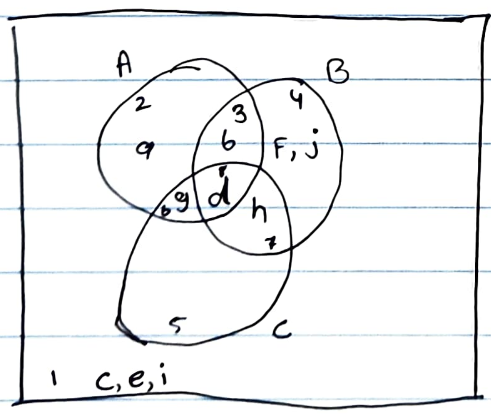
$= \{1,2,3\}$

(g) $B - \emptyset = \{3,4,5\}$

(Q7)



(Q8)



(09) given $K \subset U$, $L \subset U$, $|U| = 20$ $|\bar{K}| = 7$, $|K - L| = 10$

$|L - K| = 5$

Find $|K \cap L|$

$|\bar{K}| = 7 \rightarrow K = 13$

using the defindtion of $A - B$ in this case

$K - L = 10$

we know $|K| = 13$

we look at set k and remoe any elements in

it that also exist in L

$\{ t \in U$ and $t \in K$ and $t \notin L \}$

so $|K - L|$ will give the number of elements in

k that are not in L

that nomber is 10

we know that total nomber in $|K|$ are 13

so there are 3 elements

that are in common.

hence $|K \cap L| = 3$

→ Text book exercises

$U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,4,7,10\}$

$B = \{1,2,3,4,5\}$

$C = \{2,4,6,8\}$

1. $A \cup B = \{1,4,7,10,2,3,5\}$
2. $B \cap C = \{2,4\}$
3. $A - B = \{t \mid t \in U \text{ and } t \in A \text{ and } t \notin B\}$

   $= \{7,10\}$
5. $\bar{A} = \{2,3,5,6,8,9\}$
6. $U - C = \{t \mid t \in U \text{ and } t \in U \text{ and } t \notin C\}$

   $= \{1,3,5,7,9,10\}$
7. $\bar{U} = \{\} = \emptyset$
9. $B \cap \emptyset = \emptyset$
10. $A \cup U = \{1,4,7,10,2,3,5,6,8,9\}$
12. $A \cap (B \cup C) = A \cap \{1,2,3,4,5,6,8\}$

    $= \{1,4\}$

→ $X = \{1,2,3,4,5\}$

$Y = \{2n \mid n \in Z^+\}$

(24) $X \cap \bar{Y} = \{1,3,5\}$

(28) $|\emptyset| = 0$

(29) $|\{\emptyset\}| = 1$

(34) $A = \{1, 2, 3\}$

$B = \{n \mid n \in Z^+ \text{ and } n^2 < 10\}$

$B = \{1, 2, 3\}$

$Z^+ = \{1, 2, 3, 4 \ldots\}$

we are choosing n that are Z^+ and ~~two~~ their square is less than 10

$1^2 = 1$ ✓

$2^2 = 4$ ✓ so we will take 1, 2, 3

$3^2 = 9$ ✓ $B = \{1, 2, 3\}$

$4^2 = 16$? X

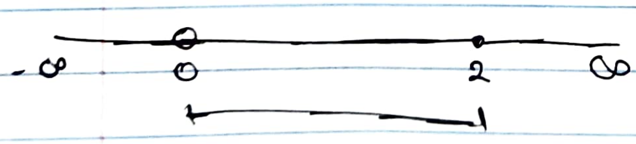
$A = \{1, 2, 3\}$ they have same elements

$B = \{1, 2, 3\}$ Hence $A = B$

(43) $\{x \mid x \in R \text{ and } 0 < x \le 2\}$, $\{1, 2\}$

we know that $\mathbb{R}$ real numbers are all numbers from $-\infty$ to ∞

Hence there will be infinatiy many Real numbers between $0 < x \le 2$



$\{0.0001, \ldots, 0.99\} \neq \{1, 2\}$

not equal

97 $A \cap B = A$

we understand that $A \cap B$ are all elements that are in both A and B

we want A and B = A

Hence we want $A \subseteq B$

so all elements in A are also in B

So if the condition $A \subseteq B$ is true

$A \cap B$ will equal A

98 $A \cup B = A$

There will be two possible condition

$B = \{\}$ or $\emptyset$. Since B is null the only elements in $A \cap B$ will be the elements in A

second condition can be

$B \subseteq A$ so B only has element that are in A.

99 $\bar{A} \cap U = \emptyset$

Condition = $A = U$.

100 

The condition is

$B \subseteq A$

101 $A \Delta B = (A \cup B) - (A \cap B)$

$A \Delta B = \{1, 2, 3, 4, 5\} - \{2, 3\}$

$A \Delta B = \{1, 4, 5\}$

102 The symmetric difference is defined by:

$A \Delta B = (A \cup B) - (A \cap B)$

this means $A \Delta B$ first we need to find

sets $(A \cup B)$ → all elemements in A or B

no doplicates

set $A \cap B$ → all elements that are in both

A & B

Once we have these two sets we perform -

$A - B = \{ t \mid t \in U \text{ and } t \in A \text{ and } t \notin B \}$

Hence now we select all elements in the universe

that are in set A but not in set B

Hence $A \Delta B = \{1, 2, 3, 4, 5\} - \{2, 3\}$

not including elements in B

$\{1, 4, 5\}$