CS 241 Home work 3

(Q1)

(a) Associative laws

$(p \vee q) \vee r = p \vee (q \vee r)$, $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

(b) commutative law

$p \vee q = q \vee p$, $p \wedge q = q \wedge p$

(c) Distributive law

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$,

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(d) Identity law

$p \vee F = p$, $p \wedge T = p$

(e) complement laws

$p \vee \neg p = T$, $p \wedge \neg p = F$

(f) idempotent laws

$p \vee p = p$, $p \wedge p = p$

(g) Bound laws

$p \vee T = T$, $p \wedge F = F$

(h) Absorption laws

$p \vee (p \wedge q) = p$, $p \wedge (p \vee q) = p$

(i) involution

$\neg\neg p = p$

(j) 0/1 laws

$\neg F = T$, $\neg T = F$

(k) $\neg(p \vee q) = \neg p \wedge \neg q$, $\neg(p \wedge q) = \neg p \vee \neg q$

(a)

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F	F
T	F	T	T	T	T	T	F	F	F	F
T	F	F	T	F	T	T	F	F	F	F
F	T	T	T	T	T	T	F	T	F	F
F	T	F	T	T	T	T	F	F	F	F
F	F	T	F	T	T	T	F	F	F	F
F	F	F	F	F	F	F	F	F	F	F

From the truth table above

$(p \vee q) \vee r = p \vee (q \vee r)$ $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

LHS = RHS LHS = RHS

(b) refer to the table before for $p \wedge r$ and other values

$p \wedge (q \vee r)$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r)$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	F	T	T	T	T
T	T	T	T	T	T
F	F	F	T	T	T
F	F	F	T	T	T
F	F	F	F	F	F
F	F	F	F	T	F
F	F	F	F	F	F

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ $\qquad$ $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

LHS = RHS $\qquad$ LHS = RHS

(c)

$\neg p$	$\neg q$	p	q	$p \vee q$	$p \wedge q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
F	T	T	F	T	F	F	F	T	T
F	F	T	T	T	T	F	F	F	F
T	T	F	F	F	F	T	T	T	T
T	F	F	T	T	F	F	F	T	T

$\therefore \neg(p \vee q) = \neg p \wedge \neg q$ $\qquad$ $\neg(p \wedge q) = \neg A \vee \neg B$

LHS = RHS $\qquad$ LHS = RHS

(Q2)

(1) $\neg(P_1 \vee P_2 \vee P_3) = \neg P_1 \wedge \neg P_2 \wedge \neg P_3$

By Morgan's laws we know

$\neg(P_1 \vee P_2 \vee P_3) = \neg(P_1 \vee P_2) \wedge \neg P_3 = -P_1 \wedge -P_2 \wedge -P_3$

$\neg(P_1 \vee P_2 \vee P_3) = \neg P_1 \wedge \neg(P_2 \vee P_3) = -P_1 \wedge -P_2 \wedge -P_3$

(2) $\neg(P_1 \wedge P_2 \wedge P_3) = \neg P_1 \vee \neg P_2 \vee \neg P_3$

By De Morgan's laws we know

$\neg(P_1 \wedge P_2 \wedge P_3) = \neg(P_1 \wedge P_2) \vee \neg P_3 = \neg P_1 \vee \neg P_2 \vee \neg P_3$

$\neg(P_1 \wedge P_2 \wedge P_3) = \neg P_1 \vee \neg(P_2 \wedge P_3) = \neg P_1 \vee \neg P_2 \vee \neg P_3$

(Q3)

A	B	C	$\neg A$	$\neg B$	$\neg C$	$p \wedge q \wedge r$	$\neg(p \wedge q \wedge r)$	$\neg p \wedge \neg p \wedge r$	$\neg p \vee q \vee \neg r$
T	T	T	F	F	F	T	F	F	T
T	T	F	F	F	T	F	T	F	T
T	F	T	F	T	F	F	T	F	F
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	F	T	F	T
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	T	F	T
						$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
						P_1	P_2	P_3	P_4

(Q4) write the logically equavalent expression

(a) $p \to r$

$\to$ opreator stands for if p then q

it can be re writen as follows:

$p \to r \equiv \neg p \vee r$

(b) $\neg(p \to r)$

$\neg(p \to r) \equiv \neg(\neg p \vee r) \equiv p \wedge \neg r$

(c) $(p \to q) \to r \equiv (\neg p \vee q) \vee r$

(d) $p \to (q \to r) \equiv \neg p \vee (\neg q \vee r)$

(e) $p \oplus r \equiv (p \wedge \neg r) \vee (\neg p \wedge r)$

(f) $p \oplus (r \wedge s) \equiv (p \wedge \neg(r \wedge s)) \vee (\neg p \wedge (r \wedge s))$

(g) $p \leftrightarrow \equiv (p \to r) \wedge (r \to q) \equiv (\neg p \vee r) \wedge (\neg p \vee q)$

(Q5) $\to$ is not associative $(p \to q) \to r$ will have same output as $p \to (q \to r)$.

table question 71

p	q	r	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T
T	T	F	F	F
⋮	⋮	⋮	⋮	⋮

(Q6) $\oplus$ is associative $(p \oplus q) \oplus r$ is equal to $p \oplus (q \oplus r)$

p	q	r	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	F
T	T	F	T	T
T	F	T	F	F
T	F	F	T	T

→ Homework continued

(Q7)

p	r	$p \downarrow r$	$p \downarrow p$	$r \downarrow r$	$(p \downarrow r) \downarrow (p \downarrow r)$	$(p \downarrow p) \downarrow (r \downarrow r)$
p	r	$\neg(p \vee r)$	$\neg(p \vee p)$	$\neg(r \vee r)$	$\neg(p \vee r) \vee (p \vee r)$	$\neg(p \vee p) \vee (r \vee r)$
T	T	F	F	F	T	T
T	F	F	F	T	T	F
F	r	F	T	F	T	F
F	P	T	T	T	F	F

we can conclude $p \downarrow p = \neg p$ & $r \downarrow r = \neg r$ $(p \downarrow r) \downarrow (p \downarrow r) = \neg(p \downarrow r)$

$p \downarrow r = \neg(p \vee r)$

∴

1.2 Exercises

(Q1) $2 + 5 = 19$ $2+5=19$ is a proposition

$2+5 \neq 19$ $2+5 \neq 19$ is the negation of the proposition

if a sentence is false its nagation is true

(Q13) 10 heads were obtained

Negation: less than 10 heads were obtained

(Q14) Some heads were obtained

Negation! no heads were obtained

(Q15) Some head and some tail were obtained

Negation: only heads were obtained

(Q16) At least 1 head was obtained

Negation: no heads were obtained

(Q20) $p = F \quad q = T \quad r = F$

$\neg p \vee \neg (q \wedge r)$

$T \vee \neg (F)$

$T \vee T$

T

(Q26) write truth table for

$(p \wedge q) \wedge \neg p$

p	q	$\neg p$	$(p \wedge q)$	$(p \wedge q) \wedge \neg p$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

Exercise 1.3

$p = T \quad q = F \quad r =$ unknown

(Q30) $(p \vee r) \leftrightarrow r$ → the value of this is unknown

this will always be true because p is true

at this point the output is unknown since if $r = T$ then they are equavalent but if its false then its not

(Q31) $(q \wedge r) \leftrightarrow r$

unknown status

q is always false so $q \wedge r$ is always false

if r is true then $(q \wedge r) \leftrightarrow r$ = false

but if r is true its true

(54) p: today is Monday $\quad$ $\neg p$: ~~t~~today is not Monday

q: it is raining $\quad$ $\neg q$: it is not raining

r: it is hot $\quad$ $\neg r$: it is cold

$\neg q \rightarrow (r \wedge p)$

it is not raining $\rightarrow$ (it is Hot $\wedge$ today is Monday)

$\rightarrow$ if it is not raining then today is Monday and its hot

(68) $P = p \rightarrow q \quad Q = \neg q \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$\neg q \rightarrow \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T
		✓			✓

using truth table colums

Hence proved $P \equiv Q$

$\therefore$ LHS = RHS Hence proved

(70) $P = (p \to q) \wedge (q \to r)$, $Q = p \to r$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	F
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T
				X		X

Hence $p \to r \not\equiv (p \to q) \wedge (q \to r)$

because the last and third last are not equal

(71) $P = (p \to q) \to r$, $Q = p \to (q \to r)$

look at the table above ↑ "$\to$" is not associative

hence they do not equal

$(p \to q) \to r$	$p \to (q \to r)$
T	T
F	F
T	T
T	T
T	T
F	T
T	T
F	T

Hence

$P \not\equiv Q$

(73) D Morgans logic for negation

Pat will use treadmill or lift weights

p = Pat will use treadmill

q = lift weight

$\neg(p \vee q) = \neg p \wedge \neg q$

Pat will not use treadmill and not use a weight lift

(74) Dale is smart and funny

p : Dale is smart

q : Dale is funny

$\neg(p \wedge q) = \neg A \vee \neg B = \neg p \vee \neg q$

Dale is not smart or not funny.

(75) Shirley will either take the bus or catch a ride to School

p = Shirley will take a bus

q = " " catch a ride

$\neg(p \vee q) = \neg p \wedge \neg q$

Shirley will not take a bus and she will not catch a ride to School

(76) Red pepper and onion are required to make chili

p = red pepper is required to make chili

q = onion are required to make chili

$\neg(p \wedge q) = \neg p \vee \neg q$

Red pepper are not required to make chili

Onion are not required to make chili