Homework 2 Meer Modi 01/29/2024

$A = \{1,2,3\}$ $B = \{2,3\}$ $C = \{A, B\}$ $D = \{'a'\}$ $E = \{\{0,4\}\}$

(Q1)

(a) $(A, B) \in C \rightarrow$ False

(b) $(1,3) \in A \times B \rightarrow$ true

$\{1,2,3\} \times \{2,3\}$

$= \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,3)\}$

$(1,3) \in \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,3)\}$

True

(c) $(1,3) \subseteq A \times B \rightarrow$ syntax error

$(1,3) \rightarrow$ tuple

(d) $(1,3) \in C \rightarrow$ False

(e) $\{2\} \in P(B) \rightarrow$ True

$\{2\} \in \{\{2,3\},\{2\},\{3\},\emptyset\}$

according to thm

$|P(B)| = 2^{|B|}$

$|P(B)| = 2^2 = 4$

$\{2\} \in \{\{2,3\},\{2\},\{3\},\emptyset\}$

(f) $2 \in P(B) \rightarrow$ False

Power of a set returns a set with sets inside of it

Hence 2 has no chances of being in $P(B)$

(g) $\{2\} \subseteq P(B) \rightarrow$ False

$\{2\} \subseteq \{\{2,3\},\{2\},\{3\},\emptyset\}$

(h) $2 \subseteq P(B) \rightarrow$ syntax error

2 is not a set

(i) $\{\{3,2\}\} \subseteq P(A) \rightarrow$ true

$\{\{3,2\}\} \subseteq \{\{1,2,3\},\{1\},\{2\},\{3\},\emptyset,\{1,2\},\{2,3\},\{1,3\}\}$

the order within a set does not matter

$\{3,2\} = \{2,3\}$

hence true

(j) $\{\{3,2\}\} \subseteq P(C) \rightarrow$ False

$P(C) = \{\{A\},\{B\},\{A,B\},\emptyset\}$

$\{\{3,2\}\} \subseteq \{\{A\},\{B\},\{A,B\},\emptyset\}$

(k) $\{\{2,3\}\} \in P(C) \rightarrow$ False

$\{\{2,3\}\} \in \{\{A\},\{B\},\{A,B\},\emptyset\}$

(Q2)

(a) $B \times A$

$\{2,3\} \times \{1,2,3\}$

$= \{(2,1),(2,2),(2,3),$
$(3,1),(3,2),(3,3)\}$

(b) $A \times D$

$\{1,2,3\} \times \{\text{'a'}\}$

$= \{(1,\text{'a'}),(2,\text{'a'}),$
$(3,\text{'a'})\}$

(c) $\emptyset \times B$

$\emptyset \times \{2,3\} = \emptyset$

(d) $C \times C$

$\{A,B\} \times \{A,B\}$

$= \{(A,A),(A,B),(B,A),$
$(B,B)\}$

(e) $E \times B \times D$

$\{\{0,4\}\} \times \{2,3\} \times \{\text{'a'}\}$

$\{(\{0,4\},2),(\{0,4\},3)\} \times \{\text{'a'}\}$

$\{((\{0,4\},2),\text{'a'}),((\{0,4\},3),\text{'a'})\}$

(f) $P(D)$ Power set of D

$D = \{\text{'a'}\}$

$P(D) = \{\{\},\{\text{'a'}\}\}$

(g) $P(E)$ ~ $E = \{\{0,4\}\}$

$P(E) = \{\{\},\{\{0,4\}\}\}$

(h) $P(B)$ $B = \{2,3\}$

$P(B) = \{\{2\},\{3\},\emptyset,\{2,3\}\}$

(Q3)

(a) $\{(x,y) \mid x \in \mathbb{N} \ \& \ x<5 \ \& \ y=0\}$

$= \{(1,0),(2,0),(3,0),(4,0)\}$

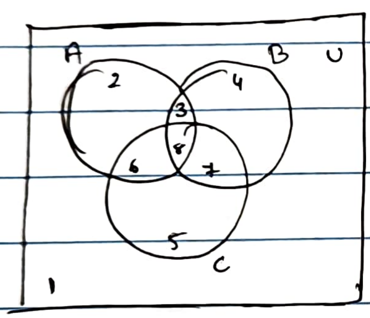
(b) $\{4x \mid x \in \mathbb{Z} \ \& \ -1 \le x \le 1\}$

$= \{-4, 0, 4\}$

(c) $\{a+b \mid a \in \mathbb{N} \ \& \ a<3 \ \& \ b \in \{5,6\}\}$

$= \{6, 7, 8\}$

(Q4)



(a) $(A \cup B) \cup C = A \cup (B \cup C)$

$\{2,3,4,5,6,7,8\} = \{2,3,4,5,6,7,8\}$

(b) $A \cup B = B \cup A$

$\{2,6,3,8,4,7\} = \{2,6,3,8,4,7\}$

(c) $(A \cap B) \cap C = A \cap (B \cap C)$

$\{3,8\} \cap \{6,8,7,5\} = \{2,3,8,6\} \cap \{8,7\}$

$\{8\} = \{8\}$

(d) $A \cap B = B \cap A$

$\{3,8\} = \{3,8\}$

(h) $A \cup \bar{A} = U$

$\{2,6,8,3\} \cup \{1,5,7,4\}$

$= U$

$A \cap \bar{A} = \emptyset$

$\{2,6,8,3\} \cap \{1,5,7,4\}$

$= \emptyset$

(e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\{2,3,8,6\} \cap \{4,3,8,7,6,5\} = \{3,8\} \cup \{6,8\}$

$\{6,8,3\} = \{3,8,6\}$

(f) $A \cup \emptyset = A$

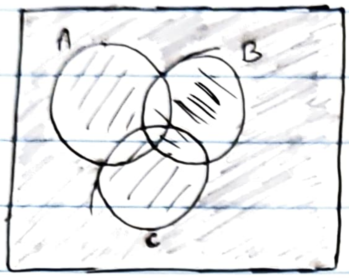
$\{2,3,8,6\} \cup \{\} = A$

(g) $A \cap U = A$

$\{2,3,8,6\} \cap U = A$

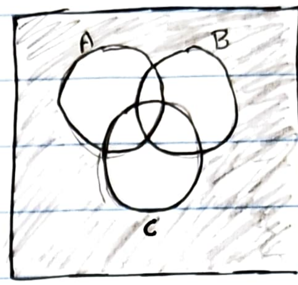
→ at the end of PDF

(05) $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$



$\equiv \rightarrow A \cup B \cup C$

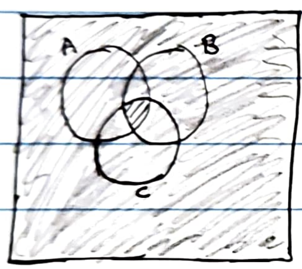
$\rightarrow \overline{A \cup B \cup C}$



$\rightarrow \bar{A} \cap \bar{B} \cap \bar{C}$

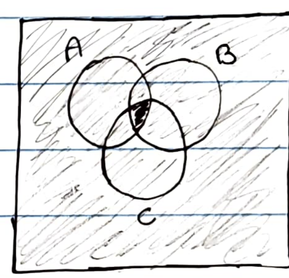
Hence $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

(b) $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$



$\equiv \rightarrow A \cap B \cap C$

$\rightarrow \overline{A \cap B \cap C}$

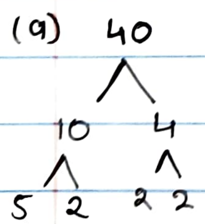


$\rightarrow \bar{A} \cup \bar{B} \cup \bar{C}$

- Primes, divisors, modulo

(01) Prime Factorization

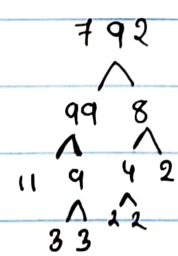
(a)



$2 \times 2 \times 2 \times 5$

$2^3 \times 5$

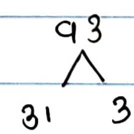
(b)



$2 \times 2 \times 2 \times 3 \times 3 \times 11$

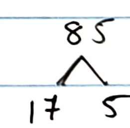
$2^3 \times 3^2 \times 11^1$

(c)



3×31

(d)



5×17

→ 98, 85 are relatively prime.

(Q2) Defination (Quotient Remainder Thm): let n, d be $\mathbb{Z}$

$d > 0$ there are intigers q, r such that

$n = d \cdot q + r$ and $0 \le r < d$

(a) $4 \mid 7$

$n = 7$ $d = 4$

$\frac{7}{4} = 1.75 \notin \mathbb{Z}$

False

(b) $4 \mid 12$

$n = 12$ $d = 4$

$\frac{12}{4} = 3 \in \mathbb{Z}$

True

(c) $4 \mid -12$

$n = -12$ $d = 4$

$\frac{-12}{4} = -3 \in \mathbb{Z}$

True

(d) $12 \mid 4$

$n = 4$ $d = 12$

$\frac{4}{12} = 0.3\bar{3} \notin \mathbb{Z}$

False

(e) $0 \mid 12$

$n = 12$ $d = 0$

$\frac{12}{0}$ → undefined

(f) $12 \mid 0$

$n = 0$ $d = 12$

$\frac{0}{12} = 0 \in \mathbb{Z}$

True

(g) $\frac{4}{12} = 0.\bar{3}$

(h) $\frac{12}{4} = 3$

(i) $\frac{0}{12} = 0$

(j) $\frac{12}{0}$ → undefined

* Homework 2 continued

(Q3) $n = d \cdot q + r$ Find intigers q and r $0 \le r < d$

(a) $n = 28$ $d = 7$

$28 = 7 \cdot q + r$

$28 = 7 \cdot 4 + 0$

$q = 4$ $r = 0$

4R0

(b) $n = -28$ $d = 7$

$-28 = 7 \cdot q + r$

$-28 = 7 \cdot (-4) + 0$

$q = -4$ $r = 0$

-4 R0

(c) $n = 31$ $d = 7$

$31 = 7 \cdot q + r$

$31 = 7 \cdot 4 + 3$

$q = 4$ $r = 3$

4 R 3

(d) $n = -31$ $d = 7$

$-31 = 7 \cdot q + r$

$-31 = 7 \cdot (-5) + 4$

$q = -5$ $R = 4$

-5 R 4

(e) $n = 0$ $d = 6$

$0 = 6 \times q + r$

$0 = 6 \times 0 + 0$

$q = 0$ $R = 0$

0R0

(f) $n = 5$ $d = 6$

$5 = 6 \times q + r$

$5 = 6 \times 0 + 5$

$q = 0$ $R = 5$

0R5

(g) $n = -5$ $d = 6$

$-5 = 6 \times q + r$

$-5 = 6 \times (-1) + 1$

$q = (-1)$ $r = 1$

-1 R 1

(h) $n = -10$ $d = 6$

$-10 = 6 \times q + r$

$-10 = 6 \times (-2) + 2$

$q = -2$ $r = 2$

-2 R 2

(Q4)

(a) 7 mod 5

$n = 7 \quad d = 5$

$7 = 5 \times q + r$

$7 = 5 \times 1 + 2$

$R = 2$

$7 \% 5 = 2$

(b) -7 mod 5

$n = -7 \quad d = 5$

$-7 = 5 \times (-2) + 3$

$R = 3$

$-7 \% 5 = 3$

(c) 3 mod 5

$n = 3 \quad d = 5$

$3 = 5 \times (0) + 3$

$R = 3$

$3 \% 5 = 3$

(d) -3 mod 5

$n = -3 \quad d = 5$

$-3 = 5(-1) + 2$

$R = 2$

$-3 \% 5 = 2$

(e) 0 mod 5

$n = 0 \quad d = 5$

$0 = 5 \cdot q + r$

$0 = 5 \cdot 0 + 0$

$R = 0$

$0 \% 5 = 0$

(f) 0 mod 3

$n = 0 \quad d = 3$

$0 = 3 \cdot 0 + 0$

$R = 0$

$0 \% 3$

(g) 48 mod 12

$n = 48 \quad d = 12$

$48 = 12 \cdot 4 + 0$

$R = 0$

$48 \% 12 = 0$

(h) 48 mod 11

$48 = 11 \times 4 + 4$

$R = 4$

$48 \quad R = 4$

$48 \% 11 = 4$

i . -48 mod 11

$-48 = 11 \times (-5) + 7$

$-48 \% 11 = 7$

(Q5)

(a) $6 | 42$

$\frac{42}{\cancel{6}} = 7 \in \mathbb{Z}$

True $q = 7$

(d) 41 mod 10 = 3

$41 = 10 \times q + 1$

$41 = 10 \times 4 + 1$

$R = 1$

$1 \neq 3$ False

(b) $6 | 40$

$\frac{40}{\cancel{6}} = \frac{20}{3} \notin \mathbb{Z}$

False $q = \frac{20}{3}$

(c) 39 mod 8 = 7

$39 = 8 \cdot q + r$

$39 = 8 \times 4 + 6$

$q = 4$ $R = 7$

$7 = 7$ True

→ section 1.1 22, 25, 56, 77, 96, 103

(Q22) $U = \{0, 1, 2, 3 \dots\}$

$x = \{1, 2, 3, 4, 5\}$

$y = \{2n \mid n \in z^{+}\} = \{0, 2, 4, 6, 8, 10 \dots\}$

$\bar{x} \cap y$

$\bar{x}$ = all elements not in x but in the universe

$\bar{x} = \{0, 6, 7, 8, 9, 10 \dots\}$

$y = \{0, 2, 4, 6, 8, 10 \dots\}$

$\bar{x} \cap y = \{t \mid t \in U \ \& \ t \overset{\in}{\cancel{\notin}} \bar{x} \ \& \ t \in y\}$

(Q25) $x \cup \bar{y}$

$\bar{y}$ = all elements not in y but in universe

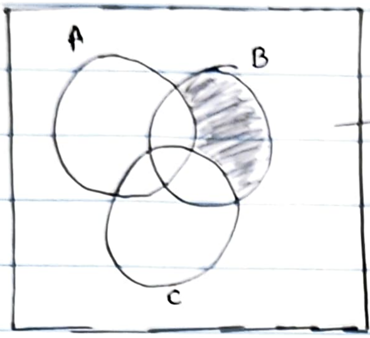
x = all element in x

$x = \{1, 2, 3, 4, 5\}$ → Finite

$\bar{y} = \{1, 3, 5, 7 \dots\}$ → infinite

$x \cup \bar{y} = \{t \mid t \in U \ \& \ (t \in x \text{ or } t \overset{\in}{\cancel{\notin}} \bar{y})\}$

(Q56) $B \cap (\overline{C \cup A})$



$\rightarrow B \cap (\overline{C \cup A})$

(Q77) $Z \times R$

Z Reprents all intigers $\{\ldots -1, 0, 1 \ldots\}$

R Represents all Real numbers $\{\ldots 0 \ldots\}$

$Z \times R$ will return a set with touples inside of it which will have coordinates points (x,n) in them

$Z \times R = \{(x,n) \mid x \in Z \text{ and } n \in R\}$

(Q96) if x has n members how many proper subsets does x have

let ω = a subset of x

Proper sub set = $\omega \subset x$ if $\omega \subseteq x$ and $\omega \neq y$

$\underline{2^n - 1}$

(Q103) Given universe U describe $A \Delta A$

$A \Delta B =$ Symmetric difference

$A \Delta B = (A \cup B) - (A \cap B)$

(a) $A \Delta A = (A \cup A) - (A \cap A)$

$= A - A$

$A - A = \{ t \mid t \in U \text{ and } t \in A \text{ and } t \notin A \}$

$= \{\} \quad \emptyset$

(b) $A \Delta \bar{A} = (A \cup \bar{A}) - (A \cap \bar{A})$

$(A \cup \bar{A}) - \emptyset$

$= U - \emptyset$

$U - \emptyset = \{ t \mid t \in U \text{ and } t \notin \emptyset \}$

(c) $U \Delta A = (U \cup A) - (U \cap A)$

$U - A$

$= \{ t \mid t \in U \text{ and } t \notin A \}$

(d) $\emptyset \Delta A = (\emptyset \cup A) - (\emptyset \cap A)$

$A - \emptyset$

$A - \emptyset = \{ t \mid t \in U \text{ and } t \in A \}$

(i) $A \cup A = A$

$\{2,3,8,6\} \cup \{2,3,8,6\} = A$

(j) $A \cap A = A$

$\{2,3,8,6\} \cap \{2,3,8,6\} = A$

(k) $A \cup U = U$

$\{2,3,8,6\} \cup \{1,2,3,4,5,6,7,8\}$

$= U$

(l) $A \cap \emptyset = \emptyset$

$\{2,3,8,6\} \cap \{\} = \emptyset$

(m) $A \cup (A \cap B) = A$

$\{2,3,8,6\} \cup \{3,8\}$

$\{2,3,8,6,\cancel{3,8}\} = A$

(n) $A \cap (A \cup B) = A$

$\{2,3,8,6\} \cap \{2,3,8,6,4,7\}$

$= A$

(o) $\bar{\bar{A}} = A$

$\{1,4,5,7\} = \bar{A}$

$\bar{\bar{A}} = \{2,3,8,6\} = A$

(p) $\bar{\emptyset} = U$

$\overline{\{\}} = \{1,2,3,4,5,6,7,8\}$

(q) $\bar{U} = \emptyset$

$\overline{\{1,2,3,4,5,6,7,8\}} = \{\}$

(r) $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$\{1,5\} = \{4,7,5,1\} \cap \{2,6,5,1\}$

$\{1,5\} = \{1,5\}$

(s) $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

$\{1,2,6,4,7,5\} = \{1,2,6,4,7,5\}$